Standard Model

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IMPRS Elementary Particle Physics
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- Elements of Gauge Field Theories
- Formulation of the Standard Model
- Tests of the Standard Model
- Higgs Bosons
- Outlook: SUSY extension of the SM

QED as U(1) gauge theory:

Lagrangian $\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f$ has global phase symmetry:

$$\psi_f \to \psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}\psi_f, \quad \overline{\psi_f} \to \overline{\psi'_f} = \overline{\psi_f}\exp\{+\mathrm{i}Q_f e\theta\}$$

with space-time-independent group parameter θ

"Gauging the symmetry": demand local symmetry, $\theta \to \theta(x)$

To maintain local symmetry, extend theory by "minimal substitution":

$$\partial^\mu \to D^\mu = \partial^\mu + \mathrm{i}Q_f e A^\mu(x) = \text{"covariant derivative"},$$

$A^\mu(x)$ = spin-1 gauge field (photon).

Transformation property of photon $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)$ ensures

- $D_\mu\psi_f \to (D_\mu\psi_f)' = D'_\mu\psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}(D_\mu\psi_f)$
- gauge invariance of field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Gauge-invariant Lagrangian of QED:

$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - Q_f e A\!\!\!/ - m_f)\psi_f - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian gauge theory (Yang–Mills theory):

Starting point:

Lagrangian $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi)$ of free or self-interacting fields with "internal symmetry":

- $\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$ = multiplet of a compact Lie group G:

  $\Phi \to \Phi' = U(\theta)\Phi, \quad U(\theta) = \exp\{-\mathrm{i}gT^a\theta^a\}$ = unitary,

  T^a = group generators, $\quad [T^a, T^b] = \mathrm{i}C^{abc}T^c, \quad \mathrm{Tr}\left\{T^aT^b\right\} = \frac{1}{2}\delta^{ab}$

- $\mathcal{L}_\Phi$ is invariant under G: $\quad \mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) = \mathcal{L}_\Phi(\Phi', \partial_\mu\Phi')$

Example: self-interacting (complex) boson multiplet

$$\mathcal{L}_\Phi = (\partial_\mu\Phi)^\dagger(\partial^\mu\Phi) - m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

(m = common boson mass, λ = coupling strength)

Gauging the symmetry by minimal substitution:

$$\mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) \to \mathcal{L}_\Phi(\Phi, D_\mu\Phi) \quad \text{with } D_\mu = \partial_\mu + \mathrm{i}gT^aA^a_\mu(x),$$

g = gauge coupling, T^a = generator of G in Φ representation, $A^a_\mu(x)$ = gauge fields

Transformation property of gauge fields:

- $\mathcal{L}_\Phi(\Phi, D_\mu\Phi)$ local invariant if $D_\mu\Phi \;\to\; (D_\mu\Phi)' = D'_\mu\Phi' = U(\theta)(D_\mu\Phi)$

  $\Rightarrow\; T^a A'^a_\mu \;=\; UT^a A^a_\mu U^\dagger - \frac{\mathrm{i}}{g}U(\partial_\mu U^\dagger),$ $\quad A^a_\mu A^{a,\mu}$ = not gauge invariant

  infinitesimal form: $\delta A^a_\mu = gC^{abc}\delta\theta^b A^c_\mu + \partial_\mu\delta\theta^a$

- covariant definition of field strength: $[D_\mu, D_\nu] = \mathrm{i}gT^a F^a_{\mu\nu}$

  $\Rightarrow\; T^a F^a_{\mu\nu} \;\to\; T^a F'^a_{\mu\nu} = UT^a F^a_{\mu\nu}U^\dagger,$ $\quad F^a_{\mu\nu}F^{a,\mu\nu}$ = gauge invariant

  explicit form: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gC^{abc}A^b_\mu A^c_\nu$

Yang–Mills Lagrangian for gauge and matter fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \mathcal{L}_\Phi(\Phi, D_\mu\Phi)$$

- Lagrangian contains terms of order $(\partial A)A^2$, A^4 in F^2 part

  $\hookrightarrow$ cubic and quartic gauge-boson self-interactions

- gauge coupling determines gauge-boson–matter and gauge-boson self-interaction $\to$ unification of interactions

- mass term $M^2(A^a_\mu A^{a,\mu})$ for gauge bosons forbidden by gauge invariance

  $\hookrightarrow$ gauge bosons of unbroken Yang–Mills theory are massless

- Why unification of weak and elmg. interaction ?
  - similiarity: spin-1 fields couple to matter currents formed by spin-$\frac{1}{2}$ fields
  - elmg. coupling of charged $\mathrm{W}^{\pm}$ bosons
- $\gamma, \mathrm{W}^{+}, \mathrm{W}^{-}$ as gauge bosons of group SU(2) ? – No!

  Reason: charge operator Q cannot be SU(2) generator, since Tr $\{Q\} \neq 0$

  for fermion doublets: $Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ for $\begin{pmatrix} \nu_\mathrm{e} \\ \mathrm{e}^- \end{pmatrix}$, etc.

  Possible way out: additional heavy fermions like E^+ as partner to e^- ?

  $\hookrightarrow$ no experimental confirmation !
- Minimal solution: $\mathrm{SU(2)_I} \times \mathrm{U(1)_Y}$
  - $\mathrm{SU(2)_I}$ $\longrightarrow$ weak isospin group with gauge bosons $\mathrm{W}^{+}, \mathrm{W}^{-}, \mathrm{W}^{0}$
  - $\mathrm{U(1)_Y}$ $\longrightarrow$ weak hypercharge with gauge boson B

  W^0 and B carry identical quantum numbers

  $\hookrightarrow$ two neutral gauge bosons γ, Z as mixed states

  Experiment: 1973 discovery of neutral weak currents at CERN
  1983 discovery of Z boson at CERN

Electroweak interaction:

Fermion fields:

$$\text{quarks: } \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R, \quad \text{leptons: } \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, e_R$$

3 generations: $\begin{matrix} u, d, & s, c & t, b \\ \nu_e, e & \nu_\mu, \mu & \nu_\tau, \tau \end{matrix}$

gauge bosons: γ, Z, W^+, W^-

Gauge group: $\mathrm{SU(2)_I} \times \mathrm{U(1)_Y} \supset \mathrm{U(1)_{em}}$

Strong interaction: QCD

quarks: q_r, q_g, q_b

gauge bosons: $g_1, \ldots g_8$: gluons, $\mathrm{SU(3)_C}$

All postulated fermions and gauge bosons experimentally verified

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\mathrm{ferm}} = \mathrm{i}\overline{\psi_f}\partial\!\!\!/\psi_f = \mathrm{i}\overline{\Psi^{\mathrm{L}}_L}\partial\!\!\!/\Psi^{\mathrm{L}}_L + \mathrm{i}\overline{\Psi^{\mathrm{L}}_Q}\partial\!\!\!/\Psi^{\mathrm{L}}_Q + \mathrm{i}\overline{\psi^{\mathrm{R}}_l}\partial\!\!\!/\psi^{\mathrm{R}}_l + \mathrm{i}\overline{\psi^{\mathrm{R}}_u}\partial\!\!\!/\psi^{\mathrm{R}}_u + \mathrm{i}\overline{\psi^{\mathrm{R}}_d}\partial\!\!\!/\psi^{\mathrm{R}}_d$$

Minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g_2 T^a_{\mathrm{I}} W^a_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y B_\mu = D^{\mathrm{L}}_\mu \omega_- + D^{\mathrm{R}}_\mu \omega_+,$$

$$D^{\mathrm{L}}_\mu = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}}\begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} - \frac{\mathrm{i}}{2}\begin{pmatrix} g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu & 0 \\ 0 & -g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu \end{pmatrix}$$

$$D^{\mathrm{R}}_\mu = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}} B_\mu$$

Photon identification:

"Glashow rotation": $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$, $c_{\mathrm{W}} = \cos\theta_{\mathrm{W}}$, $s_{\mathrm{W}} = \sin\theta_{\mathrm{W}}$, θ_{W} = mixing angle

$$D^{\mathrm{L}}_\mu\Big|_{A_\mu} = -\frac{\mathrm{i}}{2} A_\mu \begin{pmatrix} -g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} & 0 \\ 0 & g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} \end{pmatrix} \stackrel{!}{=} \mathrm{i}e A_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1$ $\quad\to\quad$ $g_2 = \dfrac{e}{s_{\mathrm{W}}}$
- normalize $Y^{\mathrm{L/R}}$ such that $g_1 = \dfrac{e}{c_{\mathrm{W}}}$

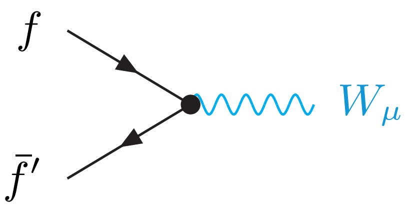
  $\hookrightarrow$ Y fixed by "Gell-Mann–Nishijima relation": $Q = T^3_{\mathrm{I}} + \dfrac{Y}{2}$

Fermion–gauge-boson interaction:

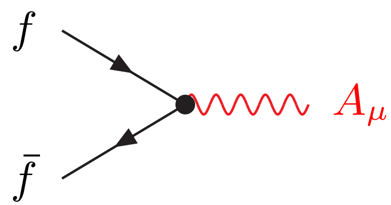
$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix}\Psi_F^{\mathrm{L}} + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\sigma^3\not{Z}\Psi_F^{\mathrm{L}}$$
$$- e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\psi_f}\not{Z}\psi_f - eQ_f\overline{\psi_f}\not{A}\psi_f$$

(f=all fermions, F= all doublets)

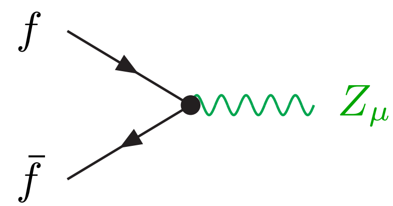
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-$$



$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_f^+\omega_+ + g_f^-\omega_-) = \mathrm{i}e\gamma_\mu(v_f - a_f\gamma_5)$$

with $g_f^+ = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f, \quad g_f^- = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{c_{\mathrm{W}}s_{\mathrm{W}}},$

$$v_f = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}, \quad a_f = \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}$$

Yang–Mills Lagrangian for gauge fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

$$W^a_{\mu\nu} \;=\; \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} \;=\; \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

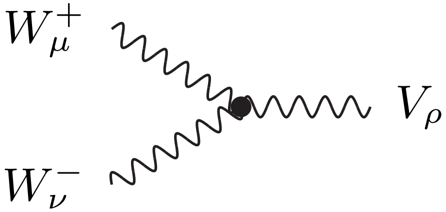
$$\begin{aligned}
\mathcal{L}_{\mathrm{YM}} = &-\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\
&- \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)
\end{aligned}$$

$+$ (trilinear interaction terms involving AW^+W^-, ZW^+W^-)

$+$ (quadrilinear interaction terms involving AAW^+W^-, AZW^+W^-, ZZW^+W^-, $W^+W^-W^+W^-$)

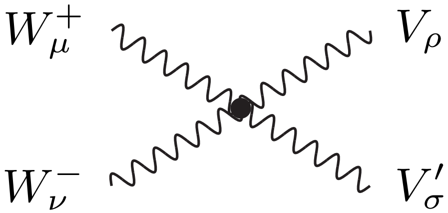
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$\mathrm{i}eC_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\dfrac{c_\mathrm{W}}{s_\mathrm{W}}$



$$\mathrm{i}e^2C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \dfrac{c_\mathrm{W}}{s_\mathrm{W}},$

$$C_{WWZZ} = -\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{W}^2}$$

Problem:

gauge fields Z, W^+, W^- are massive

explicit mass terms $\Leftrightarrow$ gauge invariance broken

$\Rightarrow$ **Higgs mechanism**

scalar field postulated, gauge-invariant
mass terms from coupling to Higgs field

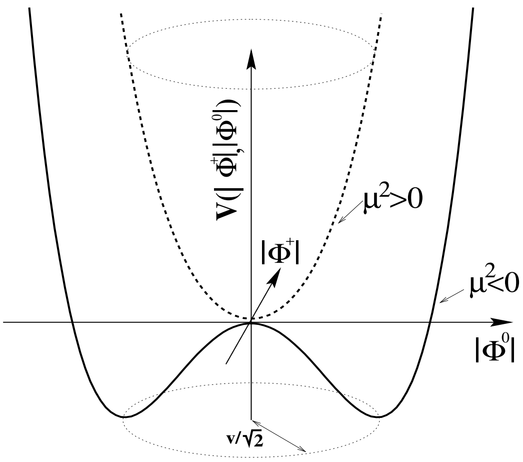
Higgs sector of the Standard Model:

scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential:

$$V(\phi) = \mu^2 \left|\Phi^\dagger \Phi\right| + \lambda \left|\Phi^\dagger \Phi\right|^2, \quad \lambda > 0$$

$\mu^2 < 0$: spontaneous symmetry breaking



minimum of the potential at $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\sqrt{\frac{-\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$

Gauge-invariant interaction with gauge fields:

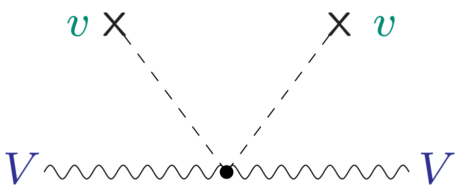
$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$

$\Rightarrow$ mass terms

Unitary gauge:

$$\Phi = \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$VV\Phi\Phi$ coupling:



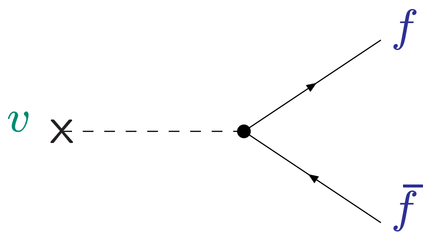
$\Rightarrow$ VV mass terms

$$\frac{1}{2} g_2^2 v^2 \equiv M_{\text{W}}^2, \quad \frac{1}{2}(g_1^2 + g_2^2) v^2 \equiv M_{\text{Z}}^2$$

3 components of Higgs doublet $\longrightarrow$ longitudinal components of $W^\pm$, Z

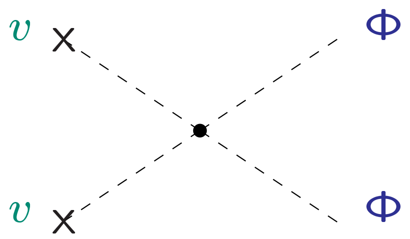
H: elementary scalar field, Higgs boson

Fermion mass terms: Yukawa couplings



$m_f = v\, g_f$ free parameters

Mass of the Higgs boson: self-interaction

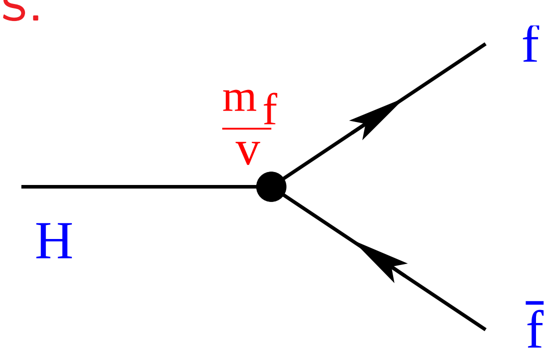


$M_{\mathsf{H}} = v\sqrt{\lambda}$ free parameter

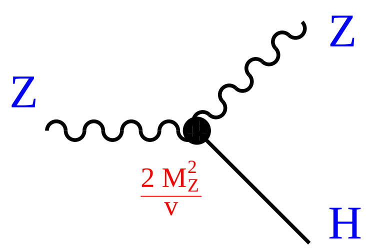
$\Rightarrow$ Higgs couplings proportional to masses of the particles

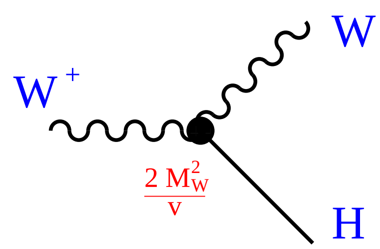
Interactions of the Higgs boson

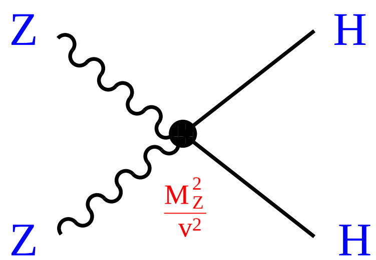
with fermions:

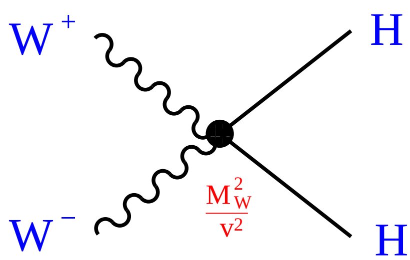


with gauge bosons: $\left(\frac{2M_W}{v} = g\right)$









Quark mixing and $\mathcal{CP}$-violation

The weak eigenstates (d', s', b') of quarks differ from the mass eigenstates d, s, b:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} .$$

V_{CKM}: unitary transformation, Cabibbo-Kobayashi-Maskawa (CKM) matrix

GIM Mechanism:

unitarity of CKM-matrix $\Rightarrow$ no flavor changing neutral current transitions at tree level

$b \to s\gamma, \ldots$ are loop-induced in SM $\Rightarrow$ high sensitivity to new physics effects

$V^{\dagger}_{\text{CKM}} V_{\text{CKM}} = 1$

$\Rightarrow$ parameterized by 3 angles $\underbrace{+\ 1 \text{ phase}}_{\text{gives rise to } \mathcal{CP}\text{-violation in SM}}$

$\Rightarrow$ weak interaction violates $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{CP}$

Physical parameters of the Standard Model

- gauge sector (2 parameters)
  elementary charge e
  weak mixing angle θ_W, $\cos\theta_\mathrm{W} = \frac{M_\mathrm{W}}{M_\mathrm{Z}}$

- Higgs sector (2 parameters)

  Higgs-boson mass M_H
  W-boson mass M_W

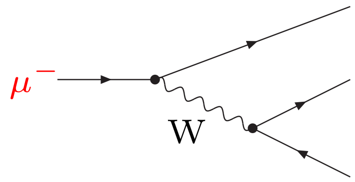
- fermion sector (9+4 parameters)
  fermion masses $m_\mathrm{e}, m_\mu, m_\tau$
  $m_\mathrm{u}\, m_\mathrm{d}, m_\mathrm{c}, m_\mathrm{s}, m_\mathrm{t}, m_\mathrm{b}$
  quark-mixing matrix (V_{ij}) $\theta_{12}, \theta_{23}, \theta_{13}, \delta_\mathrm{CP}$

  if right-handed neutrino is included in addition
  neutrino masses $m_{\nu_\mathrm{e}}, m_{\nu_\mu}, m_{\nu_\tau}$
  lepton-mixing matrix $\theta^l_{12}, \theta^l_{23}, \theta^l_{13}, \delta^l_\mathrm{CP}$
  $\Rightarrow$ 12+8 parameters

$\Rightarrow$ most parameters originate from fermion sector

Important electroweak experiments

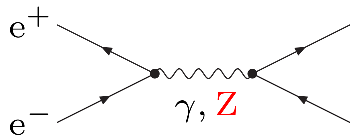
- Muon decay: $\mu^- \rightarrow \nu_\mu \mathrm{e}^- \bar{\nu}_\mathrm{e}$



determination of the Fermi constant

$$G_\mu = \frac{\pi \alpha M_\mathrm{Z}^2}{\sqrt{2} M_\mathrm{W}^2 (M_\mathrm{Z}^2 - M_\mathrm{W}^2)} + \ldots$$

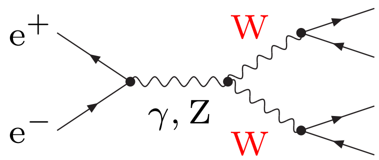
- Z production (LEP1/SLC): $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{Z} \rightarrow f\bar{f}$

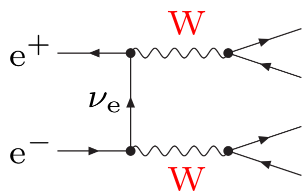


various precision measurements at the Z resonance: $M_\mathrm{Z}, \Gamma_\mathrm{Z}, \sigma_\mathrm{had}, A_\mathrm{FB}, A_\mathrm{LR}, \mathrm{etc.}$

$\Rightarrow$ good knowledge of the $\mathrm{Z}f\bar{f}$ sector

- W-pair production (LEP2/ILC): $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{WW} \rightarrow 4f(+\gamma)$





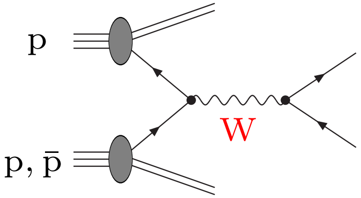
– measurement of M_W

– $\gamma\mathrm{WW}$/ZWW couplings

– quartic couplings: $\gamma\gamma\mathrm{WW}$, $\gamma\mathrm{ZWW}$

Important electroweak experiments (continued)

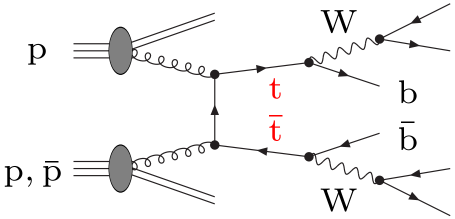
- W production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{W} \to l\nu_l(+\gamma)$



– measurement of M_W

– bounds on $\gamma\mathrm{WW}$ coupling

- top-quark production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{t\bar{t}} \to 6f$



– measurement of m_t

Theoretical predictions

parametrized by $\alpha(M_\mathrm{Z})$, M_W, M_Z, m_t, m_f, $\alpha_\mathrm{s}(M_\mathrm{Z})$ and M_H

$\hookrightarrow$ global fit of SM to data yields bounds on M_H

But: high precision necessary,
since M_H sensitivity weak $\sim \frac{\alpha}{\pi}\log(M_\mathrm{H}/M_\mathrm{W})$

Gauge-invariant Lagrangian:

$$\mathcal{L}_{\mathrm{EW}}(\underbrace{g_2, g_1, v}_{M_W,\ M_Z,\ \alpha}, \underbrace{\lambda}_{M_H}, \underbrace{g_f}_{m_f}) + \mathcal{L}_{\mathrm{QCD}}(\alpha_s)$$

gauge invariance $\Rightarrow$ **theory is renormalizable**

[G. 't Hooft '71]

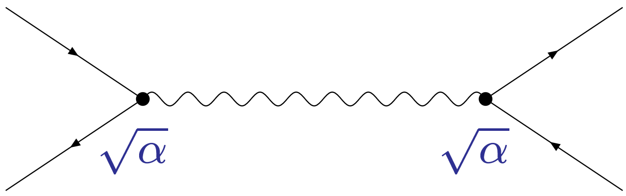
[G. 't Hooft, M. Veltman '72]

Nobel price '99

$\Rightarrow$ quantum field theory: quantum effects calculable

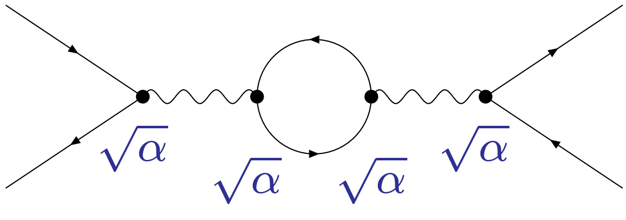
expansion in coupling constant:

lowest order, classical limit



$\alpha \approx \frac{1}{137}$

quantum corrections: loop diagrams

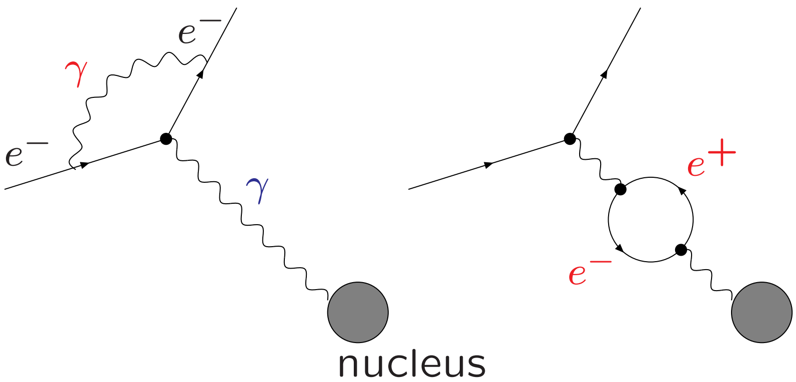


via loop corrections: all particles of the model enter

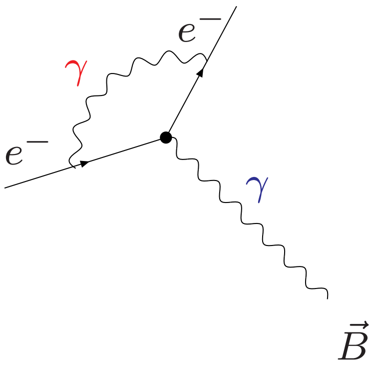
$\boxed{\text{Electroweak precision tests}}$

a theoretical concept becomes precision physics

Lamb shift:



$g-2$:



$$a = \frac{1}{2}(g-2)$$

$$a_{\text{exp}} = 1\,159\,652\,188(\pm 4)\times 10^{-12}$$

$$a_{\text{theo}} = 1\,159\,652\,157(\pm 28)\times 10^{-12}$$

- LEP1/SLC: $e^+e^- \to Z \to f\bar{f}$
  LEP1: $\sim 4 \times 10^6$ events/experiment
  4 experiments (1989 – 1995)

- LEP2: $e^+e^- \to W^+W^-$
  $\mathcal{O}(10^4)$ W pairs (1996 – 2000)

- Tevatron: $q\bar{q}' \to W \to l\nu, q\bar{q}'$
  (p$\bar{\text{p}}$) $q\bar{q}' \to t\bar{t},\ t \to W^+b \to \ldots$

- low-energy experiments (μ decay, νN scattering, νe scattering, atomic parity violation, ...)

exp. results

M_Z [GeV]	$= 91.1875 \pm 0.0021$	0.002%
Γ_Z[GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2\theta_{\text{eff}}^{\text{lept}}$	$= 0.23148 \pm 0.00017$	0.07%
M_W [GeV]	$= 80.410 \pm 0.032$	0.04%
m_t [GeV]	$= 172.7 \pm 2.9$	1.7%
G_F [GeV^{-2}]	$= 1.16637(1)10^{-5}$	0.001%

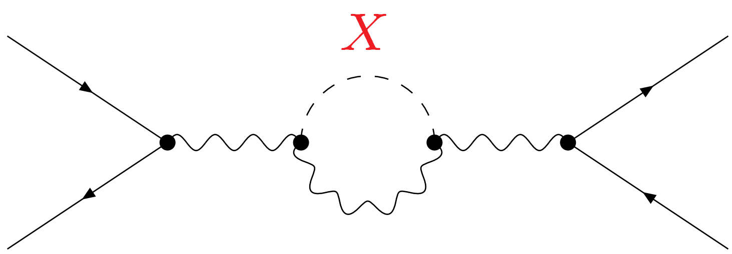
Comparison of electro-weak precision observables with theory:

EW Precision data: $\Delta\rho, \Delta r, M_W, \sin^2\theta_{\text{eff}}$	$\leftrightarrow$	Theory: SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



$\Downarrow$

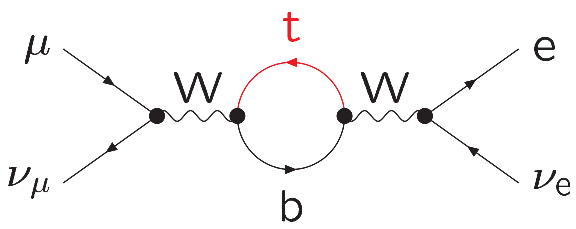
sensitivity to internal particles (X)

$\boxed{\text{Loop contributions}}$

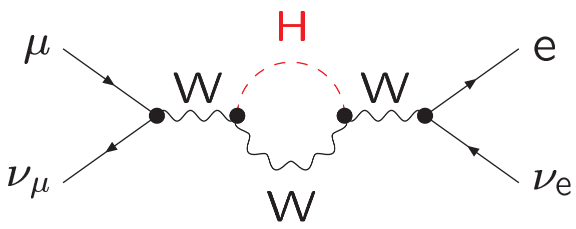
quantum corrections, of $\mathcal{O}(1\%)$

contain all details of the theory

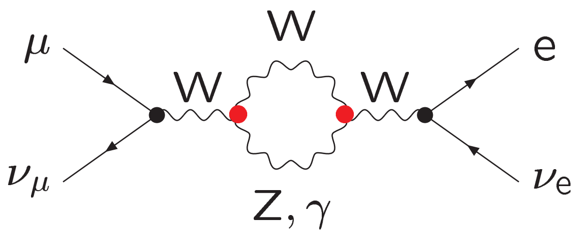
- top quark



- Higgs boson

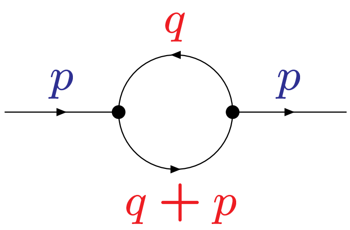


- gauge-boson self-couplings



$\Rightarrow$ allow for indirect experimental tests of not directly accessible quantities

Example of loop integral:



$$\sim \int d^4q \frac{1}{(q^2 - m_1^2)\left[(q+p)^2 - m_2^2\right]}$$

$$q \to \infty : \quad \sim \int^{\infty} \frac{q^3 dq}{q^4} = \int^{\infty} \frac{dq}{q} \quad \to \infty$$

$\Rightarrow$ integral diverges for large q

$\Rightarrow$ theory in this form not physically meaningful

$\Rightarrow$ further concept needed: renormalization

Renormalizable theories: infinities can consistently be absorbed into parameters of theory

Two step procedure:

Regularization:

theory modified such that expressions become mathematically meaningful

$\Rightarrow$ "regulator" introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^\infty d^4q \;\to \int_0^\Lambda d^4q; \quad \Lambda \to \infty \text{ at the end}$$

technically more convenient: dimensional regularization

$$\int d^4q \;\to \int d^Dq, \quad D = 4 - \varepsilon; \quad D \to 4 \text{ at the end}$$

Renormalization:

original "bare" parameters replaced by renormalized parameters + counterterms

reparameterization:

$$\underbrace{g_0}_{\substack{\text{bare}\\ \text{parameter}}} = \underbrace{g}_{\substack{\text{renormalized}\\ \text{parameter}}} + \underbrace{\delta g}_{\text{counterterm}}$$

Renormalizable theory:
divergencies compensated by counterterms

Renormalization:

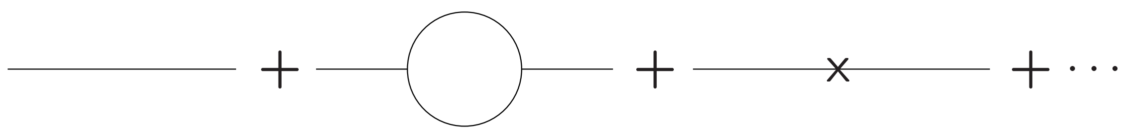
- absorption of divergencies
- determination of physical meaning of parameters order by order in perturbation theory

<u>Example:</u>

mass renormalization, $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:



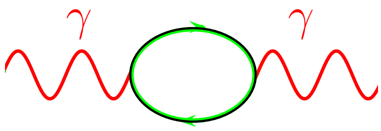
$p^2 - m^2$ $\qquad \Sigma(p^2) \qquad$ $-\,\delta m^2$

on-shell renormalization: $\delta m^2 = \mathrm{Re}\,\Sigma(m^2)$

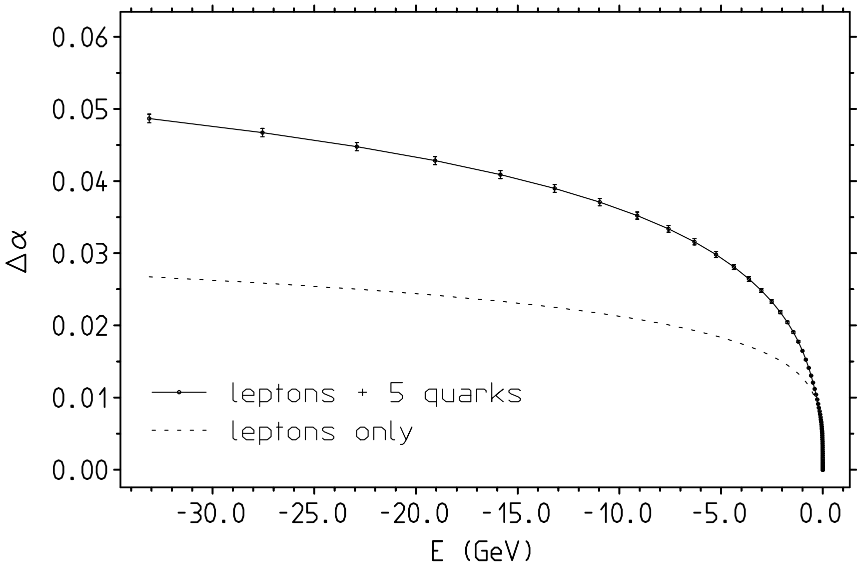
charge renormalization: $e + \delta e$

δe for $q^2 = 0$ (real photons) involves

photon vacuum polarization



virtual pairs



$$\Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \equiv \Delta\alpha$$

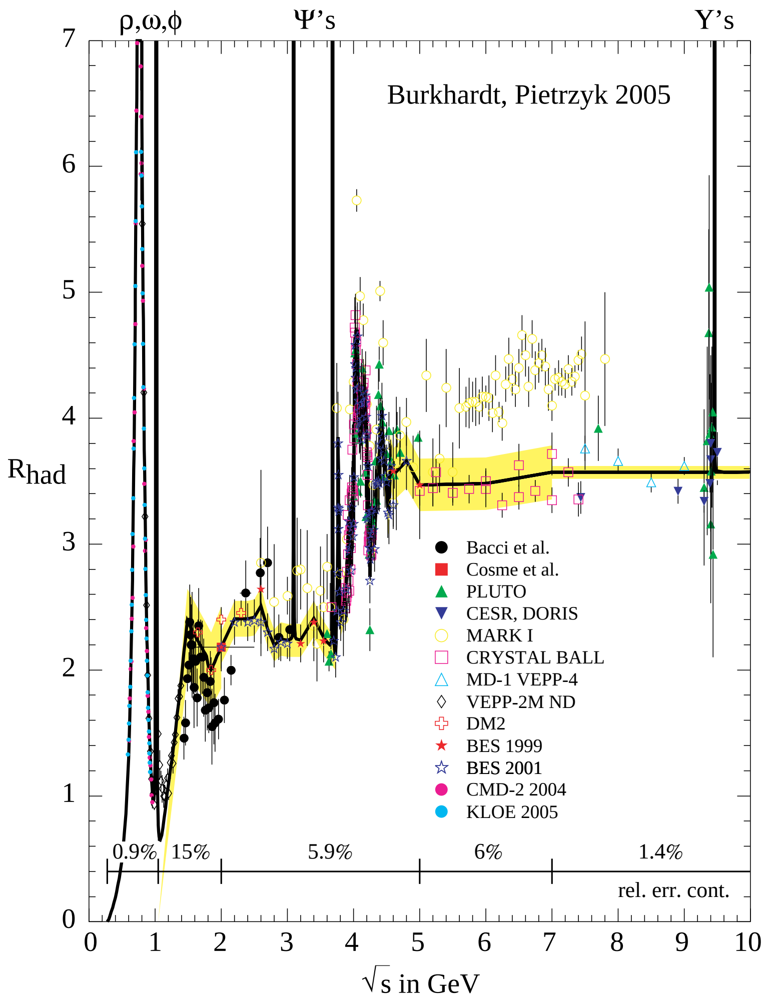
$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha}$$ effective charge

$$\Delta\alpha = \Delta\alpha_{\text{lept}} + \Delta\alpha_{\text{had}},$$

$$\Delta\alpha_{\text{lept}} = 0.031498 \quad (3 - \text{loop})$$

$$\Delta\alpha_{\text{had}} = 0.02758 \pm 0.00035$$

$$\Delta\alpha_{\text{had}} = -\frac{\alpha}{3\pi} M_Z^2 \ \text{Re} \int_{4m_\pi^2}^{\infty} ds' \ \frac{R_{\text{had}}(s')}{s'(s' - M_Z^2 - i\epsilon})$$

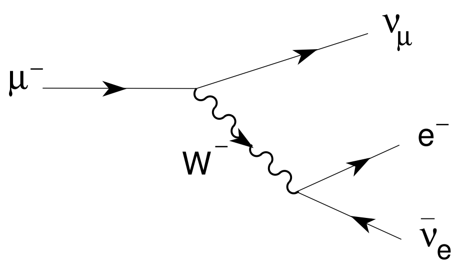


$\boxed{M_W - M_Z \text{ correlation}}$

Definition of Fermi constant G_F via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta q)$$

Δq: QED corrections in Fermi Model,
included in definition



SM prediction:

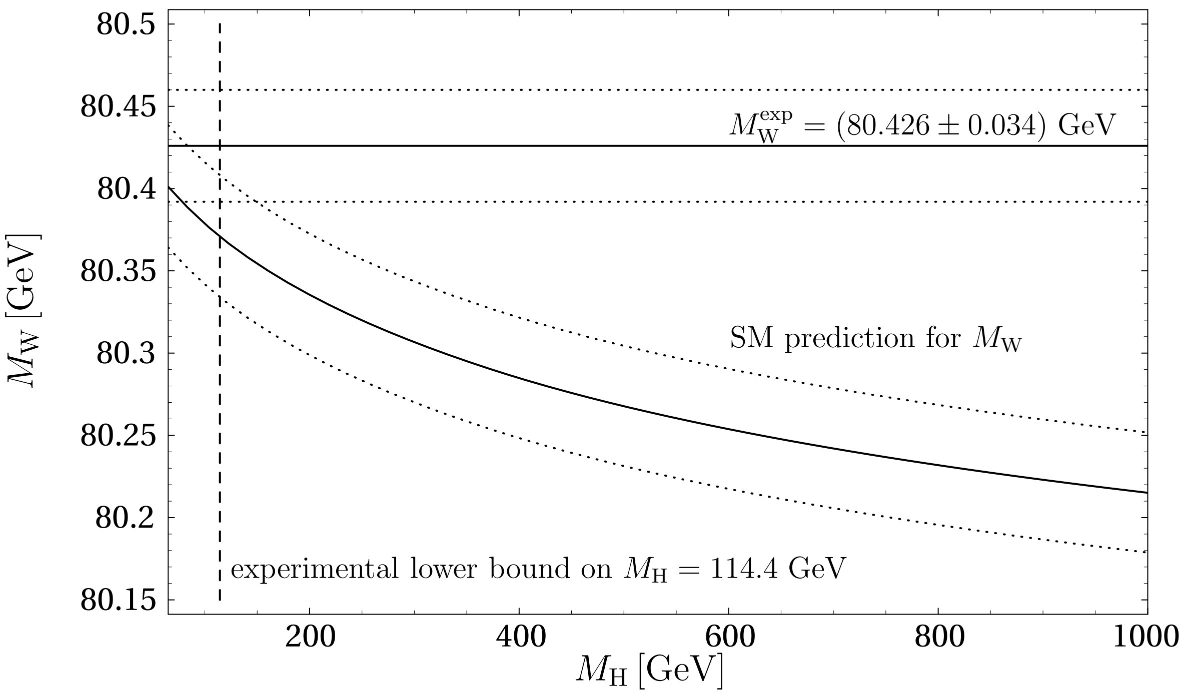
$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2\left(1 - M_W^2/M_Z^2\right)}(1 + \Delta r)$$

Δr: quantum correction, $\Delta r = \Delta r(m_t, M_H, \dots)$

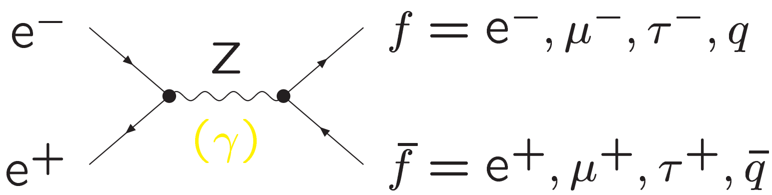
complete at 1-loop and 2-loop order

$\rightarrow \quad M_W = M_W(\alpha, G_F, M_Z, m_t, M_H)$

[Awramik, Czakon, Freitas, Weiglein]



Z-boson resonance



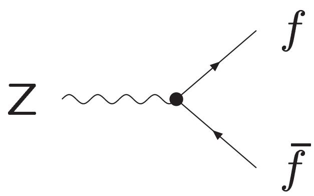
LEP1: $\sim 16 \cdot 10^6$ events (1989–1995)

resonance cross-section

(approximate Breit-Wigner) $\qquad s = E^2_{\mathrm{CMS}}$

$$\sigma_f(s) = 12\pi \frac{s}{M_Z^2} \frac{\Gamma(Z \to e^+e^-)\,\Gamma(Z \to f\bar{f})}{(s - M_Z^2)^2 + s^2(\Gamma_Z)^2/M_Z^2}$$

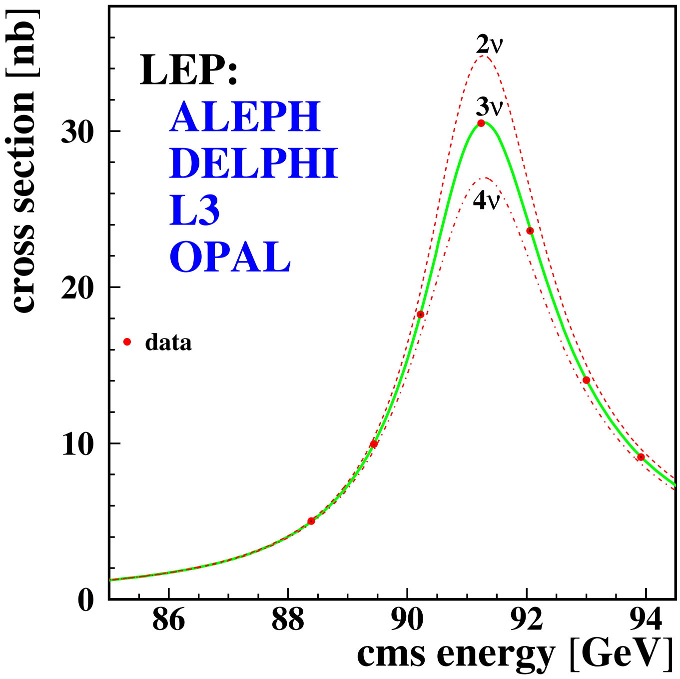
Z-boson width



$$\Gamma_Z = \underbrace{\Gamma(e^-e^+) + \Gamma(\mu^-\mu^+) + \Gamma(\tau^-\tau^+)}_{\text{leptonic}}$$
$$+ \underbrace{\sum_q \Gamma(q\bar{q})}_{\text{hadronic}} + \underbrace{N_\nu \Gamma(\nu\bar{\nu})}_{\text{invisible}}$$

- Line shape $\Rightarrow M_Z, \Gamma_Z$
- peak cross section
  $\Rightarrow \Gamma(Z \to l^+l^-), \Gamma(Z \to \text{hadrons})$

Z resonance



Z-boson observables can be expressed in terms of

- effective Z boson couplings:

$$g_V^f \to g_V^f + \Delta g_V^f, \qquad g_A^f \to g_A^f + \Delta g_A^f$$

with higher order contributions in $\Delta g_{V,A}^f$

- effective ew mixing angle (for $f = e$):

$$\sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\,\frac{g_V^e}{g_A^e}\right)$$

complete at 1-loop order, 2-loop fermionic contributions

$\boxed{\text{Z Observables}}$

- Z width, partial widths

$$\Gamma_f = \Gamma(Z \to f\bar{f}) \sim [(g_V^f)^2 + (g_A^f)^2] \cdot (1 + K_{\mathrm{QED}} + K_{\mathrm{QCD}})$$

$$\Gamma_{\mathrm{had}} = \sum_q \Gamma_q, \qquad R_{\mathrm{had}} = \frac{\Gamma_{\mathrm{had}}}{\Gamma_e}, \qquad R_{\mathrm{b}} = \frac{\Gamma_b}{\Gamma_{\mathrm{had}}}$$

- Asymmetries in $e^+e^- \to f\bar{f}$

$$A_{\mathrm{FB}}^f = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}} \qquad \text{with}$$

$$\sigma_{\mathrm{F}} = \int_{\pi/2}^{\pi} \mathrm{d}\theta \frac{\mathrm{d}\sigma}{\mathrm{d}\theta}, \qquad \sigma_{\mathrm{B}} = \int_0^{\pi/2} \mathrm{d}\theta \frac{\mathrm{d}\sigma}{\mathrm{d}\theta}$$

$$A_{\mathrm{LR}} = \frac{\sigma(e_{\mathrm{L}}) - \sigma(e_{\mathrm{R}})}{\sigma(e_{\mathrm{L}}) + \sigma(e_{\mathrm{R}})}$$

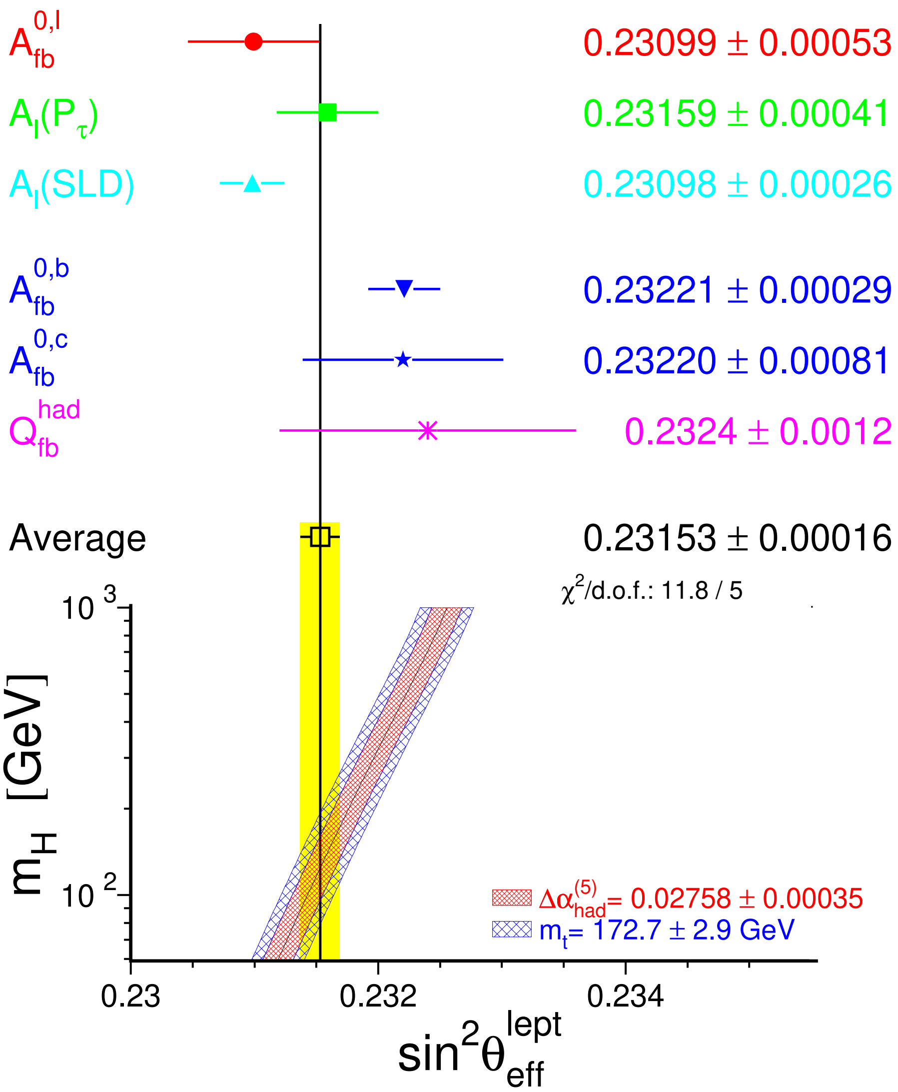
$$A_{\mathrm{FB}}^f = \frac{3}{4} A_e \cdot A_f, \qquad A_f = \frac{2 g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}$$

$$A_{\mathrm{LR}} = A_e$$

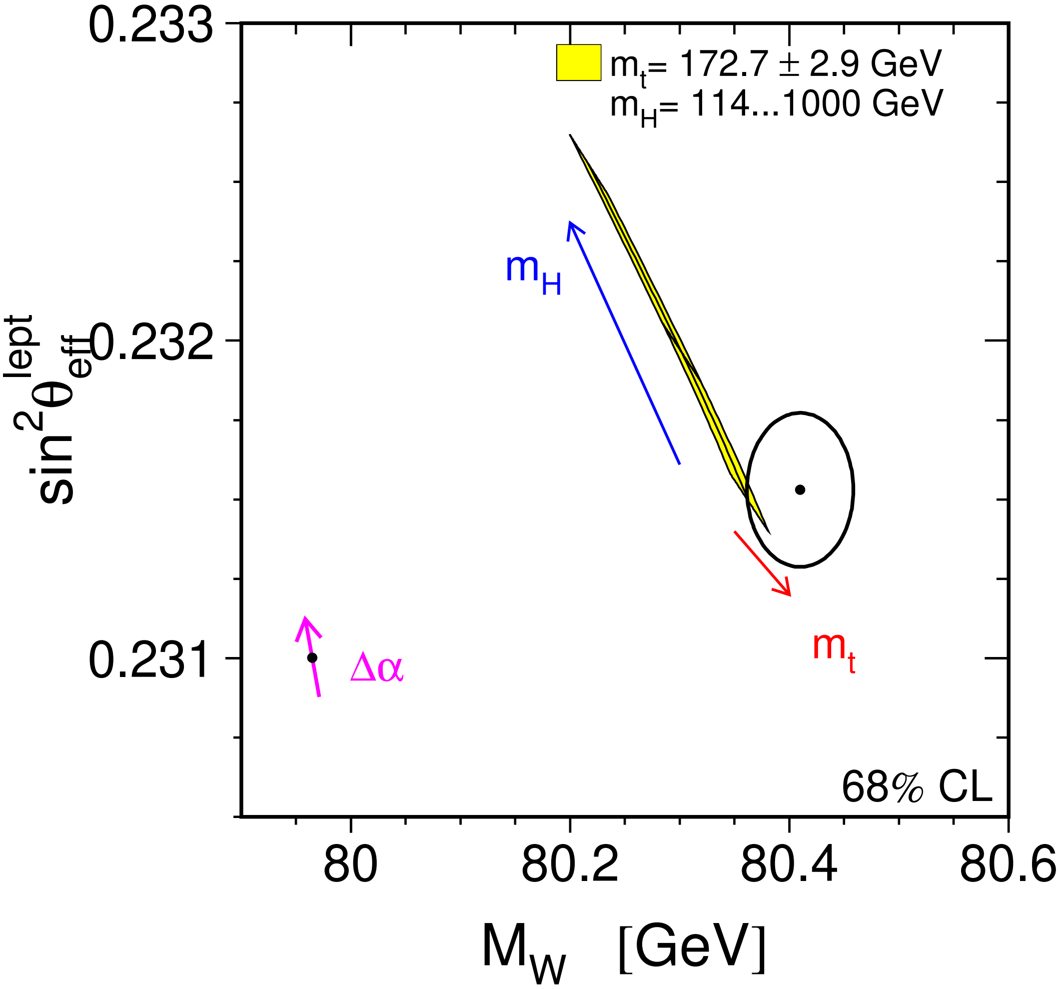
special :

$$A_{\mathrm{FB}}^b \simeq \text{const} \cdot A_e \quad \text{since} \quad A_b \simeq 0.935 \;\; \text{(SM)}$$

LEP Electroweak Working Group [Summer 2005]



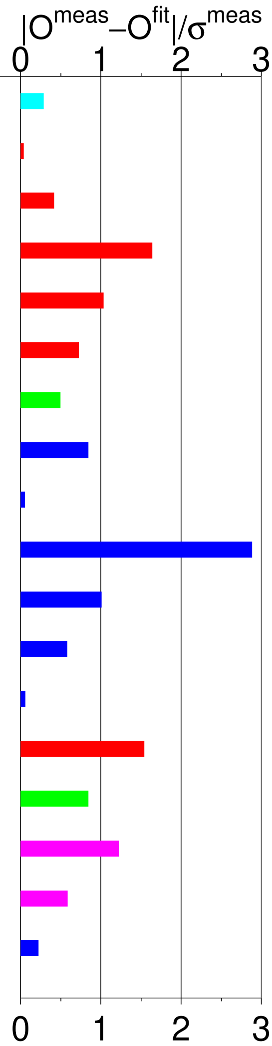
LEP Electroweak Working Group



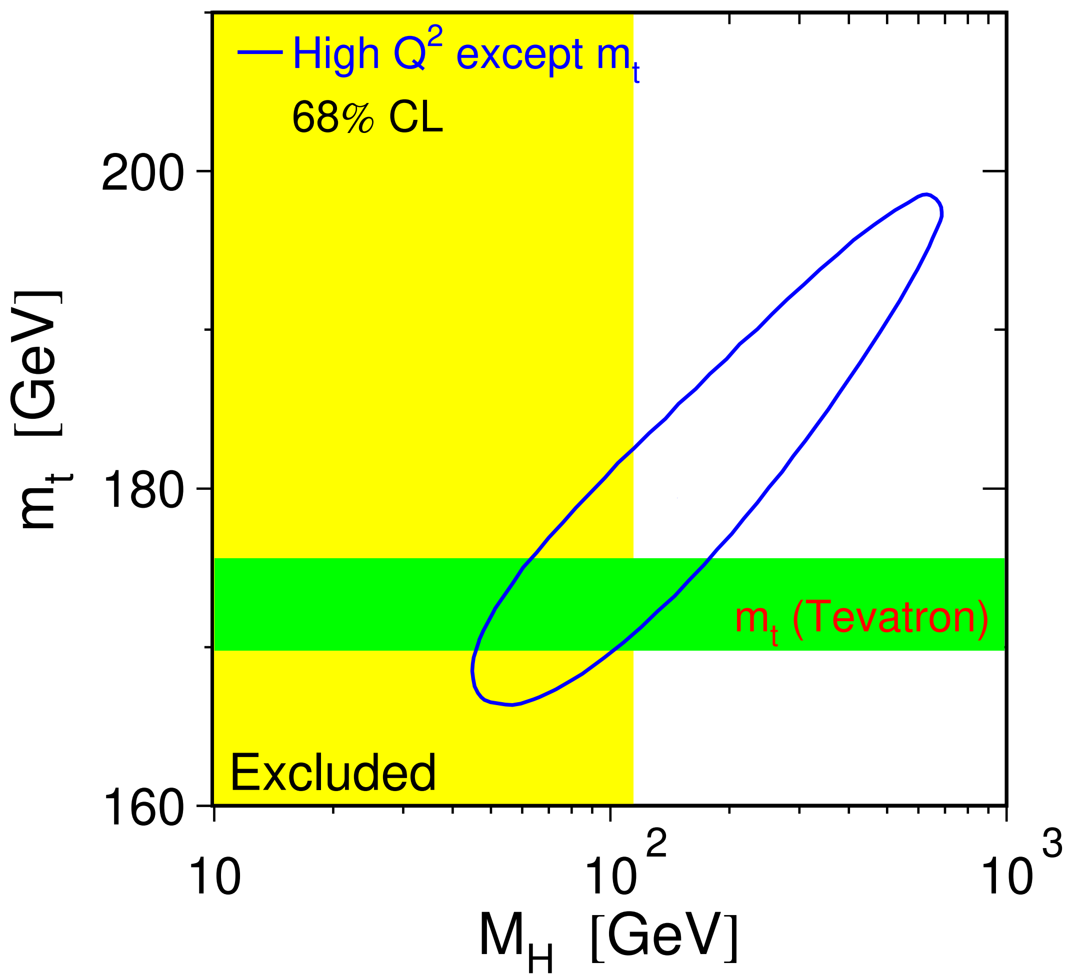
Global fit [M. Grünewald, EPS Lisbon 2005]

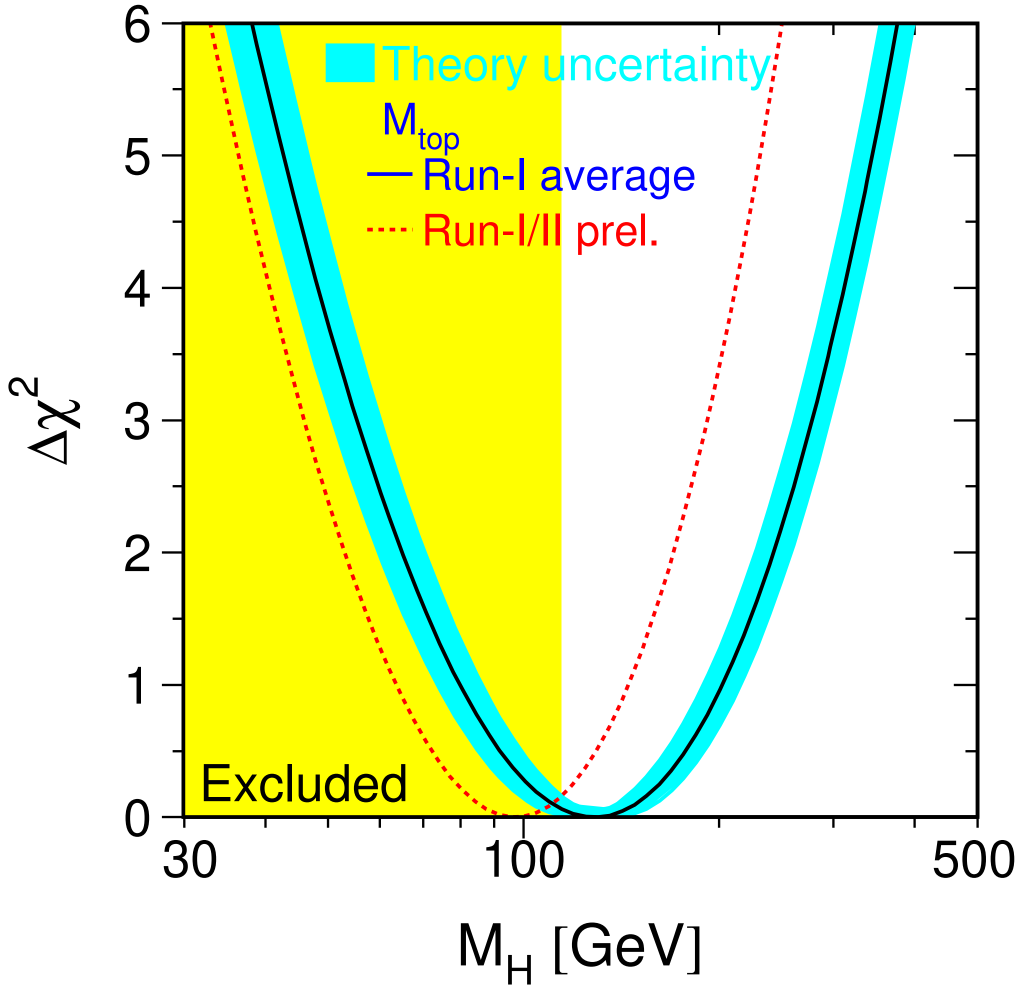
Preliminary

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4962
σ_{had}^0 [nb]	41.540 ± 0.037	41.479
R_l	20.767 ± 0.025	20.741
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21573
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.383
Γ_W [GeV]	2.133 ± 0.069	2.092
m_t [GeV]	174.3 ± 3.4	175.1



Bounds on m_t and M_H



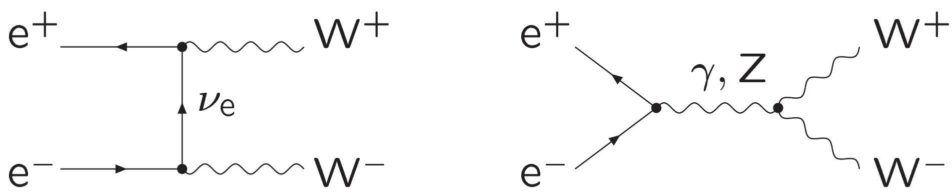


$$M_{\mathrm{H}} < 186\ \text{GeV} \quad (95\%\text{C.L.})$$

renormalized probability for $M_{\mathrm{H}} > 114$ GeV to 100%:

$$M_{\mathrm{H}} < 219\ \text{GeV} \quad (95\%\text{C.L.})$$

W-pair production



LEP2: $\mathcal{O}(10^4)$ events (1996-2000)

study of W-pair production allows

- precise measurement of M_W

  $\Delta M_W \approx 40\,\mathrm{MeV}$, $\Delta M_W / M_W \approx 0.05\%$

- measurement of triple-gauge-boson couplings

  total cross-section

  $\Delta\sigma_{WW} / \sigma_{WW} \sim 1\%$

  triple-gauge-boson couplings

  $\sim 3\%$

$\boxed{\text{anomalous gauge couplings}}$

generalization of gauge boson self couplings

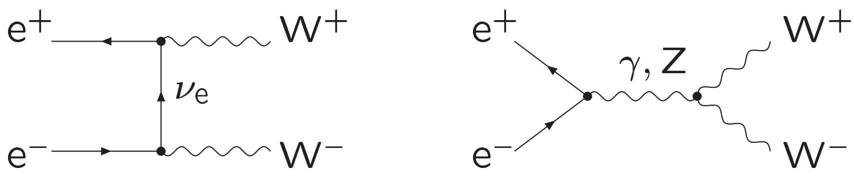
($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\quad Z_{\mu\nu}$, $W^\pm_{\mu\nu}$ analogously)

$$
\begin{aligned}
\mathcal{L}_{WW\gamma/Z} \;=\; & e\left[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)\, W^{-\mu} A^\nu \right.\\
& + \kappa_\gamma\, W^+_\mu W^-_\nu\, F^{\mu\nu} \\
& + \frac{\lambda_\gamma}{M_W^2}\, W^+_{\rho\mu} W^{-\mu}_\nu\, F^{\rho\nu} + \text{h. c.}] \\
& + e \cot\theta_W \left[(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)\, W^{-\mu} Z^\nu \right.\\
& + \kappa_Z\, W^+_\mu W^-_\nu\, Z^{\mu\nu} \\
& \left. + \frac{\lambda_Z}{M_W^2}\, W^+_{\rho\mu} W^{-\mu}_\nu\, Z^{\rho\nu} + \text{h. c.}\right]
\end{aligned}
$$

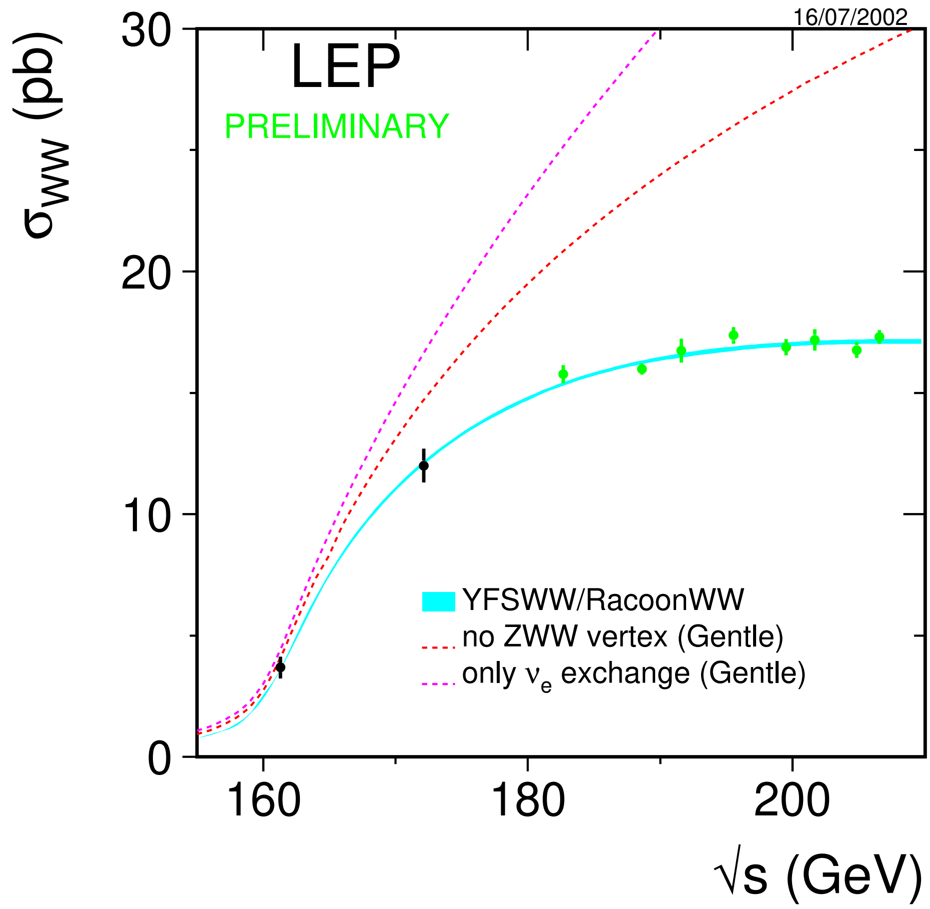
Standard Model:

$$\kappa_\gamma = \kappa_Z = 1, \quad \lambda_\gamma = \lambda_Z = 0$$

W-pair-production cross-section



LEPEWWG



contributions of non-abelian couplings
relevance of gauge invariance

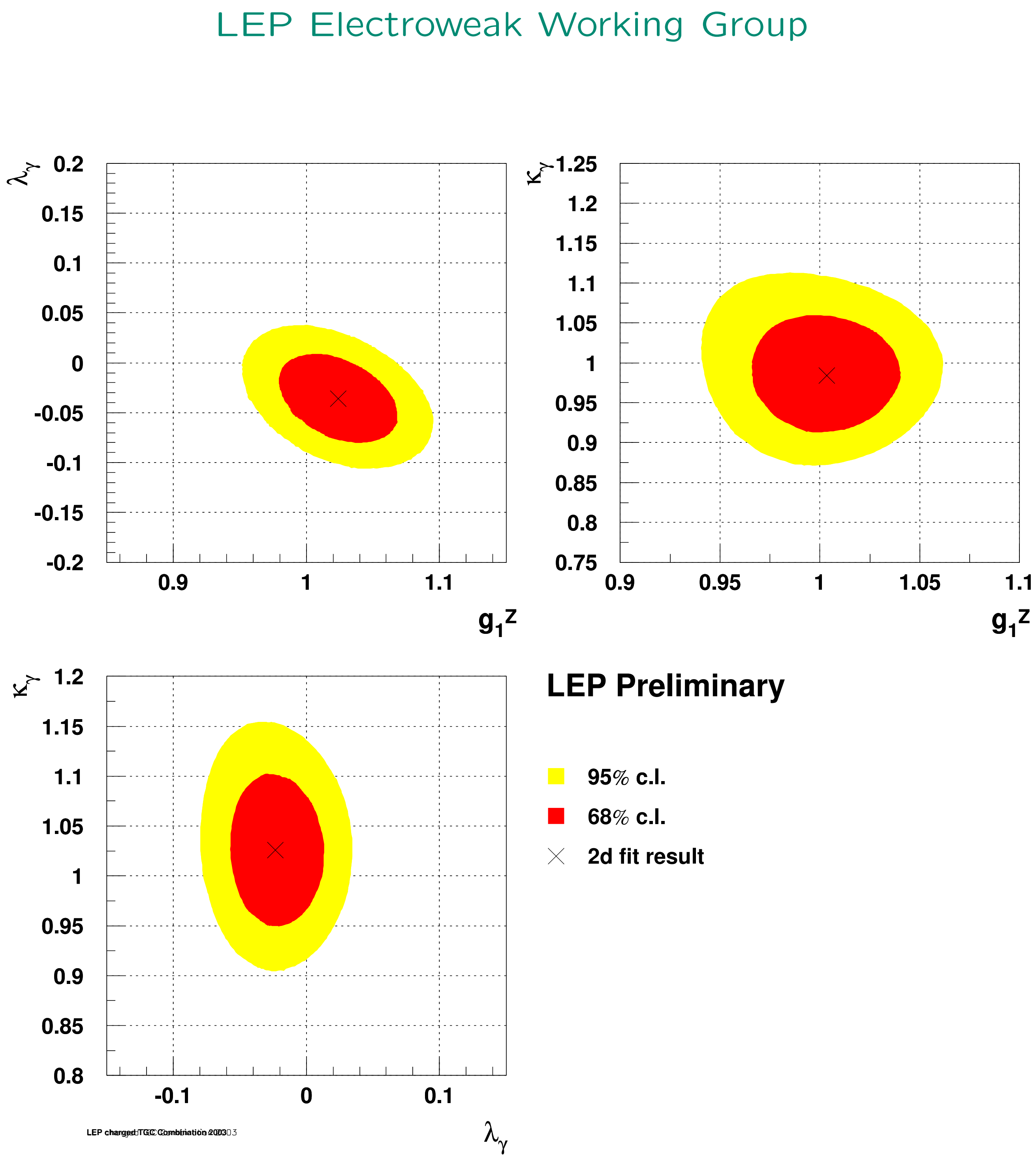


Figure 11.2: The 68% and 95% confidence level contours for the three two-parameter fits to the charged TGCs g_1^Z-λ_γ, g_1^Z-κ_γ and λ_γ-κ_γ. The fitted coupling value is indicated with a cross; the Standard Model value for each fit is in the centre of the grid. The contours include the contribution from systematic uncertainties.

Anomalous g-factor of the muon

- Dirac theory: $g = 2$

- QED, 1-loop order: $g = 2 + \frac{\alpha}{\pi}$

- Standard Model prediction

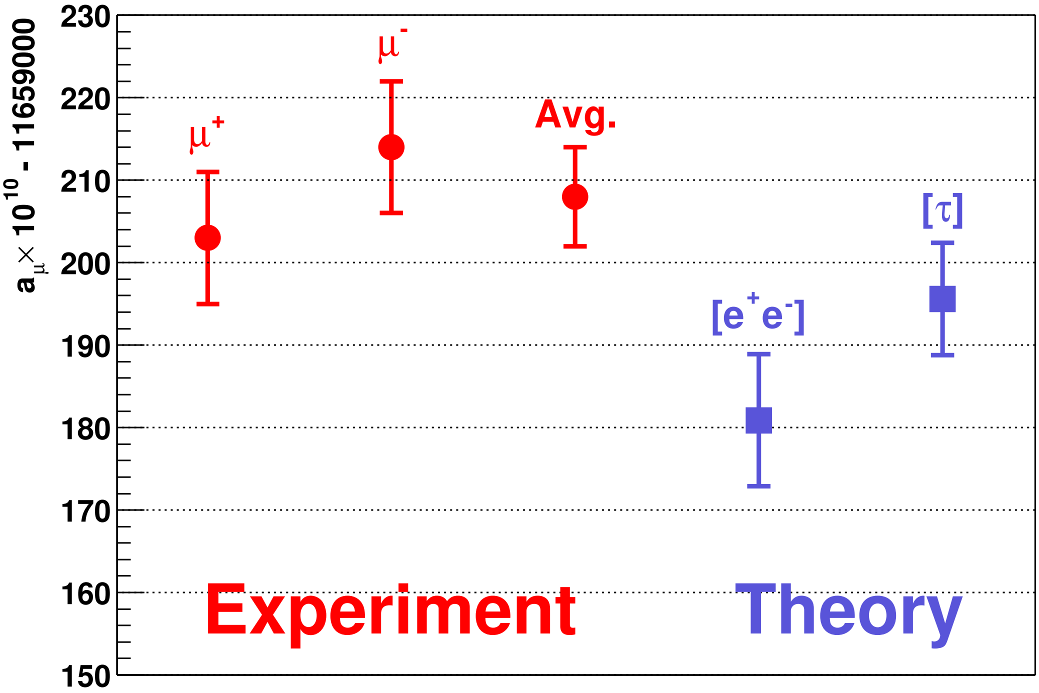
  QED part: 4-loop (5-loop estimate)
  Electroweak part: 2-loop

- **Experiment 2004:** Brookhaven E821

$$a_\mu = \frac{g-2}{2} = 11659208(6) \cdot 10^{-10}$$

above the SM prediction

Theory versus experiment



e^+e^- data based prediction:
2.7 σ below exp. value

τ data based prediction:
0.7 σ below exp. value

uncertainty mainly from hadronic vacuum polarization

Summary of precision tests

- Electroweak precision physics

  $\Rightarrow$ Sensitivity to quantum effects of the theory

  $\Rightarrow$ test consistency of the model

  constraints on unknown parameters

- Precision tests of the SM

  $\Rightarrow$ light Higgs preferred, $M_H \lesssim 200$ GeV

  preference for light Higgs is not an artefact of observables deviating by $\approx 3\sigma$ from SM prediction

- Prospects for next generation of colliders:

  improved accuracy of precision observables
  M_W, $\sin^2\theta_{eff}$, m_h, ... and
  input parameters m_t, ...

  $\Rightarrow$ Highly sensitive test of electroweak theory

(expected) experimental precision

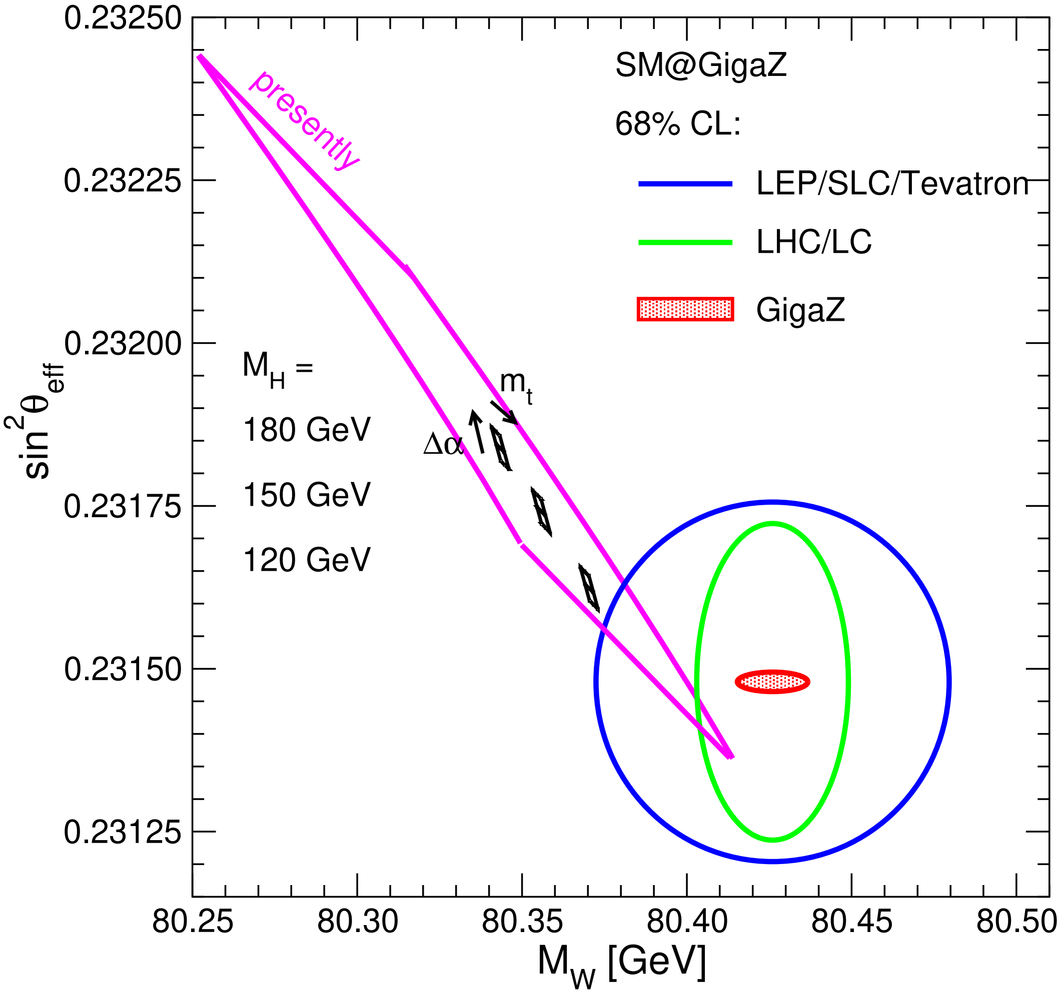
error for	LEP/Tev	Tev/LHC	LC	GigaZ
M_W [MeV]	33	15	15	7
$\sin^2\theta_{\text{eff}}$	0.00017	0.00021		0.000013
m_{top} [GeV]	4.3	2	0.2	0.13
M_{Higgs} [GeV]	–	0.1	0.05	0.05

together with

$\delta M_Z = 2.1$ MeV (LEP)

$\delta G_F/G_F = 1 \cdot 10^{-5}$ (μ lifetime)

[Erler, Heinemeyer, Hollik, Weiglein, Zerwas]



$\boxed{\text{Higgs bosons}}$

Higgs boson is the only missing ingredient of the SM

$\Rightarrow$ Higgs search (& Higgs physics) is one of the main goals of collider physics

$\leq$ 2000: LEP:
e^+e^- collider, $E_{\text{CM}} \lesssim 206$ GeV

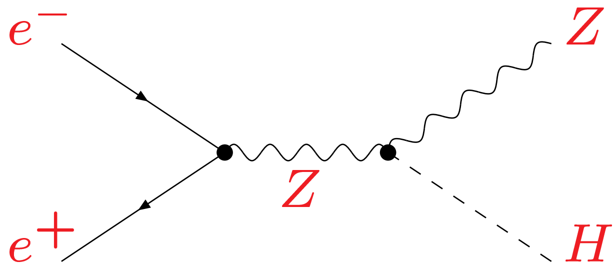
$\geq$ 2001: Tevatron, Run II:
$p\bar{p}$ collider, $E_{\text{CM}} \approx 2$ TeV

$\gtrsim$ 2007: LHC:
pp collider, $E_{\text{CM}} \approx 14$ TeV

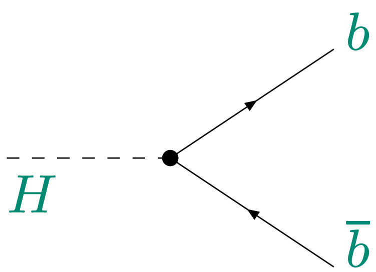
$\gtrsim$ 2012: ? LC:
e^+e^- collider, $E_{\text{CM}} \approx$ 500–1000 GeV

Search for the Standard Model Higgs at LEP

Dominant production process: $e^+e^- \to ZH$



Dominant decay process: $H \to b\bar{b}$



exclusion limit (95% C.L.):

$$M_{\mathrm{H}} > 114.4 \text{ GeV}$$

Theoretical bounds on Higgs boson mass from

- perturbativity
  $\longrightarrow$ upper bound

- unitarity
  $\longrightarrow$ upper bound

- triviality (Landau pole)
  $\longrightarrow$ upper bound

- vacuum stability
  $\longrightarrow$ lower bound

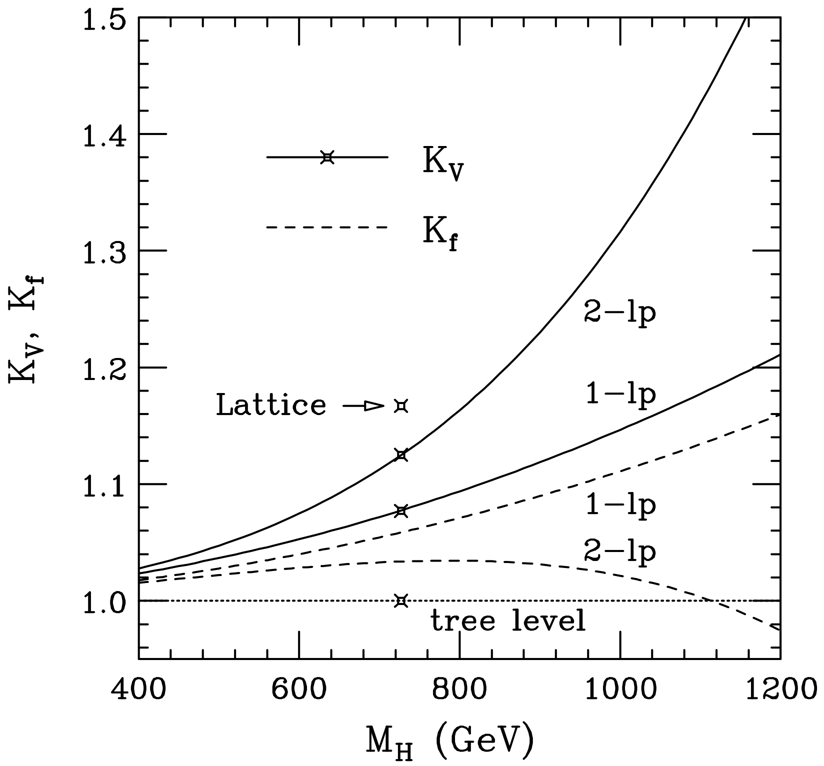
perturbativity

Higgs decay widths into fermions:

$$\Gamma(H \to f\bar{f}) = \Gamma_{\text{tree}} \cdot K_f$$
$$K_f = 1 + (1-\text{loop}) + (2-\text{loop}) + \cdots$$

Higgs decay widths into vector bosons:

$$\Gamma(H \to V\bar{V}) = \Gamma_{\text{tree}} \cdot K_V$$
$$K_V = 1 + (1-\text{loop}) + (2-\text{loop}) + \cdots$$



[Ghinculov; Frinck, Kniehl, Riesselmann]

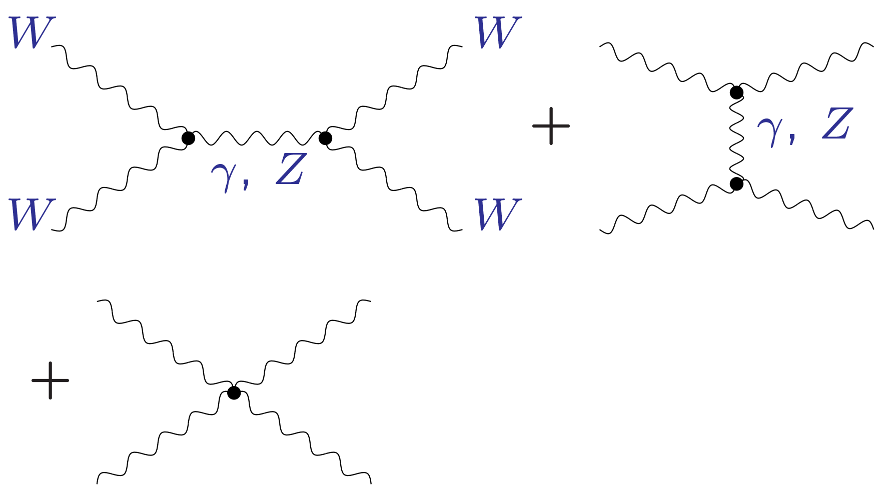
(1-loop) = (2-loop) for $M_H = 930$ GeV

$\boxed{\text{unitarity}}$

scattering of longitudinally polarized W bosons:

$$W_L W_L \to W_L W_L$$

$\mathcal{M}_V =$

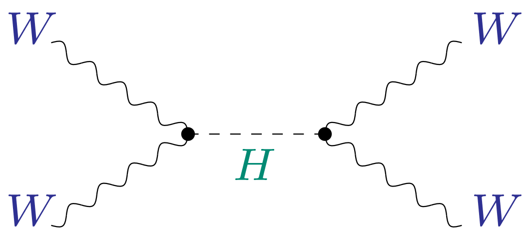


$$= -g^2 \frac{E^2}{M_{\mathrm{W}}^2} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

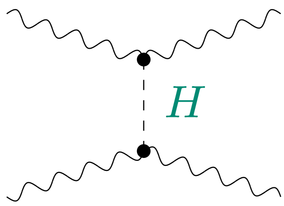
$\Rightarrow$ violation of probability conservation

Extra contribution from scalar particle:

$\mathcal{M}_S =$



$+$



$$= g_{WWH}^2 \frac{E^2}{M_{\mathrm{W}}^4} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

$$\mathcal{M} \quad = \mathcal{M}_V + \mathcal{M}_S$$

$\Rightarrow$ terms with bad high-energy behavior cancel for

$$g_{WWH} = g\, M_{\mathrm{W}}$$

$$\text{for } s >> M_W^2, \qquad \text{with } t = -\tfrac{s}{2}(1 - \cos\theta),$$

$$\mathcal{M} \approx \frac{M_H^2}{v^2}\left(2 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2}\right)$$

partial wave expansion:

$$\mathcal{M}(s,t) \;=\; 8\pi \sum_{l=0}^{\infty} (2l+1)\, P_l(\cos\theta)\, a_l$$

unitarity condition:

$$|a_l| < 1$$

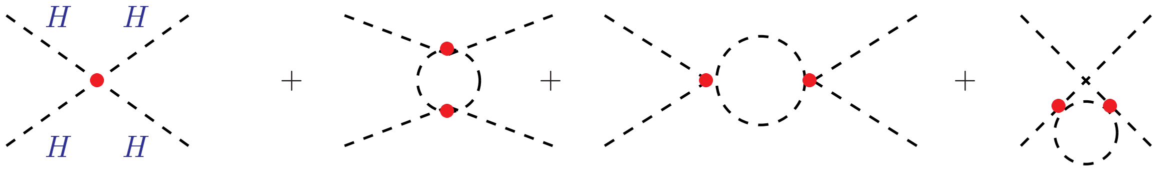
project on $l = 0$ partial wave:

$$\begin{aligned} a_0 &= \frac{1}{16\pi} \int_{-1}^{1} d\cos\theta\, \mathcal{M}(s,t) \\ &= \frac{M_H^2}{8\pi v^2} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log\left(1 + \frac{s}{M_H^2}\right) \right] \\ &\approx \frac{M_H^2}{4\pi v^2} \quad \text{for} \quad s >> M_H^2 \end{aligned}$$

$$\boxed{a_0 < 1 \quad \to \quad M_H < 872\,\text{GeV}}$$

triviality (Landau pole)

Higgs self coupling is scale dependent, $\lambda(Q)$



variation with scale Q described by RGE

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}\lambda^2, \qquad t = \log\frac{Q^2}{v^2}$$

solution:

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^2}\lambda(v)\log\frac{Q^2}{v^2}} \quad \text{with} \quad \lambda(v) = \frac{M_H^2}{2v^2}$$

diverges at scale $Q = \Lambda_C$ (Landau pole)

$$\Lambda_C = v \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

maximum Higgs mass by condition $\Lambda_C > M_H$

$$\rightarrow \qquad M_H < 800\,\text{GeV}$$

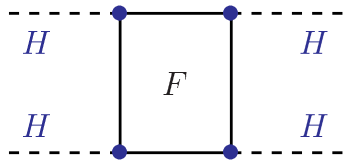
vacuum stability

top-quark Yukawa coupling

$$g_t = \frac{\sqrt{2} m_t}{v}$$

contributes to the running Higgs self coupling $\lambda(Q)$ through

top loop $\quad \sim g_t^4$



variation with scale Q described by RGE

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}\left(\lambda^2 - \frac{m_t^4}{v^4}\right)$$

approximate solution:

$$\lambda(Q) = \lambda(v) - \frac{3m_t^4}{2\pi^2 v^4} \log \frac{Q}{v}$$

$$\lambda(Q) < 0 \text{ for } Q > \Lambda_C \rightarrow \text{vacuum not stable}$$

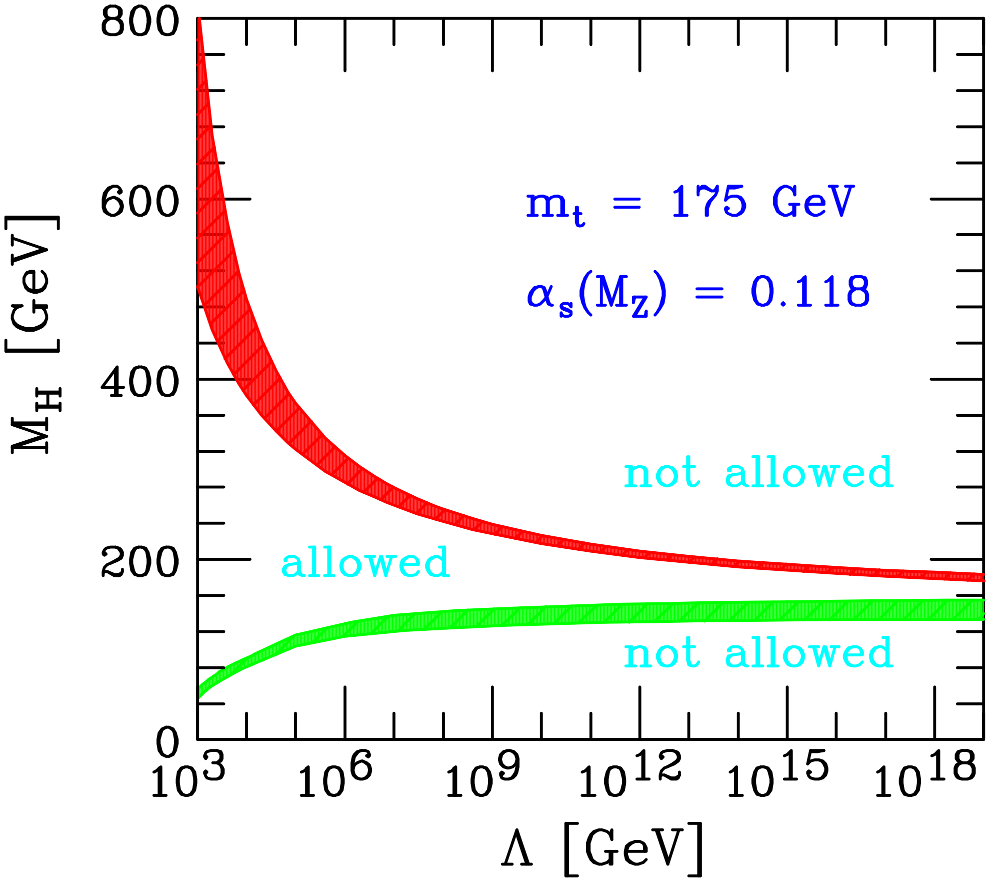
high value of Λ_C needs M_H large enough

$$\Lambda_C \sim 10^{16}: \quad M_H > 130\,\text{GeV}$$

$$\Lambda_C \sim 10^{3}: \quad M_H > 70\,\text{GeV}$$

combined effects, RGE in two-loop order:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(12\lambda^2 - 3\,g_t^2 + 6\,\lambda\,g_t^2 + \cdots\right)$$



[Hambye, Riesselmann]

$\boxed{\text{Higgs boson(s)}}$

- questions to be answered:

– numbers of Higgs particles

– masses and quantum numbers
(spin, partity, charges, CP, ...)

– couplings to fermions / gauge bosons

– self couplings $\rightarrow$ Higgs potential

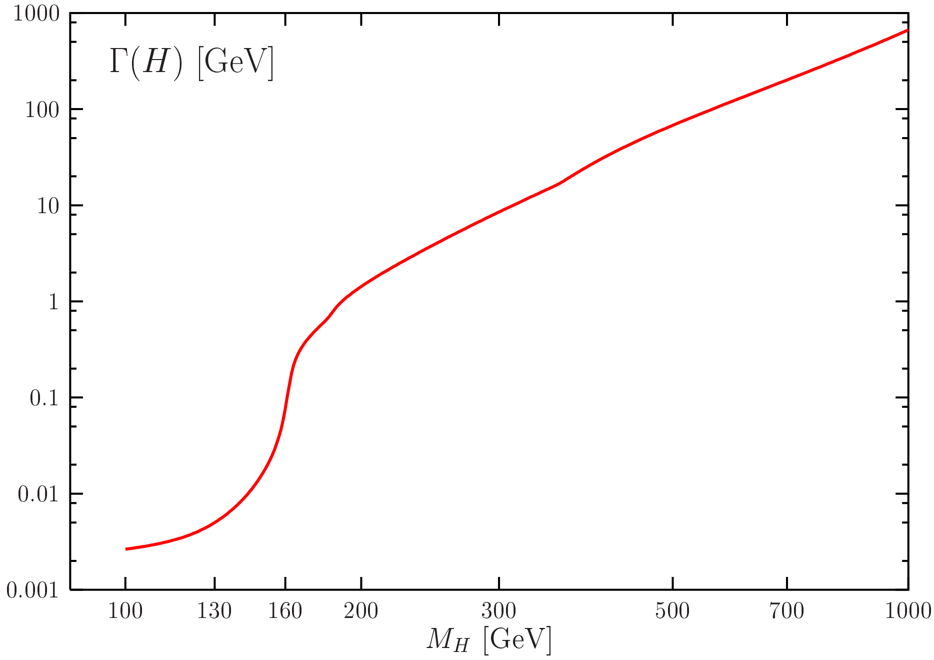
- needs precise determination of mass(es) and coupling constants

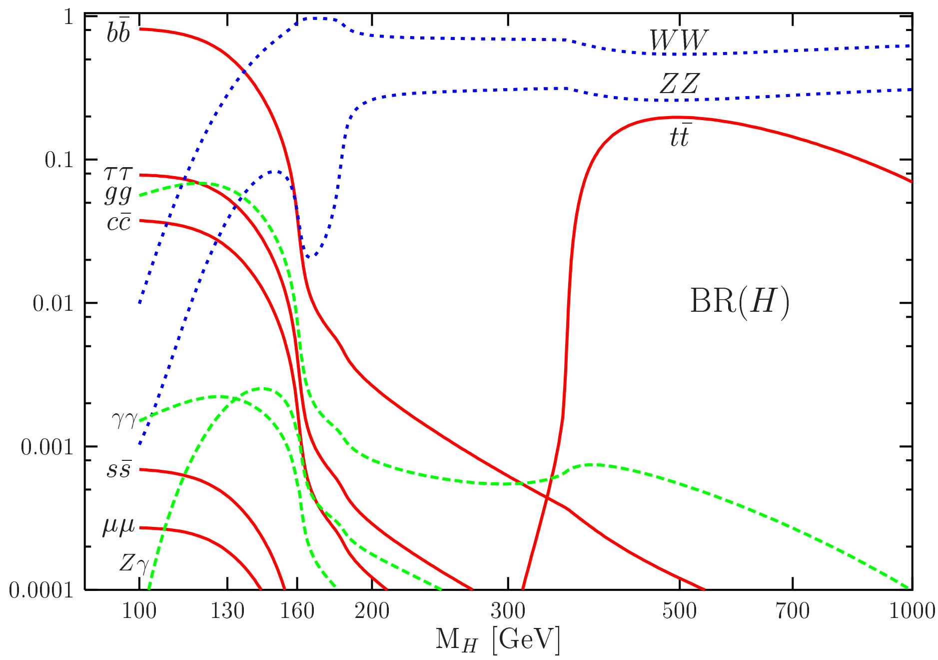
– production cross sections

– decay rates/ branching ratios

– inclusion of higher-order effects

total width and branching ratios





Production mechanisms

- gluon-gluon fusion:

$$gg \to H$$

NNLO QCD [Harlander, Kilgore]
NL EW [Degrassi, Maltoni]

- WW (ZZ) fusion:

$$qq \to Hq'q'$$

NLO QCD [Figy, Oleari, Zeppenfeld]

- Higgs-strahlung processes:

$$qq' \to WH$$
$$q\bar{q} \to ZH$$

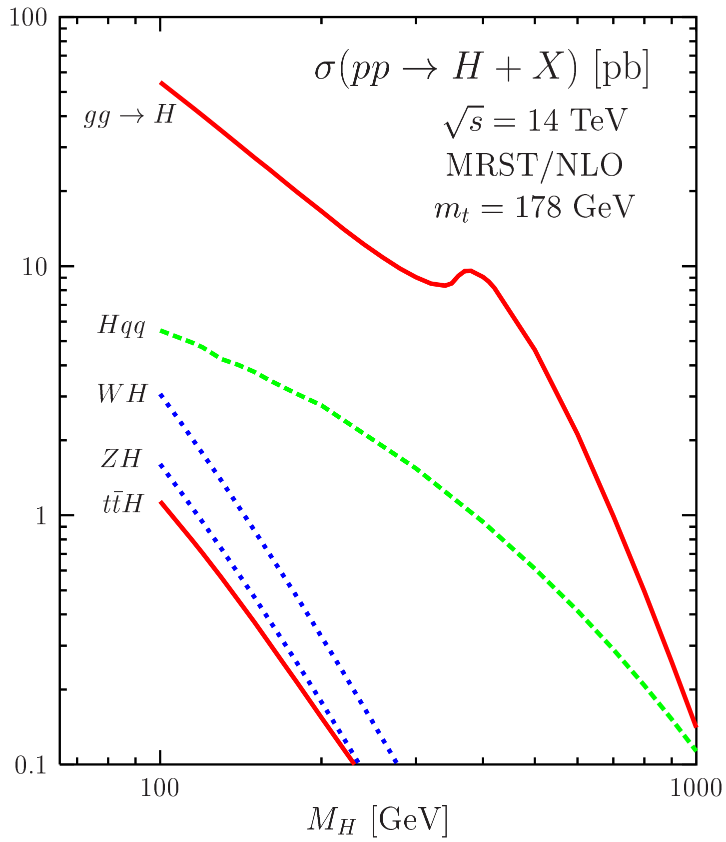
NNLO QCD + NLO EW [Brein et al.]

- radiation from heavy quarks:

$$gg,\ q\bar{q} \to t\bar{t}H\ (b\bar{b}H$$

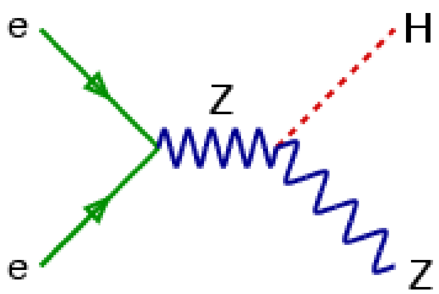
NLO QCD [Beenakker et al., Dawson et al.]
NLO EW [Denner et al.]

Higgs production at the LHC

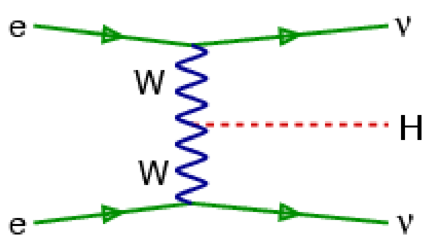


The Profile of the Higgs Boson

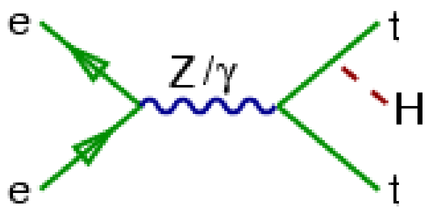
Production Processes



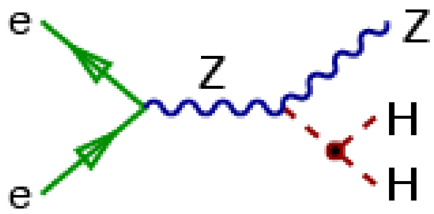
	500 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
	350 GeV	500 GeV	800 GeV
m_H = 120	74000	35000	27000
m_H = 160	52000	29000	24000
m_H = 250	5500	16500	19000



	500 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
	350 GeV	500 GeV	800 GeV
m_H = 120	15500	37500	158000
m_H = 160	7500	25000	126000
m_H = 250	6500	8000	71000

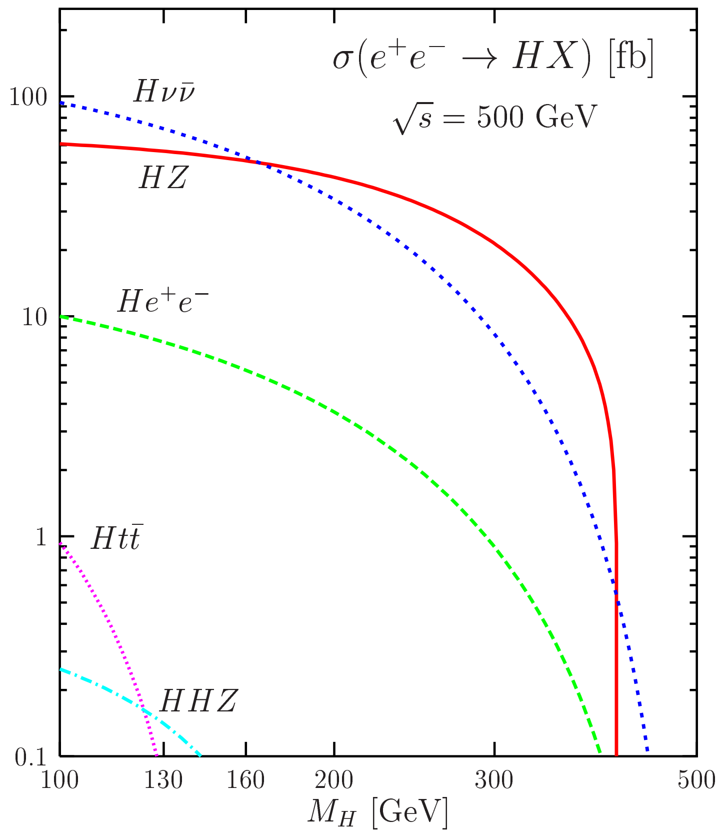


	500 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
	350 GeV	500 GeV	800 GeV
m_H = 120	–	90	2600
m_H = 160	–	–	1500
m_H = 250	–	–	390



	500 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
	350 GeV	500 GeV	800 GeV
m_H = 120	–	80	160
m_H = 160	–	20	120
m_H = 250	–	–	30

Higgs production at a Linear Collider



Standard Model established as a quantum field theory

- in agreement with (almost) all experiments (accuracy $\gtrsim 0.1\%$)
- quantum corrections are established
- indirect and direct determination of m_{t} agree
- constraints on the Higgs-boson mass $\Rightarrow$ light Higgs boson
- triple-gauge boson self-interactions established at per-cent level

not yet directly tested

- existence of Higgs boson
- Higgs-boson self-interaction $\Rightarrow$ Higgs potential
- Yukawa interaction

$\Rightarrow$ future experiments

Open questions of the Standard Model

- Large number of free parameters, in particular in fermion sector
- origin of gauge group $SU(3) \times SU(2) \times U(1)$ with three different gauge couplings
- origin of charge quantization
- origin and number of fermion generations
- origin of mass pattern
- origin of baryon asymmetry in universe
- inclusion of gravity ($\Rightarrow$ string theories)

Minimal Supersymmetric Standard Model (MSSM)

SM		Spin	SUSY		Spin
leptons	$\ell,\ \nu_\ell$	$\frac{1}{2}$	sleptons	$\tilde{\ell},\ \tilde{\nu}_\ell$	0
quarks	q	$\frac{1}{2}$	squarks	$\tilde{q}$	0
gluons	g	1	gluinos	$\tilde{g}$	$\frac{1}{2}$
EW bosons	γ, Z, W	1	charginos	$\tilde{\chi}^{\pm}_{1,2}$	$\frac{1}{2}$
Higgs	$h, H, A, H^{\pm}$	0	neutralinos	$\tilde{\chi}^{0}_{1,2,3,4}$	$\frac{1}{2}$

lightest SUSY particle stable LSP $= \tilde{\chi}^0_1$

- masses of SUSY partners > 100 GeV (experimentally)

- lightest Higgs boson < 135 GeV (theoretically)

$\boxed{\text{SUSY Higgs sector}}$

SM Higgs:

- $\lambda\Phi^4$ term ad hoc
- Higgs boson mass: free parameter
- no a-priori reason for a light Higgs boson
- SM (perturbatively) unstable at some high energy

SUSY Standard Model avoids these questions

minimal model: MSSM

$$H_2 = \begin{pmatrix} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} v_1 + H_1^0 \\ H_1^- \end{pmatrix}$$

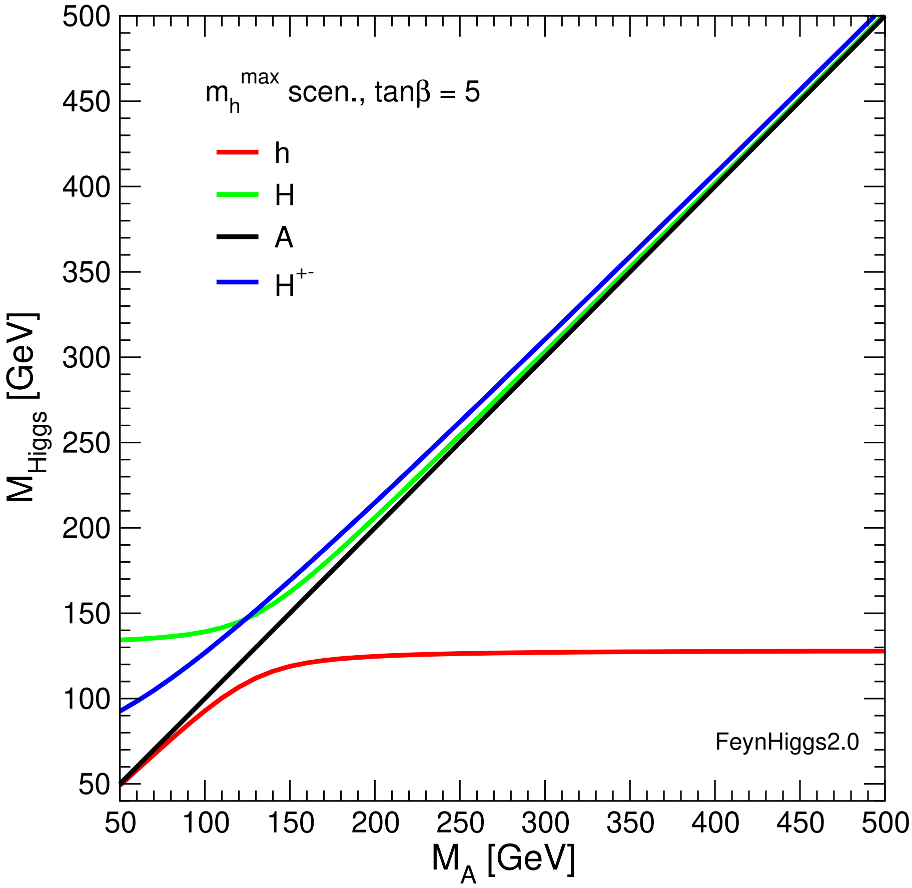
couples to u couples to d

- SUSY gauge interaction $\rightarrow$ H^4 terms
- self coupling remains weak

physical Higgs bosons: $h^0,\ H^0,\ A^0,\ H^\pm$

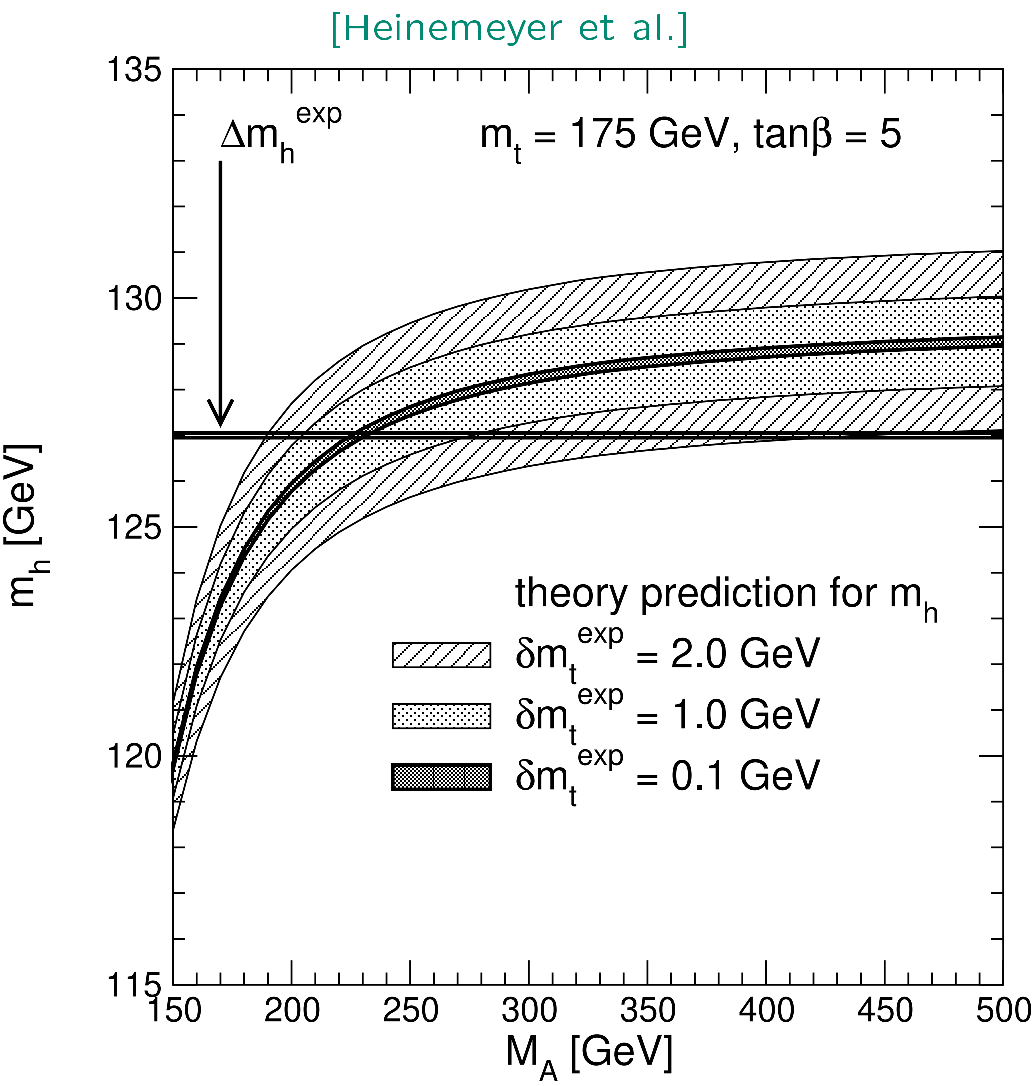
2 vacuum expectation values: $\frac{v_2}{v_1} = \tan\beta$

Spectrum of Higgs bosons in the MSSM (example)



large M_A: h^0 like SM Higgs boson

m_h^0 strongly influenced by quantum effects



[Heinemeyer et al.]
Δm_h^{exp}
m_t = 175 GeV, tanβ = 5
theory prediction for m_h
δm_t^{exp} = 2.0 GeV
δm_t^{exp} = 1.0 GeV
δm_t^{exp} = 0.1 GeV
m_h [GeV]
M_A [GeV]
135
130
125
120
115
150
200
250
300
350
400
450
500

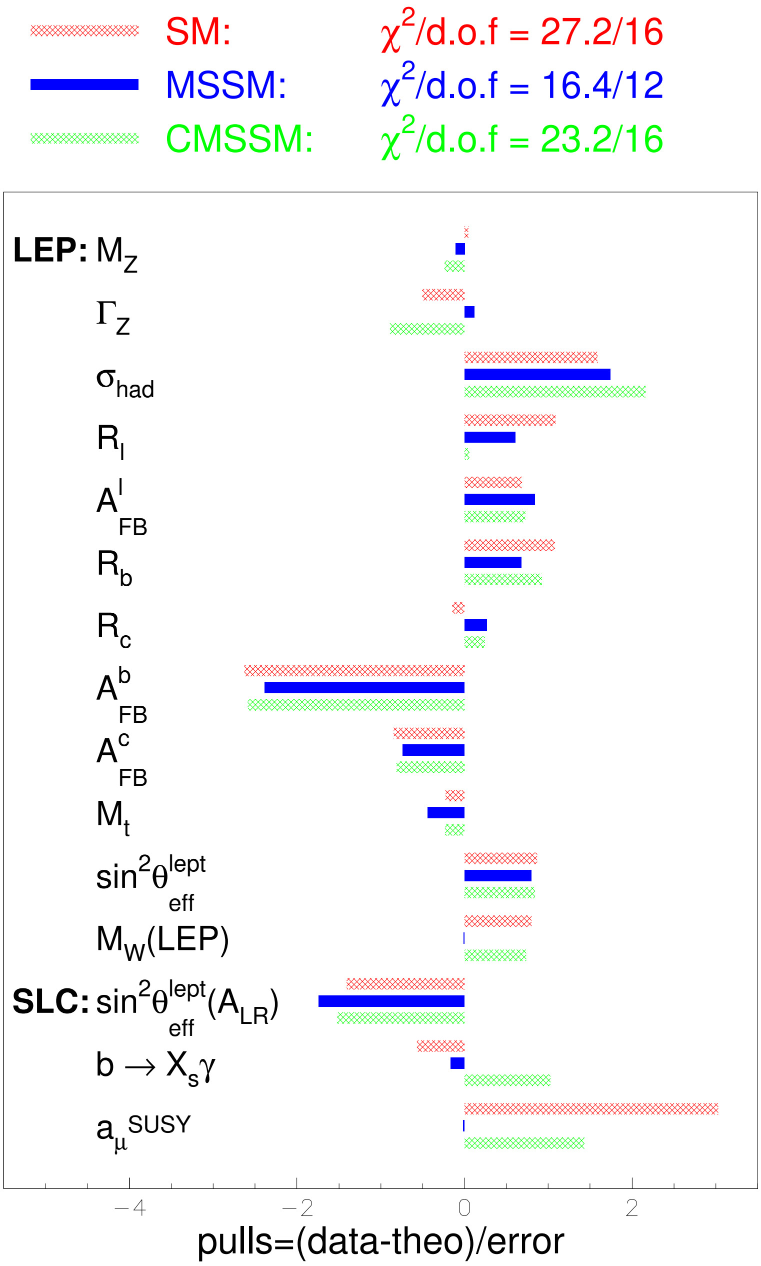
Possible scenarios

- a single light Higgs boson
  - SM Higgs boson?
  - SUSY light Higgs boson?
    with H, A, $H^{\pm}$ heavy (decoupling scenario)
    $h \sim H_{\rm SM}$
- a light Higgs boson + more $(H, A, H^{\pm})$
  - SUSY Higgs?
  - non-SUSY 2-Higgs-Doublet model?
- a single heavy Higgs boson ($\gg 200$ GeV)
  - SUSY ruled out
  - SM + (?)   strong interaction?
- no Higgs boson
  - strongly interacting weak interaction
    new strong force $\sim$ TeV scale

Global fits in the MSSM

[de Boer, Dabelstein, WH, Mösle, Schwickerath]

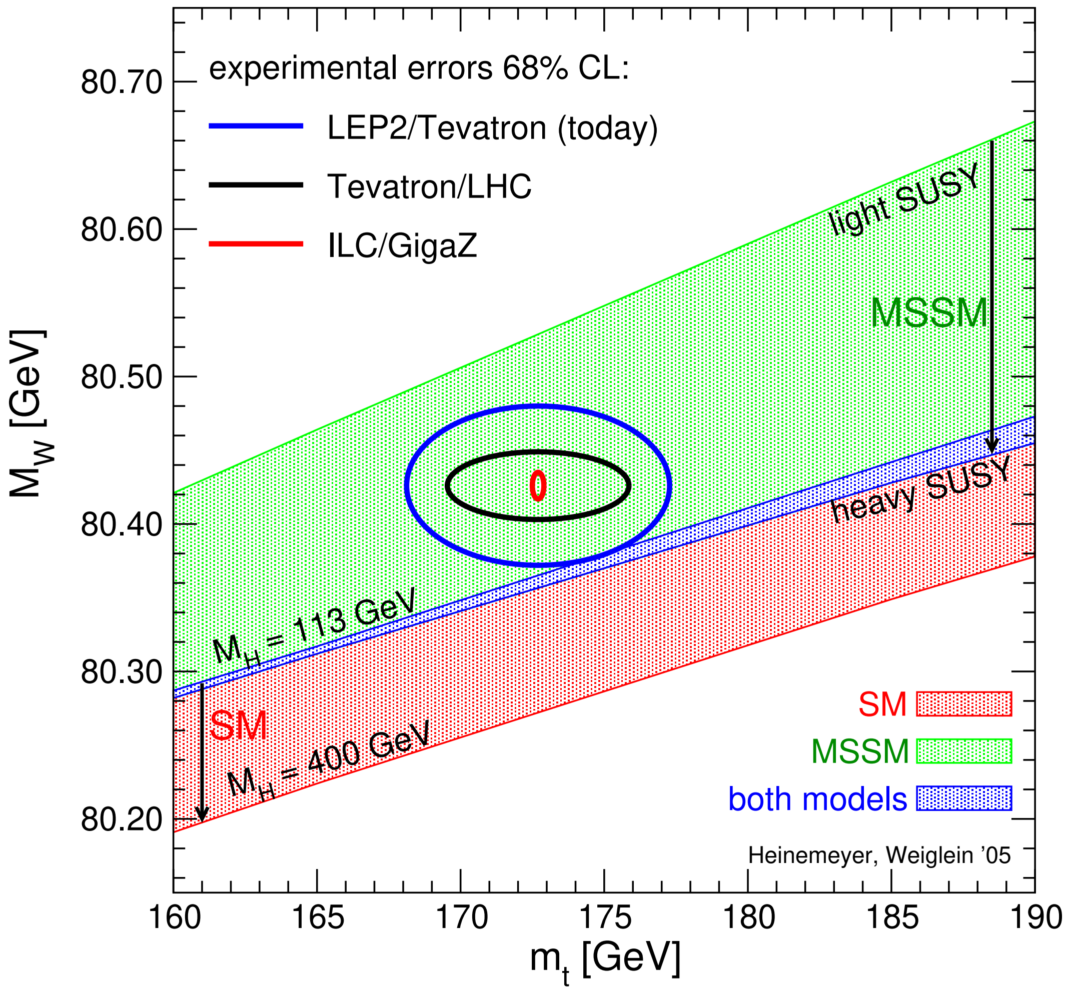
[de Boer, Sander]



special: M_W and $a_\mu = (g-2)/2$ for muon

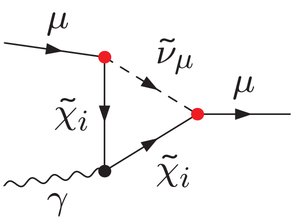
[Chankowski, Dabelstein, WH, Mösle, Pokorski, Rosiek]

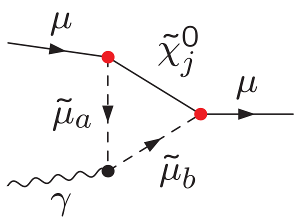
[update: Heinemeyer, Weiglein]



$g-2$

Feynman diagrams for MSSM 1L corrections:





- Diagrams with chargino/sneutrino exchange
- Diagrams with neutralino/smuon exchange

Enhancement factor as compared to SM:

$$\begin{aligned} \mu - \tilde{\chi}_i^{\pm} - \tilde{\nu}_\mu &: \ \sim m_\mu \ \tan\beta \\ \mu - \tilde{\chi}_j^{0} - \tilde{\mu}_a &: \ \sim m_\mu \ \tan\beta \end{aligned}$$

SM, EW 1L: $\frac{\alpha}{\pi} \frac{m_\mu^2}{M_W^2}$

MSSM, 1L: $\frac{\alpha}{\pi} \frac{m_\mu^2}{M_{\text{SUSY}}^2} \times \tan\beta$

Beyond the Standard Model

further substructure	elementary fundamental fields
effects from new strong interaction	interactions remain weak
	Grand Unified Theories
new strong dynamics at high enery scale	new symmetry supersymmetry