Top quark physics at hadron collider
Theoretical aspects

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Preliminary remark:

This lecture is meant to be an introduction into top quark physics, it is not a complete survey of the field!

! In particular, I will concentrate on the Standard Model !

Outline:

- Basic concepts
  - The top quark and the SM
  - The mass of a quark
  - SM properties of the top quark
- Top production at hadron collider
  - leading-order part. cross sections
  - PDF’s and hadron cross sections
  - Beyond LO
- Observables
  - The top spin and spin-correlations
  - W-Polarisation in top decays

The Standard Model

Building blocks:

Quarks:

$$\begin{pmatrix} u_L, u_L, u_L \\ d_L, d_L, d_L \end{pmatrix}$$

$$u_R, u_R, u_R$$
$$d_R, d_R, d_R$$

Leptons:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

$$e_R$$

- Left-handed fermions, appear in „weak“ isospin-doublets à la Heisenberg (→ SU(2))
- Quarks exist in 3 colours (R,G,B) (→ SU(3))
- Particles carry hyper charge $Y=Q-T_3$ (→ U(1))

Symmetry group SU(3) x SU(2) x U(1)

The Standard Model (cont´d)

Matter structure

	1. family	2. family	3. family	T_3 $(SU(2)_L)$	Y $(U(1)_Y)$	$Q=T_3+Y$
Quarks	u_L, u_L, u_L	c_L, c_L, c_L	t_L, t_L, t_L	1/2	1/6	2/3
	d_L, d_L, d_L	s_L, s_L, s_L	b_L, b_L, b_L	-1/2	1/6	-1/3
	u_R, u_R, u_R	c_R, c_R, c_R	t_R, t_R, t_R	0	2/3	2/3
	d_R, d_R, d_R	s_R, s_R, s_R	b_R, b_R, b_R	0	-1/3	-1/3
Leptons	ν_{eL}	$\nu_{\mu L}$	$\nu_{\tau L}$	1/2	-1/2	0
	e_L	μ_L	τ_L	-1/2	-1/2	-1
	e_R	μ_R	τ_R	0	-1	-1

heavier →

Families exact copies with respect to the quantum numbers!

Why do we need the top quark?

Top quark is required to make the SM anomaly free !

Anomaly: A current which is conserved at the classical level is no longer conserved when quantum corrections are taken into account

Famous example:

Adler-Bell-Jackiw anomaly of the axial vector current

$$\partial_\mu j^{5\mu} = \partial_\mu \bar{\Psi} \gamma^5 \gamma^\mu \Psi = \; ?$$

At the classical level:

$$\begin{aligned} \partial_\mu j^{5\mu} &= (\partial_\mu \bar{\Psi})\gamma^5\gamma^\mu\Psi + \bar{\Psi}\gamma^5\gamma^\mu(\partial_\mu\Psi) \\ &= -\bar{\Psi}\gamma^5 \overleftarrow{\not{\partial}}\,\Psi - \bar{\Psi}\gamma^5 \overrightarrow{\not{\partial}}\,\Psi \\ &= -im\bar{\Psi}\Psi - im\bar{\Psi}\Psi \\ &= -2im\bar{\Psi}\Psi \end{aligned}$$

Dirac equation:

$$(i\gamma_\mu\partial^\mu - m)\Psi = 0,$$
$$(-i\partial^\mu\bar{\Psi}\gamma_\mu - m\bar{\Psi}) = 0$$

For vanishing masses, current conserved at the classical level !

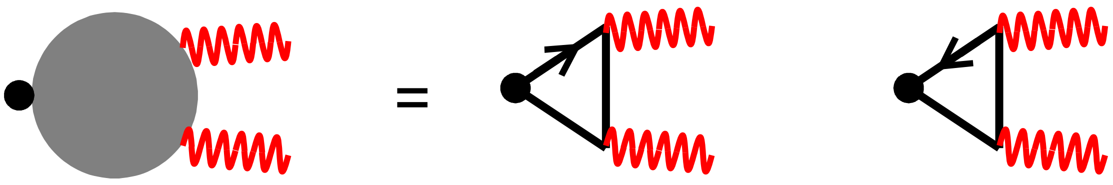
Including quantum corrections one finds:

[Adler, Bell, Jackiw]

$$\partial_\mu j^{5\mu} \sim \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} F^{\mu\nu} \neq 0$$

↖ field strength tensor $\partial_\mu A_\nu - \partial_\nu A_\mu$

→ Divergence of axial-vector current has non-zero matrix element of creating two "photons":



Serious problem in gauge theories where gauge fields are coupled to chiral currents → violation of Ward identities

Note: ABJ anomaly is a fundamental property in QFT, no regular./renorm. scheme exists that conserves the vector and the axial vector current at the same time

Solution in the Standard Model:

Arrange couplings of different fermions in such a way, that in the sum of all contr. anomaly cancels.

→ family structure of the SM

→ without the top quark, SM would be inconsistent

→ be careful when studying 5 flavour QCD

Important consequences for the top quark

Gauge couplings of top quark are fixed by the structure of the Standard Model !

What are the free parameters in the top sector?

- top quark mass or alternatively Yukawa coupling
- Cabibbo-Kobayashi-Maskawa (CKM) matrix elements

Top quark properties in the SM are functions of only two parameters:

mass and CKM matrix

→Top quark properties are testable predictions in the SM

Experimental Status:

Cabibbo-Kobayashi-Maskawa (CKM) Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

eigenstates of the interaction — mass eigenstates

Unitarity + assumption of only 3 families + Exp.:

$$|V_{td}| = 0.004 - 0.014, |V_{ts}| = 0.035 - 0.043,$$
$$|V_{tb}| = 0.9990 - 0.9993$$ **[PDG04]**

Top mass:

$$m_t = (172.7 \pm 2.9)\ \text{GeV}/\text{c}^2$$ **[hep-ex/0507091]**

Input precisely known, precise prediction of top quark properties in the SM possible !

What do we mean by the mass of a quark?

Like α_s the quark mass is not an observable,
the quark mass is a parameter defined in the context of a specific model!

Important consequence:

Numerical value depends on the prescription chosen to define/measure this parameter, in particular the renormalization conditions

How do we measure the mass?

By comparing the theoretical prediction (in a specific scheme) for a measurable quantity with the experimentally measured value

Examples: Measurement of the b-quark mass at LEP from 3-jet cross sections, determination of m_t from electroweak fits.

Note:

In the context of perturbation theory we have to go to next-to-leading order to give a meaningful definition of the parameters, i.e. quark masses, couplings

In LO the renormalization scheme is not determined!

(… this is what pure theory tells us…)

Renormalization of the quark field

bare quantities

$$m_0, \Psi_0$$

$$\mathcal{L}(m_0, \Psi_0)$$

renormalized quantities

$$m_0 = Z_0 Z_\Psi^{-1} m_R, \Psi_0 = \sqrt{Z_\Psi}\Psi_R$$

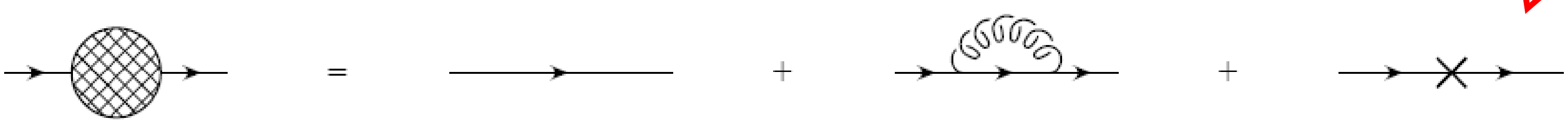
$$\begin{aligned} \mathcal{L}(m_0, \Psi_0) &= \mathcal{L}(m_R, \Psi_R) \\ &+ \underbrace{\mathcal{L}(m_0, \Psi_0) - \mathcal{L}(m_R, \Psi_R)}_{\text{counter term}} \end{aligned}$$

$$i((Z_\Psi^R - 1)\not{k} - (Z_0^R - 1)m_R)$$

→ two renormalization constants

→ two renormalization conditions needed

To determine *Z*´s calculate corrections to the free quark propagator and impose renormalization conditions



Common renormalization schemes

1) Modified Minimal Subtraction (MS):

Chose Z´s such that the poles together with – γ + ln(4π) are cancelled

$$Z_\Psi^{\overline{\rm MS}} = 1 - \frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} - \gamma + \ln(4\pi) \right)$$

$$Z_0^{\overline{\rm MS}} = 1 - \frac{\alpha_s}{\pi} C_F \left(\frac{1}{\epsilon} - \gamma + \ln(4\pi) \right)$$

2.) On-Shell/Pole-Mass Scheme:

- The pole of the propagator is at $\not{p} = m_R$
- The residue of the propagator is 1

$$Z_\Psi^{\rm on} = 1 - \frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} - \frac{2}{\epsilon_{ir}} - 3\gamma - 3\ln\left(\frac{m_{on}^2}{4\pi\mu^2} \right) + 4 \right)$$

$$Z_0^{\rm on} = 1 - \frac{\alpha_s}{\pi} C_F \left(\frac{1}{\epsilon} - \frac{1}{2\epsilon_{ir}} - \frac{3}{2}\gamma + \frac{3}{2}\ln\left(\frac{m_{on}^2}{4\pi\mu^2} \right) + 2 \right)$$

MS mass depends on renormalization scale:

$$\mu \frac{d\overline{m}(\mu)}{d\mu} = -\overline{m}(\mu)\frac{\alpha_s(\mu)}{2\pi}3C_F \equiv -\overline{m}(\mu)\gamma_m$$

Conversion between MS and Pole-mass scheme

$$m_{\rm on}\frac{Z_0^{\rm on}}{Z_\Psi^{\rm on}} = m_0 = \overline{m}(\mu)\frac{Z_0^{\overline{\rm MS}}}{Z_\Psi^{\overline{\rm MS}}}$$

$$m_{\rm on} = \overline{m}(\mu)\left(1 + \frac{\alpha_s}{\pi}C_F\left[1 - \frac{3}{4}\ln\left(\frac{m^2}{\mu^2}\right)\right]\right)$$

$$C_F = \frac{1}{2N}(N^2-1) \to \frac{4}{3}$$

Which renormalization scheme should we use ?

Pole mass versus $\overline{\text{MS}}$ mass:

$$\overline{m}(\mu = \overline{m}) = m_{\text{on}} \left(1 - \frac{4}{3}\frac{\alpha_s(m)}{\pi}\right)$$

4.2%

$$m_{\text{on}} = 172.7GeV \rightarrow \overline{m}(m) = 165.3GeV$$

Including higher order corrections the difference is about 6%

Pole mass ←→ MS mass differ by ~10 GeV

→In general NLO needed unless there is (very) good argument, why the corrections are small in a specific mass scheme.

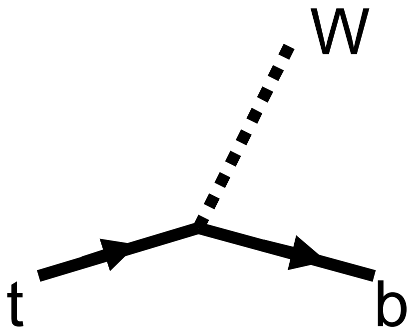
Top quark properties:

– **Top quark is extremely heavy:** $m_t \approx 36 m_b,\ m_t \approx m_{Au}$

– **Main decay in the SM:**

$$|V_{tb}| \approx 1 \quad \rightarrow \quad t \rightarrow Wb$$

$W \rightarrow q\bar{q}', \bar{\ell}\nu_\ell$



Decay width calculable in the SM:

$$\Gamma_t = \frac{G_F m_t^3}{8\pi\sqrt{2}}\left(1-\frac{m_W^2}{m_t^2}\right)^2\left(1+\frac{2m_W^2}{m_t^2}\right)\left(1-\frac{2\alpha_s}{3\pi}\left(\frac{2\pi^2}{3}-\frac{5}{2}\right)\right)$$
$$\approx 1.48 \quad GeV$$

Theoretical accuracy including known corrections at the 1% level

Top quark extremely short lived:

$$\Gamma_t = 1.48 GeV \rightarrow \tau_t = 0.44 \times 10^{-24} s < \tau_{QCD} \approx 3 \times 10^{-24} s$$

Top decays before it can form bound states!

Top quark decays essentially as a quasi free quark

[Bigi, Dokshitzer, Khoze, Kühn, Zerwas ´86]

Top quark is the only quark with this property, all the lighter quarks hadronize before they decay

Unique possibility to study a quasi free quark

→Spin of the top quark is a good observable, can be studied due to the parity violating decay t→Wb

Why is top quark physics interesting/important ?

1.) Interesting in itself as signal process

- Is the top quark just another quark, i.e. is the mass generated by the usual Higgs mechansim?
- not very well studied so far → confirm that top quark has indeed quantum numbers as predicted by SM
- top quark properties precisely calculable in the SM → any observed deviation is a signal for new physics
- in many extensions of SM, top quark plays special role
- Top quark may decay in new heavy particles
- unique possibility to study a quasi free quark

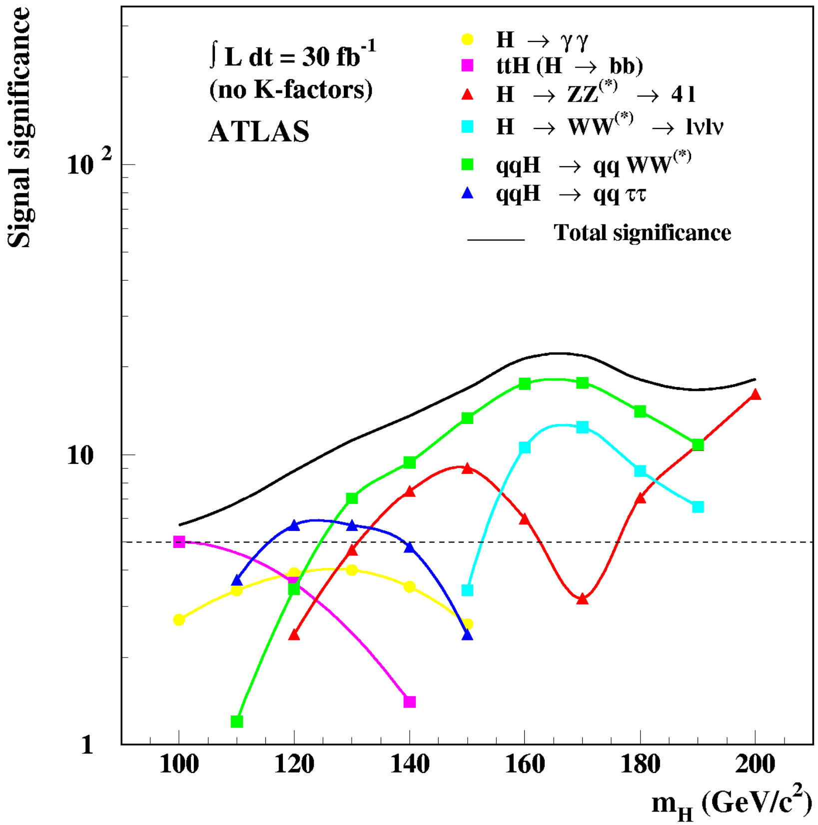
Important observables:

- ✓ tt cross section — Precise determination of top mass, consistency checks with theo. predictions, search for new physics in the tt invariant mass spectrum
- ✓ W-Polarization in top decay — Test of the V-A structure in top decay
- ✓ ttH cross section — Measurement of the Yukawa coupling
- ✓ Single top production — Direct measurement of the CKM matrix element Vtb, top polarization, search for anomalous Wtb couplings
- ✓ Spin correlations — Weak decay of a `free' quark, bound on the top width and V_{tb}, search for anomalous couplings
- (✓) tt+Jet(s) production — Search for anomalous couplings, important background
- $t t\gamma$ cross section — Measurement of the electric charge

Why is top quark physics interesting/important (cont´d)

2.) Top quark is important background for many processes

Higgs searches at LHC



[Atlas `03]

The VBF process

$$qq \to WW\,qq \to qqH$$

$H \to$ WW

is important over a wide Higgs mass range

Important backgrounds:

channel	$e^{\pm}\mu^{\mp}$	$e^{\pm}\mu^{\mp}$ w/minijet veto	$e^{\pm}e^{\mp}, \mu^{\pm}\mu^{\mp}$	$e^{\pm}e^{\mp}, \mu^{\pm}\mu^{\mp}$ w/minijet veto
$70 < m_h < 300$ GeV	1.90	1.69	1.56	1.39
SM, $m_h = 155$ GeV	5.60	4.98	4.45	3.96
$t\bar{t}$	0.086	0.025	0.086	0.025
$t\bar{t}j$	7.59	2.20	6.45	1.87
$t\bar{t}jj$	0.83	0.24	0.72	0.21
single-top (tbj)	0.020	0.015	0.016	0.012
$b\bar{b}jj$	0.010	0.003	0.003	0.001
QCD $WWjj$	0.448	0.130	0.390	0.113
EW $WWjj$	0.269	0.202	0.239	0.179
QCD $\tau\tau jj$	0.128	0.037	0.114	0.033
EW $\tau\tau jj$	0.017	0.013	0.016	0.012
QCD $\ell\ell jj$	–	–	0.114	0.033
EW $\ell\ell jj$	–	–	0.011	0.008
total bkg	9.40	2.87	8.04	2.49
S/B	1/5.0	1/1.7	1/5.1	1/1.8
$\mathcal{L}_{5\sigma}^{obs}$ [fb^{-1}]	65	25	82	32

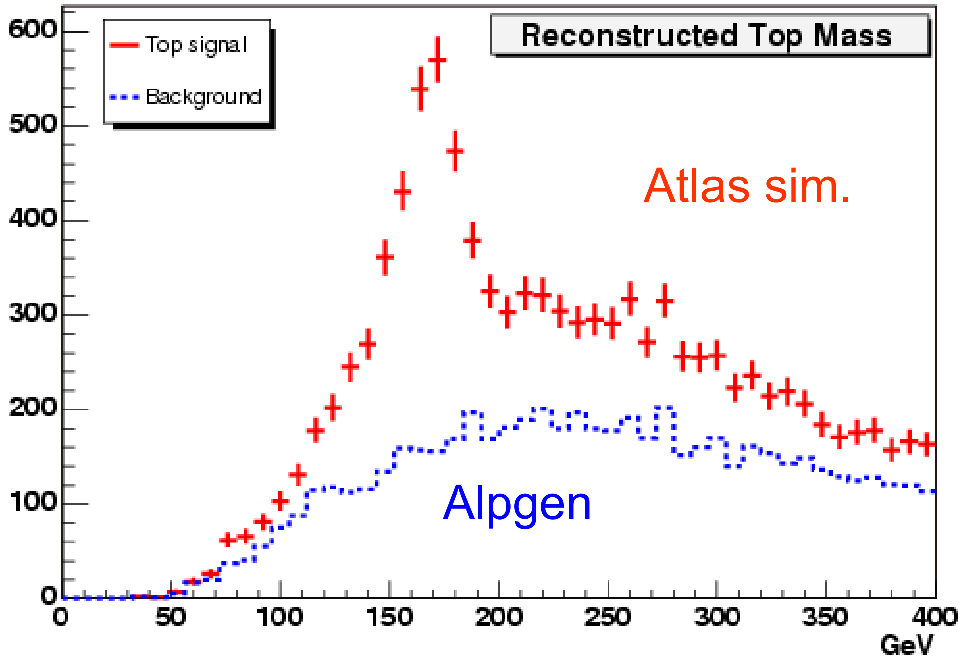
[Alves, Eboli, Plehn, Rainwater ’04]

→ Precise predictions for pp→ tt + jet are necessary

Why is top quark physics interesting/important (cont´d)

3.) Useful to test our theoretical and experimental tools

Use $p_\perp$ cut to obtain tt samples (semileptonic channel):



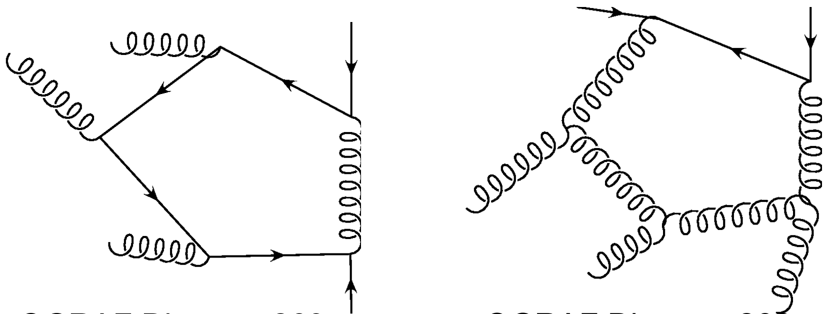
– electron or muon with $p_\perp$ > 20 GeV
– Four jets with $p_\perp$ > 40 GeV

[Gianotti,Mangano ´05]

Events can be used to:

- measure the b-tagging efficiency
- determine the jet energy scale from
  - W→jj
  - top mass reconstruction

How do we calculate amplitudes like ?



QGRAF Diagram 263 QGRAF Diagram 266

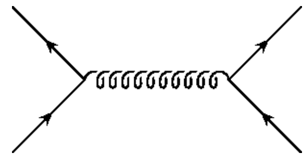
Diagrams encountered in gg→ttg,

sub-process required for pp → tt + 1Jet

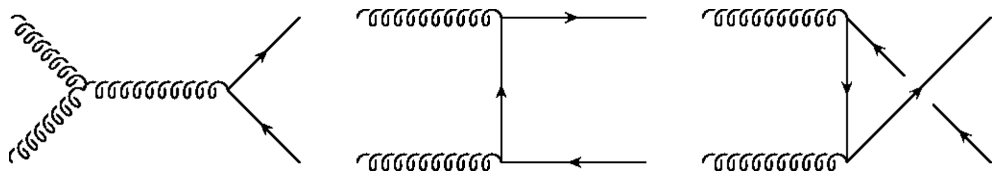
Apart from phenomenological significance, top quark physics interesting test case for NLO calculations due to rich structure of IR divergencies + additional scale

Top quark production at hadron collider

Top quark pair production



Quark-Antiquark annihilation

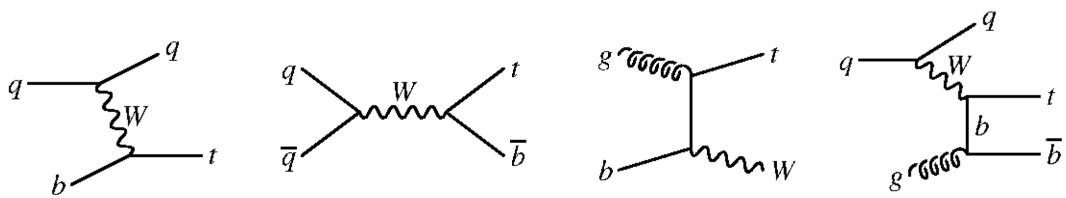


Gluon fusion

NLO corrections known!

[Dawson, Ellis, Nason ’89, Beenakker et al ’89,’91]

Single top quark production



One-loop corrections also known! [Tait ´00, Belayev 01, Harris ´02, Cao et al 04, Campbell et al 04]

Top quark pair production in detail

Partonic cross sections

$$\hat{\sigma}_{q\bar{q}} = \frac{8\pi\alpha_s^2}{27\hat{s}}\beta(1+\frac{\rho}{2})$$

$$\hat{\sigma}_{g\bar{g}} = \frac{4\pi\alpha_s^2}{12\hat{s}}\left[\left(1+\rho+\frac{\rho^2}{16}\right)\ln\left(\frac{1+\beta}{1-\beta}\right)-\beta\left(\frac{7}{4}+\frac{31}{16}\rho\right)\right]$$

$$\rho = 4m_t^2/\hat{s} \quad \beta = \sqrt{1-\rho}$$

Close to threshold:

$$\hat{\sigma}_{q\bar{q}} = \frac{4\pi\alpha_s^2}{9\hat{s}}\beta \qquad \hat{\sigma}_{g\bar{g}} = \frac{7\pi\alpha_s^2}{48\hat{s}}\beta$$

Hadronic cross section

Parton distribution functions (PDF's)

$$\sigma_{\rm Had} = \int_0^1 \int_0^1 F_{i/H_1}(x_1,\mu) F_{j/H_2}(x_2,\mu) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 s, \mu)$$

depends only on the product of x_1 and x_2,

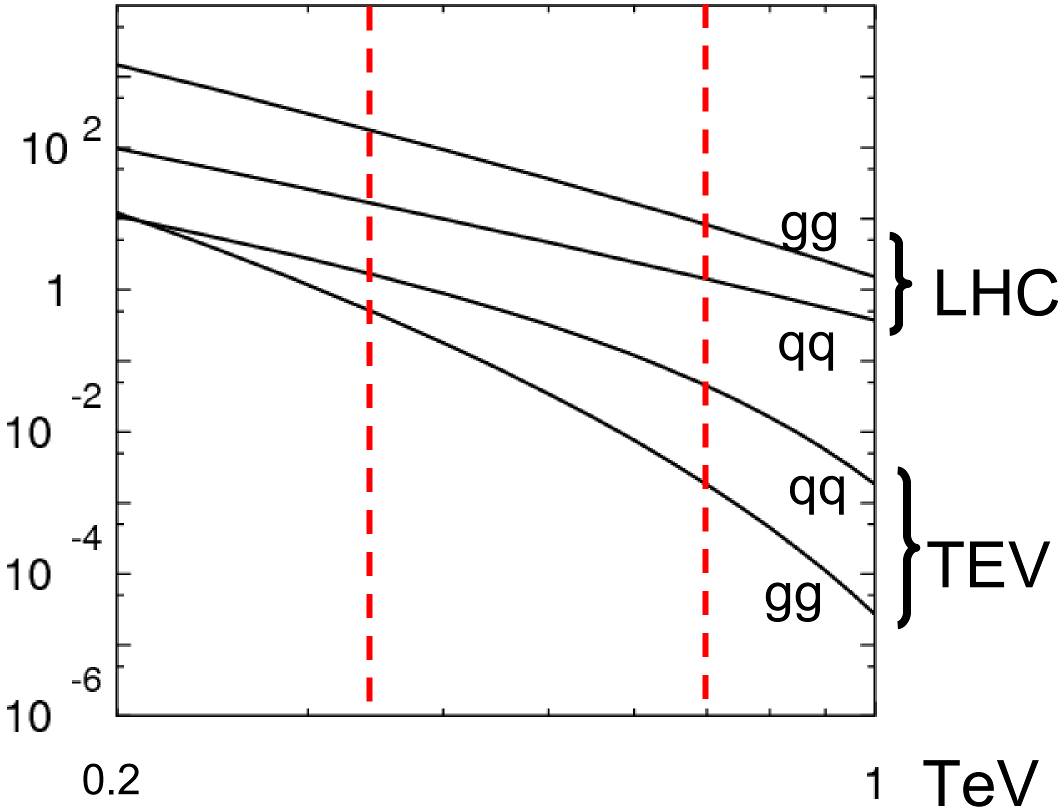
→ problem can be reduced to one-dimensional integration

$$\begin{aligned} \sigma_{\rm Had} &= \int_0^1 dx_1 \int_0^1 dx_2 \int d\tau \delta(\tau - x_1 x_2) \\ &\quad \times F_{i/H_1}(x_1,\mu) F_{j/H_2}(x_2,\mu) \hat{\sigma}_{ij}(x_1 x_2 s, \mu) \\ &= \int d\tau L_{ij}(\tau,\mu) \hat{\sigma}_{ij}(\tau s, \mu) \end{aligned}$$

$$L_{ij}(\tau,\mu) = \int_\tau^1 \frac{1}{x_1} F_{i/H_1}(x_1,\mu) F_{j/H_2}(\frac{\tau}{x_1},\mu)$$

Luminosity function

$$L_{ij}(\tau,\mu) = \int_\tau^1 \frac{1}{x_1} F_{i/H_1}(x_1,\mu) F_{j/H_2}(\frac{\tau}{x_1},\mu)$$



$$\frac{1}{s} L_{ij}\ [nb]$$

$$\sigma_{\text{Had}} = \int_{4m_t^2}^{s} d\hat{s} \frac{1}{s} L_{ij}(\hat{s},\mu) \hat{\sigma}_{ij}(\hat{s},\mu)$$

→ top quark pairs are produced “close” to threshold

→ at the Tevatron quark-antiquark annihilation dominates (~90%)
at the LHC gluon fusion is the dominant channel (~90%)

Next-to-leading order corrections

General form:

$$\hat{\sigma}_{ij}(\hat{s}, m_t, \mu) = \frac{\alpha_s(\mu)}{m_t^2}\left(f_{ij}^{(0)}(\rho) + \overbrace{4\pi\alpha_s(\mu)\left(f_{ij}^{(1)}(\rho) + \bar{f}_{ij}^{(1)}(\rho)\ln\left(\frac{\mu^2}{m_t^2}\right)\right)}^{\text{QCD corrections}} \right)$$

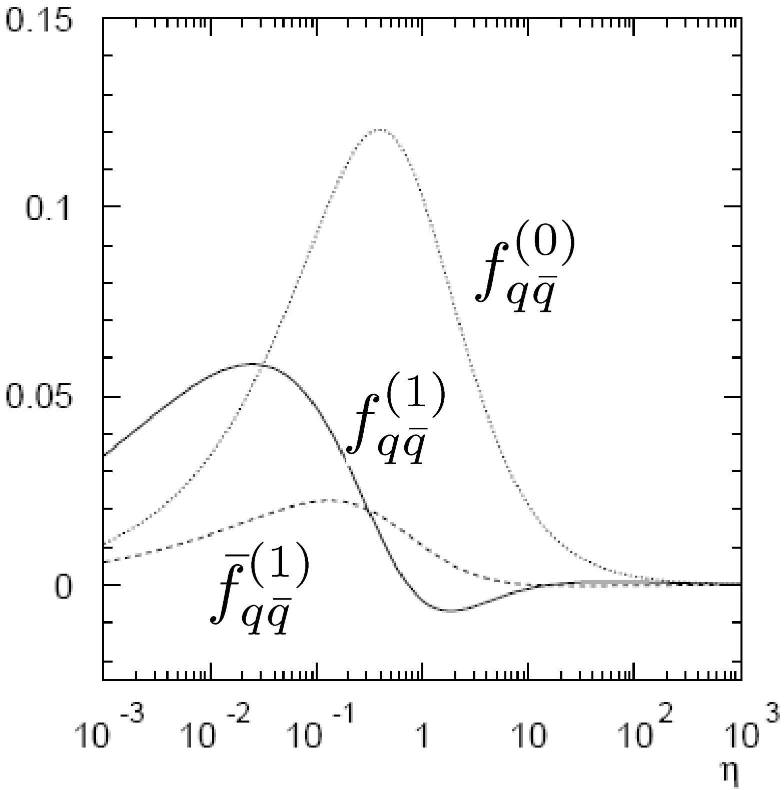
$\bar{f}_{ij}^{(1)}$ **determined by the renormalization group:**

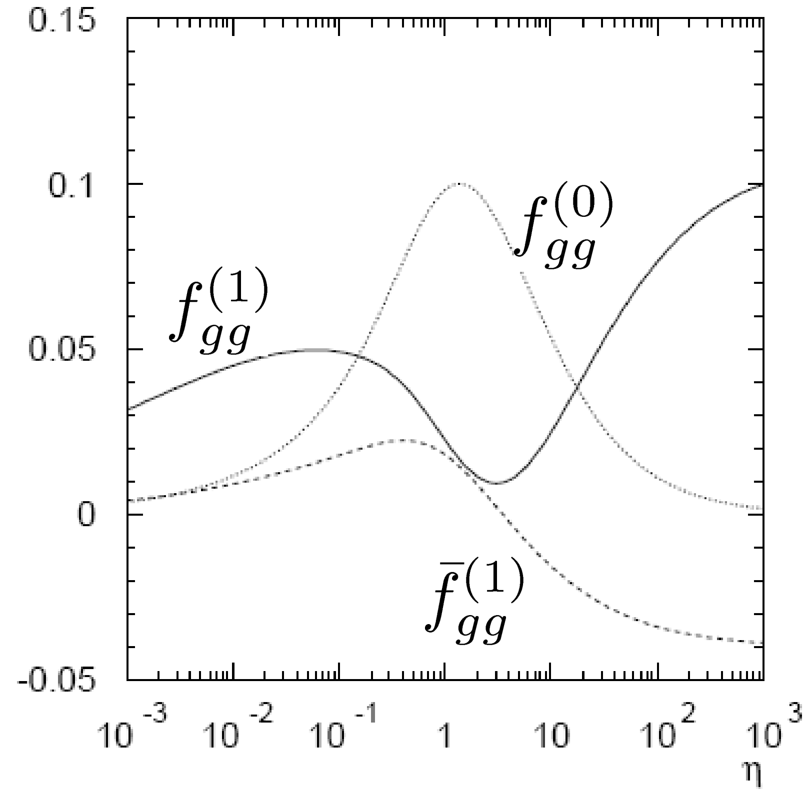
$$0 = \frac{d}{d\mu}\int dx_1 dx_2 F_i(x_1,\mu)F_j(x_2,\mu)\hat{\sigma}_{ij}(\rho = 4m_t/s/x_1/x_2, \mu)$$

$$\bar{f}_{ij}^{(1)}(\rho) = \frac{1}{8\pi^2}\left[\beta_0 f_{ij}^{(0)}(\rho) - \int_\rho^1 dz P_{kj}(z) f_{ik}^{(1)}\left(\frac{\rho}{z}\right) - \int dz P_{ki}(z) f_{jk}^{(1)}\left(\frac{\rho}{z}\right)\right.$$

$$\beta_0 = \frac{1}{3}(11N - 2n_f)$$

Functions $f_{ij}^{(1)}$ are obtained from complete NLO calculation





$$\eta = \frac{1}{\rho} - 1$$

[Dawson et al 88, Beenakker et al 89]

Functions are available as fits, for specific observables!

Corrections are important!

Resummation

Close to threshold NLO corrections are given by:

$$f_{q\bar{q}}^{(1)}(\rho) + \overline{f}_{q\bar{q}}^{(1)}(\rho) \ln\frac{\mu^2}{m^2} = \frac{1}{4\pi^2} f_{q\bar{q}}^{(0)}(\rho) \left\{ \left(C_F - \frac{1}{2}C_A\right) \frac{\pi^2}{2\beta} + 2C_F \ln^2(8\beta^2) - (8C_F + C_A)\ln(8\beta^2) - 2C_F \ln(4\beta^2)\ln\frac{\mu^2}{m^2} + \overline{C}_2\left(\frac{\mu^2}{m^2}\right) + \mathcal{O}(1-\rho) \right\}, \quad (12)$$

$$f_{gg}^{(1)}(\rho) + \overline{f}_{gg}^{(1)}(\rho) \ln\frac{\mu^2}{m^2} = \frac{1}{4\pi^2} f_{gg}^{(0)}(\rho) \left\{ \frac{N_c^2+2}{N_c(N_c^2-2)} \frac{\pi^2}{4\beta} + 2C_A \ln^2(8\beta^2) - \frac{(9N_c^2-20)C_A}{N_c^2-2}\ln(8\beta^2) - 2C_A \ln(4\beta^2)\ln\frac{\mu^2}{m^2} + \overline{C}_3\left(\frac{\mu^2}{m^2}\right) + \mathcal{O}(1-\rho) \right\}, \quad (13)$$

[Nason, Dawson, Ellis 88,
Beenakker, kuijf, vNeerven,Smith ‘89]

large logarithmic corrections in threshold region:

$$f_{ij}^{(n)}(\rho; \mu^2/m^2) \sim f_{ij}^{(0)}(\rho) \ln^{2n} \beta^2$$

→sum large logarithmic corrections to improve perturbation theory

[Bonciani, Cacciari, Catani, Kidonakis, Laenen, Mangano, Moch, Nason, Ridolfi, Sterman…]

Resummation (cont'd)

$\mu_R = \mu_F$	$p\bar{p}$ at $\sqrt{S} = 1.8$ TeV		$p\bar{p}$ at $\sqrt{S} = 2$ TeV		pp at $\sqrt{S} = 14$ TeV	
	NLO	NLO+NLL	NLO	NLO+NLL	NLO	NLO+NLL
$m_t/2$	5.17	5.19	7.10	7.12	893	885
m_t	4.87	5.06	6.70	6.97	803	833
$2m_t$	4.31	4.70	5.96	6.50	714	794

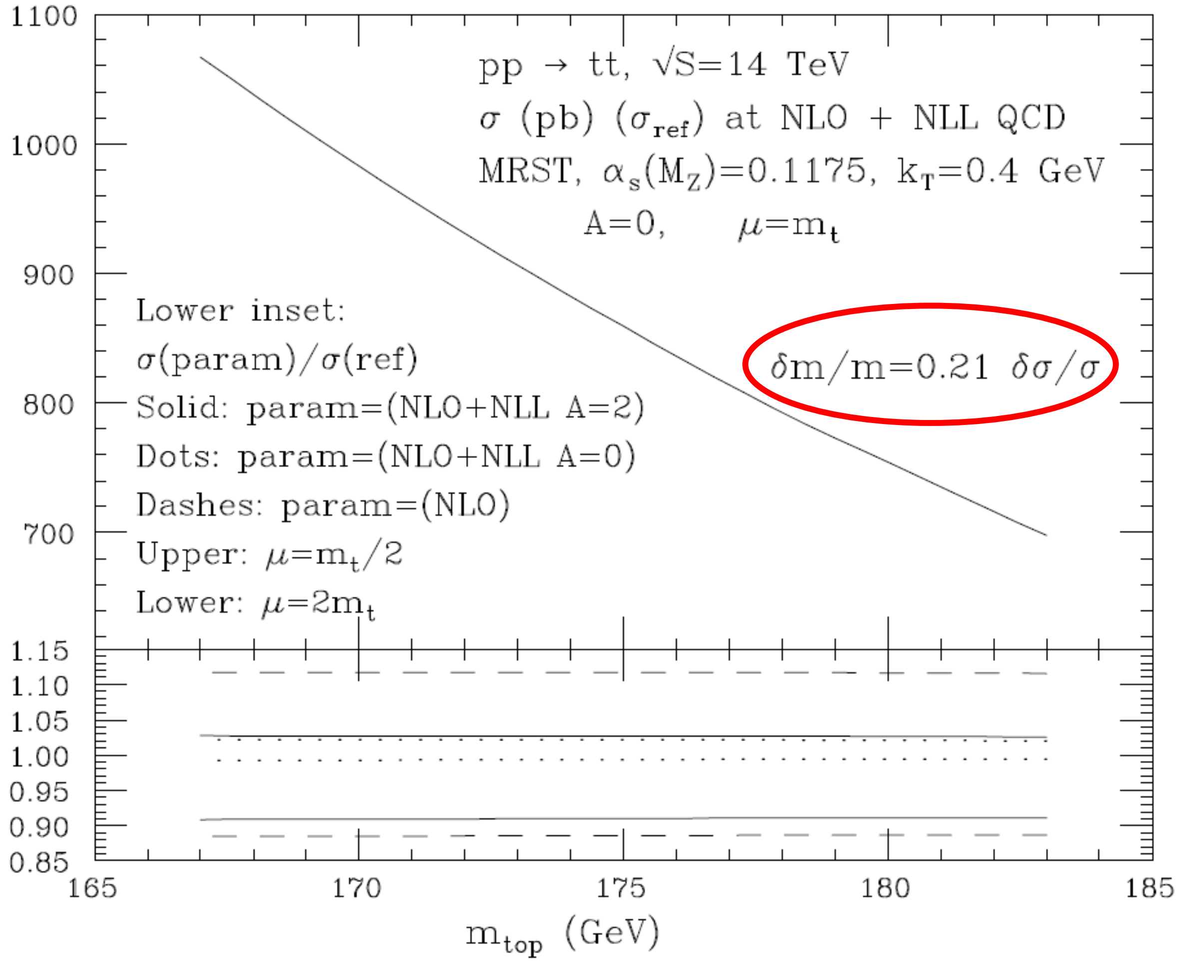
[Bonciani, Catani, Mangano, Nason 98]

→ **Resummation shifts slightly central value of NLO prediction**

→ **improves scale uncertainty**

Including uncertainties from PDF's, cross section known to 10%

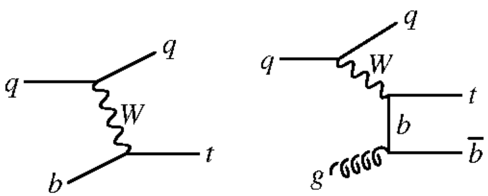
Mass determination from cross section measurement:



$$\delta m/m \approx 0.21\delta\sigma \rightarrow \delta m/m \approx 0.21 * 10 \approx 2\%$$

Single-top production

Most important production channel



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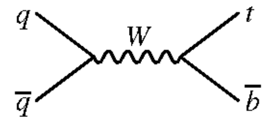


~ 10 pb

Large cross section due to:

- high energy behaviour

$$\sigma(ub \to dt) \to \frac{G_F^2 m_W^2}{2\pi}$$

- sensitive to parton luminosity at lower τ

Experimentally challenging due to large backgrounds

Top production at hadron collider

LHC is a top factory:

	Tevatron Run I	Tevatron Run II	LHC
Type Time	$p\bar{p}$ 1992-1996	$p\bar{p}$ 2001-2009	pp 2007-?
$\sqrt{s}$ (TeV) $\int \mathcal{L}dt$ (fb^{-1})	1.80 0.125	1.96 4 - 8	14 10/year, 100/year
$\sigma(t\bar{t})$ (pb) σ(single t) (pb)	~ 5 ~ 1	~ 7 ~ 1.5	~ 800 ~ 300

LHC: ~8,000,000 tt pairs per year, ~1 pair / sec

Current status Tevatron: 350/pb analysed, 1/fb on tape

Measurements will in general not be restricted by statistics!

Top quark physics can be done „from the first day“ on!

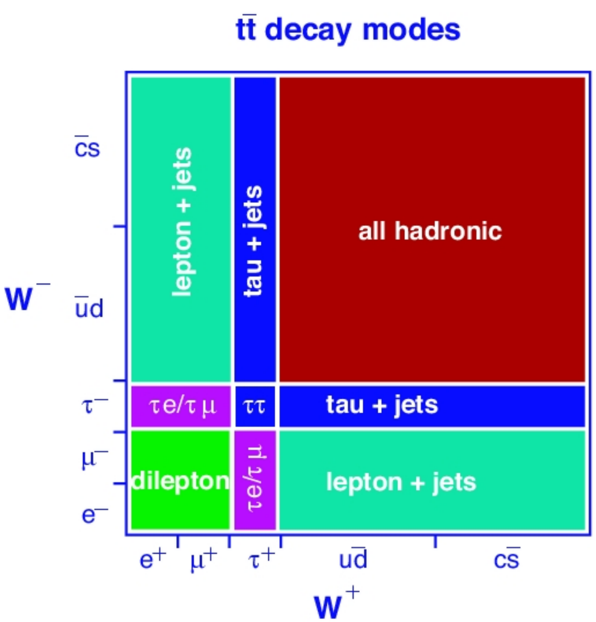
Experimental signature von top quark pairs

dominant decay in the Standard Model $t \rightarrow Wb$

depending on the decay of the W´s

we call t-decay hadronic or leptonic

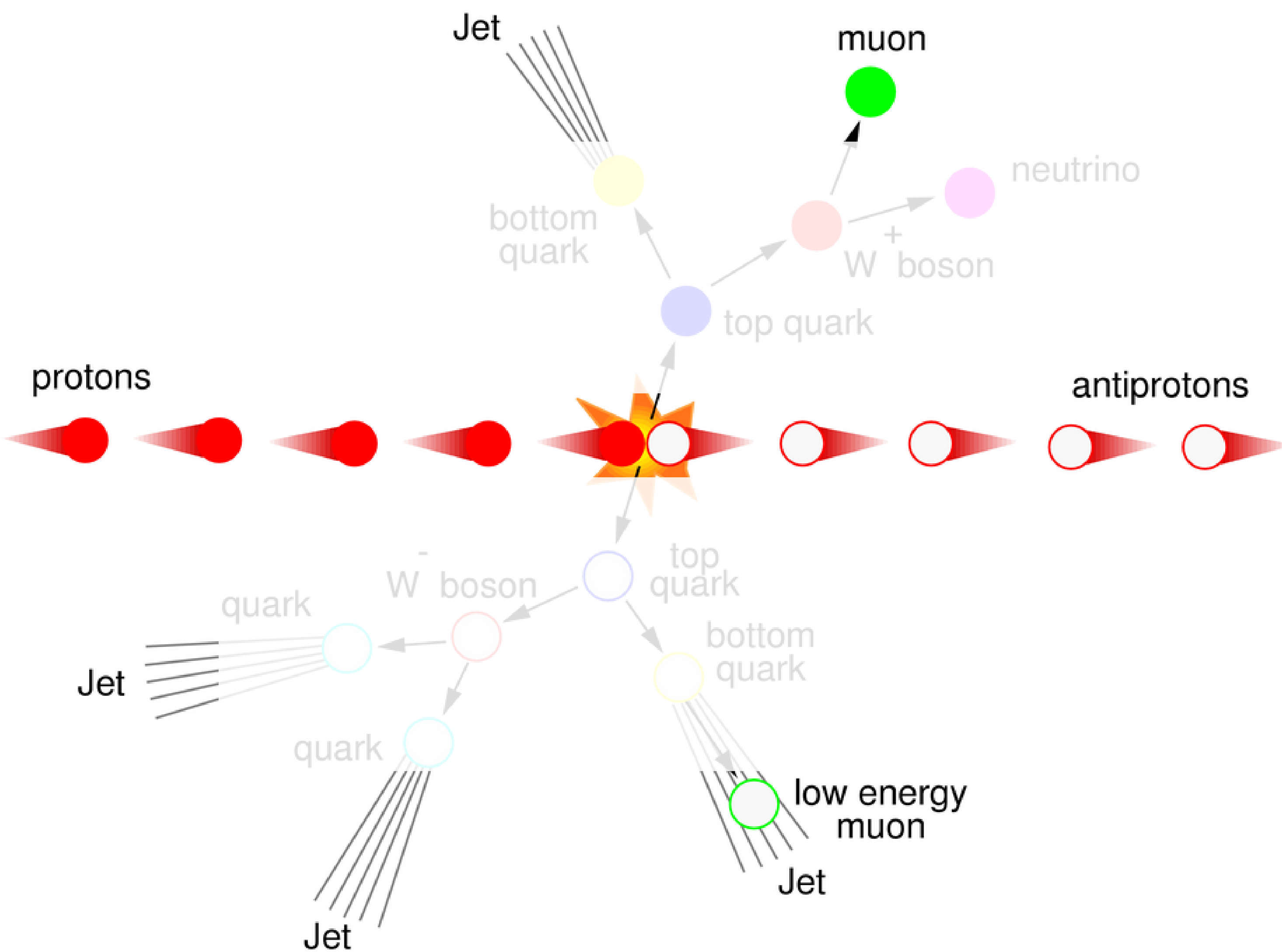
For tt have different possible combinations



Most important channels:

- **tt → lνlνbb** **Di-lepton** **5% e+μ**
- **tt → lνqqbb** **Leptons+Jets** **30% e+μ**
- **tt → qqqqbb** **hadronic** **45%**

Experimental signature von top quark pairs



The top quark mass

For the details about the measurement see lecture by Sven Menke on Monday

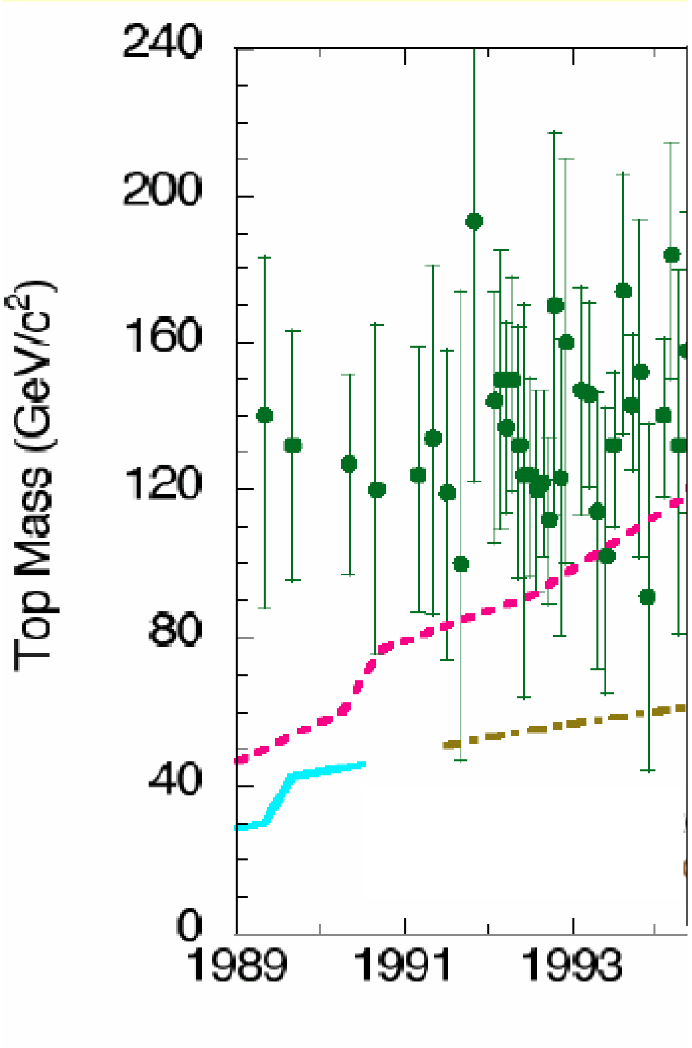
Why is the top quark mass important?

0.) Fundamental parameter of the Standard Model

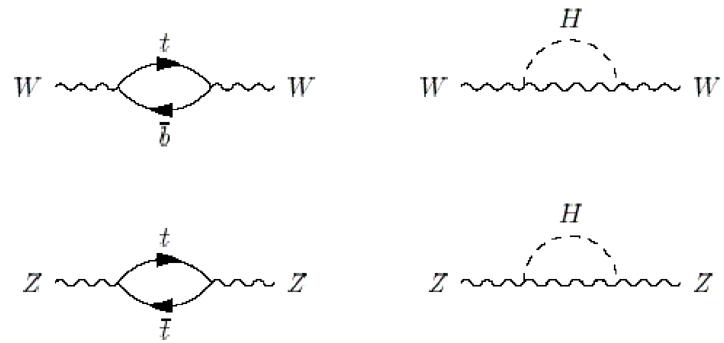
1.) Helps to constrain the Higgs mass, consistency check of the Standard Model

2.) Important input in many extensions of the SM, where the top plays an important role

Indirect measurment of the top quark mass



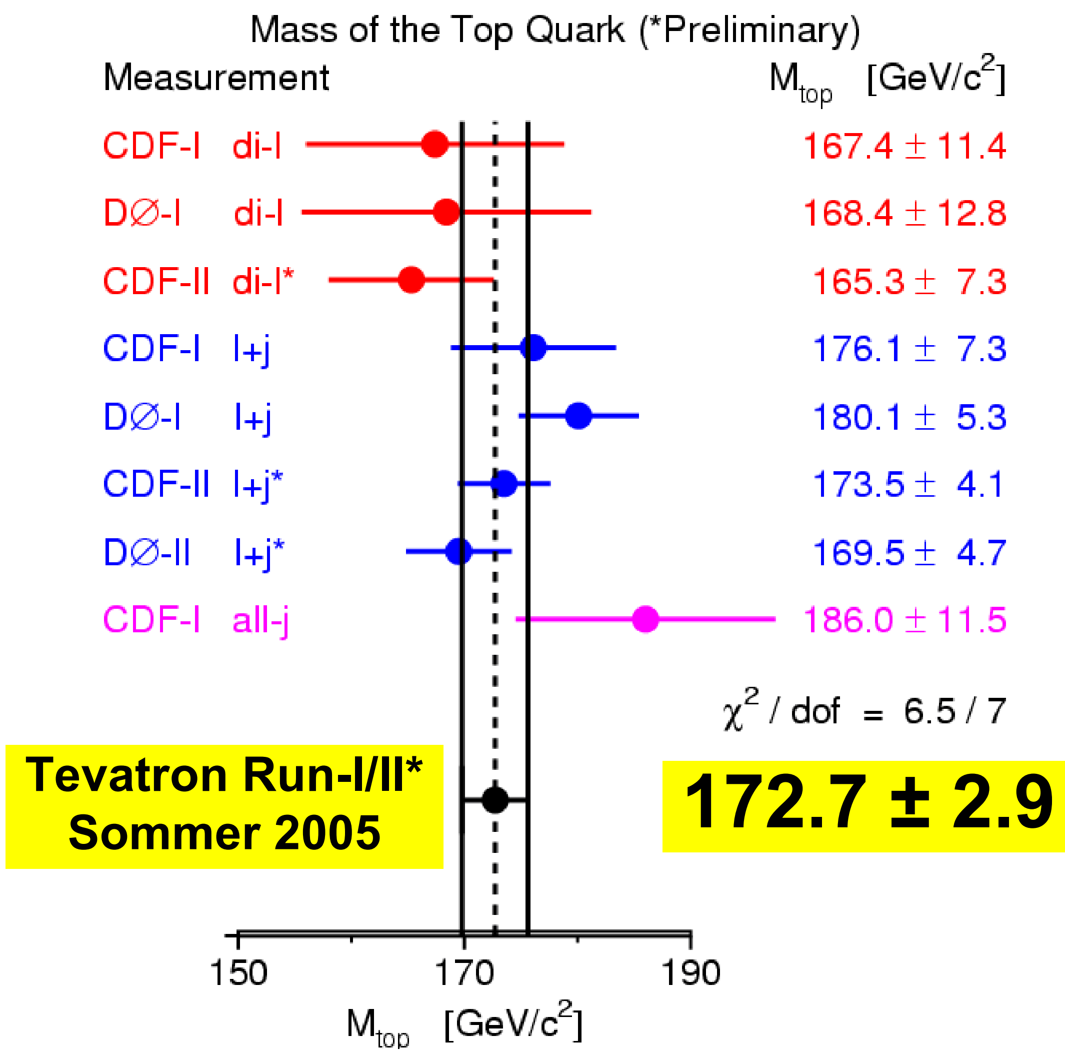
Sensitivity to top quark mass through virtual corrections:



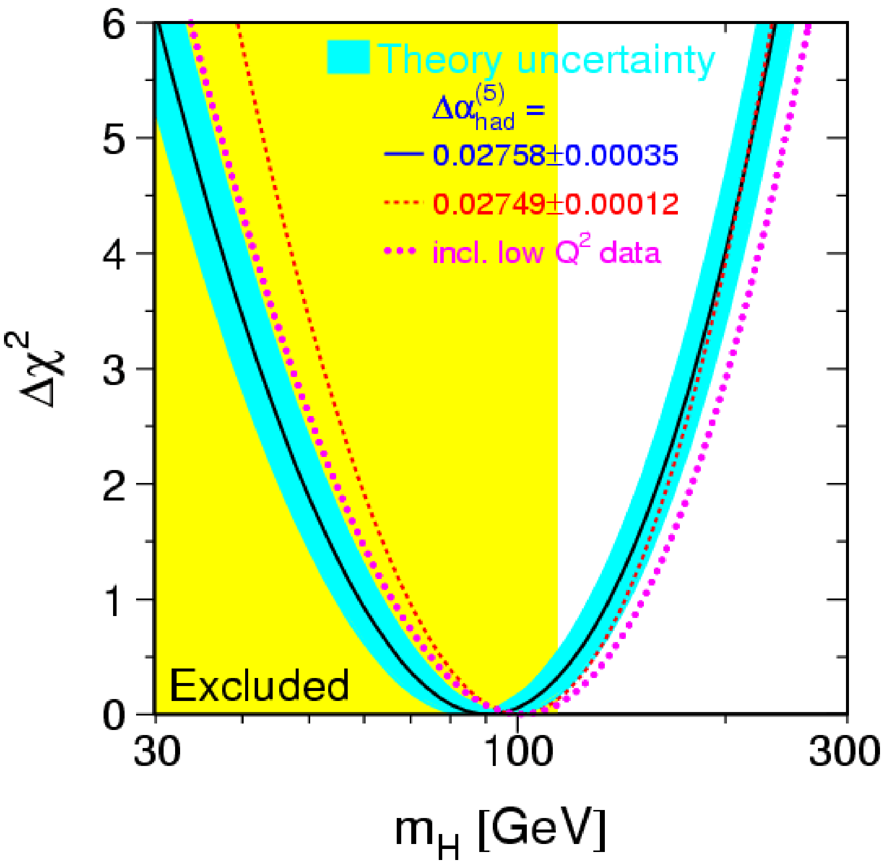
$$\rho = \frac{M_W^2}{M_Z^2(1-\sin^2\theta_W)} \equiv 1 + \Delta r$$

$$\Delta r = \frac{3G_F}{8\pi^2\sqrt{2}} m_t^2 + \frac{\sqrt{2}G_F}{16\pi^2} M_W^2 \left[\frac{11}{3}\ln\left(\frac{M_H^2}{M_W^2}\right) + \ldots\right] + \ldots$$

Direct measurement at Tevtron



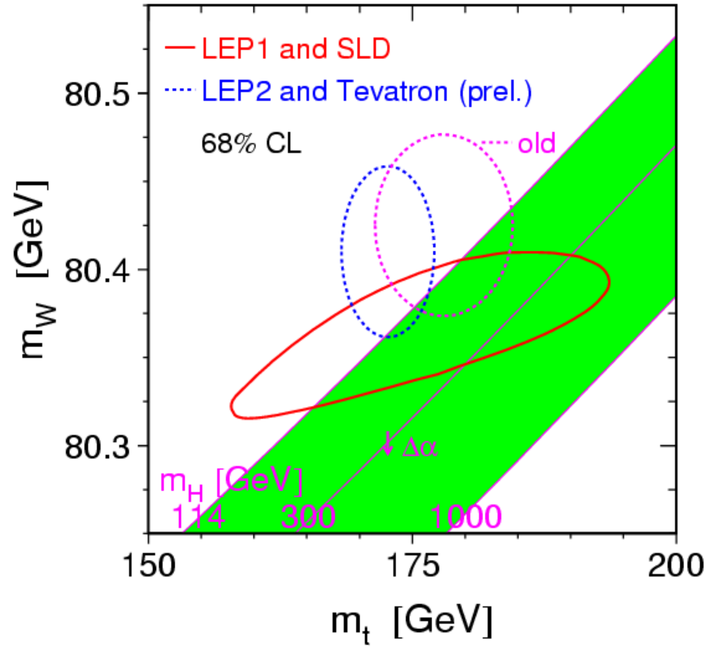
Top quark mass...

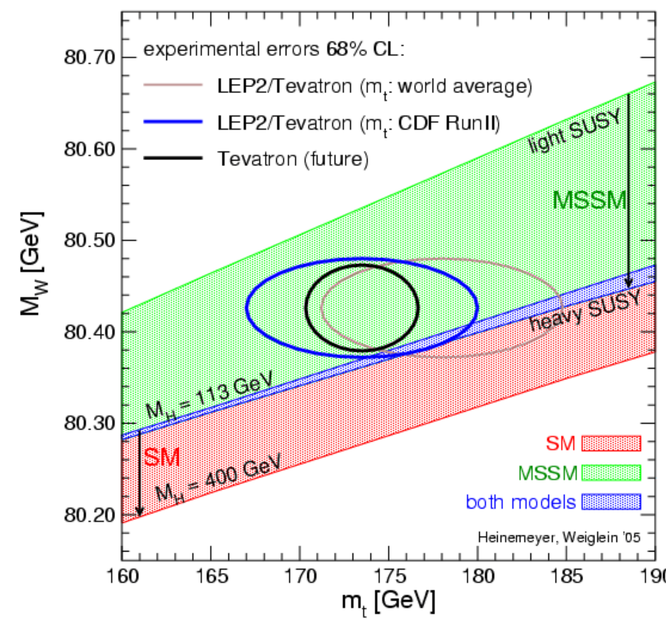


$$=91\pm^{45}_{32}\,\mathrm{GeV}$$

¿186 GeV@95 C.L.

<219 GeV with LEP Excluded





Spin correlations: Polarization versus correlations

Due to parity invariance of QCD, top's produced in qq→tt and gg → tt are essentially unpolarized *)

But: **Spins of top quark and antiquark are correlated**

[Bernreuther,Brandenburg 93,
Mahlon, Parke 96,
Stelzer,Willenbrock 96,
Bernreuther, Brandenburg, Si, P.U. 01-04]

Quantum mechanics: $^{(2s+1)}l_j$

close to threshold:

$^3S_1$: $q\bar{q} \rightarrow t\bar{t}$

$^1S_0$: $gg \rightarrow t\bar{t}$

→ Spins are parallel or anti-parallel close to threshold

*) absorptive parts at the one-loop level induce a small polarization (~1%) transverse to the scattering plane [Dharmaratna, Goldstein '96, Bernreuther,Brandenburg, P.U. ,96]

Why are spin correlations interesting?

- What is the spin of the top?
- Search for new physics
  - i.e. CP violating interactions, Higgs with undefined parity, properties of s-channel resonance
- Affect the angular distributions of the decay products
  - important for event selection
- Test of the idea that Top decays as a quasi free quark
  - precise test of the production and decay mechanism

Theoretical framework: Spin density matrix

Quantum mechanics: study spin density matrix

Single-top production:

$$R^t = A^t \, 1\!\!1 + B_i^t \, \sigma_i$$

Pair production:

$$R^{t\bar{t}} = A^{t\bar{t}} \, 1\!\!1 \otimes 1\!\!1 + B_i^{t\bar{t},+} \, \sigma_i \otimes 1\!\!1 + 1\!\!1 \otimes \sigma_i \, B_i^{t\bar{t},-} + C_{ij}^{t\bar{t}} \, \sigma_i \otimes \sigma_j$$

To evaluate R, consider:

$$|\mathcal{T}(X \to t(k_t, s_t)\bar{t}(k_{\bar{t}}, s_{\bar{t}}))|^2 = \hat{A} + \hat{B}_\mu^+ s_t^\mu + \hat{B}_\mu^- s_{\bar{t}}^\mu + \hat{C}_{\mu\nu} s_t^\mu s_{\bar{t}}^\nu$$

Expectation values:

$$\langle O \rangle = \frac{\int Tr[R \cdot O]}{\int Tr[R]}$$

Symmetries

$$R = A\ \mathbb{1} \otimes \mathbb{1} + B_i^t \sigma_i \otimes \mathbb{1} + \mathbb{1} \otimes \sigma_i B_i^{\bar{t}} + C_{ij}\ \sigma_i \otimes \sigma_j$$

$$\mathbf{B}^{t,\bar{t}} = b_1^{t,\bar{t}} \hat{\mathbf{p}} + b_2^{t,\bar{t}} \hat{\mathbf{k}} + b_3^{t,\bar{t}} \hat{\mathbf{n}},$$

$$\begin{aligned} C_{ij} \quad = \quad & c_0 \delta_{ij} + \varepsilon_{ijk}(c_1 \hat{\mathsf{p}}_k + c_2 \hat{\mathsf{k}}_k + c_3 \hat{\mathsf{n}}_k) + c_4 \hat{\mathsf{p}}_i \hat{\mathsf{p}}_j + c_5 \hat{\mathsf{k}}_i \hat{\mathsf{k}}_j + \\ & c_6(\hat{\mathsf{p}}_i \hat{\mathsf{k}}_j + \hat{\mathsf{p}}_j \hat{\mathsf{k}}_i) c_7(\hat{\mathsf{p}}_i \hat{\mathsf{n}}_j + \hat{\mathsf{p}}_j \hat{\mathsf{n}}_i) + c_8(\hat{\mathsf{k}}_i \hat{\mathsf{n}}_j + \hat{\mathsf{k}}_j \hat{\mathsf{n}}_i), \end{aligned}$$

Symmetries

	P	CP	T ($\mathcal{A}$=0)	CPT ($\mathcal{A}$=0)	Bose
$A(z)$	$A(z)$	$A(z)$	$A(z)$	$A(z)$	$A(-z)$
$b_1^{t,\bar{t}}(z)$	$-b_1^{t,\bar{t}}(z)$	$b_1^{\bar{t},t}(z)$	$b_1^{t,\bar{t}}(z)$	$b_1^{\bar{t},t}(z)$	$-b_1^{t,\bar{t}}(-z)$
$b_2^{t,\bar{t}}(z)$	$-b_2^{t,\bar{t}}(z)$	$b_2^{\bar{t},t}(z)$	$b_2^{t,\bar{t}}(z)$	$b_2^{\bar{t},t}(z)$	$b_2^{t,\bar{t}}(-z)$
$b_3^{t,\bar{t}}(z)$	$b_3^{t,\bar{t}}(z)$	$b_3^{\bar{t},t}(z)$	$-b_3^{t,\bar{t}}(z)$	$-b_3^{\bar{t},t}(z)$	$-b_3^{t,\bar{t}}(-z)$
$c_0(z)$	$c_0(z)$	$c_0(z)$	$c_0(z)$	$c_0(z)$	$c_0(-z)$
$c_1(z)$	$-c_1(z)$	$-c_1(z)$	$-c_1(z)$	$c_1(z)$	$-c_1(-z)$
$c_2(z)$	$-c_2(z)$	$-c_2(z)$	$-c_2(z)$	$c_2(z)$	$c_2(-z)$
$c_3(z)$	$c_3(z)$	$-c_3(z)$	$c_3(z)$	$-c_3(z)$	$-c_3(-z)$
$c_4(z)$	$c_4(z)$	$c_4(z)$	$c_4(z)$	$c_4(z)$	$c_4(-z)$
$c_5(z)$	$c_5(z)$	$c_5(z)$	$c_5(z)$	$c_5(z)$	$c_5(-z)$
$c_6(z)$	$c_6(z)$	$c_6(z)$	$c_6(z)$	$c_6(z)$	$-c_6(-z)$
$c_7(z)$	$-c_7(z)$	$c_7(z)$	$-c_7(z)$	$-c_7(z)$	$c_7(-z)$
$c_8(z)$	$-c_8(z)$	$c_8(z)$	$-c_8(z)$	$-c_8(z)$	$-c_8(-z)$

[Bernreuther, Brandenburg]

Spin correlations: The spin density matrix

$$C_{ij}^{gg} = \delta_{ij} c_{g0} + \hat{p}_i \hat{p}_j c_{g4} + \hat{k}_i \hat{k}_j c_{g5} + (\hat{p}_i \hat{k}_j + \hat{p}_j \hat{k}_i) c_{g6}$$

$$\begin{aligned}
A_g &= 2 f_g \left[1 + 2\beta^2 (1 - z^2)(1 - \beta^2) - \beta^4 z^4\right], \\
c_{g0} &= -2 f_g \left[(1 - \beta^2)^2 + \beta^4 (1 - z^2)^2\right], \\
c_{g4} &= 4 f_g (1 - z^2) \beta^2, \\
c_{g5} &= -4 f_g \beta^2 \left(1 - 2\beta^2 + z^2 \beta^2 - \frac{z^2 \beta^4 (1 - z^2)}{(1 + \sqrt{1 - \beta^2})^2}\right), \\
c_{g6} &= -4 f_g z (1 - z^2) \beta^2 (1 - \sqrt{1 - \beta^2}),
\end{aligned}$$

$$f_g = \frac{\pi^2 \alpha_s^2}{(1 - z^2 \beta^2)^2} \frac{(N^2 - 2 + N^2 z^2 \beta^2)}{N(N^2 - 1)}$$

for SU(N)

$$z = \cos(\vartheta)$$

$$\beta = \sqrt{1 - \frac{4 m_t^2}{s}}$$

Spin correlations: Observables…

Knowledge of R allows the calculation of arbitrary spin observables, i.e.:

$$\langle S_t \cdot S_{\bar t}\rangle \qquad \langle k_t (S_t \times S_{\bar t})\rangle$$

~~CP~~ [Bernreuther, Brandenburg '93]

Observables of the form $(a\cdot S_t)(b\cdot S_{\bar t})$ have simple interpretation:

$$C_{t\bar t} = 4\langle (a\cdot S_t)(b\cdot S_{\bar t})\rangle = \frac{\int \text{dLips}\ a_i C_{ij} b_j}{\int \text{dLips}\ A}$$

$$= \frac{\sigma_{t\bar t}(\uparrow\uparrow)+\sigma_{t\bar t}(\downarrow\downarrow)-\sigma_{t\bar t}(\uparrow\downarrow)-\sigma_{t\bar t}(\downarrow\uparrow)}{\sigma_{t\bar t}(\uparrow\uparrow)+\sigma_{t\bar t}(\downarrow\downarrow)+\sigma_{t\bar t}(\uparrow\downarrow)+\sigma_{t\bar t}(\downarrow\uparrow)}$$

where ↑/↓ denote spin up/down with respect to a,b as quantization axis

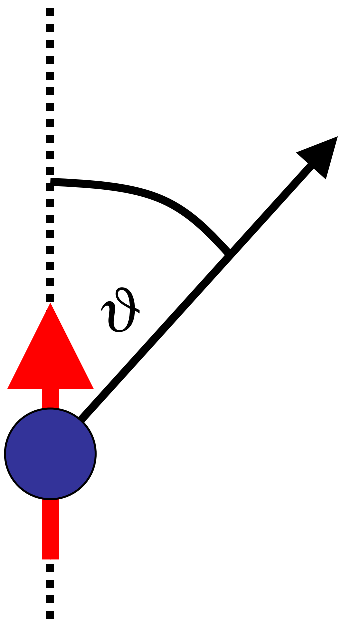
To measure C study for example double differential distributions.

Measurement of the top quark polarization

How can we measure the "spin" of the top quark?

Basic ingredients:

- Top quark decays before hadronization
- Parity violating decay $t \rightarrow Wb$



The top quark polarization can be studied through the angular distribution of the decay products!

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\vartheta} = \frac{1}{2}(1 + \kappa_f \cos\vartheta)$$

	$\ell^+, \bar{d}$	ν_ℓ^+, u	b	W^+	least energetic jet from $q\bar{q}'$
κ_f	1	-0.31	-0.41	0.41	0.51

QCD NLO corrections known!

[Czarnecki, Jezabek, Kühn 91, Brandenburg, Si, P.U. '02]

Spin correlations: LO Standard Model predictions

Double differential distribution:

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\vartheta_\ell \, d\cos\vartheta_{\bar{\ell}}} = \frac{1}{4}\left(1 - C\cos\vartheta_\ell\cos\vartheta_{\bar{\ell}}\right)$$

$$C = \kappa_\ell \kappa_{\bar{\ell}} C_{t\bar{t}}$$

Size of C depends on the "quantization axis":

		$\ell^+ + \ell^- + X$	$\ell + Jet + X$	$Jet + Jet + X$
Tevatron	C_{hel}	-0.471	-0.240	-0.123
	C_{beam}	**0.928**	0.474	0.242
LHC	C_{hel}	**0.319**	0.163	0.083

$\mu_R=\mu_F=m_t=175$ GeV, CTEQ6L

Spin correlations can be very large!

Spin correlations: LO SM predictions (2)

Interesting features:

- For the qq sub process an optimal quantization axis yielding 100% correlation exists:

$$d_{\text{off}} = a = b = \frac{-\hat{p}_q + (1-\gamma_t)(\hat{p}_q \cdot \hat{k}_t)\hat{k}_t}{\sqrt{1-(\hat{p}_q \cdot \hat{k}_t)^2(1-\gamma_t^2)}}$$

[Mahlon, Parke '97]

near threshold:

$$a = b \approx \hat{p}_q$$

→ Beam axis is a good choice at the Tevatron

- For gg no axis exists for which the correlation is 100%

  Without cuts the maximal value for the correlation in gluon fusion is 48% [P.U. '04]

Spin correlations: LO SM predictions (3)

For the qq sub-process we have in LO:

$$\langle S_t \cdot S_{\bar{t}} \rangle = \langle (d_{\text{off}} \cdot S_t)(d_{\text{off}} \cdot S_{\bar{t}}) \rangle$$

In NLO this relation still holds up to tiny corrections

The combination

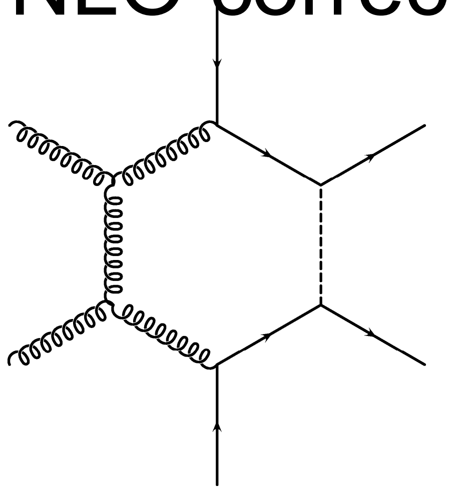
$$\langle S_t \cdot S_{\bar{t}} \rangle - \langle (d_{\text{off}} \cdot S_t)(d_{\text{off}} \cdot S_{\bar{t}}) \rangle$$

is thus only sensitive to the gluon PDF!

Can we use spin correlations to constrain the gluon PDF?

Spin correlation: NLO corrections

In general very complicated task:



Approximation:
- double pole approximation
- calculate only factorizable contributions

NLO corrections calculable:

		$\ell^+ + \ell^- + X$	$\ell + Jet + X$	$Jet + Jet + X$
Tevatron	C_{hel}	-0.352	-0.168	-0.080
	C_{beam}	**0.777**	0.370	0.176
LHC	C_{hel}	**0.326**	0.158	0.076

[Bernreuther, Brandenburg, Si, P.U. 03,04]

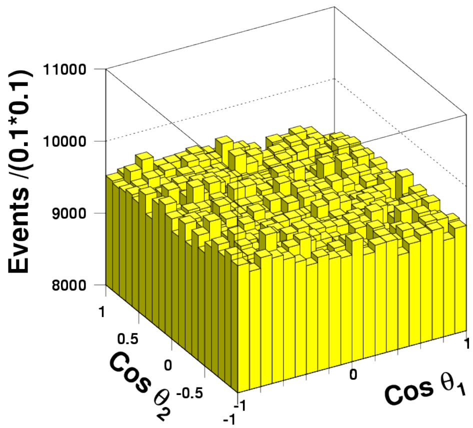
$\mu_R=\mu_F=m_t=175$ GeV, $\alpha_s(\mu=m_t)=0.1074$, CTEQ6.1M

Scale dep.: Tevatron: $\Delta C_{\text{beam}} = 5\%$ LHC: $\Delta C_{\text{hel}} = 1\%$

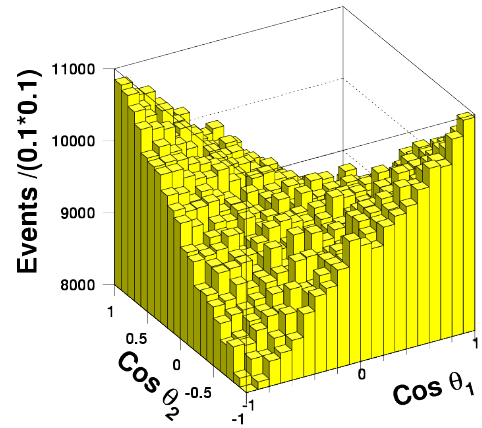
A realistic analysis…

no correlation

with correlation

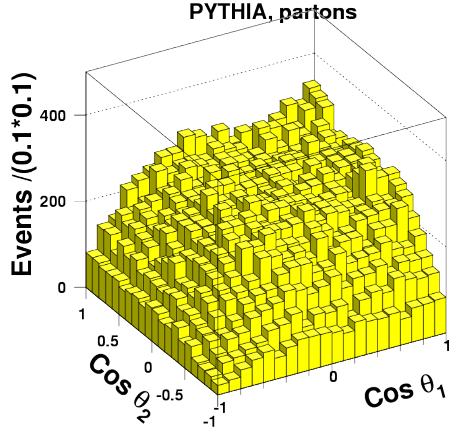


PYTHIA, partons



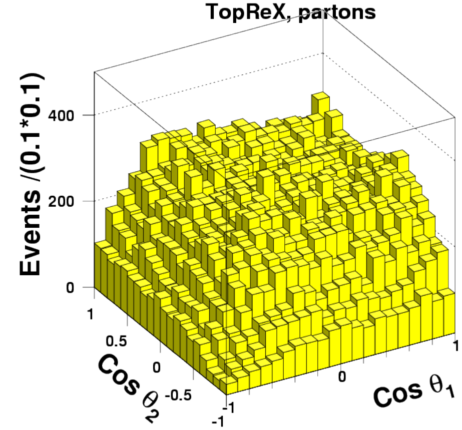
TopReX, partons

parton level



PYTHIA, reconstruction + cuts

after reconstruction



TopReX, reconstruction + cuts

Very difficult analysis, dominated by systematic uncertainties

[ATLAS collaboration, P. Pralavorio, F.Hubaut, E.Monnier]

Atlas Simulation

[Hubaut, Monnier, Pralavorio,Smolek,Simak 05]

$A \sim \langle (a \cdot S_t)(b \cdot S_{\bar{t}}) \rangle$ ←→ double differential distribution

$A_D \sim \langle (S_t \cdot S_{\bar{t}}) \rangle$ ←→ single differential distribution

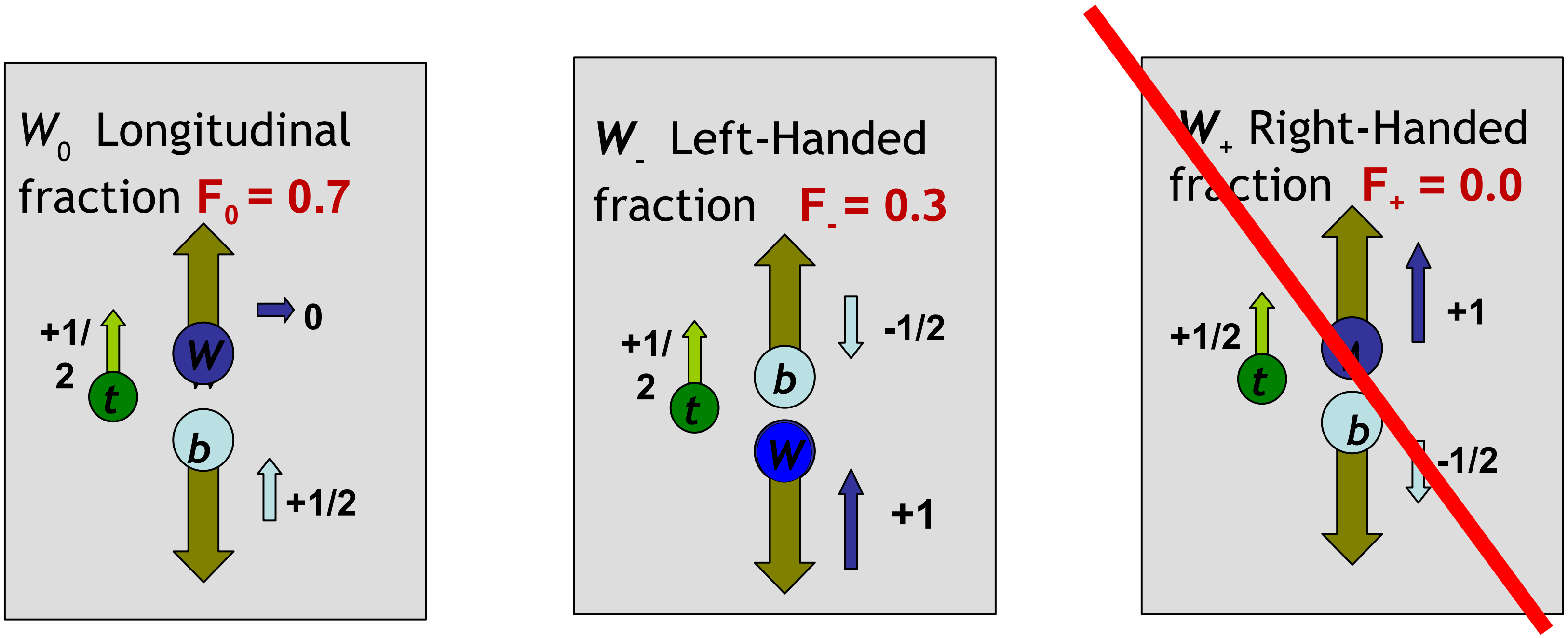
	Semileptonic (±stat±syst)	Dileptonic (±stat±syst)	Semilep+Dilep
A	0.422 ± 0.020 ± 0.081	0.404 ± 0.020 ± 0.024	0.406 ± 0.014 ± 0.023
A_D	-0.288 ± 0.012 ± 0.036	-0.290 ± 0.011 ± 0.010	-0.290 ± 0.008 ± 0.010

4-5 % accuracy **!**

Current accuracy at the Tevatron: ~20%

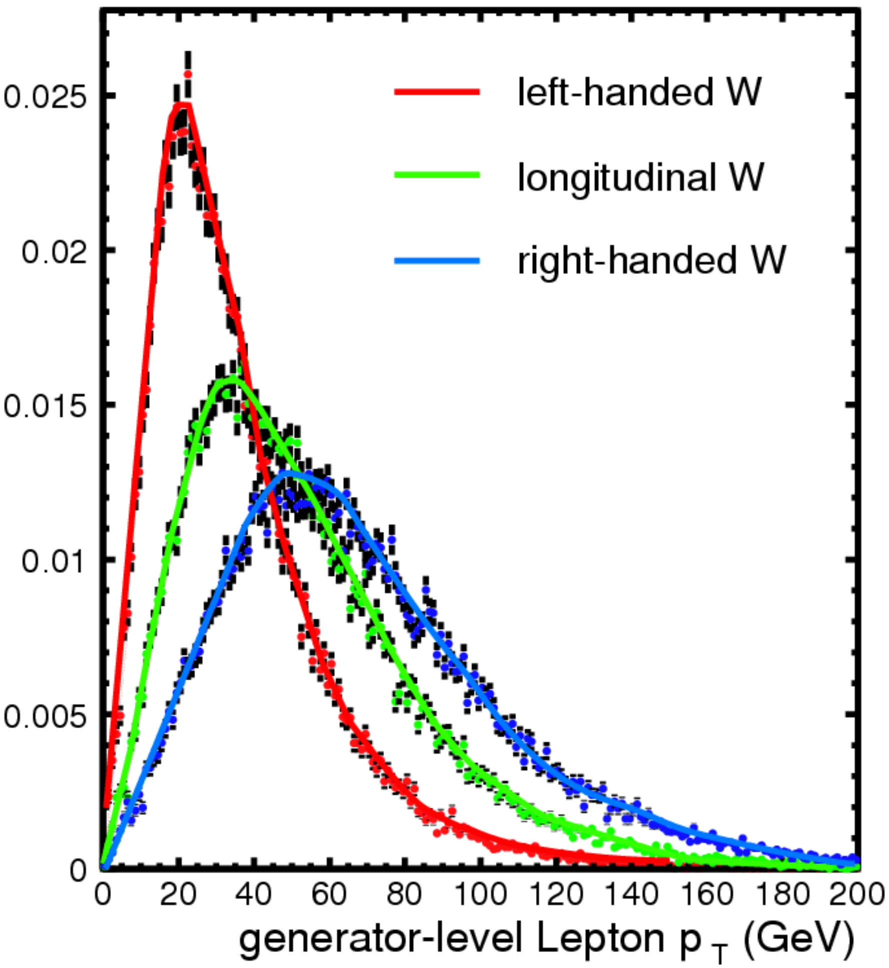
W-Polarization in top quark decay

Test of the V-A structure of the Wtb vertex

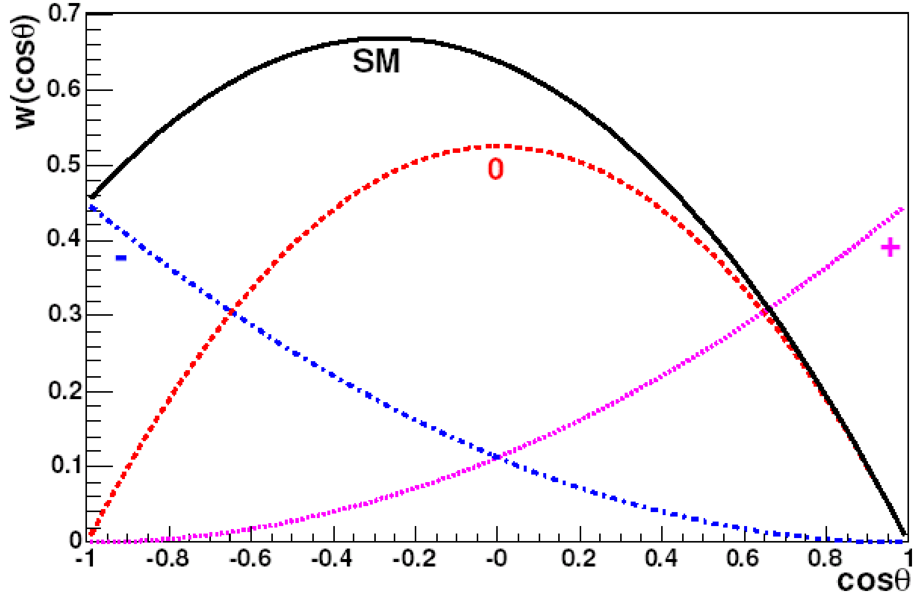


Measurement of the W-Polarization

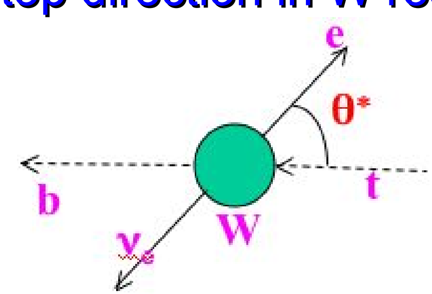
Lepton p_T



cos(θ*)



Angle between charged lepton and top direction in W rest frame.



Nice side effect of the spin correlations studies:

Precise measurement of the W-polarization possible using tt events

[Pralavario, Hubaut, Monnier ‘05]

Using the same tt events as in the previous analysis:

Simulation!

$$\frac{\Gamma_0}{\Gamma} = 0.699 \pm 0.005(\text{stat.}) \pm 0.023(\text{sys.})$$

$$\frac{\Gamma_L}{\Gamma} = 0.299 \pm 0.003(\text{stat.}) \pm 0.028(\text{sys.})$$

$$\frac{\Gamma_R}{\Gamma} = 0.002 \pm 0.003(\text{stat.}) \pm 0.013(\text{sys.})$$

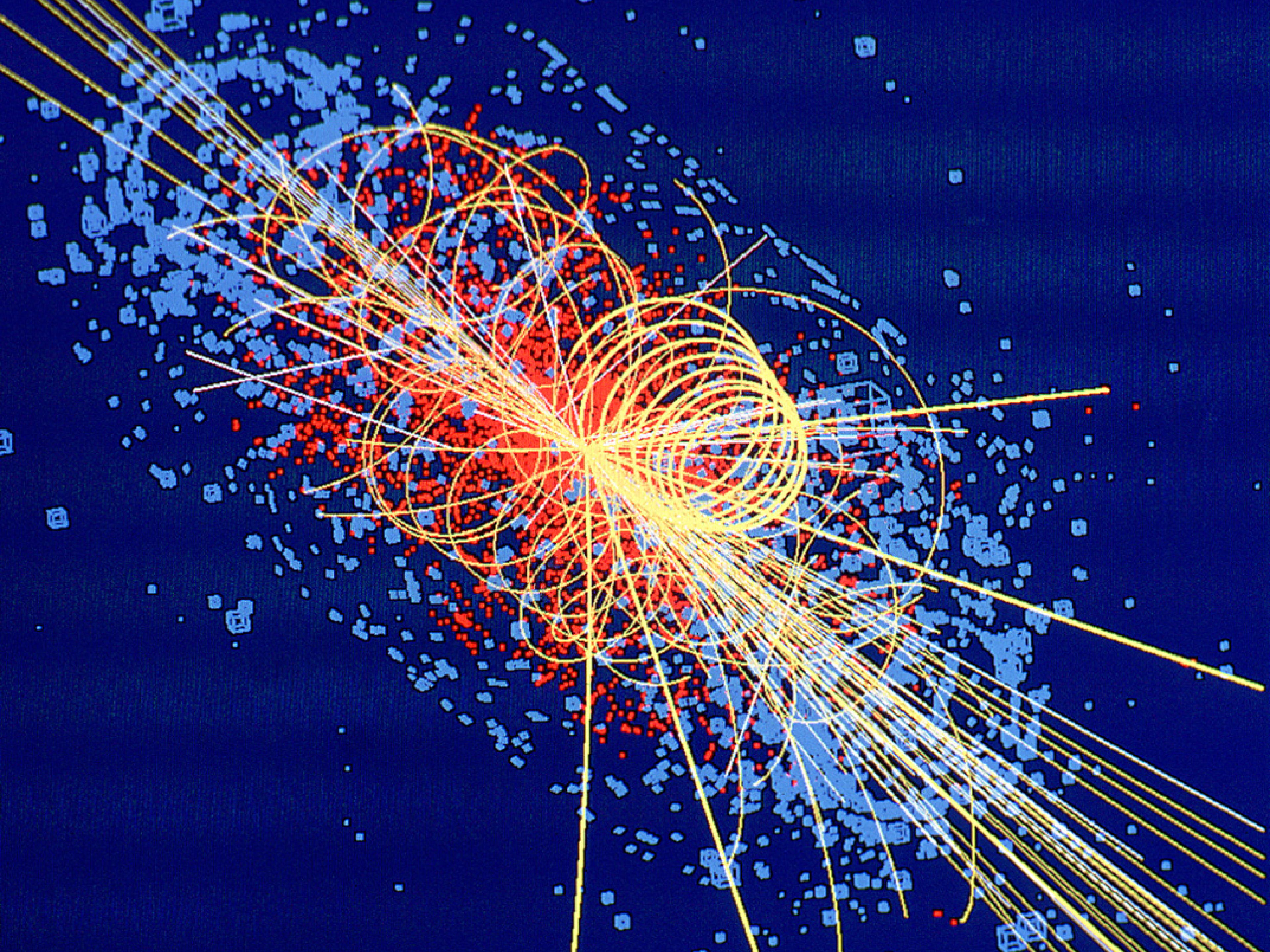
Simulated data sample corresponds to one year at low luminosity.

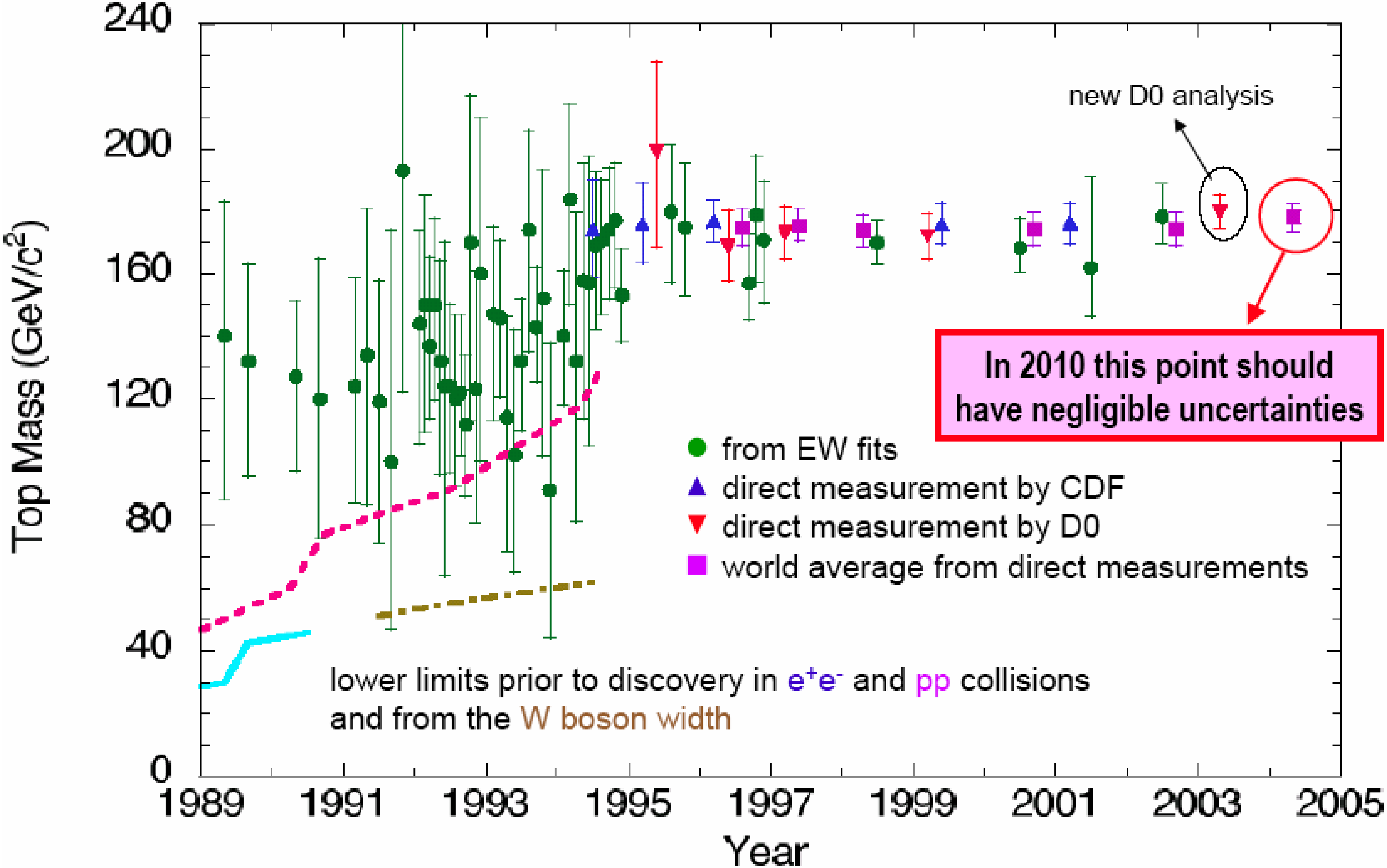
Tevatron: $\Gamma_R/\Gamma = 0.04 \pm 0.11(\text{stat}) \pm 0.06(\text{syst.})$

Conclusion:

Top quark physics is an interesting and important subject

Many interesting measurements possible





Top Mass (GeV/c²)
240
200
160
120
80
40
0
1989
1991
1993
1995
1997
1999
2001
2003
2005
Year
new D0 analysis
In 2010 this point should have negligible uncertainties
from EW fits
direct measurement by CDF
direct measurement by D0
world average from direct measurements
lower limits prior to discovery in e+e- and pp collisions
and from the W boson width