CP Violation and Rare Decays in the Standard Model and Beyond

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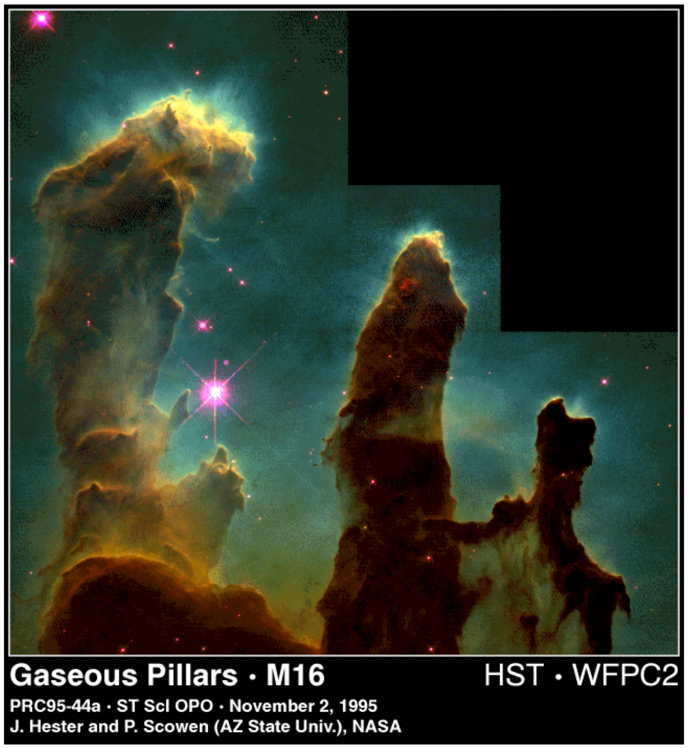
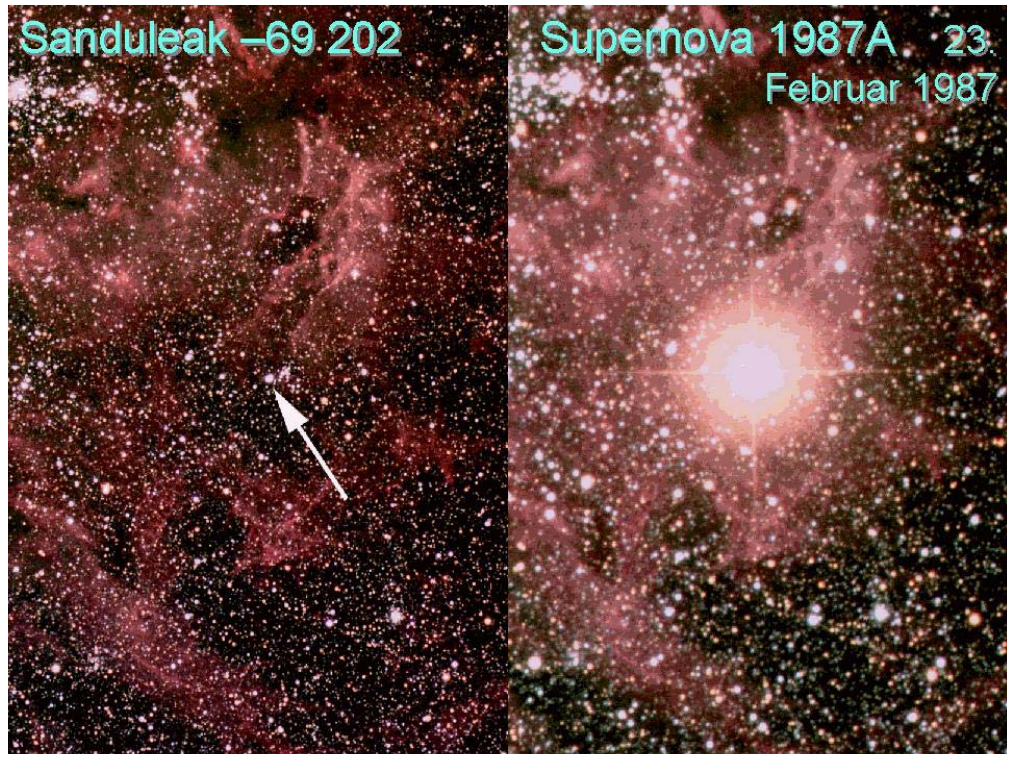
(Technical University Munich)

Munich, February 17th-22nd, 2006
Ringberg Castle, July 2006

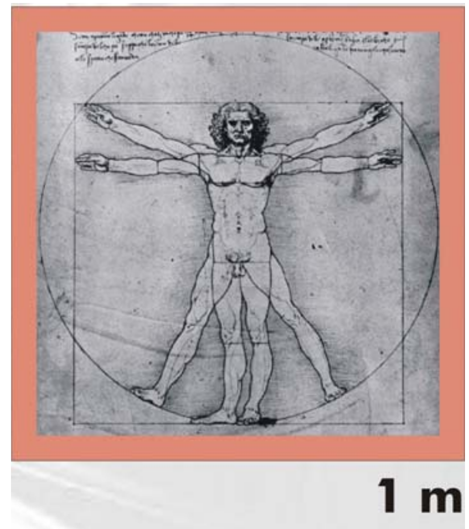
Good Afternoon !

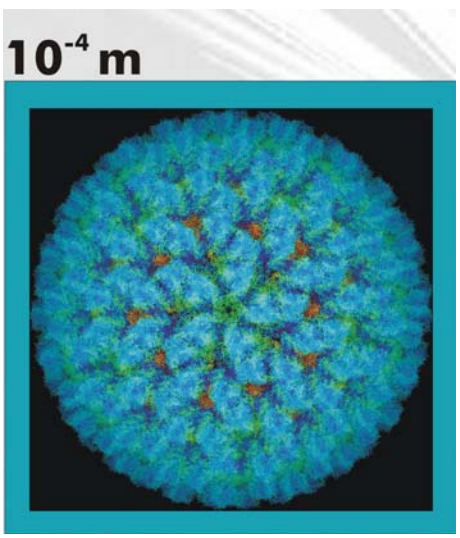
Overture 1

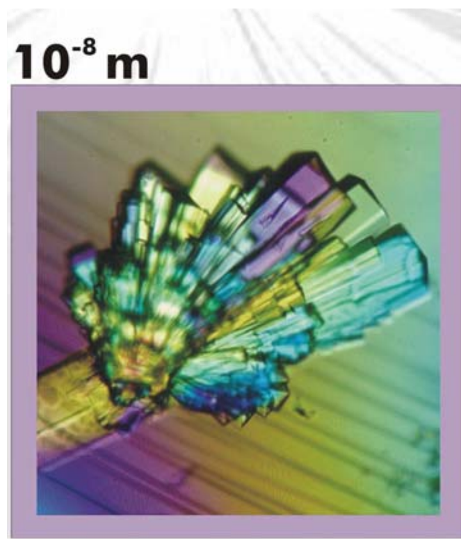

Gaseous Pillars · M16
HST · WFPC2
PRC95-44a · ST ScI OPO · November 2, 1995
J. Hester and P. Scowen (AZ State Univ.), NASA

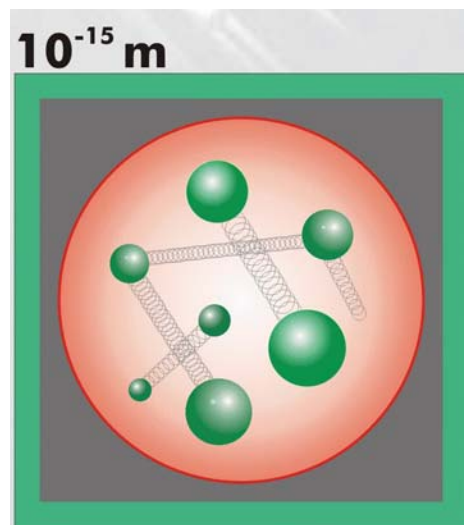

Sanduleak –69 202
Supernova 1987A
23. Februar 1987

Vastly Different Scales





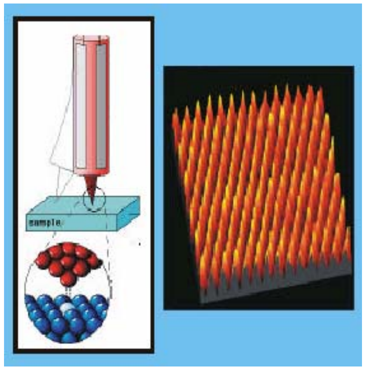




Quarks bound in the Proton

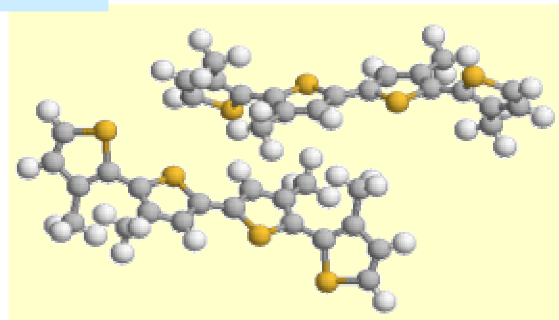
(Femtouniverse)

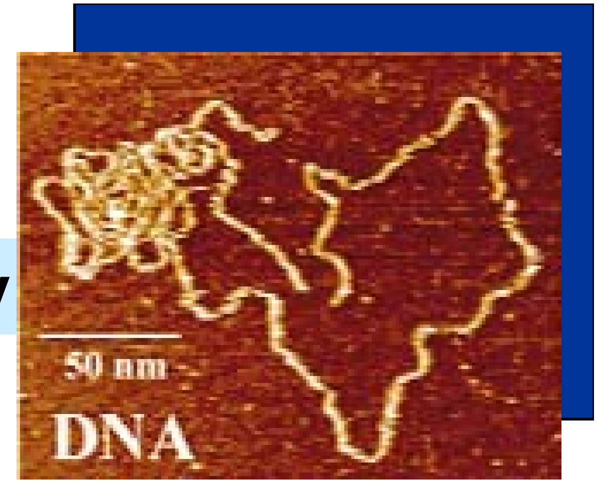
Nanotechnology



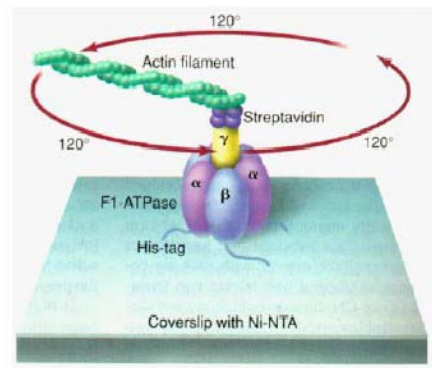
Physics

Chemistry

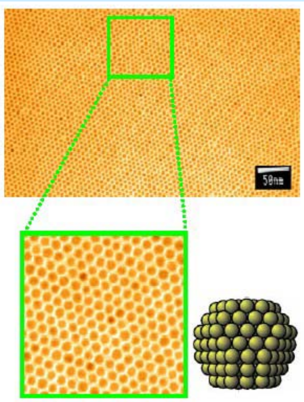


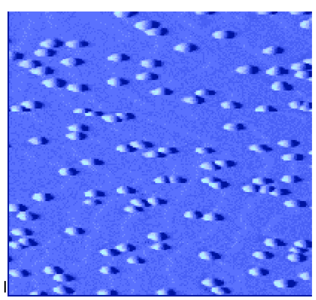


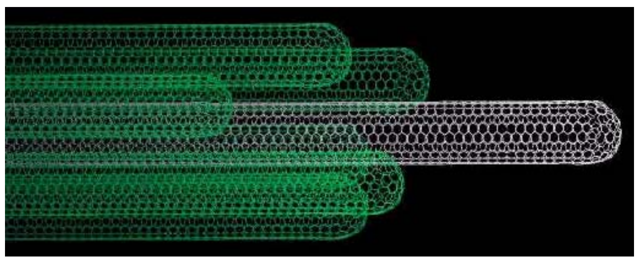
Biology



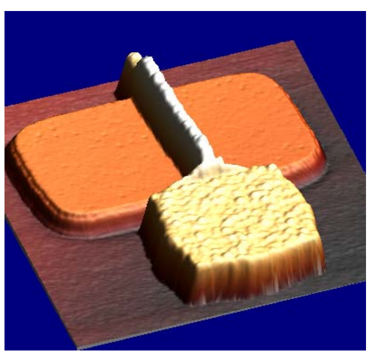
Materials

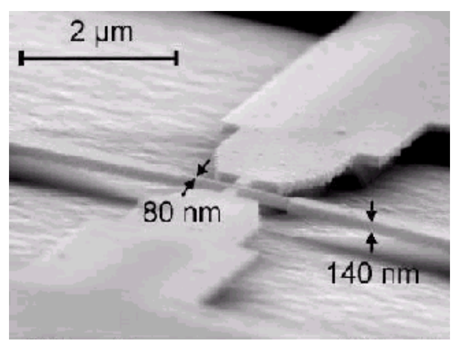




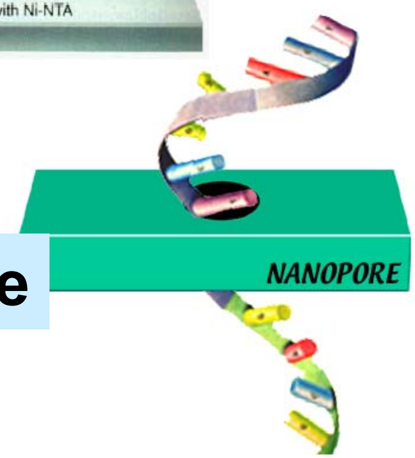


Technology





Medicine



Attouniverse

10^{-18}m

1000 times smaller than the Femtouniverse

10^{-15}m

1000000000 times smaller than the Nanouniverse

10^{-9}m

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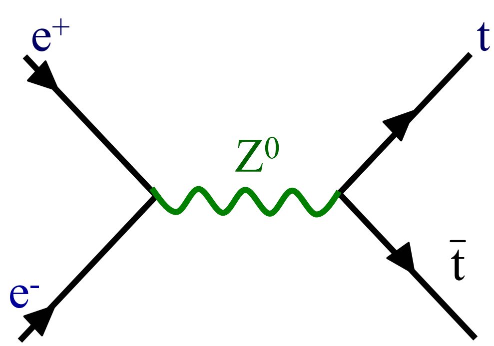
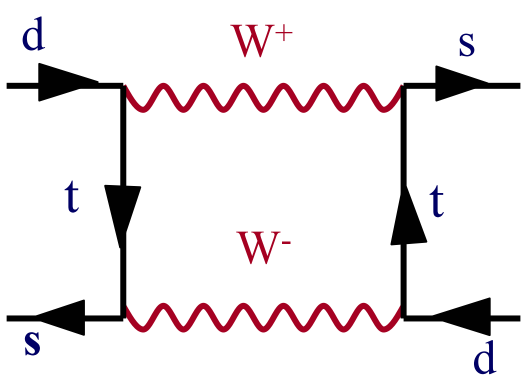




How to explore the Attouniverse ?

1. Directly :

Very high energy colliders:

CERN, SLAC, DESY, Fermilab, NLC, ...

2. Indirectly :

Use Femtouniverse and Quantum Effects to study Attouniverse and smaller universes.

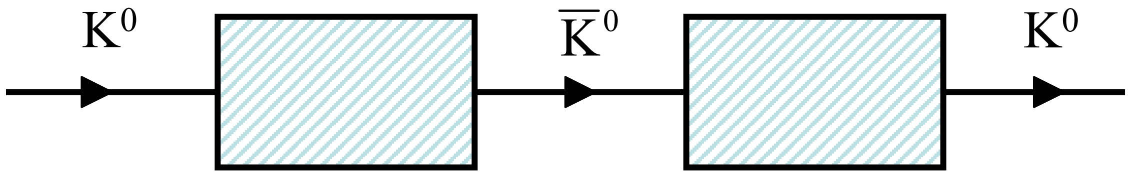
★

"Kaon-Factories" , "B-Meson Factories"

$K^0 - \overline{K}^0$ Mixing (Oscillations)

$K^0 = d\bar{s}$

$\overline{K}^0 = \bar{d}s$



(discovered in 1960)

$\Rightarrow$ K^0 and $\overline{K}^0$ are not Mass Eigenstates

Mass Eigenstates :

$$K_L = \frac{K^0 + \overline{K}^0}{\sqrt{2}} \qquad K_S = \frac{K^0 - \overline{K}^0}{\sqrt{2}}$$

(L = Long) (S = Short)

$M(K_L) \cong M(K_S) = 0.5\ \text{GeV}$

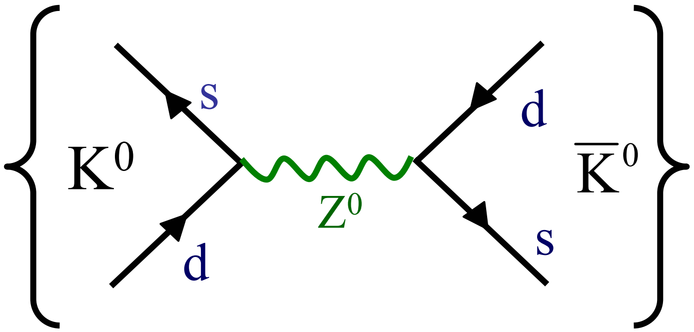
★ $M(K_L) - M(K_S) = 3.5 \cdot 10^{-15}\,\text{GeV}$ $\equiv \Delta M_K$

$$\frac{\tau(K_L)}{\tau(K_S)} \cong 600$$ ★

$M \equiv \text{mass}$

$\tau \equiv \text{Life Time}$

Could ordinary Weak Interactions explain ΔM_K ?



$$\left\{ \begin{array}{l} \Delta M_K = \left[2\cdot 10^{-2}\,\mathrm{GeV}^3\right] G_F \left[\dfrac{M_W^2}{M_Z^2}\right] \\ G_F \cong 1.1\cdot 10^{-5}\,\mathrm{GeV}^{-2} \end{array} \right\}$$

$M_Z \cong 91\ \mathrm{GeV}$

$$\Delta M_K \cong 2\cdot 10^{-7}\,\mathrm{GeV}$$

Disaster !!!
Missed by
8 orders of
magnitude !!!

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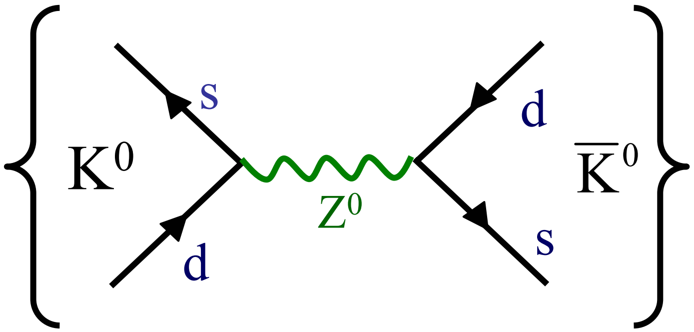

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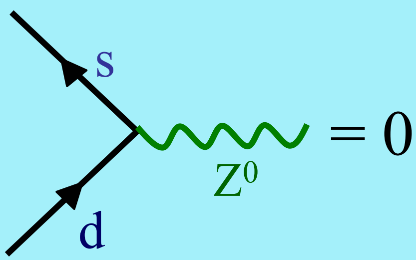

GIM Mechanism

(1971)

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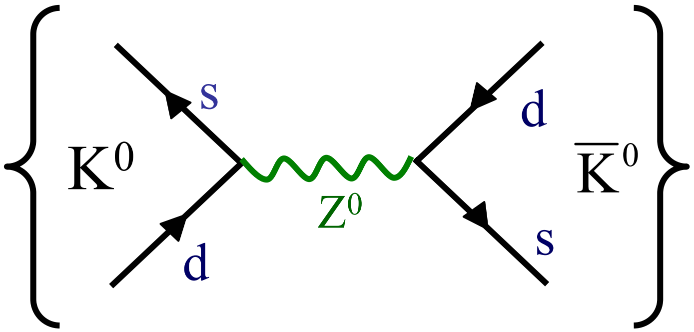

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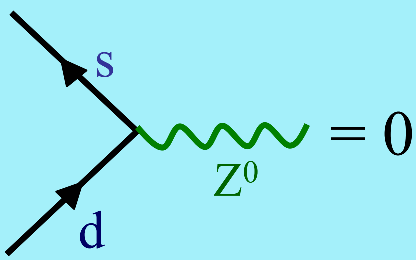

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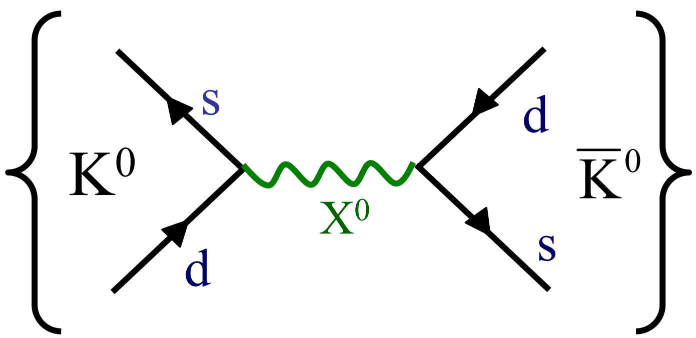


$$\Rightarrow \left\{ \Delta M_K \cong 2\cdot 10^{-7}\left[\frac{M_Z^2}{M_X^2}\right]\mathrm{GeV} = 3.5\cdot 10^{-15}\,\mathrm{GeV} \right\}$$

New very heavy neutral boson !

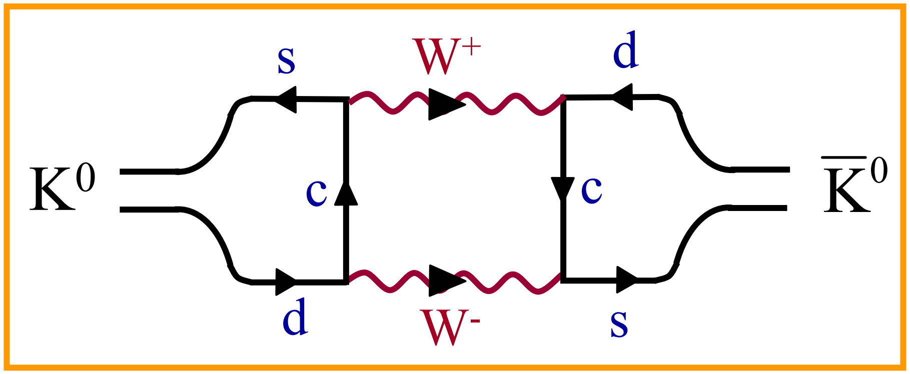
$$\left\{ M_X \cong 10^6\,\mathrm{GeV} \right\} \Rightarrow$$

Testing
10^{-22}m !

ΔM_K in the Standard Model

Gaillard-Lee (March 1974)

$\lambda \cong 0.22$



$$\left\{ \begin{array}{l} \Delta M_K = \left[1.4\ \mathrm{GeV}^5\right] G_F^2 \lambda^2 \left[\dfrac{m_c^2}{M_W^2}\right] \\ G_F \cong 1.2 \cdot 10^{-5}\,\mathrm{GeV}^{-2} \end{array} \right\}$$

$$\Delta M_K = 10^{-11}\,\mathrm{GeV}\left[\frac{m_c^2}{M_W^2}\right] = 3.5 \cdot 10^{-15}\,\mathrm{GeV}$$

$$m_c = \sqrt{3.5} \cdot 10^{-2} M_W = 1.5\ \mathrm{GeV}$$

(Prediction !!)

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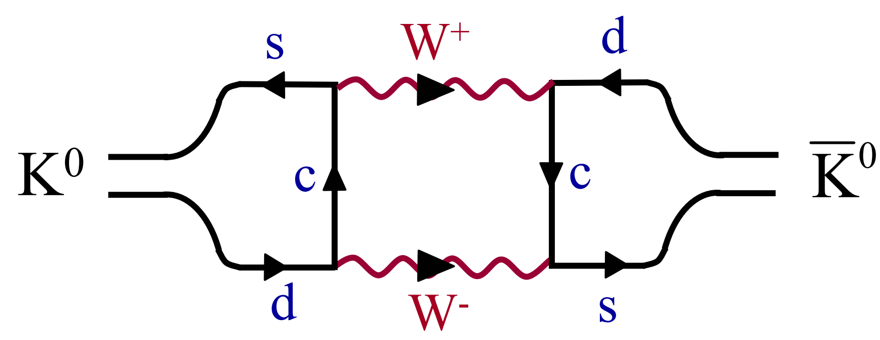

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(Prediction !!)

$$\left\{ \begin{array}{c} \text{November Revolution} \\ 1974 \\ \text{Discovery of } \bar{c}c \text{ State} \\ \text{(SLAC, Brookhaven)} \end{array} \right\}$$

$$\boxed{M_{\bar{c}c} \cong 3.1\ \mathrm{GeV}}$$

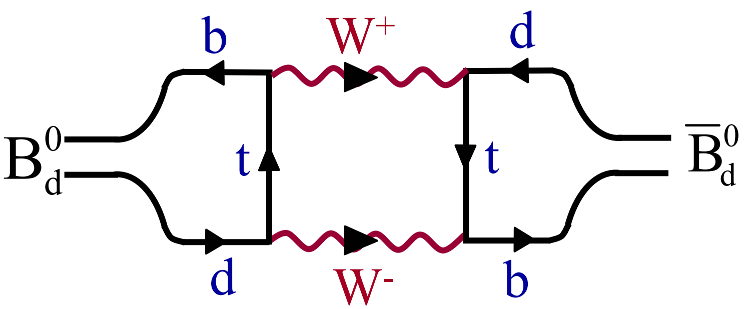
$$\boxed{m_c \cong 1.5\ \mathrm{GeV}} \quad !!$$

Prediction confirmed !

Similar Studies: 1974-1994

$B_d^0 - \overline{B}_d^0$ Oscillations

DESY 87



$+$ $\Delta M_{B_d} \cong 4 \cdot 10^{-13}\,\mathrm{GeV}$

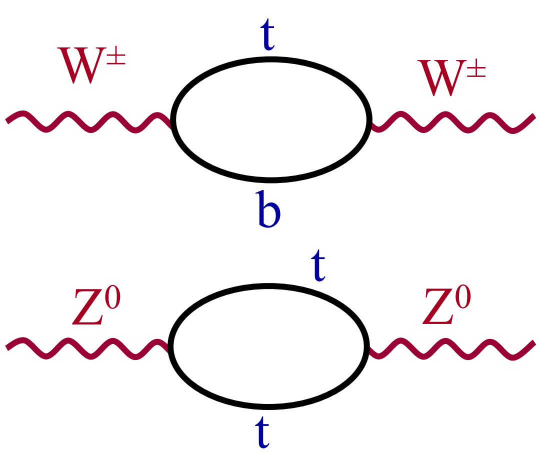
(Prediction)

$m_t \approx 150$ GeV ± 30

Electroweak Precision Studies

CERN, SLAC (1989-1994)

1994 Discovery of the Top Quark (Fermilab)



$m_t \approx 150$ GeV ± 20

(Prediction)

$m_t = 172.7 \pm 2.9$ GeV

First Lessons

1. Very rare processes allow to probe very short distance scales.

2. Before claiming New Physics it is essential to make precise calculations (higher order corrections).

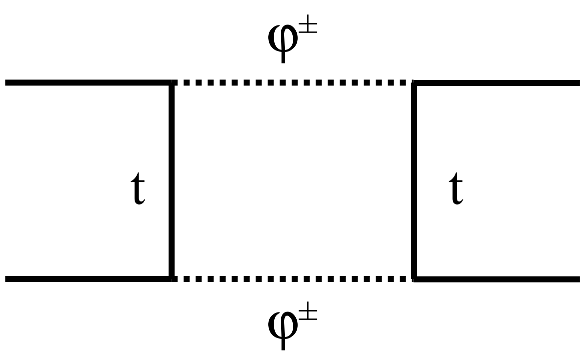
3. Low Energy Processes can give information about heavy particles prior to their discovery.

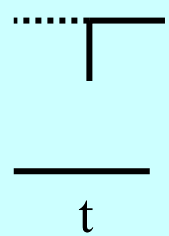
Non-Decoupling of the Top Quark from Low Energy Processes

In QCD and QED very heavy particles ($m_H \to \infty$) do not influence low energy processes: Appelquist-Carazzone Decoupling Theorem

In the $SU(2)_L \otimes U(1)_Y$ the decoupling can be violated by couplings of heavy particles that increase with the heavy particle mass.

Goldstone-Boson



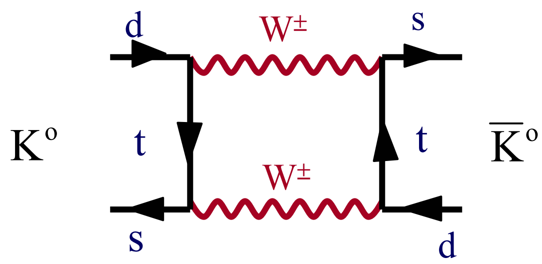


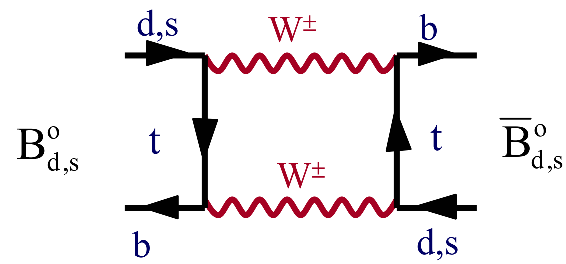
$\sim m_t$

$\sim \frac{1}{m_t}$

$$m_t^4 \cdot \frac{1}{m_t^2} = m_t^2$$

View at Short Distance Scales

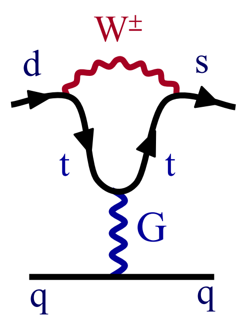


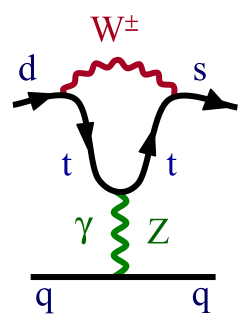


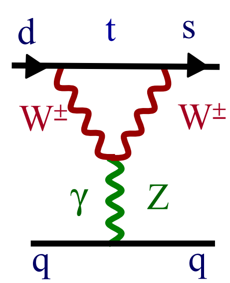
★ $\not{C}P$ ε_K-Parameter
$\Delta M\ (K_L\text{-}K_S)$

$B_d^o - \overline{B}_d^o$ Mixing ★

★ ε'

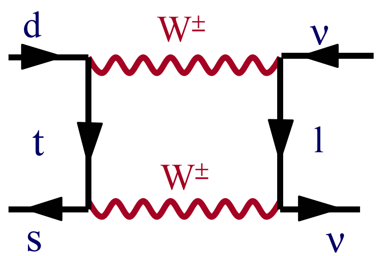


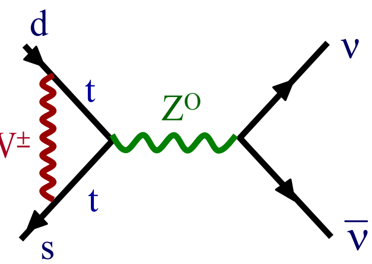


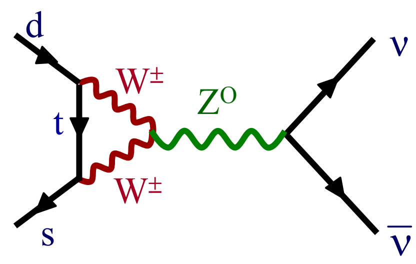


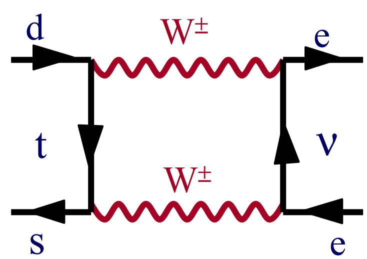
View at Short Distance Scales

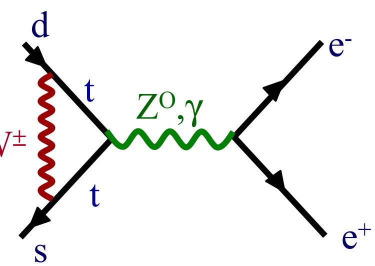
$K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$ $K_L \to \mu\bar{\mu}$, $B \to \mu\bar{\mu}$, $B \to X_S \nu\bar{\nu}$

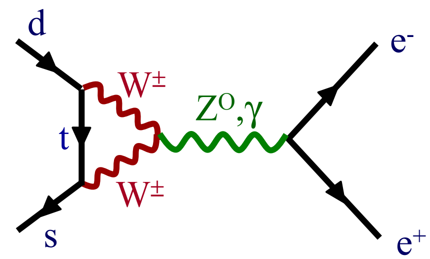




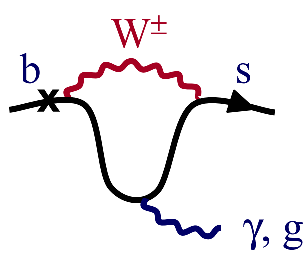








$K_L \to \pi^0 e^+ e^-$ $B \to X_s e^+ e^-,\ X_s \mu\bar{\mu}$



$B \to X_s \gamma$ $B \to K^* \gamma$

$B \to X_d \gamma$ $b \to s$ gluon

Goals for these Lectures

1. Develop Formalism for Rare Processes:
   CP-Violating Transitions, CP-Asymmetries and
   Rare Decays within Gauge Theories
2. Apply this Formalism to the Standard Model and its simplest Extensions
3. Develop a systematic Procedure for Probing New Physics with these Processes
4. Identify most interesting Problems and Questions

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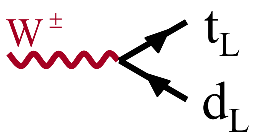
3. Develop a systematic Procedure for Probing New Physics with these Processes
4. Identify most interesting Problems and Questions
5. Make these Lectures enjoyable to Students as much as possible

Overture 2

Four Basic Properties in the SM

1. Charged Current Interactions only between left-handed Quarks



$$\frac{g_2}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)\cdot V_{td}$$

2. Quark Mixing

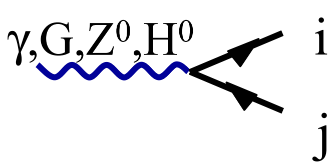
{ Weak Eigenstates } ≠ {Mass Eigenstates }

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{ub} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

(Weak Eigenstates) (Unitarity CKM-Matrix) (Mass Eigenstates)

3. GIM Mechanism

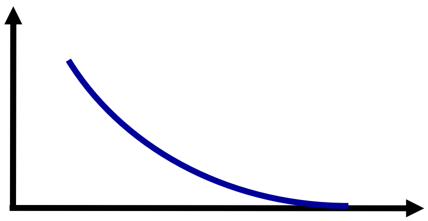
Natural suppression of FCNC

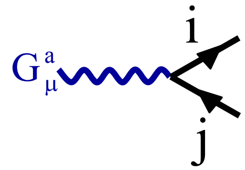
{ 

$= 0$ } ⇒ {Loop Induced Decays, sensitive to short distance flavour dynamics}

4. Asymptotic Freedom

α_{QCD}





$g_{QCD} T^a_{ij} \gamma_\mu$

LD $\Leftarrow$ RG $\Rightarrow$ SD

Q

$$\alpha_{QCD} = \frac{g^2_{QCD}}{4\pi}$$

$$\alpha_{QCD}(Q) = \frac{4\pi}{\beta_0 \ln\left(Q^2/\Lambda^2_{\overline{MS}}\right)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln\ln\left(Q^2/\Lambda^2_{\overline{MS}}\right)}{\ln\left(Q^2/\Lambda^2_{\overline{MS}}\right)} + \ldots\right]$$

$$\Lambda^{(5)}_{\overline{MS}} = 235 \pm 30\ \mathrm{MeV} \qquad \alpha^{(5)}_{\overline{MS}}(M_Z) = 0.1187 \pm 0.0020$$

SD = Short Distances (Perturbation Theory)

$\Uparrow$

RG = Renormalization Group Effects

$\Downarrow$

LD = Long Distances (Non-Perturbative Physics)

Kobayashi-Maskawa Picture of CP Violation

CP Violation arises from a single phase δ
in $W^{\pm}$ interactions of Quarks

ud $c_{12}c_{13}$	us $s_{12}c_{13}$	ub $s_{13}e^{-i\delta}$
cd $-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta}$	cs $c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta}$	cb $s_{23}c_{13}$
td $s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta}$	ts $-s_{23}c_{12}-s_{12}s_{23}s_{13}e^{i\delta}$	tb $c_{23}c_{13}$

Four Parameters: $(\theta_{12} \approx \theta_{cabibbo})$

$$s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}|, \quad \delta$$

$$c_{ij} \equiv \cos\theta_{ij} \;;\; s_{ij} \equiv \sin\theta_{ij} \;;\; c_{13} \cong c_{23} \cong 1$$

Wolfenstein Parametrization

Parameters: λ, A, ρ, η

	d	s	b
u	$1-\frac{\lambda^2}{2}$	λ	V_{ub}
c	$-\lambda$	$1-\frac{\lambda^2}{2}$	V_{cb}
t	V_{td}	V_{ts}	1

$\lambda=0.22$

$V_{us}=\lambda+0(\lambda^7)$

$V_{cb}=A\lambda^2+0(\lambda^8)$

$V_{ts}=-A\lambda^2+0(\lambda^4)$

$(A=0.83\pm0.02)$

$$V_{ub} \equiv A\,\lambda^3(\rho - i\eta) \qquad V_{td} = A\lambda^3(1-\bar{\rho}-i\bar{\eta})$$

$$\bar{\rho} = \rho\left(1-\frac{\lambda^2}{2}\right) \qquad \bar{\eta} = \eta\left(1-\frac{\lambda^2}{2}\right)$$

(AJB, Lautenbacher, Ostermaier, 94)

$$R_b \equiv \sqrt{\bar{\rho}^2+\bar{\eta}^2} = \left(1-\frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

Circle around $(\bar{\rho},\bar{\eta}) = (0,0)$

$$R_t \equiv \sqrt{(1-\bar{\rho})^2+\bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|$$

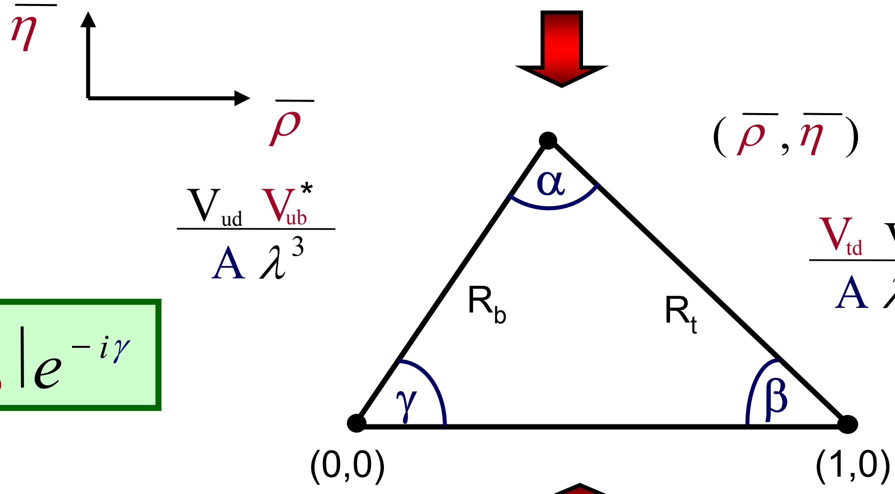
Circle around $(\bar{\rho},\bar{\eta}) = (1,0)$

Unitarity Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$\overline{\eta} \neq 0$ Signals CP Violation

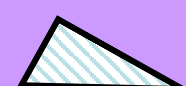
$$\frac{V_{ud}V_{ub}^*}{A\lambda^3} \qquad \frac{V_{td}V_{tb}^*}{A\lambda^3}$$



$$V_{ub} = |V_{ub}|e^{-i\gamma}$$

$$V_{td} = |V_{td}|e^{-i\beta}$$

An Important Target of Particle Physics

$$J_{CP} = \lambda^2 |V_{cb}|^2 \overline{\eta} = 2 \cdot$$ 

Area of unrescaled UT

Particular Definition of λ, A, ρ, η

$$s_{12} \equiv \lambda$$
$$s_{23} \equiv A\,\lambda^2$$
$$s_{13}e^{i\delta} \equiv A\,\lambda^3(\rho - i\eta)$$

BLO: Phys.Rev. (94); (Schmidtler, Schubert)

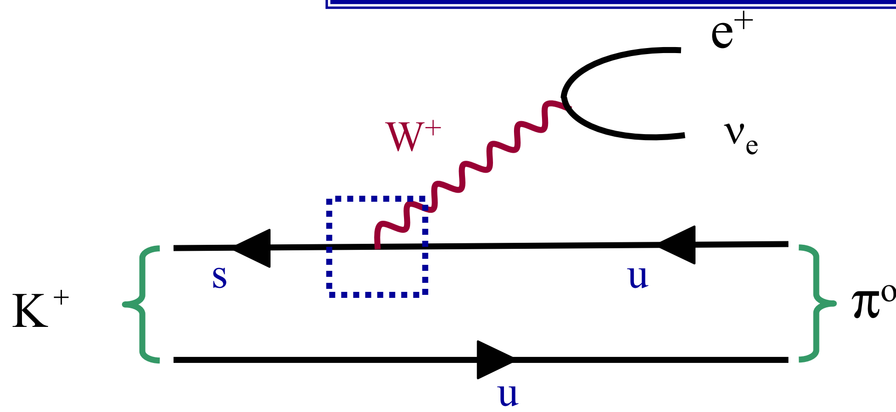
At $0(\lambda^5)$ equivalent to (Branco, Lavoura, 88)

Basic Virtues of this Definition:

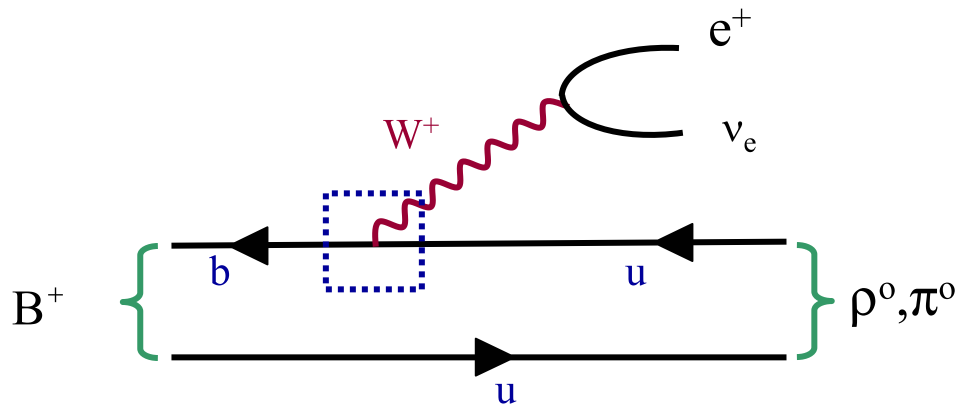
$$V_{us} = \lambda + 0\left(\lambda^7\right)$$
$$V_{ub} = A\lambda^3\left(\rho - i\eta\right)$$
$$V_{cb} = A\lambda^2 + 0\left(\lambda^8\right)$$
$$V_{td} = A\lambda^3\left(1 - \bar{\rho} - i\bar{\eta}\right)$$

The apex of UT given by $\left(\bar{\rho}, \bar{\eta}\right)$ (BLO)

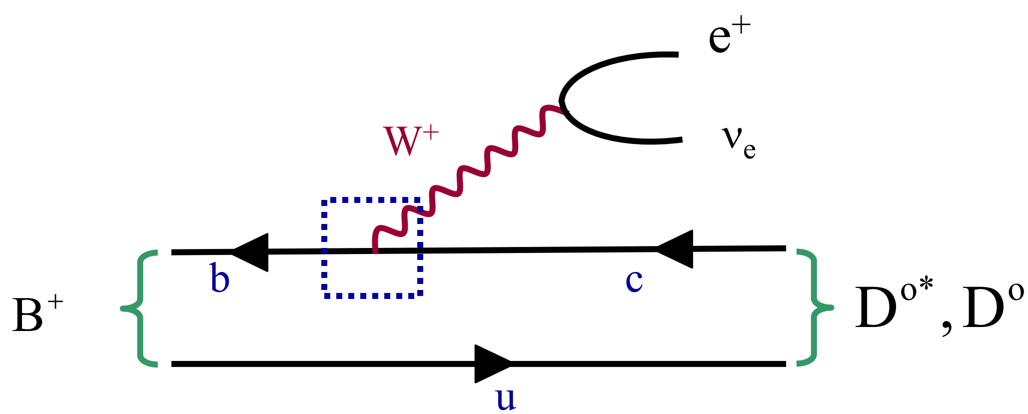
Tree Level Decays



V_{us}



V_{ub}



V_{cb}

Information from Tree Level Decays

$$|V_{us}| = 0.225 \pm 0.002 \qquad = \lambda$$

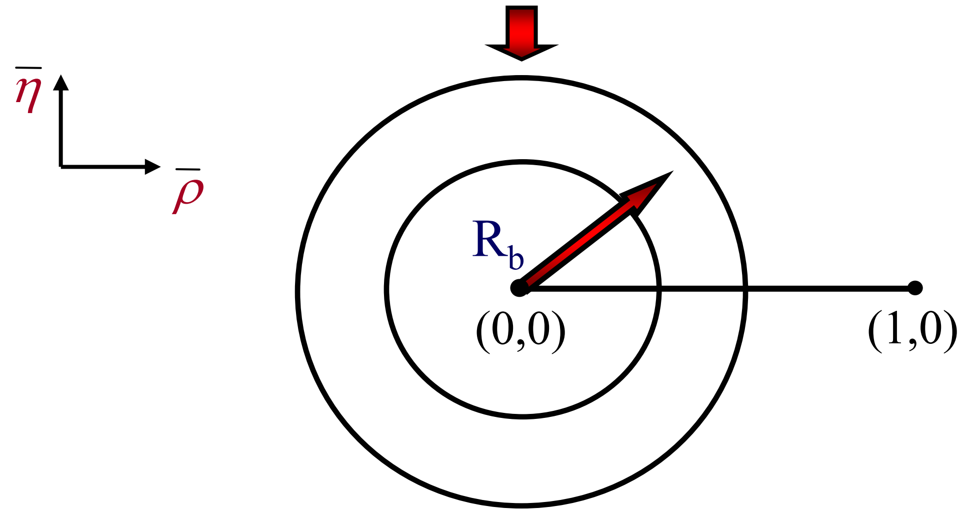
$$|V_{cb}| = (41.5 \pm 0.8)\cdot 10^{-3} \qquad (A = 0.83 \pm 0.02)$$

2004:

$$\left|\frac{V_{ub}}{V_{cb}}\right| = (0.092 \pm 0.012) \qquad (R_b = 0.40 \pm 0.06)$$

2005:

$$(0.102 \pm 0.005) \qquad (0.44 \pm 0.02)$$



Unitarity Clock

Apex of Unitarity Triangle somewhere on this Band

To find it **GO TO**

Loop Induced Decays

CP-Violation in K-Decays

CP-Violation in B-Decays

$$F_1^{th}\left(\lambda, A, \overline{\eta}, \overline{\rho}\right) = F_1^{exp}$$

$$F_2^{th}\left(\lambda, A, \overline{\eta}, \overline{\rho}\right) = F_2^{exp}$$

$$F_3^{th}\left(\lambda, A, \overline{\eta}, \overline{\rho}\right) = F_3^{exp}$$

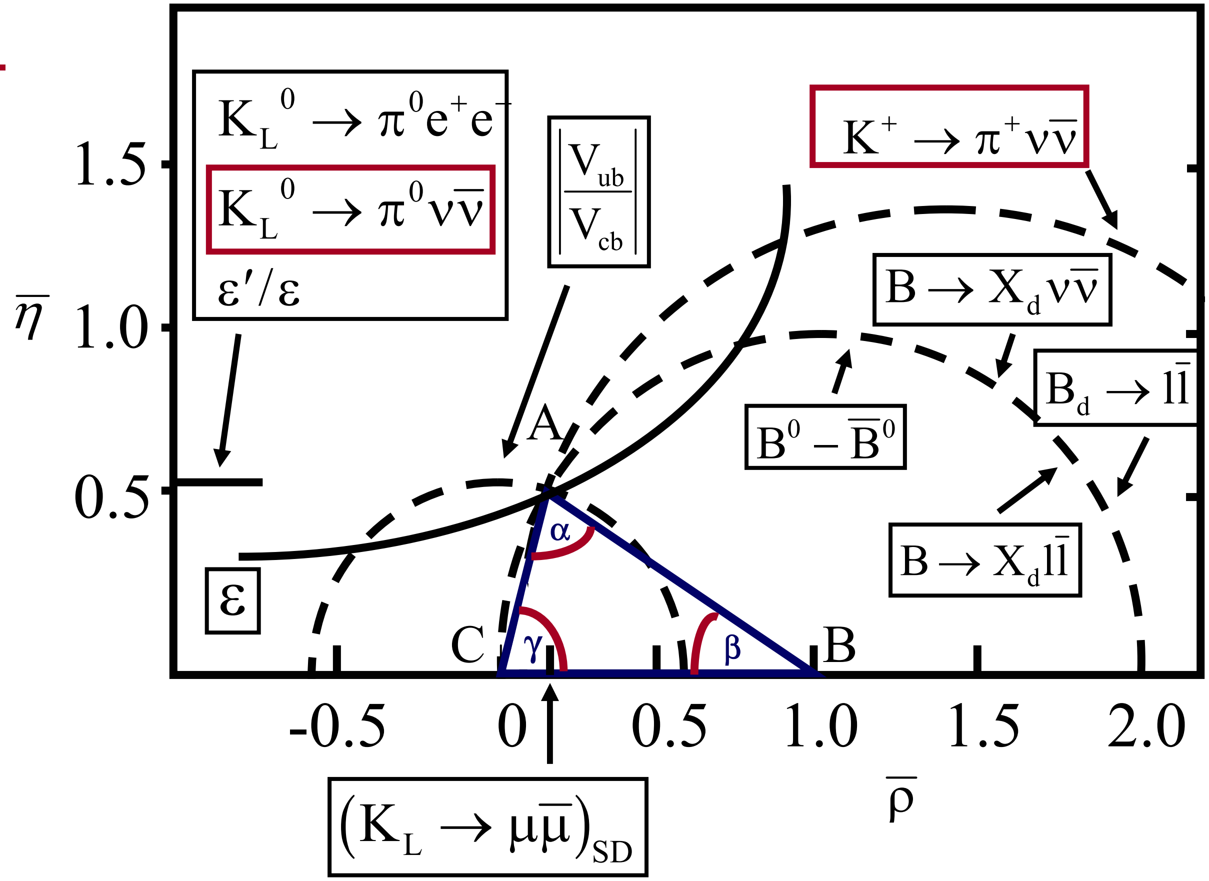
etc.

$$\Downarrow$$

Determination of
the Unitarity Triangle

Hunting Δ with Rare and ~~CP~~ Decays

2012:



★ **Quark Mixing and CP Violation closely related in the St. Model**

★ $\left\{\begin{array}{l}\text{CP Asymmetries}\\ \text{in}\\ \text{B} - \text{Decays}\end{array}\right\} \Rightarrow \left\{\begin{array}{c}\alpha\\ \beta\\ \gamma\end{array}\right\}$

Lecture I

1. TH Framework
2. Various Types of ~~CP~~

Lecture II

3. Standard Analysis of Δ
4. α, β, γ from B´s
5. $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$

Lecture III

6. Rare B- and K-Decays
7. Models with MFV

Lecture IV

8. Going Beyond MFV
9. Probing New Physics in 10 Steps
10. Outlook

Literature

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in Heavy Flavours II (World Scientific) (1998) (hep-ph / 9704376)

AJB

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Erice Lectures (2000) (hep-ph / 0101336)
★ Spain Lectures (2004) (hep-ph / 0505175)

Y. Nir

Scottish Universities Summer School (hep-ph / 0109090)

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B-Physics at the LHC (hep-ph / 0003238)

Books: Branco, Lavoura, Silva;
Bigi, Sanda

B Physics at the Tevatron (Run II and Beyond) (hep-ph/0201071)

Fleischer:

Physics Reports (hep-ph/0207108)

1.

Theoretical Framework

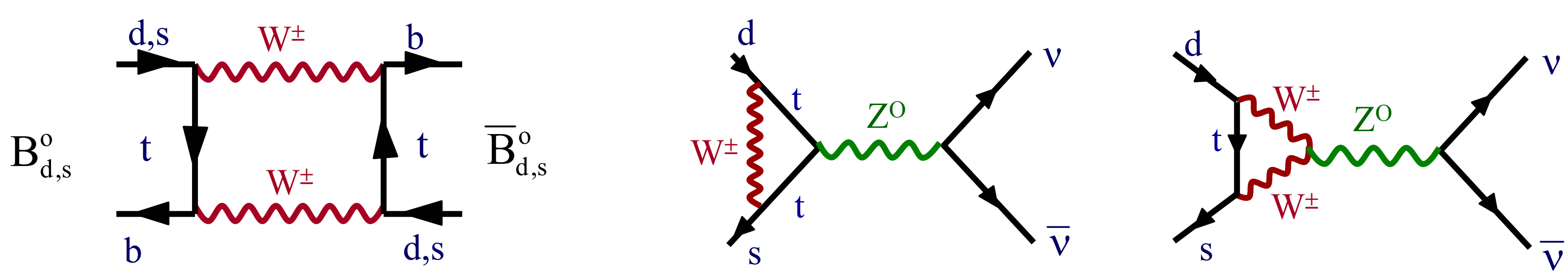
Starting Point :

$$\mathcal{L} = \mathcal{L}_{SM}\left(g_i, m_i, V^i_{CKM}\right) + \mathcal{L}_{NP}\left(g^{NP}_i, m^{NP}_i, V^i_{NP}\right)$$

Goal :

Identify the effects of $\mathcal{L}_{NP}$ in weak decays in the presence of the background from $\mathcal{L}_{SM}$

First Implication from $\mathcal{L}$: Feynman Diagrams



$B^o_d - \overline{B}^o_d$ Mixing

$K^+ \rightarrow \pi^+ \nu\overline{\nu}, \ K_L \rightarrow \pi^0 \nu\overline{\nu}$

+ NP

Two challenges :

1. Theory formulated in terms of quarks but experiments involve their bound states (K, B, D)

2. NP takes place at very short distance scales (10^{-19}-10^{-18}m), while K, B, D live at 10^{-16}-10^{-15}m.

Two challenges :

1. Theory formulated in terms of quarks but experiments involve their bound states (K, B, D)

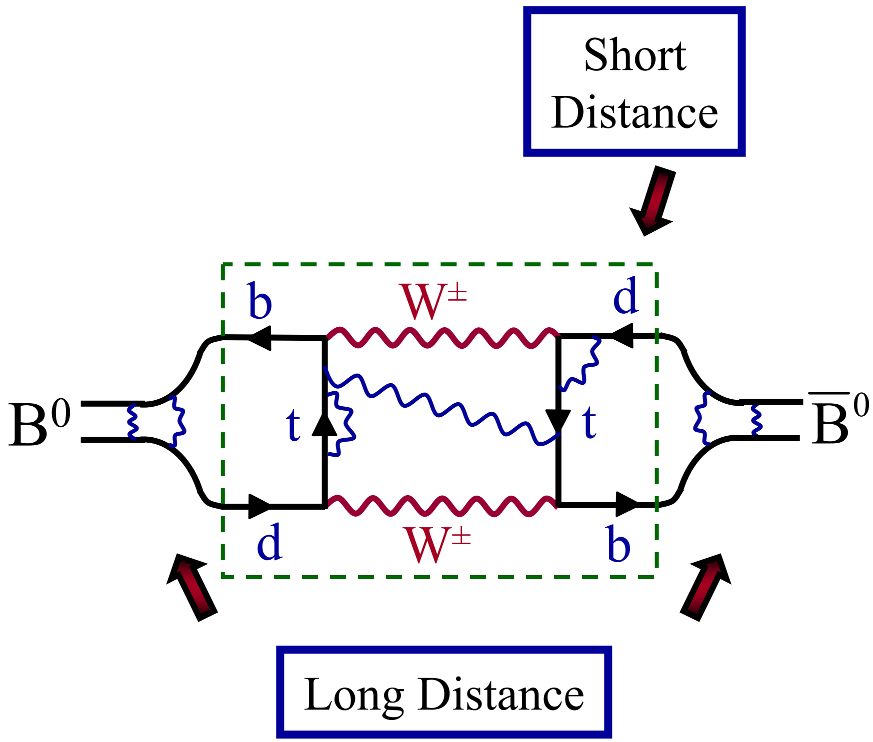
2. NP takes place at very short distance scales (10^{-19}-10^{-18}m), while K, B, D live at 10^{-16}-10^{-15}m.

Solution : Effective Theories, OPE, Renormalization Group

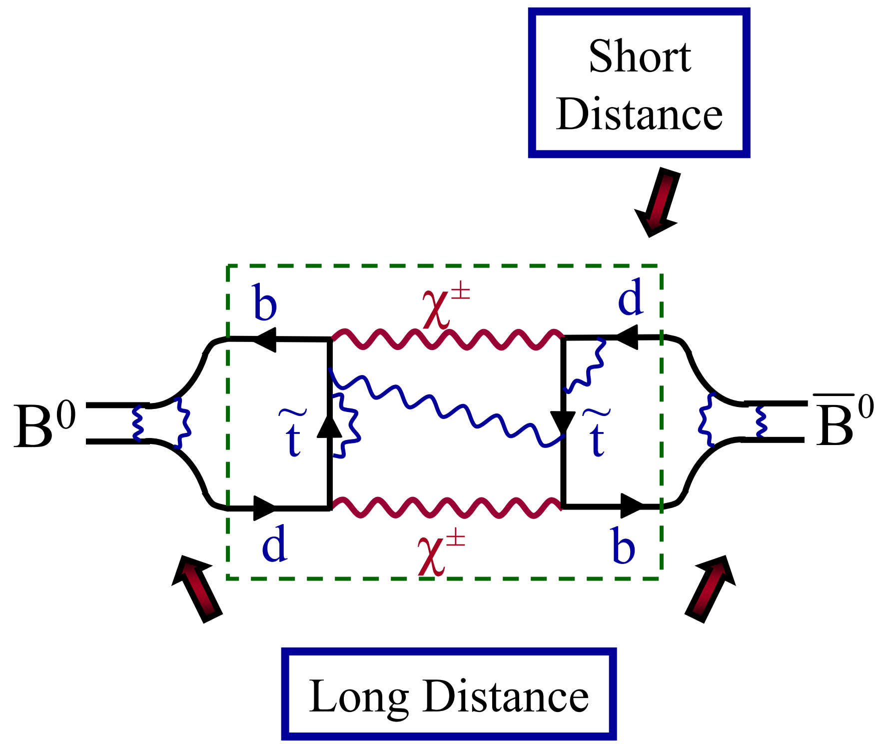
→ Separation of SD from LD
+ Summation of large $\log(\mu_{SD} / \mu_{LD})$

The Problem of Strong Interactions

$B_d^0 - \overline{B}_d^0$ Mixing (SM)



$B_d^0 - \overline{B}_d^0$ Mixing (MSSM)



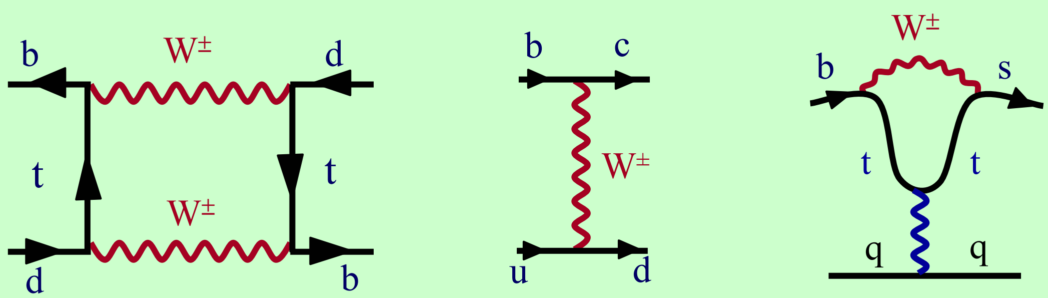
SD : Perturbative
(Asymptotic Freedom)

LD : Non-Perturbative
(Confinement)

Effective Field Theory

Full Theory

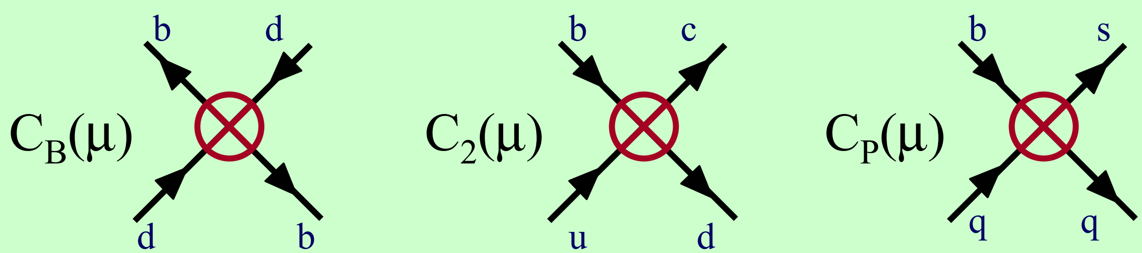
($W^{\pm}$, Z^0, G, γ, t, H^0, b, u, d, s, c, l)



$\mu \geq M_W$

Effective Theory

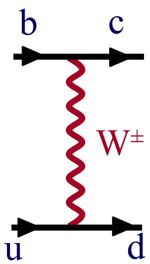
(G, γ, b, u, d, s, c, l)



$\mu \cong 0(m_b)$

"Generalized Fermi Theory" with calculable "couplings" $C_B(\mu)$, $C_2(\mu)$,...

Simplest Example of Operator Product Expansion



$$= \left[\bar{c}\gamma_\mu(1-\gamma_5)b\right]\left[V_{cb}\, i\frac{g_2}{2\sqrt{2}}\right]\left[\frac{-i}{k^2-M_W^2}\right]\left[\bar{d}\gamma^\mu(1-\gamma_5)u\right]\left[V_{ud}^*\, i\frac{g_2}{2\sqrt{2}}\right]$$

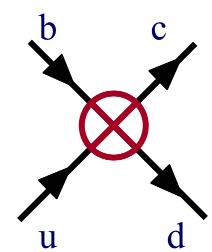
$$= \frac{g_2^2}{8}\frac{i}{k^2-M_W^2}V_{cb}V_{ud}^*\left(\bar{c}b\right)_{V-A}\left(\bar{d}u\right)_{V-A}$$

$$\frac{i}{k^2-M_W^2} = -\frac{i}{M_W^2} + 0\left(\frac{k^2}{M_W^2}\right)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}$$

To get H_{eff} multiply by i

$$H_{eff} = \underbrace{\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*}_{\text{Wilson Coefficient}}\cdot 1\cdot \underbrace{\left(\bar{c}b\right)_{V-A}\left(\bar{d}u\right)_{V-A}}_{\text{Local Operator}}$$



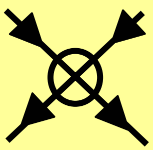
Important: **QCD corrections generate new operators and make WC μ-dependent : $C_i(\mu)$**

Operator Product Expansion

$$\begin{Bmatrix}\text{Wilson}\\ \text{Coefficients}\end{Bmatrix} \quad \begin{Bmatrix}\text{Local}\\ \text{Operators}\end{Bmatrix}$$

$$H_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu)\, Q_i$$

Q_i ⟺



Four Quark Interaction Vertex $(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$

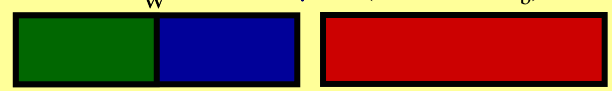
$C_i(\mu)$ ⟺ **Coupling Constants** $C(\mu) = \left[\dfrac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{6}{23}}$

$\{K, B, D, \ldots\}$

$$A(M \to F) = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) \langle F | Q_i(\mu) | M \rangle$$

$$\begin{Bmatrix}\pi\pi,\ \pi\nu\bar{\nu}\\ \mu\bar{\mu},\ K^*\gamma, \ldots\end{Bmatrix}$$

M_W $\quad$ $\mu = 0(1\ \mathrm{GeV}, m_b)$ $\quad$ 0



Short $\quad$ RG $\quad$ Long Distance

$$\begin{Bmatrix}\text{Top}\\ \text{SUSY}\\ H^{\pm}\ldots\end{Bmatrix} \quad \begin{Bmatrix}\text{Renormalization Group}\\ \sum\left(\alpha_s \log \dfrac{M_w}{\mu}\right)^n\end{Bmatrix} \quad \begin{Bmatrix}\text{Lattice, } 1/N\\ \text{HQET, QCDS}\\ \text{ChPTh}\end{Bmatrix}$$

$$\langle \overline{K}^0 | (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} | K^0 \rangle = \frac{8}{3} \hat{B}_K F_K^2 m_K^2 [\alpha_s(\mu)]^{2/9}$$

Operators

Current-Current

$$Q_1 = \left(\bar{s}_\alpha u_\beta\right)_{V-A}\left(\bar{u}_\beta d_\alpha\right)_{V-A} \qquad Q_2 = \left(\bar{s}u\right)_{V-A}\left(\bar{u}d\right)_{V-A}$$

QCD-Penguins

$$Q_3 = \left(\bar{s}d\right)_{V-A}\sum_{q=u,d,s}\left(\bar{q}q\right)_{V-A} \qquad Q_4 = \left(\bar{s}_\alpha d_\beta\right)_{V-A}\sum_{q=u,d,s}\left(\bar{q}_\beta q_\alpha\right)_{V-A}$$

$$Q_5 = \left(\bar{s}d\right)_{V-A}\sum_{q=u,d,s}\left(\bar{q}q\right)_{V+A} \qquad \boxed{Q_6 = \left(\bar{s}_\alpha d_\beta\right)_{V-A}\sum_{q=u,d,s}\left(\bar{q}_\beta q_\alpha\right)_{V+A}}$$

Electroweak-Penguins

$$Q_7 = \frac{3}{2}\left(\bar{s}d\right)_{V-A}\sum_{q=u,d,s}e_q\left(\bar{q}q\right)_{V+A} \qquad \boxed{Q_8 = \frac{3}{2}\left(\bar{s}_\alpha d_\beta\right)_{V-A}\sum_{q=u,d,s}e_q\left(\bar{q}_\beta q_\alpha\right)_{V+A}}$$

$$Q_9 = \frac{3}{2}\left(\bar{s}d\right)_{V-A}\sum_{q=u,d,s}e_q\left(\bar{q}q\right)_{V-A} \qquad Q_{10} = \frac{3}{2}\left(\bar{s}_\alpha d_\beta\right)_{V-A}\sum_{q=u,d,s}e_q\left(\bar{q}_\beta q_\alpha\right)_{V-A}$$

Basic Structure of Decay Amplitudes

$A =$ Long Distance Contribution · QCD Renormalization Group Factor η^{QCD} · Short Distance Contributions $0(M_W, m_t)$

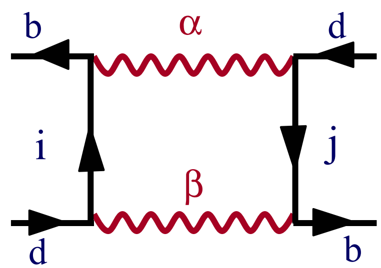
Long Distance Contribution	QCD Renormalization Group Factor η^{QCD}	Short Distance Contributions $0(M_W,m_t)$
Hadronic Matrix Element	QCD – Effects $0(m_b) \leq \mu \leq 0(M_W)$ $0(m_c)$	Tree Diagrams Penguin Diagrams Box Diagrams
(Non-Perturbative)	(RG improved Perturbation Theory)	(Perturbation Theory)

Deriving $H_{\rm eff}\left(B_d^0 - \overline{B}_d^0\right)$ and $\Delta M_d\left(B_d^0 - \overline{B}_d^0\right)$ in 7 Steps

Step 1 : Calculate Box Diagrams

$\Phi^\pm$ - Goldstone Bosons Must be taken into account except in a Unitarity Gauge

$$\sum_{i,j=u,c,t} \quad \sum_{\alpha,\beta=W^\pm,\Phi^\pm}$$



$$\equiv \sum_{i,j=u,c,t} F\left(x_i, x_j\right)\left(V_{ib}^* V_{id}\right)\left(V_{jb}^* V_{jd}\right)Q$$

UG : see AJB, Poschenrieder, Uhlig; hep-ph/0410309

Step 2 : Use

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Multiply by i and keep only $V_{td}V_{tb}^*$ part

$$x_i \equiv \frac{m_i^2}{M_W^2}$$

$$H_{\rm eff}^{(\Delta B=2)} = \frac{G_F^2}{16\pi^2} M_w^2 \left(V_{tb}^* V_{td}\right)^2 \quad S_0(x_t) \quad Q(\Delta B = 2)$$

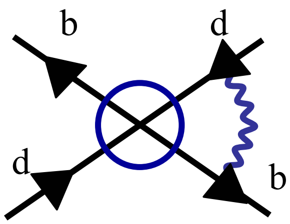
$$S_0(x_t) = \tilde{F}(x_t, x_t) + \tilde{F}(x_u, x_u) - 2\tilde{F}(x_t, x_u)$$

$$Q(\Delta B = 2) = \left(\bar{b}d\right)_{V-A}\left(\bar{b}d\right)_{V-A}$$

$$x_u = 0$$

Step 3 : Include QCD Corrections in the Leading Logarithmic Approximation

 = Gluon



$$\left\{ \quad + \ldots \right\} \Rightarrow \left\{ S_0(x_t) \rightarrow \left[\frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)}\right]^{6/23} S_0(x_t) \equiv C(\mu_b) \right\}$$

Wilson Coefficient of $Q(\Delta B=2)$

$$\mu_W = 0(M_W) \qquad \mu_b = 0(m_b)$$

Problems with LO ≡ LLA : Sensitivity to the choices of

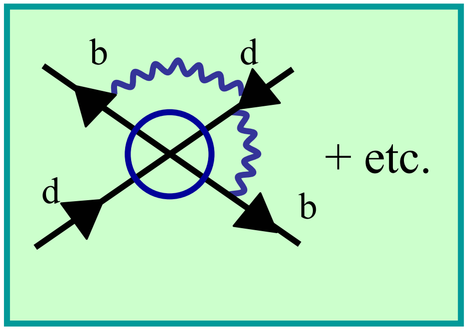
i)	μ_W	$80\ \text{GeV} < \mu_W < 300\ \text{GeV}$
ii)	μ_b	$2.5\ \text{GeV} < \mu_b < 5\ \text{GeV}$
iii)	μ_t	$x_t = \dfrac{m_t^2(\mu_t)}{M_W^2}$
		$80\ \text{GeV} < \mu_t < 300\ \text{GeV}$

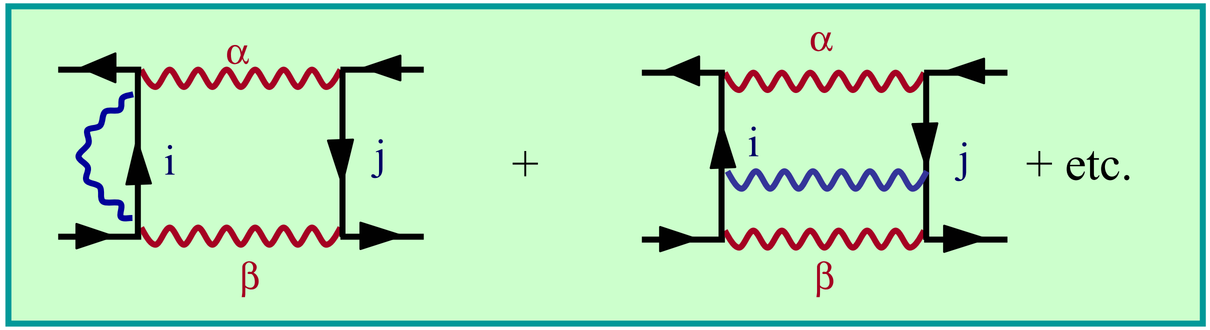
Step 4 : Include Next to Leading QCD Corrections

(AJB, Jamin, Weisz 1990)

Requires:


= Gluon





Pages 101-103 Les Houches Lectures

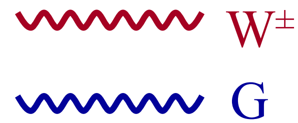
$$H_{\text{eff}}^{(\Delta B=2)} = \frac{G_F^2}{16\pi^2} M_W^2 \left(V_{tb}^* V_{td}\right)^2 \underbrace{S_0(x_t)\ \tilde{\eta}_B^{QCD} \left[\frac{\alpha_S(\mu_W)}{\alpha_S(\mu_b)}\right]^{6/23} \left(1 + J_5 \frac{\alpha_S(\mu_b) - \alpha_s(\mu_W)}{4\pi}\right)}_{\text{Independent of } \mu_W \text{ and } \mu_t \text{ but still dependent on } \mu_b} Q(\Delta B = 2)$$

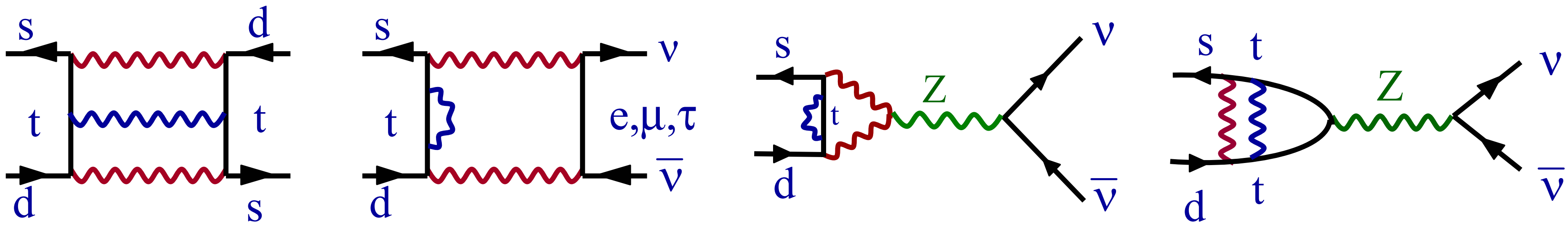
Independent of μ_W and μ_t but still dependent on μ_b

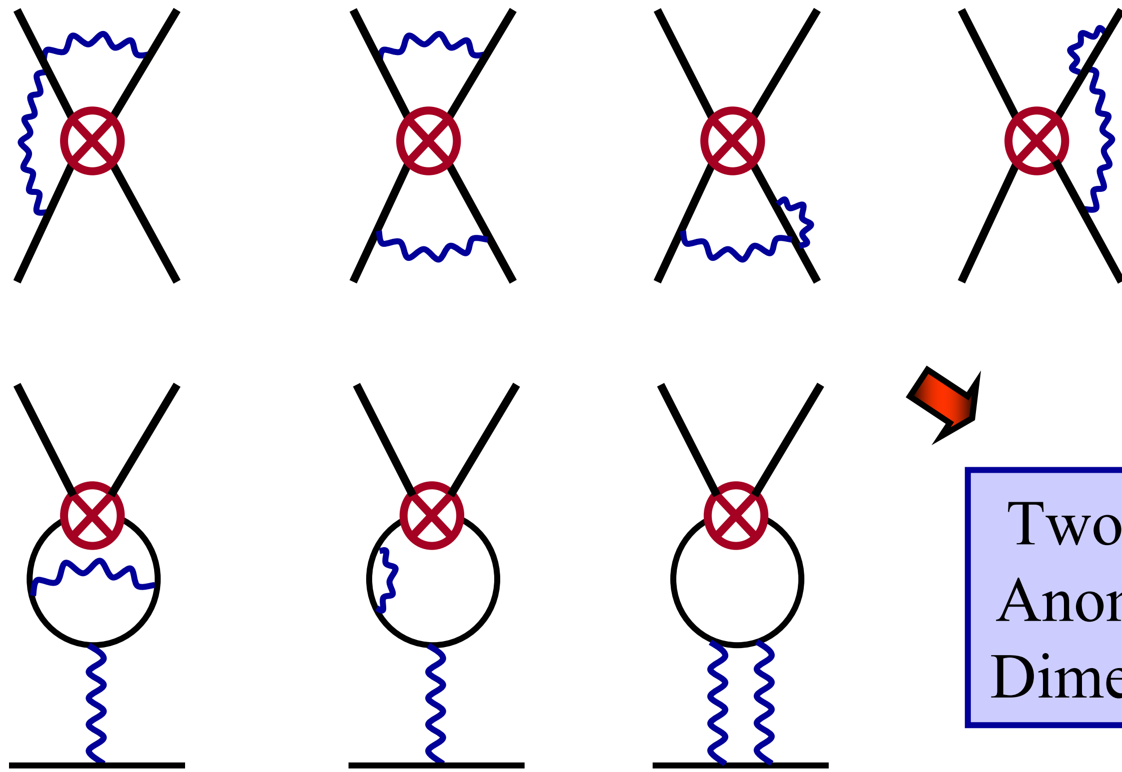
$$\tilde{\eta}_B^{QCD} = 1 + \frac{\alpha_s(\mu_W)}{4\pi} G(\mu_W, \mu_t)$$

$$J_5 = 1.627$$

Typical Two-Loop Diagrams







Two-Loop Anomalous Dimensions

Step 5 : Calculate the Matrix Element $\langle Q(\Delta B = 2)\rangle$

$$\left\langle \bar{B}_d^0 \middle| Q(\Delta B = 2) \middle| B_d^0 \right\rangle = \frac{8}{3} B_B(\mu_b) F_B^2 m_B^2$$

F_B = B-Meson Decay Constant

This μ_b – dependence cancels the one in $H_{\text{eff}}^{\Delta B=2}$

Step 6 : Put $\langle H_{\text{eff}}^{\Delta B=2}\rangle$ in a manifestly μ_w, μ_t, μ_b independent Form

$$\eta_B^{QCD} \equiv \tilde{\eta}_B^{QCD} \left[\alpha_s(\mu_w)\right]^{6/23} \left(1 - J_5 \frac{\alpha_s(\mu_w)}{4\pi}\right)$$

with $\mu_t = m_t$

μ_w - independent

$$\hat{B}_B \equiv B_B(\mu_b)\left[\alpha_s(\mu_b)\right]^{-6/23} \left(1 + J_5 \frac{\alpha_s(\mu_b)}{4\pi}\right)$$

μ_b – independent

$S_0(x_t)$ evaluated at $\mu_t = m_t$

Step 7 : Calculation of $\Delta M_d\left(B_d^0 - \overline{B}_d^0\right)$

Use $$\Delta M_d = \frac{1}{m_b}\left|\left\langle \overline{B}_d^0 \middle| H_{eff}^{(\Delta B=2)} \middle| B_d^0 \right\rangle\right|$$

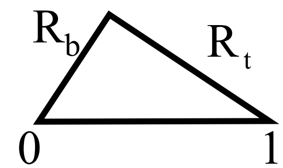
$$\Rightarrow \quad \Delta M_d = \frac{G_F^2}{6\pi^2} m_b M_w^2 \underbrace{\left(\hat{B}_d F_{B_d}^2\right)}_{\text{independent of } \mu_b} \underbrace{\eta_B^{QCD}\; S_0(x_t)}_{\text{independent of } \mu_w,\, \mu_t} \left|V_{td}\right|^2$$

$$\sqrt{\hat{B}_d}\, F_{B_d} = \left(235^{+33}_{-41}\right) \text{MeV}$$

$$\eta_B^{QCD} = 0.551 \pm 0.006$$

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t \log x_t}{2(1-x_t)^3}$$
$$\approx 2.46\left(\frac{m_t}{170\text{GeV}}\right)^{1.52}$$

$(\Delta M)_{d,s}$, $|V_{td}|/|V_{ts}|$ and R_t



$$(\Delta M)_d = \frac{0.50}{\text{ps}}\left[\frac{\sqrt{\hat{B}_d}F_{Bd}}{230\text{MeV}}\right]^2\left[\frac{|V_{td}|}{7.8\cdot 10^{-3}}\right]^2\left[\frac{\eta_B}{0.55}\right]\left[\frac{S(x_t)}{2.34}\right]$$

$$(\Delta M)_s = \frac{18.4}{\text{ps}}\left[\frac{\sqrt{\hat{B}_s}F_{Bs}}{270\text{MeV}}\right]^2\left[\frac{|V_{ts}|}{0.040}\right]^2\left[\frac{\eta_B}{0.55}\right]\left[\frac{S(x_t)}{2.34}\right]$$

$$S(x_t) = 2.42 \pm 0.12$$

$$\eta_B = 0.55 \pm 0.01$$

AJB, Jamin, Weisz

$$|V_{td}| = \lambda |V_{cb}| R_t$$

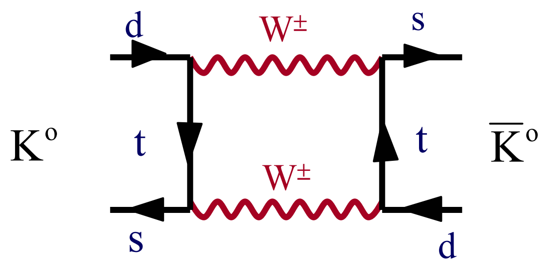
$$|V_{ts}| = |V_{cb}|\left(1 - \frac{\lambda^2}{2} + \bar{\rho}\lambda^2\right)$$

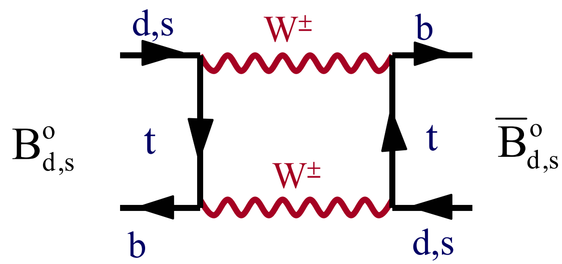
$$\xi = \frac{\sqrt{\hat{B}_s}F_{B_s}}{\sqrt{\hat{B}_d}F_{B_d}} = 1.22 \pm 0.07$$

$$\frac{|V_{td}|}{|V_{ts}|} = 1.01\,\xi\sqrt{\frac{\Delta M_d}{\Delta M_s}}$$

$$R_t = 0.90\sqrt{\frac{\Delta M_d}{0.50/\text{ps}}}\sqrt{\frac{18.4/\text{ps}}{\Delta M_s}}\left[\frac{\xi}{1.22}\right]$$

View at Short Distance Scales

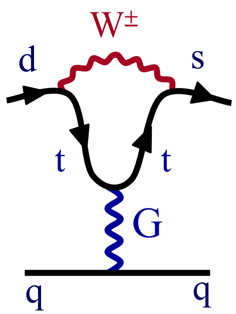


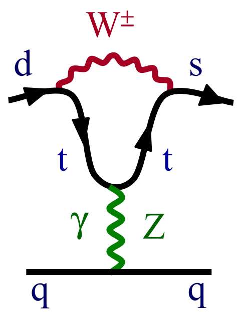


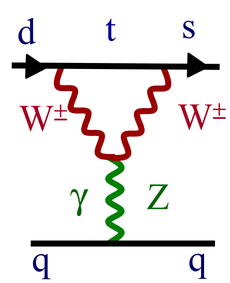
~~C~~P ε_K-Parameter
$\Delta M\ (K_L - K_S)$

$B^o_d - \overline{B}^o_d$ Mixing

ε'

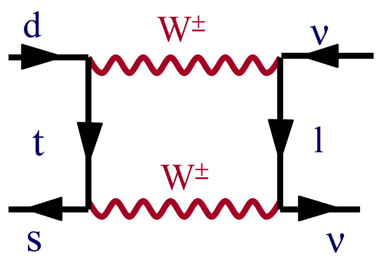


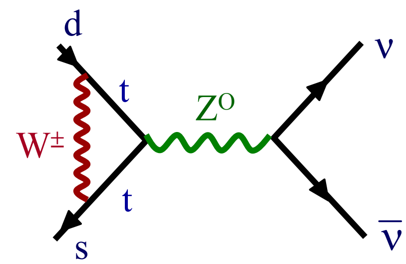


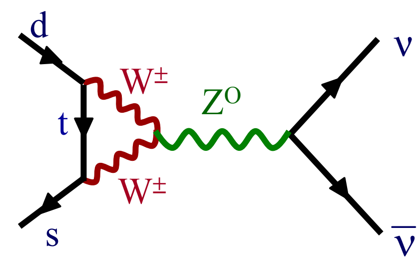


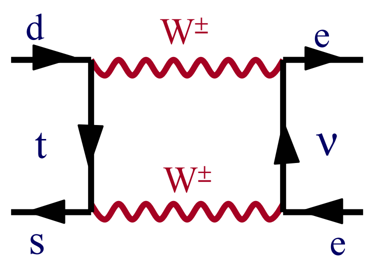
View at Short Distance Scales

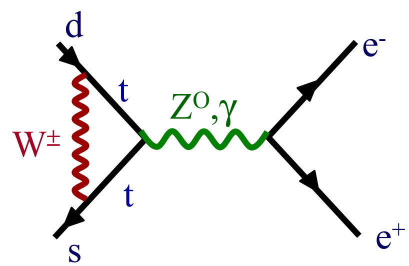
$K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$ $K_L \to \mu\bar{\mu}$, $B \to \mu\bar{\mu}$, $B \to X_S \nu\bar{\nu}$

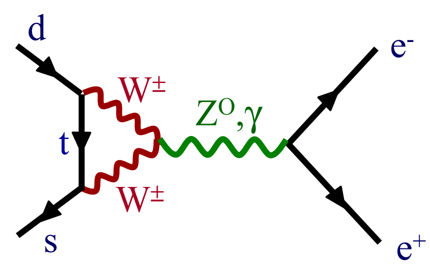




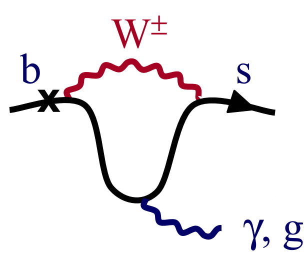








$K_L \to \pi^0 e^+ e^-$ $B \to X_s e^+ e^-$, $X_s \mu\bar{\mu}$



$B \to X_s \gamma$ $B \to K^* \gamma$

$B \to X_d \gamma$ $b \to s$ gluon

Penguin-Box Expansion (SM)

Buchalla, AJB, Harlander (90)

The m_t dependence of all K and B Decays resides in 7 Basic Universal Functions $F_i(x_t)$

$$x_t = \frac{m_t^2}{M_W^2}$$

$$A(\mathrm{Decay}) = \sum_i B_i \eta^i_{QCD} V^i_{CKM} F_i(x_t)$$

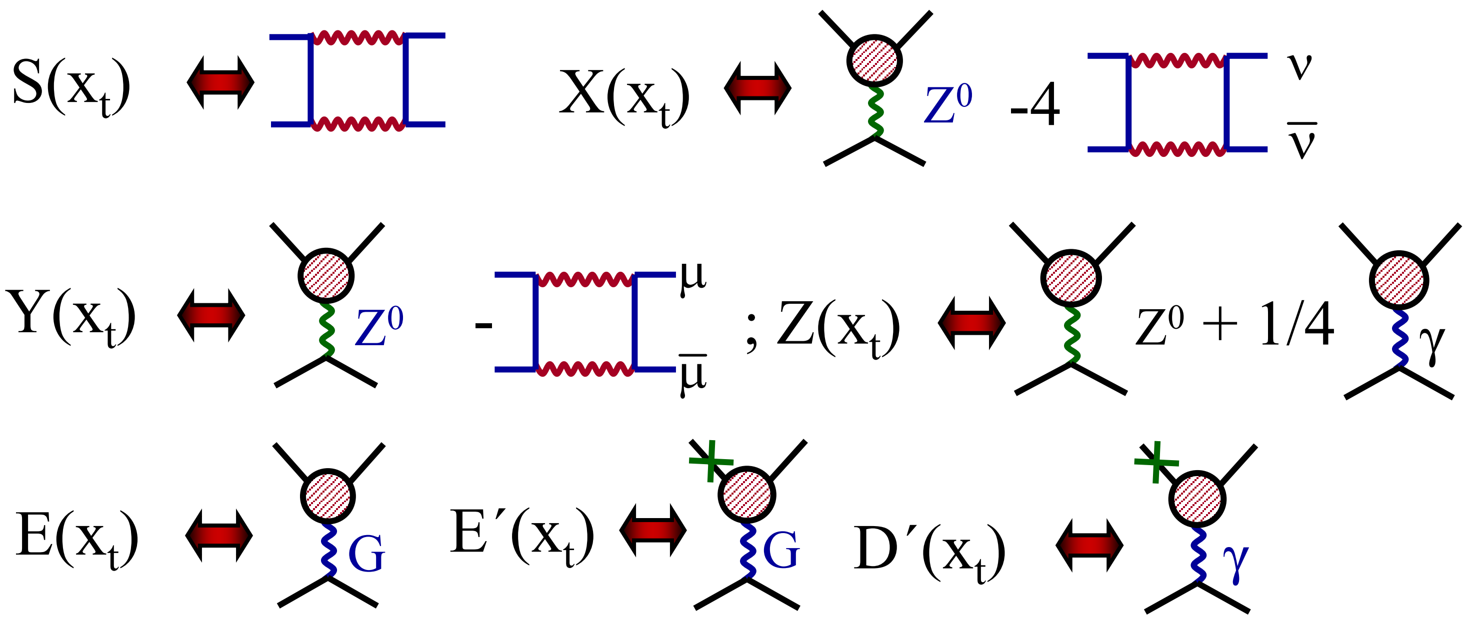
F_i : S, X, Y, Z, E, E', D'

(Gauge Invariant set of functions)

Relation to OPE:

$$C_i(\mu) = \sum_j \overset{\text{RG}}{U_{ij}(\mu, M_W)} \underbrace{\left[\sum_r H_{jr} F_r(x_t)\right]}_{C_j(M_W)} = \sum_r \eta^{QCD}_{ir} F_r(x_t)$$

Decay	Contributing Functions
$B_d^0-\overline{B}_d^0,\ B_s^0-\overline{B}_s^0,\ \varepsilon$	$S(x_t)$
$K\to\pi\nu\overline{\nu},\ B\to X_s\nu\overline{\nu}$	$X(x_t)$
$K_L\to\mu\overline{\mu},\ B\to l\overline{l}$	$Y(x_t)$
ε'	$Z(x_t),\ X(x_t),\ Y(x_t),\ E(x_t)$
$K_L\to\pi^0e^+e^-$	$Y(x_t),\ Z(x_t),\ E(x_t)$
$B\to X_se^+e^-$	$Y(x_t),\ Z(x_t),\ D'(x_t), E'(x_t), E(x_t)$
$B\to X_s\gamma$	$D'(x_t),\ E'(x_t)$



m_t Dependence of Basic Universal Functions

$$S(x_t) \equiv S_0(x_t) = 2.46\left[\frac{m_t}{170\text{GeV}}\right]^{1.52}$$

$$X(x_t) = 1.57\left[\frac{m_t}{170\text{GeV}}\right]^{1.15} \qquad Y(x_t) = 1.02\left[\frac{m_t}{170\text{GeV}}\right]^{1.56}$$

$$Z(x_t) = 0.71\left[\frac{m_t}{170\text{GeV}}\right]^{1.86} \qquad E(x_t) = 0.26\left[\frac{m_t}{170\text{GeV}}\right]^{-1.02}$$

$$D'(x_t) = 0.38\left[\frac{m_t}{170\text{GeV}}\right]^{0.60} \qquad E'(X_t) = 0.19\left[\frac{m_t}{170\text{GeV}}\right]^{0.38}$$

Master Formula for Weak Decays

AJB (2001)
hep-ph/0101336
hep-ph/0109197

Non-Perturbative Factors in the SM

QCD RG Factors

Short Distance Loop Functions (Penguins, Boxes)

New Flavour-Changing Parameters

Represent different Dirac and Colour Stuctures

$$A(\mathrm{Decay}) = B_i \ \eta^i_{QCD} \ V^i_{CKM} \left[F^i_{SM} + F^i_{New}\right] + B_i^{New} \left[\eta^i_{QCD}\right]^{New} V^i_{New} \left[G^i_{New}\right]$$

(Summation over i)

$\mathrm{New} \equiv \mathrm{NP}$

Non-Perturbative Factors beyond SM

Short Distance Loop Functions Penguins, Boxes

F^i_{SM}, F^i_{New}, G^i_{New} : **Fully calculable in Perturbation Theory**

η^i_{QCD}, $\left[\eta^i_{QCD}\right]^{New}$: **Fully calculable in RG improved Perturbation Theory**

B_i, B_i^{New} : **Require Non-Perturbative Methods or can be extracted from leading decays**

(represent $\langle Q_i \rangle$)

$\left.\right\}$ **Fully calculable in the SM**

Possible Dirac Structures in $K^0 - \bar{K}^0$ and $B^0_{d,s} - \bar{B}^0_{d,s}$

SM:

$$\gamma_\mu(1-\gamma_5) \otimes \gamma^\mu(1-\gamma_5)$$

Beyond SM:

$$\gamma_\mu(1-\gamma_5) \otimes \gamma^\mu(1+\gamma_5)$$

$$(1-\gamma_5) \otimes (1+\gamma_5)$$

$$(1-\gamma_5) \otimes (1-\gamma_5)$$

$$\sigma_{\mu\nu}(1-\gamma_5) \otimes \sigma^{\mu\nu}(1-\gamma_5)$$

MSSM with large tanβ
General Supersymmetric Models
Models with complicated Higgs System

$\text{NLO}\left[\eta^i_{QCD}\right]^{New}$: Ciuchini, Franco, Lubicz, Martinelli, Scimemi, Silvestrini

AJB, Misiak, Urban, Jäger

Three Simple Scenarios

Inami Lim Functions

SM : $$A\left(\text{Decay}\right) = \sum_i B_i\, \eta^i_{QCD}\, V^i_{CKM} \underbrace{F^i_{SM}}_{\text{real}} \left(m_t\right)$$

MFV : $$A\left(\text{Decay}\right) = \sum_i B_i\, \eta^i_{QCD}\, V^i_{CKM} \underbrace{\left[F^i_{SM} + F^i_{New}\right]}_{\text{real}}$$

(Minimal Flavour Violation)

AJB, Gambino, Gorbahn, Jäger, Silvestrini
D'Ambrosio, Giudice, Isidori, Strumia

Enhanced Z^0-Penguins : $$A\left(\text{Decay}\right) = \sum_i B_i\, \eta^i_{QCD}\, V^i_{CKM} \left[\underbrace{F^i_{SM}}_{\text{real}} + \underbrace{\Delta^i_{New}}\right]$$

Dominated by Z^0-Penguins with a New Complex Phase

AJB, Colangelo, Isidori, Romanino, Silvestrini
Buchalla, Hiller, Isidori; Atwood, Hiller
AJB, Fleischer, Recksiegel, Schwab

Two more complicated Scenarios

MSSM (MFV)
(large tanβ)

(Higgs penguin)

$$A\left(\text{Decay}\right) = \sum_i B_i\, \eta^i_{QCD}\, V^i_{CKM}\left[F^i_{SM} + \overbrace{F^i_{New}}^{\text{real}}\right] + \sum_i B^{New}_i \left[\eta^i_{QCD}\right]^{New} V^i_{CKM} \underbrace{\left[G^i_{New}\right]}_{\text{real}}$$

General
MSSM

Z'-Models
L-R Models
Multi-Higgs
Models

$$A\left(\text{Decay}\right) = \sum_i B_i\, \eta^i_{QCD}\, V^i_{CKM}\left[F^i_{SM} + \overbrace{F^i_{New}}^{\text{complex}}\right] + \sum_i B^{New}_i \left[\eta^i_{QCD}\right]^{New} V^i_{New} \underbrace{\left[G^i_{New}\right]}_{\text{complex}}$$

Inclusive Decays

(Generally TH cleaner than Exclusive Decays)

Examples: $B \to X_s\gamma,\ B \to X_s\mu^+\mu^-,\ B \to X_s\nu\bar{\nu}$

$X_s \equiv$ all final states with $\Delta S = 1$ quantum number

1. Construction of H_{eff} as in the case of Exclusive Decays
2. The branching ratios calculated perturbatively from b-quark decay diagrams
3. Non-Perturbative Effects $\sim \Lambda^2_{QCD} / m_b^2$ (~ 5%)

Heavy Quark Expansions:

Chay, Georgi, Grinstein (1990)
Bigi, Shifman, Uraltsev, Vainshstein (1992)
Manohar, Wise (1993); Mannel (1993)

2.

Particle Mixing and Various Types of CP Violation

Express Review of $B^0 - \overline{B}^0$ Mixing

Flavour Eigenstates

$$B^0 = (\bar{b}d)$$

$$\overline{B}^0 = (b\bar{d})$$

$$CP|B^0\rangle = -|\overline{B}^0\rangle$$
$$CP|\overline{B}^0\rangle = -|B^0\rangle$$

In the absence of $B^0 - \overline{B}^0$ Mixing:

$$|B^0(t)\rangle = |B^0(0)\rangle \exp[-i\,Ht]$$
$$|\overline{B}^0(t)\rangle = |\overline{B}^0(0)\rangle \exp[-i\,Ht]$$

$$H = \underbrace{M}_{\text{Mass}} - i\underbrace{\frac{\Gamma}{2}}_{\text{Width}}$$

Time Evolution in the Presence of Mixing

$$i\frac{d\psi(t)}{dt} = \hat{H}\,\psi(t) \qquad \psi(t) = \begin{pmatrix} |B^0(t)\rangle \\ |\overline{B}^0(t)\rangle \end{pmatrix}$$

Hermitian Matrices with positive (real) eigenvalues

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{21} - i\frac{\Gamma_{21}}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix}$$

M_{ij}-transition with virtual intermediate states

Γ_{ij}- transition with physical intermediate states

$$M_{21} = M_{12}^* \qquad \Gamma_{21} = \Gamma_{12}^*$$

Express Review of B^0-$\overline{B}^0$ Mixing

Flavour Eigenstates

$$B_d^0 = \left(\overline{b}d\right)$$
$$\overline{B}_d^0 = \left(b\overline{d}\right)$$
$$B_s^0 = \left(\overline{b}s\right)$$
$$\overline{B}_s^0 = \left(b\overline{s}\right)$$

see:
Erice (2000)
Spain (2004)
$\left(K^0 - \overline{K}^0\right)$

Mass Eigenstates

$$B_{H,L} = p\,B^0 \pm q\,\overline{B}^0$$
$$\frac{q}{p} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta M - i\,\dfrac{\Delta\Gamma}{2}}$$
$$\Delta M = M\left(B_H\right) - M\left(B_L\right)$$
$$\Delta\Gamma = \Gamma\left(B_H\right) - \Gamma\left(B_L\right)$$

All exact formulae from $K^0 - \overline{K}^0$ system apply
but now: $\boxed{\left|M_{12}\right| >> \left|\Gamma_{12}\right|}$ ⇒

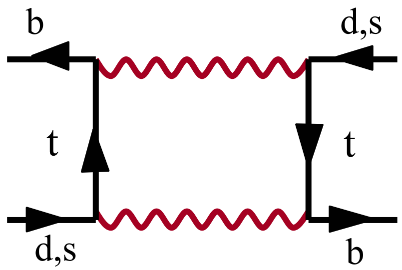
Master Formulae (B^0-$\bar{B}^0$)

$$\Delta M = 2|M_{12}|$$

$$\Delta\Gamma = 2\frac{\mathrm{Re}\left(M_{12}\Gamma^*_{12}\right)}{|M_{12}|}$$

$$\frac{q}{p} \cong \frac{M^*_{12}}{|M_{12}|}\left[1 - \frac{1}{2}\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]$$

$$M^*_{12} = \langle \overline{B}^0 | H_{eff} | B^0 \rangle \approx$$



$$\left(M^*_{12}\right)_d \sim \left(V_{td}V^*_{tb}\right)^2 \qquad \left(M^*_{12}\right)_s \sim \left(V_{ts}V^*_{tb}\right)^2$$

$$V_{td} = |V_{td}|e^{-i\beta} \qquad V_{ts} = |V_{ts}|e^{-i\beta_s} \qquad (\beta_s \cong 0)$$

$$\frac{q}{p} \cong e^{i2\varphi_M} \qquad \varphi_M = \begin{cases} -\beta & B^0_d - \overline{B}^0_d \\ -\beta_s & B^0_s - \overline{B}^0_s \end{cases}$$

(Pure Phase)

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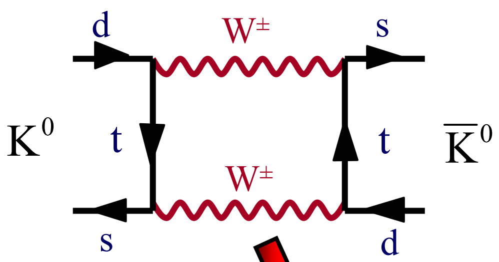
Indirect and Direct $\not{C}\!P$ in $K_L \to \pi\pi$



$(K^0 - \overline{K}^0$ Mixing$)$

Mass Eigenstates are not CP Eigenstates

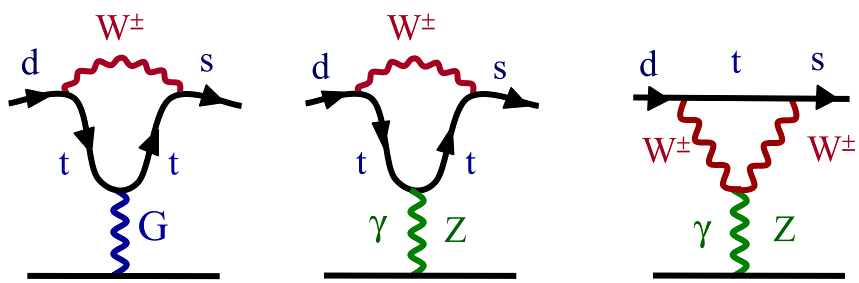
indirect CP violation (ε)

$$K_{1,2} = \frac{K^0 \mp \overline{K}^0}{\sqrt{2}}$$

$$CP\,|K^0\rangle = -|\overline{K}^0\rangle$$

$$K_L = \overset{(-)}{K}_2 + \varepsilon \overset{(+)}{K}_1 \xrightarrow{\varepsilon} \overbrace{\pi^+\pi^-,\ \pi^0\pi^0}^{CP\,(+)}$$

direct CP-Violation (ε′)



$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon'$$

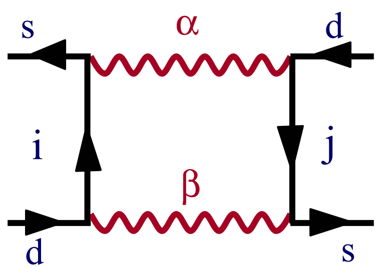
$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\varepsilon'$$

ε′=0 in Superweak Models
Wolfenstein (64)

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right)$$

Master Formulae for $K^0 - \overline{K}^0$ and ε_K

$$\sum_{i,j=u,c,t} \quad \sum_{\alpha,\beta=W^\pm,\Phi^\pm}$$



$$\equiv \sum_{i,j=u,c,t} F\left(x_i, x_j\right)\left(V_{is}^* V_{id}\right)\left(V_{js}^* V_{jd}\right) Q$$

$$M_{12} = \frac{G_F^2}{12\pi^2} F_K^2 \hat{B}_K m_K M_W^2 \Big[\underbrace{\tilde{\lambda}_c^2 \eta_1^{QCD} S_0(x_c)}_{\text{Dominates } \Delta M_K} + \underbrace{\tilde{\lambda}_t^2 \eta_2^{QCD} S_0(x_t) + 2\tilde{\lambda}_c \tilde{\lambda}_t \eta_3^{QCD} S_0(x_c, x_t)}_{\text{Dominates } \varepsilon_K} \Big]$$

$$\tilde{\lambda}_i = V_{is} V_{id}^*$$

$$\Delta M_K = 2\,\mathrm{Re}\, M_{12}$$

$$\varepsilon_K = \frac{\exp(i\pi/4)}{\sqrt{2}\,\Delta M_K}\,\mathrm{Im}\, M_{12}$$

$\eta_2^{QCD} = 0.57 \pm 0.01$ (AJB, Jamin, Weisz, 90)

$$\left.\begin{aligned} \eta_1^{QCD} &= 1.32 \pm 0.32 \\ \eta_3^{QCD} &= 0.47 \pm 0.05 \end{aligned}\right\} \text{(Herrlich, Nierste, 94, 95)}$$

February 2006

$$\Delta M_K = (0.5301 \pm 0.0016) \cdot 10^{-2} \, / \, ps$$

$$\Delta M_d = (0.503 \pm 0.006) / ps$$

$$\Delta M_s > \quad 16 / ps \quad (95\% \text{ C.L.})$$

$$1/ps = 6.582 \cdot 10^{-13} \text{GeV}$$

$$\varepsilon = (2.280 \pm 0.013) \cdot 10^{-3} e^{i\pi/4}$$

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (16.6 \pm 1.6) \cdot 10^{-4}$$

Modern Classification of CP Violation

We have:

Particle-Antiparticle Mixing

and

Decay

1. CP Violation in Mixing
2. CP Violation in Decay
3. CP Violation in the Interference of Mixing and Decay

Classification of $\cancel{CP}$ in B- and K-Decays

(Nir 99),...

1. CP Violation in Mixing

$$B_{H,L} = p\left|B^0\right\rangle \pm q\left|\overline{B}^0\right\rangle$$

Mass Eigenstates

$$\cancel{CP}: \quad \boxed{|q/p| \neq 1} \Rightarrow \text{(Not CP Eigenstates)}$$

$$a_{SL} = \frac{\Gamma\left(\overline{B}^0(t) \to l^+\nu X\right) - \Gamma\left(B^0(t) \to l^-\nu X\right)}{\Gamma\left(\overline{B}^0(t) \to l^+\nu X\right) + \Gamma\left(B^0(t) \to l^-\nu X\right)}$$

$$a_{SL} = \frac{1-|q/p|^4}{1+|q/p|^4} = \mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}}$$

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2}$$

Observed in K-system: $\mathrm{Re}\,\varepsilon_K \neq 0$

$$\overline{B}^0 \to B^0 \to l^+\nu X$$

(Phase Difference)

$$B^0 \to \overline{B}^0 \to l^-\nu X$$

"wrong charge" leptons

$$a_{SL} \approx 0\left(10^{-3}\right)$$

Hadronic Uncertainties in Γ_{12}, M_{12}

2. CP Violation in Decay

$$A_f = \langle f | H^{weak} | B \rangle \qquad \bar{A}_{\bar{f}} = \langle \bar{f} | H^{weak} | \bar{B} \rangle$$

$$\cancel{CP}: \quad \boxed{\left| \bar{A}_{\bar{f}} / A_f \right| \neq 1} \qquad f \xrightarrow{CP} \bar{f}$$

$$a^{Decay}_{f^\pm} = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)} = \frac{1 - \left| \bar{A}_{f^-} / A_{f^+} \right|^2}{1 + \left| \bar{A}_{f^-} / A_{f^+} \right|^2}$$

Requires at least two different contibutions with different <u>weak</u> (φ_i) and <u>strong</u> (δ_i) phases

$$A_f = \sum_i A_i e^{i(\delta_i + \varphi_i)} \qquad \bar{A}_{\bar{f}} = \sum_i A_i e^{i(\delta_i - \varphi_i)} \qquad (A_2 << A_1) \qquad r \equiv \frac{A_2}{A_1} << 1$$

$$i = 1,\ 2$$

$$\boxed{a^{Decay}_{f^\pm} \approx -2r \sin(\delta_2 - \delta_1) \sin(\varphi_2 - \varphi_1)}$$

Observed in K-system: $\boxed{\mathrm{Re}\ \varepsilon'_K \neq 0}$

Hadronic Uncertainties in A_i , δ_i

B^0-Decays into CP-Eigenstate

$$B^0 \to \overline{B}^0 \to f$$
$$B^0 \to B^0 \to f$$

ΔM = Difference between Mass Eigenstates in $(B^0, \overline{B}^0)$ System

$f \equiv f_{CP}$ = CP eigenstate
η_f = CP-parity = ± 1

Time-dependent asymmetry:

$$a_{CP}(t,f) = \frac{\Gamma\left(B^0(t) \to f\right) - \Gamma\left(\overline{B}^0(t) \to f\right)}{\Gamma\left(B^0(t) \to f\right) + \Gamma\left(\overline{B}^0(t) \to f\right)}$$

$$a_{CP}(t,f) = a_{CP}^{\text{Decay}} \cos(\Delta M t) + a_{CP}^{\text{"mix-ind"}} \sin(\Delta M t)$$

$$a_{CP}^{\text{Decay}} = \frac{1-|\xi_f|^2}{1+|\xi_f|^2} \equiv C_f \qquad a_{CP}^{\text{"mix-ind"}} = \frac{2\,\mathrm{Im}\,\xi_f}{1+|\xi_f|^2} \equiv -S_f$$

$$\xi_f = \underbrace{\exp[i2\varphi_M]}_{\text{Mixing}} \frac{\overline{A}_f\left(\overline{B}^0 \to f\right)}{A_f\left(B^0 \to f\right)}$$

Decay Amplitudes

For a **single** decay contribution or sum of contributions with **the same weak phase**

$$\xi_f = -\eta_f \exp[i2\varphi_M] \cdot \exp[-i2\varphi_D]$$

$|\xi_f|^2 = 1$ $\qquad \varphi_D$: weak phase in the B^0 decay

$\Rightarrow$ ξ_f = given only in terms of CKM phase

$$a_{CP}^{\text{decay}} = 0$$

Dominance of a single CKM Amplitude

A_{Tree}, A_P	- hadronic matrix elements
δ_T, δ_P	- final state interaction phases
φ_T, φ_P	- weak CKM phases

$$\frac{\overline{A}_f\left(\overline{B}^0 \to f\right)}{A_f\left(B^0 \to f\right)} = -\eta_f\left[\frac{A_{Tree}e^{i(\delta_T-\varphi_T)} + A_P e^{i(\delta_P-\varphi_P)}}{A_{Tree}e^{i(\delta_T+\varphi_T)} + A_P e^{i(\delta_P+\varphi_P)}}\right]$$

Tree Dominance

$$\frac{\overline{A}_f\left(\overline{B}^0 \to f\right)}{A_f\left(B^0 \to f\right)} = -\eta_f e^{-i2\varphi_T}$$

(Pure Phase)
Very Clean !

Penguin Dominance

$$\frac{\overline{A}_f\left(\overline{B}^0 \to f\right)}{A_f\left(B^0 \to f\right)} = -\eta_f e^{-i2\varphi_P}$$

(Pure Phase)
Very Clean !

Also pure phase if $\varphi_T=\varphi_P$!! $\left(\text{Example: } B_d^0 \to J/\psi\, K_S\right)$

3. CP Violation in the Interference of Mixing and Decay

Misnomer: ("Mixing induced CP-Violation")

$$a_{CP}(t,f) = \operatorname{Im}\xi_f \sin(\Delta M t)$$

$$\operatorname{Im}\xi_f = \eta_f \sin(2\varphi_D - 2\varphi_M) \equiv -S_f$$

Very clean
TH

Measures the difference between the phases of B^0-$\bar{B}^0$ mixing ($2\varphi_M$) and of decay amplitude ($2\varphi_D$)

Examples:

$B_d^0 \to \psi K_S$: $\varphi_D = 0 \quad \varphi_M = -\beta \quad \eta_f = -1$

$$\operatorname{Im}\xi_{\psi K_S} = -\sin 2\beta$$

$B_d^0 \to \pi^+\pi^-$: $\varphi_D = \gamma \quad \varphi_M = -\beta \quad \eta_f = +1$

$$\operatorname{Im}\xi_{\pi\pi} = \sin(2(\gamma+\beta)) = -\sin 2\alpha$$

$K_L \to \pi^0\nu\bar{\nu}$: Measures the difference between the phases in K^0-$\bar{K}^0$ mixing and $\bar{s} \to \bar{d}\bar{\nu}\nu$ amplitude

B^0-Decays into CP Eigenstates

$$\left(\text{Two Contributions } \; r = \frac{A_2}{A_1} << 1\right)$$

$$a_{CP}(t,f) = C_f \cos(\Delta M t) - S_f \sin(\Delta M t)$$

$$C_f = -2r \sin(\varphi_1 - \varphi_2)\sin(\delta_1 - \delta_2)$$

$$S_f = -\eta_f \left[\sin 2(\varphi_1 - \varphi_M) + 2r\cos 2(\varphi_1 - \varphi_M)\sin(\varphi_1 - \varphi_2)\cos(\delta_1 - \delta_2)\right]$$

φ_i = weak phases $\qquad$ δ_i = strong phases

$$\{r = 0\} \Rightarrow \quad C_f = 0 \qquad S_f = -\eta_f \sin 2(\varphi_1 - \varphi_M)$$

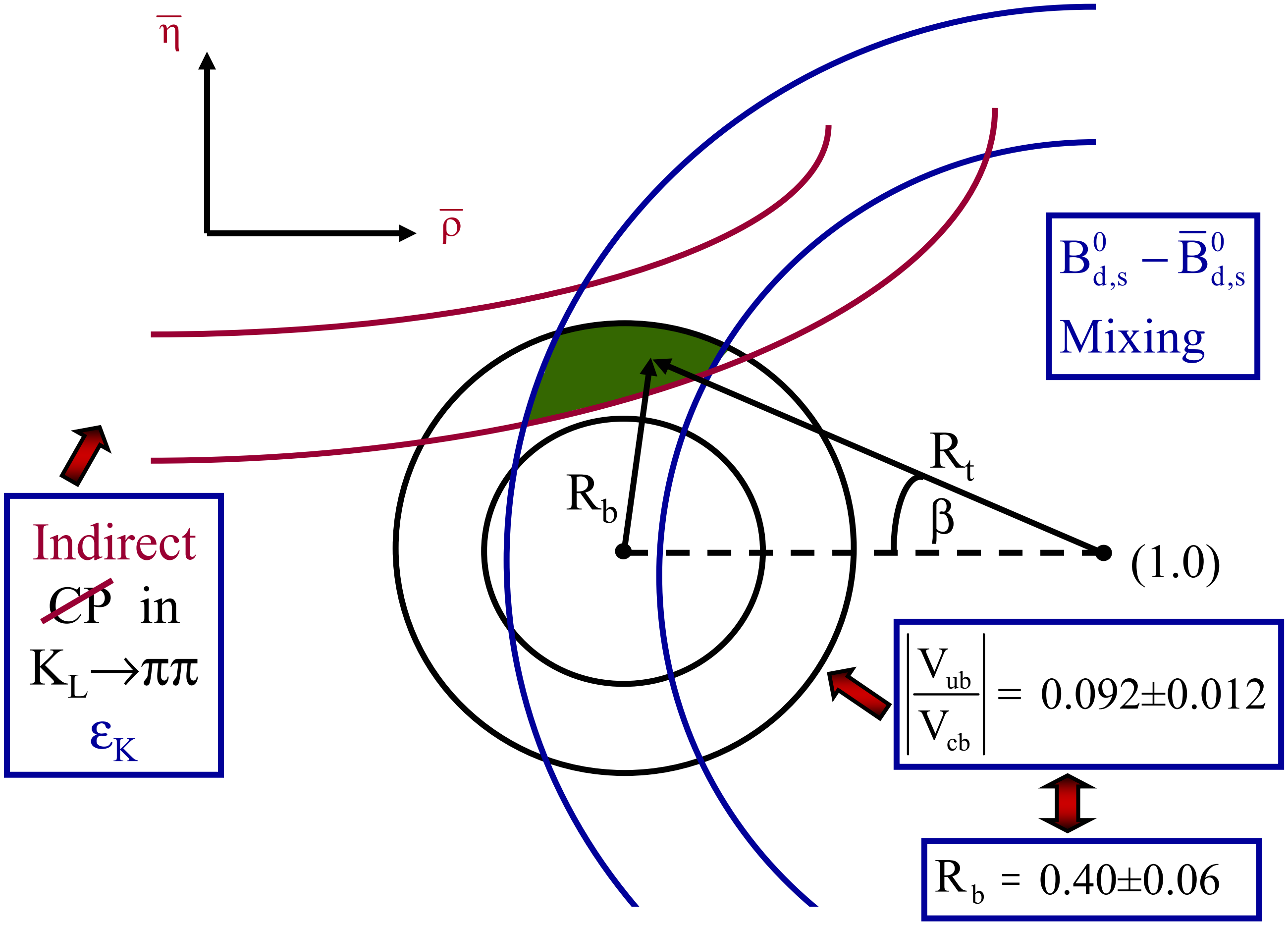
Comparison of Two-Languages

CP violation in mixing	$\equiv$	Manifestation of indirect $\not{CP}$
CP violation in decay	$\equiv$	Manifestation of direct $\not{CP}$
CP violation in interference of mixing and decay	$\equiv$	With a single decay it is impossible to state whether $\not{CP}$ in mixing or decay. But $\mathrm{Im}\,\xi_{f_1} \neq \mathrm{Im}\,\xi_{f_2}$ signals CP violation in decay (Direct $\not{CP}$)

3.

Standard Analysis of Unitarity Triangle

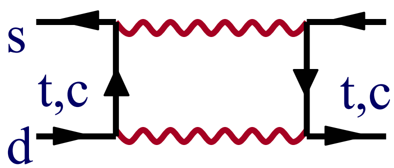
Standard Analysis of UT



Relevant Parameters

$$\hat{B}_K,\ F_{B_d}\sqrt{\hat{B}_{B_d}},\quad \xi = F_{B_s}\sqrt{\hat{B}_{B_s}}\,/\,F_{B_d}\sqrt{\hat{B}_{B_d}} \iff \varepsilon_K \qquad \Delta M_d \qquad \Delta M_s / \Delta M_d$$

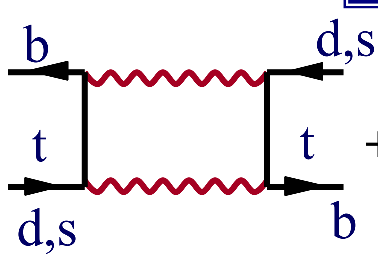
Indirect CP in $K_L \to \pi\pi$

$\Big\{ \mathrm{Im}$ 

$+\ \mathrm{QCD} \Big\} \Rightarrow \varepsilon_K \Leftarrow \{\text{Experiment}\}$

exp: $$\varepsilon_K = (2.280 \pm 0.013) \cdot 10^{-3}\, e^{i\frac{\pi}{4}}$$

$B^0_{d,s} - \overline{B}^0_{d,s}$ Mixing

$\Big\{$ 

$+\ \mathrm{QCD} \Big\} \Rightarrow (\Delta M)_{d,s} \Leftarrow$ {CLEO, ARGUS, 4*LEP,SLD, BaBar, Belle}

$$(\Delta M)_{d,s} \equiv M(B^0_H)_{d,s} - M(B^0_L)_{d,s}$$

Mass Eigenstates

exp:

$(\Delta M)_d = (0.503 \pm 0.006)\,/\,\mathrm{ps}$

★ $(\Delta M)_s > 16.0\,/\,\mathrm{ps}$ (95% C.L.) (LEP / SLD) (Tevatron)

Basic Formulae

1. ε_K - Hyperbola

$$\bar\eta\left[\left(1-\bar\rho\right)A^2F_{tt}\eta_{QCD}^{tt}+P_c\left(\varepsilon\right)\right]A^2\hat{B}_K=0.213$$

$\eta_{QCD}^{tt}=0.57\pm0.01;\quad P_C\left(\varepsilon\right)=0.28\pm0.05;\quad F_{tt}=2.42\pm0.12$

$\left(F_{tt}\equiv S\left(x_t\right)\right)$

2. $B_d^0-\bar{B}_d^0$ Mixing Constraint

$$R_t=0.86\left[\frac{0.041}{\left|V_{cb}\right|}\right]\sqrt{\frac{2.34}{F_{tt}}}\sqrt{\frac{\Delta M_d}{0.50/ps}}\left[\frac{230MeV}{\sqrt{\hat{B}_d}F_{B_d}}\right]\sqrt{\frac{0.55}{\eta_B^{QCD}}}$$

$\left|V_{cb}\right|=0.041\pm0.001;\quad \Delta M_d=\left(0.503\pm0.006\right)/ps;\quad \eta_B^{QCD}=0.55\pm0.01$

3. $B_s^0-\bar{B}_s^0$ Mixing Constraint ($\Delta M_d/\Delta M_s$)

$$R_t=0.90\sqrt{\frac{\Delta M_d}{0.50/ps}}\sqrt{\frac{18.4/ps}{\Delta M_s}}\left[\frac{\xi}{1.22}\right]$$

$$\xi=\frac{\sqrt{\hat{B}_s}F_{B_s}}{\sqrt{\hat{B}_d}F_{B_d}}$$

$\Delta M_s>16.0\ /\ ps\quad (95\%\ \text{C.L.})\quad \text{LEP}\ \ (\text{SLD})$

4. $\sin 2\beta$ from $A_{CP}(\psi K_S)$

$$A_{CP}(\psi K_S) \equiv -a_{\psi K_S} \sin(\Delta M_d t)$$

$$a_{\psi K_S} = \sin 2\beta \qquad \text{(SM)}$$

$$\sin 2\beta_{\psi K_S} = \begin{cases} 0.79 \pm {}^{0.41}_{0.44} & \text{(CDF)} \\ 0.741 \pm \underset{\text{(stat)}}{0.067} \pm \underset{\text{(syst)}}{0.033} & \text{(BaBar)} \\ 0.719 \pm 0.074 \pm 0.035 & \text{(Belle)} \end{cases}$$

$$\left(\text{ALEPH} : 0.84 \; {}^{+0.82}_{-1.04} \; \pm 0.16\right)$$

$$\Downarrow$$

$$\sin 2\beta = 0.726 \pm 0.037 \quad \left(a_{\psi K_S}\right)$$ (Before Summer 2005)

$$\Downarrow$$

$$\beta = \begin{cases} (23.3 \pm 1.6)^{\circ} \\ (66.7 \pm 1.6)^{\circ} & \text{(excluded in the SM)} \end{cases}$$

$$(\sin\beta \cong 0.40 \pm 0.03)$$

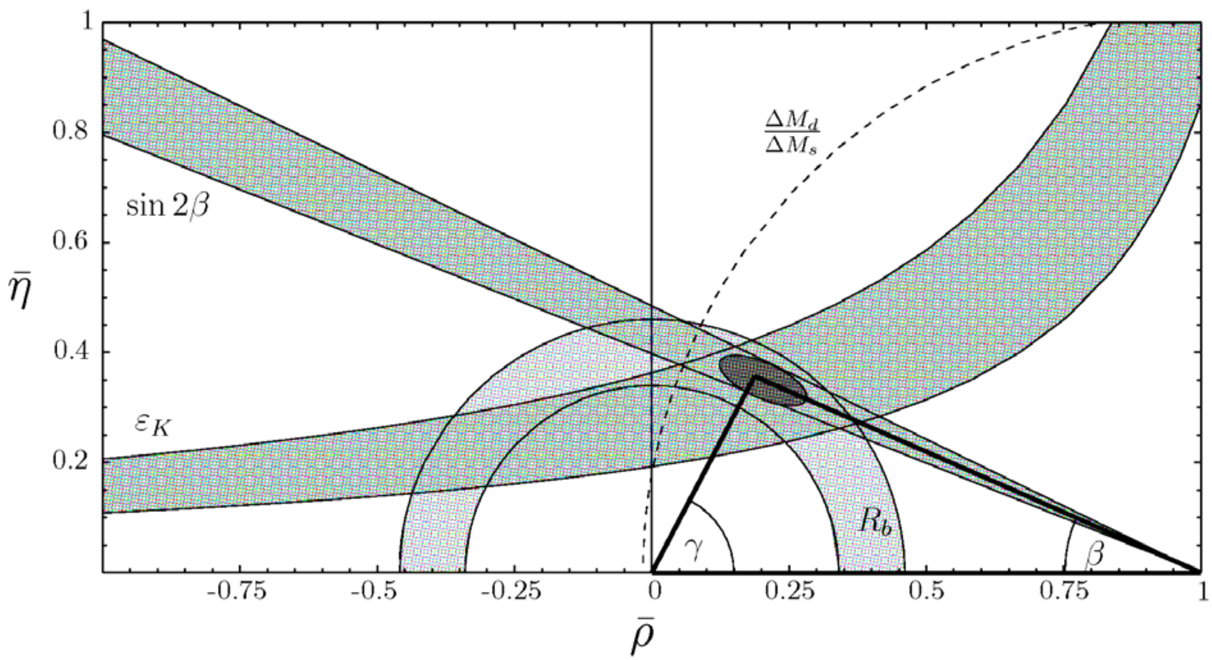
Crucial Parameters in SM and Beyond

$\lvert V_{us}\rvert = \lambda$	0.2240 ± 0.0036	
$\lvert V_{ub}\rvert$	$(3.81 \pm 0.46)\cdot 10^{-3}$	
$\lvert V_{cb}\rvert$	$(41.5 \pm 0.8)\cdot 10^{-3}$	★
$\left\lvert \frac{V_{ub}}{V_{cb}} \right\rvert$	0.092 ± 0.012	★
$m_t(m_t)$	(168 ± 4) GeV	
$\hat{B}_K$	0.86 ± 0.15	(ε_K)
$\sqrt{\hat{B}_d} F_{Bd}$	$(235 \, {}^{+33}_{-41})$ MeV	(ΔM_d)
$\xi = \frac{\sqrt{\hat{B}_s} F_{Bs}}{\sqrt{\hat{B}_d} F_{Bd}}$	1.24 ± 0.08	$\left(\frac{\Delta M_s}{\Delta M_d}\right)$

Valid for all extensions of SM !!

Unitarity Triangle 2004

AJB, Schwab, Uhlig



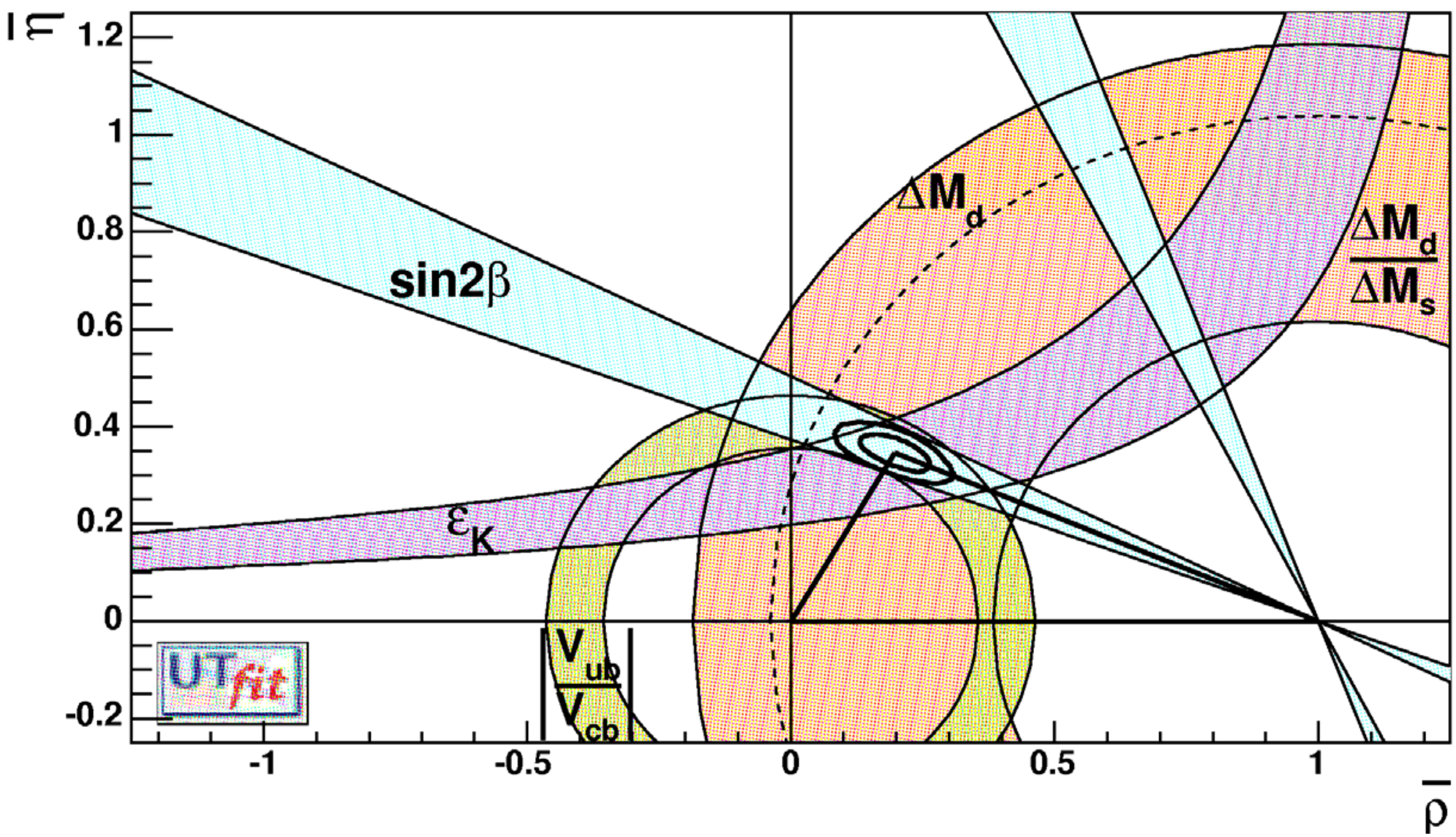
$$\bar{\eta} = 0.354 \pm 0.027$$
$$\bar{\rho} = 0.187 \pm 0.059$$

$$\gamma = (62.2 \pm 8.2)^{\circ}$$
$$R_t = 0.887 \pm 0.059$$
$$R_b = 0.400 \pm 0.039$$
$$|V_{td}| = (8.24 \pm 0.54) \cdot 10^{-3}$$

$$\mathrm{Im}\,\lambda_t = (1.40 \pm 0.12) \cdot 10^{-4}$$
$$\lambda_t = V_{ts}^{*} V_{td}$$

Unitarity Triangle 2005

UTfit collaboration : Bona et al.



Summer 2005 News

New Physics ?

$$(\sin 2\beta)_{\psi K_s} = \boxed{0.726 \pm 0.037} \Rightarrow \boxed{0.687 \pm 0.032} \bigstar$$

$$m_t(m_t) = \boxed{168 \pm 4 \text{ GeV}} \Rightarrow \boxed{163.0 \pm 2.7 \text{ GeV}}$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \boxed{0.092 \pm 0.012} \Rightarrow \boxed{0.102 \pm 0.005}$$

UT fitters

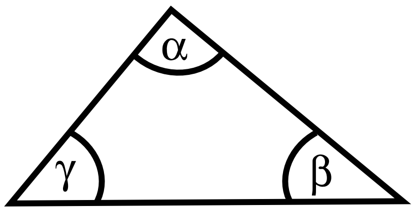
New $\left|\frac{V_{ub}}{V_{cb}}\right|, \quad m_t(m_t)$ $\Rightarrow$

$$(\sin 2\beta)_{\substack{\text{UT sides}\\ +\varepsilon_K}} = \boxed{0.791 \pm 0.034} \bigstar$$

$$(\sin 2\beta)_{\text{total}} = \boxed{0.734 \pm 0.024}$$

4.

α, β, γ from B-Decays



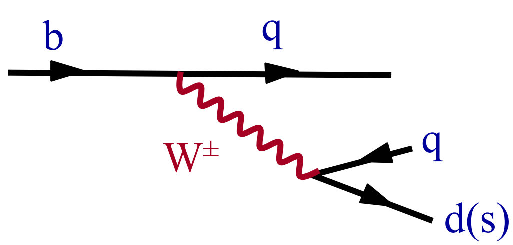
$$V_{td} = |V_{td}| e^{-i\beta}$$

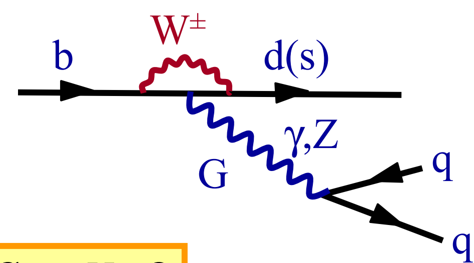
$$V_{ts} = |V_{ts}| e^{-i\beta_s}$$

$$V_{ub} = |V_{ub}| e^{-i\gamma}$$

Basic Contributions

Class I Decays with Trees and Penguins

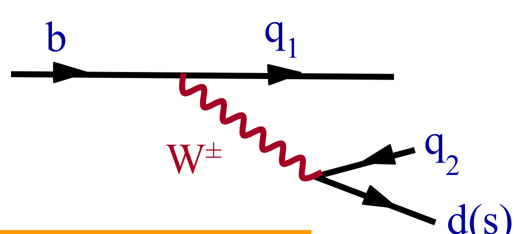




$q = u, c$

$b \to c\bar{c}s$
$b \to c\bar{c}d$
$b \to u\bar{u}s$
$b \to u\bar{u}d$

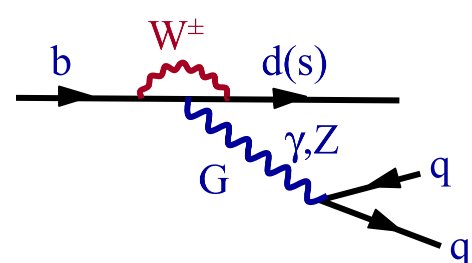
Class II Trees only



$b \to c\bar{u}s$
$b \to c\bar{u}d$
$b \to u\bar{c}s$
$b \to u\bar{c}d$

$q_1 \neq q_2 \in \{u, c\}$

Class III Penguins only



$b \to s\bar{s}s$
$b \to s\bar{s}d$
$b \to d\bar{d}s$
$b \to d\bar{d}d$

$q = d, s$

$\alpha,\ \beta,\ \gamma$ from B-Decays

$B_d^0 \to J/\psi K_s$
(β)
$\boxed{T+P}$ $\quad b \to c\bar{c}s$

$d \to s$
$B_d^0 \to B_s^0$

$B_s^0 \to J/\psi\varphi$
(β_s)
$\boxed{T+P}$ $\quad b \to c\bar{c}s$

$B_d^0 \to D^+\pi^-,\ D^-\pi^+$
$(2\beta+\gamma)$
$\boxed{T}$ $\quad b \to u\bar{c}d,\ c\bar{u}d$

$c \to s$

$c \to u$
$s \to d$

$d \to s$

$B_d^0 \to \varphi K_s$
(β)
$\boxed{P}$ $\quad b \to s\bar{s}s$

$B_d^0 \to \pi^+\pi^-$
$(\beta+\gamma)$ $\boxed{?}$
$\boxed{T+P}$ $\quad b \to u\bar{u}d$

$B_s^0 \to D_s^+K^-,\ D_s^-K^+$
$(2\beta_s+\gamma)$
$\boxed{T}$ $\quad b \to u\bar{c}s,\ c\bar{u}s$

$d \to s$

$B_s^0 \to B^+$

$B_s^0 \to K^+K^-$
(β_s, γ) $\boxed{?}$
$\boxed{T+P}$ $\quad b \to u\bar{u}s$

U-Spin Symmetry
γ and β

$B^+ \to D^0K^+,\ \bar{D}^0K^+$
(γ)
$\boxed{T}$ $\quad b \to u\bar{c}s,\ c\bar{u}s$

B^0-Decays into CP-Eigenstate

$$B^0 \to \overline{B}^0 \to f, \quad B^0 \to B^0 \to f$$

ΔM = Difference between Mass Eigenstates in $(B^0, \overline{B}^0)$ System

$f \equiv f_{CP}$ = CP eigenstate
η_f = CP-parity = ± 1

<u>Time-dependent asymmetry:</u>

$$a_{CP}(t,f) = \frac{\Gamma\left(B^0(t) \to f\right) - \Gamma\left(\overline{B}^0(t) \to f\right)}{\Gamma\left(B^0(t) \to f\right) + \Gamma\left(\overline{B}^0(t) \to f\right)}$$

$$a_{CP}(t,f) = a_{CP}^{Decay}\cos(\Delta M t) + a_{CP}^{\text{"mix-ind"}}\sin(\Delta M t)$$

$$a_{CP}^{Decay} = \frac{1-|\xi_f|^2}{1+|\xi_f|^2} \equiv C_f \qquad a_{CP}^{\text{"mix-ind"}} = \frac{2\,\mathrm{Im}\,\xi_f}{1+|\xi_f|^2} \equiv -S_f$$

$$\xi_f = \underbrace{\exp[i2\varphi_M]}_{\text{Mixing}} \frac{\overline{A}_f\left(\overline{B}^0 \to f\right)}{A_f\left(B^0 \to f\right)}$$

Decay Amplitudes

For a **single** decay contribution or sum of contributions with **the same weak phase**

$$\xi_f = -\eta_f \exp[i2\varphi_M]\cdot\exp[-i2\varphi_D]$$

$|\xi_f|^2 = 1$ $\qquad \varphi_D$: weak phase in the B^0 decay

$\Rightarrow$ ξ_f = given only in terms of CKM phase

$a_{CP}^{decay} = 0$

$B^0(\overline{B}^0)$-Decays into CP-Eigenstates

$$\begin{tikzcd}[column sep=small]
 & \overline{B}^0 \arrow[dr, "\text{(Decay)}"] & \\
B^0 \arrow[ur, "\text{(Mixing)}"] \arrow[dr] & & f \\
 & B^0 \arrow[ur] &
\end{tikzcd}
\qquad
\begin{tikzcd}[column sep=small]
 & B^0 \arrow[dr, "\text{(Decay)}"] & \\
\overline{B}^0 \arrow[ur, "\text{(Mixing)}"] \arrow[dr] & & f_{CP} = \eta_f f \\
 & \overline{B}^0 \arrow[ur] &
\end{tikzcd}$$

$\eta_f = \pm 1$ CP Parity

Basic dynamical quantity:

$$\xi_f \equiv \underbrace{\exp\left(i2\varphi_M\right)}_{\text{Mixing}} \underbrace{\frac{\overline{A}_f\left(\overline{B}^0 \to f\right)}{A_f\left(B^0 \to f\right)}}_{\text{Decay Amplitudes}} = -\eta_f \exp\left(i2\varphi_M\right) \cdot \exp\left(-i2\varphi_D\right)$$

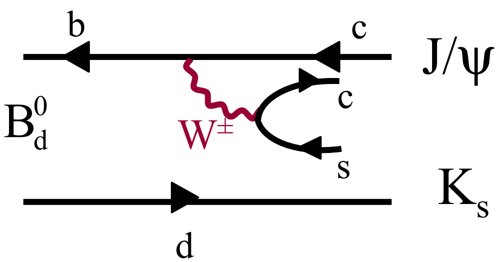
Weak phase in B^0 Decay : $\varphi_D = 0,\ \beta,\ \gamma$

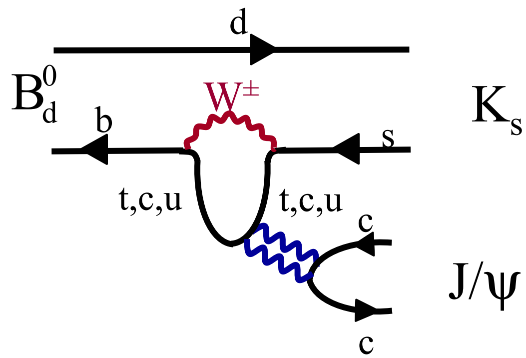
In the case of a single decay contribution or contributions with the same weak phase

$$\varphi_M = \begin{cases} -\beta & \left(B_d^0 - \overline{B}_d^0 \text{ mixing}\right) \\ \underbrace{-\beta_s}_{\approx 1^0} & \left(B_s^0 - \overline{B}_s^0 \text{ mixing}\right) \end{cases}$$

EXP $\Rightarrow$ $\operatorname{Im} \xi_f = \eta_f \sin 2\left(\varphi_D - \varphi_M\right)$ ★

$B_d^0 \to J/\psi K_S$ and β





$$\boxed{V_{td} = |V_{td}| e^{-i\beta}}$$

$$V_{cs}V_{cb}^* \cong A\lambda^2$$
$$V_{us}V_{ub}^* \cong A\lambda^4 R_b e^{i\gamma}$$
$$V_{ts}V_{tb}^* = -V_{cs}V_{cb}^* - V_{us}V_{ub}^*$$

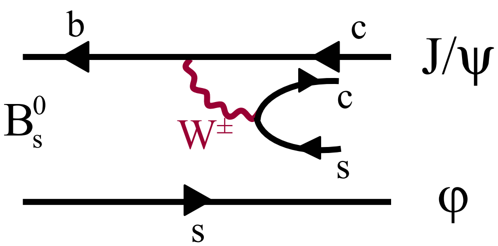
$$\begin{aligned} A\left(B_d^0 \to J/\psi K_S\right) &= V_{cs}V_{cb}^*\left(A_T + P_c\right) + V_{us}V_{ub}^* P_u + V_{ts}V_{tb}^* P_t \\ &= V_{cs}V_{cb}^*\left(A_T + P_c - P_t\right) + V_{us}V_{ub}^*\left(P_u - P_t\right) \end{aligned}$$

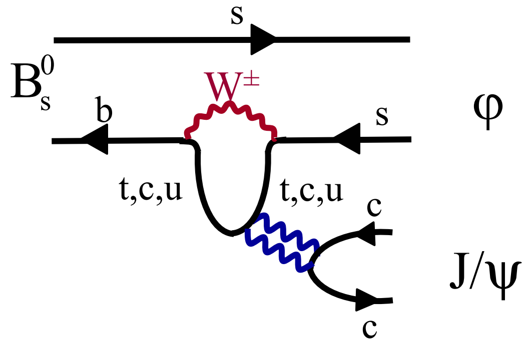
(Dominance of a single phase)

$$\left\{ \begin{array}{l} \left|\dfrac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*}\right| \le 0.02 \\ \dfrac{P_u - P_t}{A_t + P_c - P_t} << 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \varphi_D = 0 \\ \varphi_M = -\beta \\ \left|\xi_{\psi K_S}\right| = 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} a_{CP}^{mix}\left(\psi K_S\right) = \eta_{\psi K_S} \sin 2\left(\varphi_D - \varphi_M\right) = -\sin 2\beta \\ a_{CP}^{dir}\left(\psi K_S\right) = 0 \qquad a_{CP}\left(\psi K^+\right) \cong 0 \\ \boxed{C_{\psi K_S} = 0} \qquad \boxed{S_{\psi K_S} = \sin 2\beta} \end{array} \right\}$$

$B_s^0 \to J/\psi\varphi$ and β_s

$$V_{ts} = |V_{ts}| e^{-i\beta_S}$$





Differs from

$B_d^0 \to J/\psi K_S$ only by

"spectator" quark $d \to s$

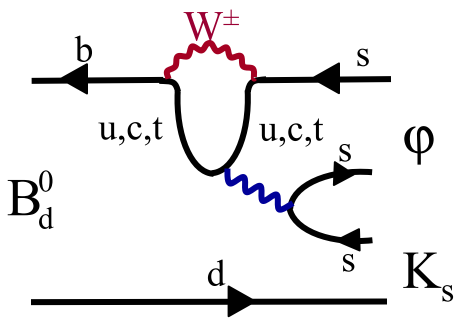
$(\varphi_D = 0)$

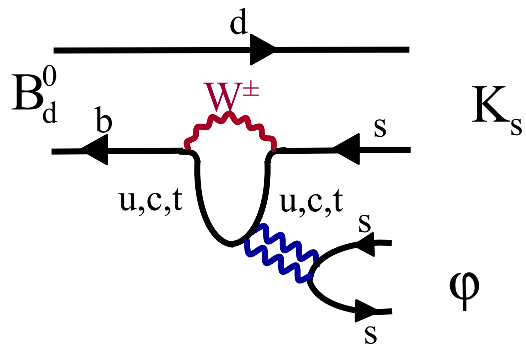
Complication: $(J/\psi\varphi)$ admixture of CP = + and CP = −

(Can be resolved: see Page 40: "B-Decays at the LHC ")

$$\left\{\begin{array}{l} \varphi_D = 0 \\ \varphi_M = -\beta_S \cong -\lambda^2\eta \\ |\xi_{\psi\varphi}| = 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} a_{CP}^{mix} = \sin 2(\varphi_D - \varphi_M) \cong \underbrace{2\lambda^2\eta}_{2\beta_S} \cong 0.03 \\ a_{CP}^{dir} \cong 0 \\ \text{A lot of room for New Physics!} \end{array}\right\}$$

$B_d^0 \to \varphi K_S$ and β (Pure Peguin Decay)





$$V_{cs}V_{cb}^* \cong A\lambda^2$$
$$V_{us}V_{ub}^* \cong A\lambda^4 R_b e^{i\gamma}$$
$$V_{ts}V_{tb}^* = -V_{cs}V_{cb}^* - V_{us}V_{ub}^*$$

$$\begin{aligned} A\left(B_d^0 \to \varphi K_S\right) &= V_{cs}V_{cb}^* P_c + V_{us}V_{ub}^* P_u + V_{ts}V_{tb}^* P_t \\ &= V_{cs}V_{cb}^*\left(P_c - P_t\right) + V_{us}V_{ub}^*\left(P_u - P_t\right) \end{aligned}$$

(Dominance of a single phase)

$$\left\{ \begin{array}{l} \left|\dfrac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*}\right| \le 0.02 \\ \dfrac{P_u - P_t}{P_c - P_t} \approx 0(1) \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} a_{CP}^{mix}\left(\varphi K_S\right) = -\sin 2\beta = a_{CP}^{mix}\left(\psi K_S\right) \\ \boxed{C_{\varphi K_S} \approx 0} \qquad \boxed{S_{\psi K_S} = S_{\varphi K_S} = \sin 2\beta} \\ \left|S_{\psi K_S} - S_{\varphi K_S}\right| \le 0.04 \ (\mathrm{SM}) \end{array} \right\}$$

(neglecting $V_{us}V_{ub}^*$)

Grossman, Isidori, Worah, London, Soni

First Results for $B_d^0 \to \varphi K_S$

$$(\sin 2\beta)_{\phi K_S} = \begin{cases} +0.45 \pm 0.43 \pm 0.07 & \text{(BaBar)} \\ -0.96 \pm 0.50 \, {}^{+0.11}_{-0.09} & \text{(Belle)} \end{cases}$$

World Averages

$$S_{\phi K_S} = -0.05 \pm 0.24$$
$$C_{\psi K_S} = -0.15 \pm 0.33$$

(Belle)

$$S_{\eta' K_S} = \quad 0.76 \pm 0.36$$
$$C_{\eta' K_S} = -0.26 \pm 0.22$$

(BaBar)

$$S_{\eta' K_s} = 0.02 \pm 0.35$$

(fully consistent with SM)

$$\left| S_{\varphi K_S} - S_{\psi K_S} \right| \cong 0.88 \pm 0.34$$

(Violation of SM by 2.6σ)

but $S_{\varphi K_S} \neq S_{\eta' K_S}$ possible
as non-leading terms
could be different

Grossman, Isidori Worah

Ciuchini Silvestrini

New Physics:

Enhanced QCD Penguins
Z^0 Penguins, ..

Hiller, Raidal, Ciuchini + Silvestrini
Fleischer, Mannel

Present Results for $B_d^0 \to \varphi K_s$

$$(\sin 2\beta)_{\phi K_s} = \begin{cases} 0.50 \pm 0.25 \pm 0.06 & \text{(BaBar)} \\ 0.06 \pm 0.33 \pm 0.09 & \text{(Belle)} \end{cases}$$

World Averages: (2005) $S_{\varphi K_s} = 0.34 \pm 0.20$ $C_{\varphi K_s} = -0.04 \pm 0.17$

To be compared with $S_{\psi K_s} \approx 0.73$

New Physics?

2006 : $S_{\varphi K_s} = 0.47 \pm 0.19$

Decays to CP non-eigenstates and γ

$\overset{(-)}{B}{}^0_d \to D^{\pm}\pi^{\mp}$

(Dunietz+Sachs)

$d \to s$

$\overset{(-)}{B}{}^0_s \to D_s^{\pm}K^{\mp}$

Aleksan, Dunietz, Kayser

- $B_d^0\left(B_s^0\right)$ and $\bar{B}_d^0\left(\bar{B}_s^0\right)$ can decay to the same final state
- Requires full time-dependent analysis: 4 time dependent rates

  $B^0_{d,s}(t) \to f, \quad \bar{B}^0_{d,s}(t) \to f,$

  $B^0_{d,s}(t) \to \bar{f}, \quad \bar{B}^0_{d,s}(t) \to \bar{f},$
- Tree diagrams only

$$\xi_f = e^{i2\varphi_M}\frac{A\left(\bar{B}^0 \to f\right)}{A\left(B^0 \to f\right)} \qquad \begin{matrix}\bar{B}^0 \searrow & \\ \updownarrow & f \\ B^0 \nearrow & \end{matrix}$$

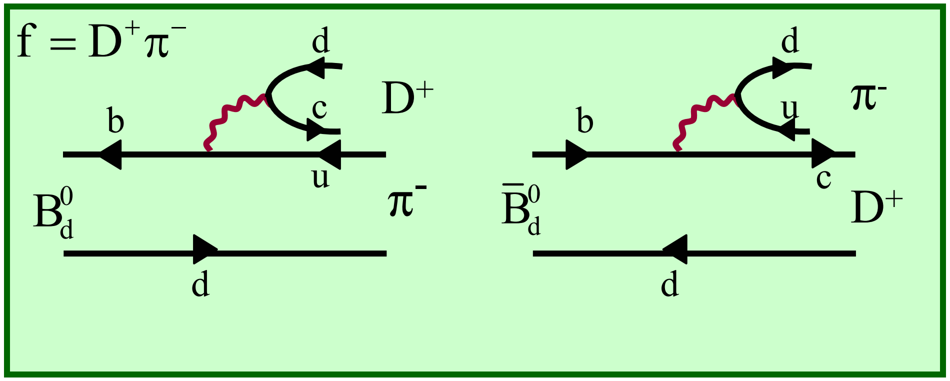
$$\xi_{\bar{f}} = e^{i2\varphi_M}\frac{A\left(\bar{B}^0 \to \bar{f}\right)}{A\left(B^0 \to \bar{f}\right)} \qquad \begin{matrix}\bar{B}^0 \searrow & \\ \updownarrow & \bar{f} \\ B^0 \nearrow & \end{matrix}$$

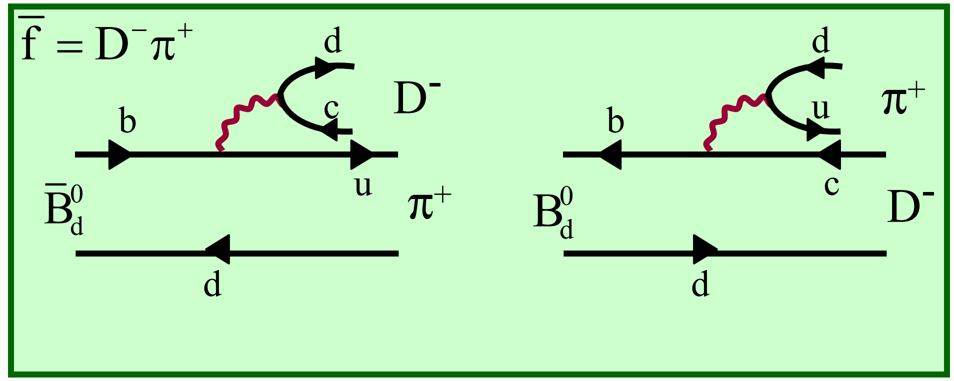
$$\varphi_M = \begin{cases} -\beta & B_d^0 \\ -\beta_s & B_s^0 \end{cases}$$

$$\boxed{\xi_f \cdot \xi_{\bar{f}} = F\left(\gamma, \beta_{(s)}\right)}$$

(Dunietz, Sachs)

$B_d^0 \to D^{\pm}\pi^{\mp}, \ \bar{B}_d^0 \to D^{\pm}\pi^{\mp}$ and γ





$\left(M_f A\lambda^4 R_b e^{i\gamma}\right)$ $\left(\bar{M}_f A\lambda^2\right)$ $\left(\bar{M}_{\bar{f}} A\lambda^4 R_b e^{-i\gamma}\right)$ $\left(M_{\bar{f}} A\lambda^2\right)$

$$\xi_f^{(d)} = e^{-i2\beta}\frac{A\left(\bar{B}_d^0 \to f\right)}{A\left(B_d^0 \to f\right)} = e^{-i(2\beta+\gamma)}\frac{1}{\lambda^2 R_b}\frac{\bar{M}_f}{M_f}$$

$$\xi_{\bar{f}}^{(d)} = e^{-i2\beta}\frac{A\left(\bar{B}_d^0 \to \bar{f}\right)}{A\left(B_d^0 \to \bar{f}\right)} = e^{-i(2\beta+\gamma)}\lambda^2 R_b\frac{\bar{M}_{\bar{f}}}{M_{\bar{f}}}$$

$$\bar{M}_f = M_{\bar{f}} \qquad M_f = \bar{M}_{\bar{f}}$$

Hadronic Matrix Elements

$$\xi_f^{(d)} \cdot \xi_{\bar{f}}^{(d)} = e^{-i2(2\beta+\gamma)}$$

$2\beta+\gamma$ without hadronic uncertainties

(β known)

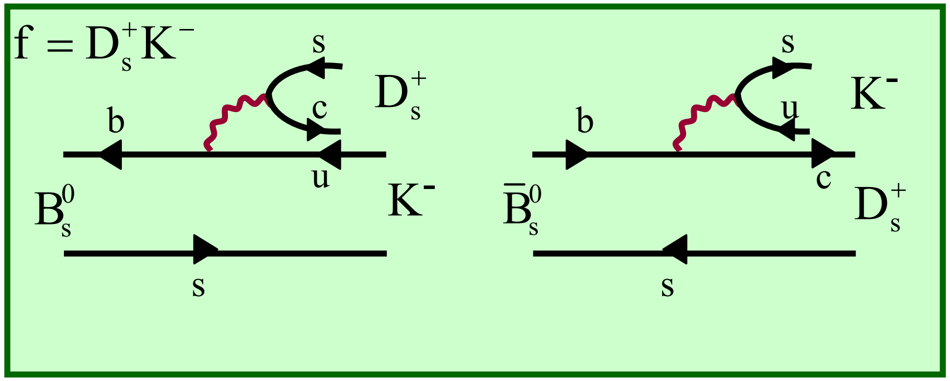
$$\gamma$$

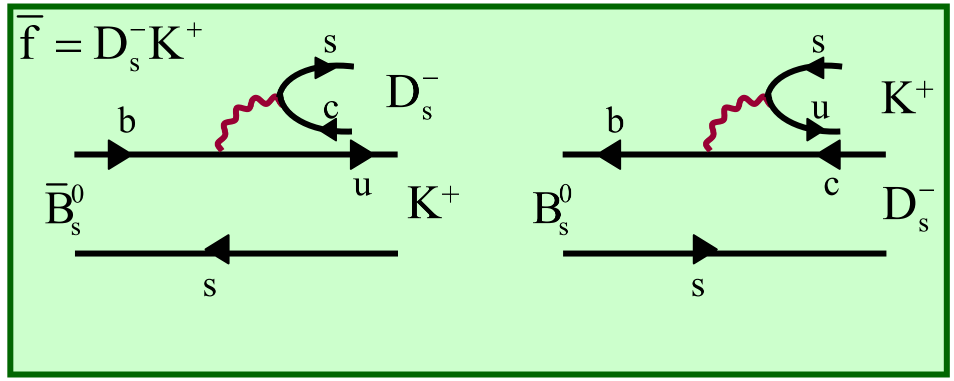
Small Interference: difficult exp. task

Aleksan
Dunietz
Kayser

$B_s^0 \to D_s^{\pm} K^{\mp},\ \bar{B}_s^0 \to D_s^{\pm} K^{\mp}$ and γ

Directly obtained from $B_d^0, \bar{B}_d^0 \to D^{\pm}\pi^{\mp}$ through $\boxed{d \to s}$





$\left(M_f A\lambda^3 R_b e^{i\gamma}\right)$ $\left(\bar{M}_f A\lambda^3\right)$ $\left(\bar{M}_{\bar{f}} A\lambda^3 R_b e^{-i\gamma}\right)$ $\left(M_{\bar{f}} A\lambda^3\right)$

In analogy to $B_d^0, \bar{B}_d^0 \to D^{\pm}\pi^{\mp}$

$$\xi_f^{(s)} \cdot \xi_{\bar{f}}^{(s)} = e^{-i2(2\beta_s+\gamma)}$$

$\Rightarrow$ $2\beta_s + \gamma$ without hadronic uncertainties $\Rightarrow$ $\boxed{\gamma}$

β_s from $B_s^0 \to \varphi\psi$

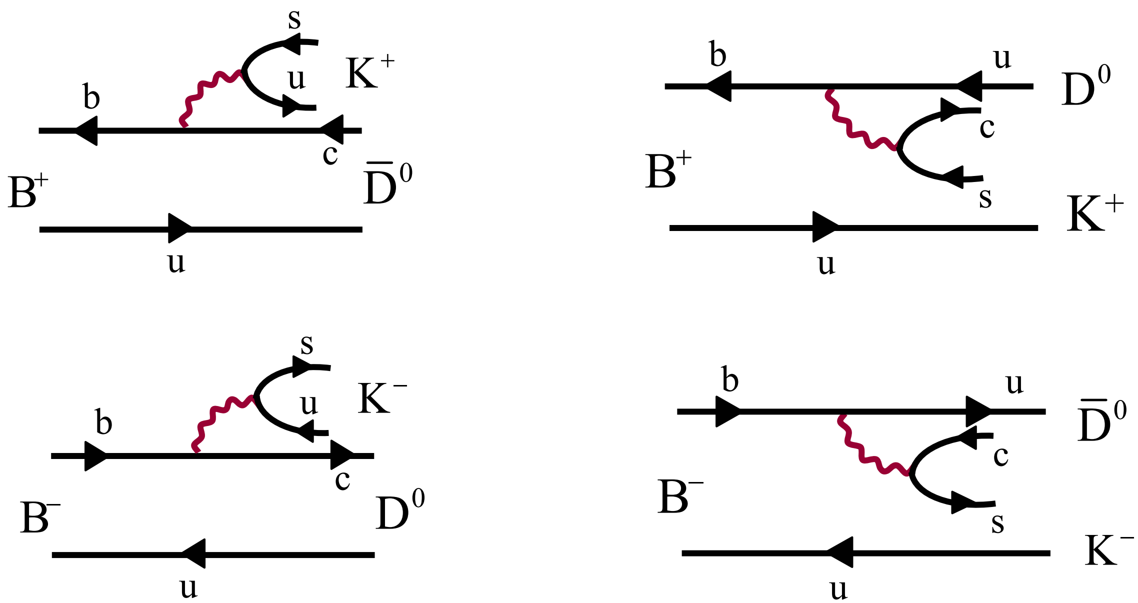
β_s – phase in $B_s^0 - \bar{B}_s^0$

Much bigger interference than in $B_d^0, \bar{B}_d^0 \to D^{\pm}\pi^{\mp}$

$B^\pm \to D^0K^\pm,\ \bar{D}^0K^\pm$ and γ

(Gronau + Wyler)

Directly obtained from $B_s^0,\ \bar{B}_s^0 \to D_s^\pm K^\pm$ through $\boxed{B_s \to B^\pm}$



$$K^+\bar{D}^0 \neq K^+D^0$$

Need

$$B^+ \to D_+^0 K^+$$

$$D_+^0 = \frac{1}{\sqrt{2}}\left(D^0 + \bar{D}^0\right)$$

To each process only single diagram contributes

$$A\left(B^+ \to \bar{D}^0K^+\right) = A\left(B^- \to D^0K^-\right)$$

$$A\left(B^+ \to D^0K^+\right) = A\left(B^- \to \bar{D}^0K^-\right)e^{2i\gamma}$$

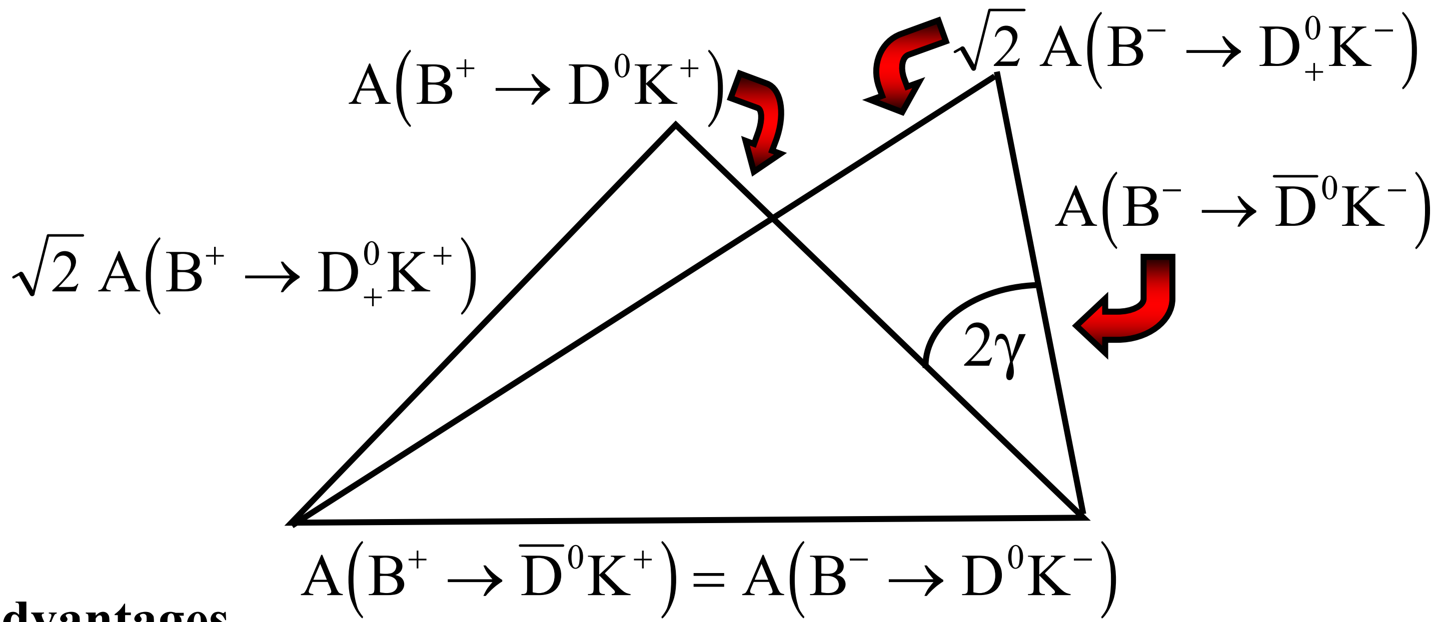
$0\left(A\lambda^3\right)$ $\qquad$ $0\left(A\lambda^3R_b\right)$ Colour suppressed

Gronau-Wyler Method for γ

$$\sqrt{2}\,A\left(B^+ \to D^0_+ K^+\right) = A\left(B^+ \to D^0 K^+\right) + A\left(B^+ \to \overline{D}^0 K^+\right)$$

$$\sqrt{2}\,A\left(B^- \to D^0_+ K^-\right) = A\left(B^- \to \overline{D}^0 K^-\right) + A\left(B^- \to D^0 K^-\right)$$

$$D^0_+ = \frac{1}{2}\left(\left|D^0\right\rangle + \left|\overline{D}^0\right\rangle\right) \qquad CP = +$$



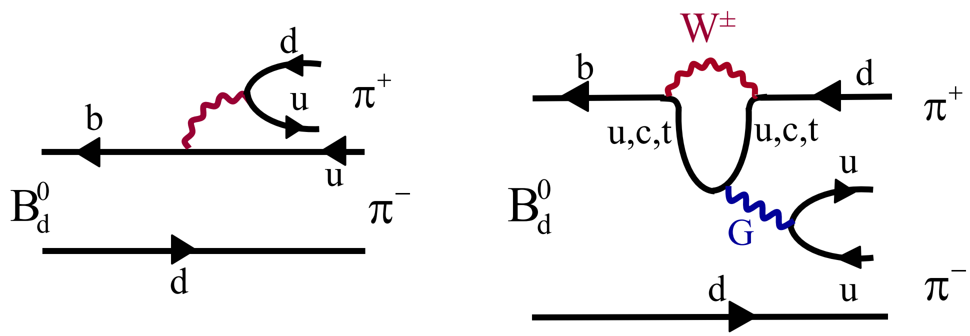
Advantages

- Pure Trees
- No tagging
- No time dependent measurements
- Only rates

Disadvantages

- $Br\left(B^+ \to D^0 K^+\right) \sim 0\left(10^{-6}\right)$
- $Br\left(B^+ \to \overline{D}^0 K^+\right) \sim 0\left(10^{-4}\right)$
- Detection of D^0_+

$B_d^0 \to \pi^+\pi^-$ and α



$$V_{ub}^* V_{ud} = A\lambda^3 R_b e^{i\gamma}$$
$$V_{cb}^* V_{cd} = A\lambda^3$$
$$V_{tb}^* V_{td} = -V_{ub}^* V_{ud} - V_{cb}^* V_{cd}$$

$$A\left(B_d^0 \to \pi^+\pi^-\right) = V_{ub}^* V_{ud}\left(A_T + P_u\right) + V_{cb}^* V_{cd} P_c + V_{tb}^* V_{td} P_t$$
$$= V_{ub}^* V_{ud}\left(A_T + P_u - P_t\right) + V_{cb}^* V_{cd}\left(P_c - P_t\right)$$

$$\left|\frac{V_{cb}^* V_{cd}}{V_{ub}^* V_{ud}}\right| = \frac{1}{R_b} \approx 0(2)$$
$$\frac{P_c - P_t}{A_T + P_u - P_t} = \frac{P_{\pi\pi}}{T_{\pi\pi}} \quad ?$$

Assuming $\frac{P_{\pi\pi}}{T_{\pi\pi}} << 1$

$\varphi_D = \gamma$ $\quad$ $\varphi_M = -\beta$ $\quad$ $|\xi_{\pi\pi}| = 1$

$$a_{CP}^{mix} = \eta_{\pi\pi} \sin 2\left(\varphi_D - \varphi_M\right) = \sin 2\left(\gamma + \beta\right) = -\sin 2\alpha$$

$a_{CP}^{dir} = 0$ $\quad$ $C_{\pi\pi} = 0$ $\quad$ $S_{\pi\pi} = \sin 2\alpha$

Dominance of a
single amplitude uncertain

Results for $B_d^0 \to \pi^+\pi^-$

$$C_{\pi\pi} = \begin{cases} -0.09 \pm 0.15 \pm 0.04 & \text{(BaBar)} \\ -0.58 \pm 0.15 \pm 0.07 & \text{(Belle)} \end{cases}$$

Consistent with 0

$\cancel{CP}$

$$S_{\pi\pi} = \begin{cases} -0.30 \pm 0.17 \pm 0.03 & \text{(BaBar)} \\ -1.00 \pm 0.21 \pm 0.07 & \text{(Belle)} \end{cases}$$

Consistent with 0

$\cancel{CP}$

World Average: $C_{\pi\pi} = -0.37 \pm 0.11 \qquad S_{\pi\pi} = -0.61 \pm 0.16$

Isospin analysis (Gronau + London)
Model independent determination of α

Model independent upper bound
(Grossman, Quinn; Charles)

$$\sin^2(\alpha_{eff} - \alpha) \leq \frac{Br(B^0 \to \pi^0\pi^0)}{Br(B^+ \to \pi^+\pi^0)}$$

$$\sin 2\alpha_{eff} \equiv \frac{\operatorname{Im}\xi_{\pi\pi}}{|\xi_{\pi\pi}|}$$

Model dependent determination of α using $(P_{\pi\pi}/T_{\pi\pi})_{TH}$

Beneke, Buchalla, Neubert, Sachrajda: small $C_{\pi\pi}$

Keum,Li,Sanda: large $C_{\pi\pi}$

Most recent:
Buchalla, Safir;
AJB, Fleischer, Recksiegel, Schwab
Gronau, Rosner et al.
Ali, Lunghi, Parkhomenko

U-Spin Strategies

$(d \leftrightarrow s)$

Fleischer:

$$\left\{\begin{array}{l} B_d^0 \to \pi^+\pi^- \\ B_s^0 \to K^+K^- \end{array}\right\} \Rightarrow \boxed{\beta, \gamma}$$

$$\left\{\begin{array}{l} B_d^0 \to J/\psi K_s \\ B_s^0 \to J/\psi K_s \end{array}\right\} \Rightarrow \boxed{\gamma}$$

$$\left\{\begin{array}{l} B_d^0 \to D^+D^- \\ B_s^0 \to D_s^+D_s^- \end{array}\right\} \Rightarrow \boxed{\gamma}$$

Uncertainty from
U-Spin breaking

Gronau + Rosner; Chiang Wolfenstein:

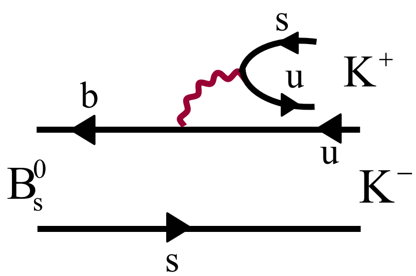
$$\left\{\begin{array}{l} B_d^0 \to \pi^-K^+ \\ B_s^0 \to \pi^+K^- \end{array}\right\} \Rightarrow \boxed{\gamma}$$

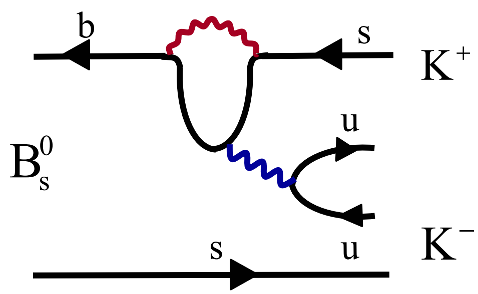
Uncertainty from U-Spin breaking,
rescattering, colour suppressed
EW-Penguins

$B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ (β and γ)

(Fleischer)

$\{\text{Replace in } B_d^0 \to \pi^+\pi^- :\ d \to s\}$





$$V_{ub}^* V_{us} = A\lambda^4 e^{i\gamma} R_b$$
$$V_{cb}^* V_{cs} = A\lambda^2$$
$$V_{tb}^* V_{ts} = -V_{ub}^* V_{us} - V_{cb}^* V_{cs}$$

$$A\left(B_s^0 \to K^+K^-\right) = V_{ub}^* V_{us}\left(A'_T + P'_u - P'_t\right) + V_{cb}^* V_{cs}\left(P'_c - P'_t\right)$$

U-Spin Symmetry:
$$\frac{P_{\pi\pi}}{T_{\pi\pi}} = \frac{P_c - P_t}{A_T + P_u - P_t} = \frac{P'_c - P'_t}{A'_T + P'_u - P'_t} = \frac{P_{KK}}{T_{KK}} \equiv d e^{i\delta}$$

strong phase

$$a_{CP}^{mix}\left(B_d^0 \to \pi^+\pi^-\right) \qquad a_{CP}^{mix}\left(B_s^0 \to K^+K^-\right)$$
$$a_{CP}^{dir}\left(B_d^0 \to \pi^+\pi^-\right) \qquad a_{CP}^{dir}\left(B_s^0 \to K^+K^-\right)$$

$\Rightarrow$

$d,\ \delta,\ \boxed{\beta,\ \gamma}$ subject to U-Spin breaking corrections

$\left(\beta_s \text{ from } B_s \to J/\psi\varphi\right)$

β present in $B_d^0 - \bar{B}_d^0$ mixing

$$V_{td} = |V_{td}| e^{-i\beta}$$
$$V_{ub} = |V_{ub}| e^{-i\gamma}$$
$$V_{ts} = |V_{ts}| e^{-i\beta_s}$$

Golden Measurements

(Essentially no TH uncertainties)

1. Decays into CP-Eigenstates (time evolution)

$B_d^0 \left(\overline{B}_d^0\right) \to \psi K_s \Rightarrow \beta$ $\qquad$ $B_s^0 \left(\overline{B}_s^0\right) \to \psi \varphi \Rightarrow \beta_s$

2. Decays into CP non-Eigenstates (time evolution)

$B_d^0 \left(\overline{B}_d^0\right) \to D^{\pm} \pi^{\mp} \Rightarrow 2\beta + \gamma$ $\qquad$ $B_s^0 \left(\overline{B}_s^0\right) \to D_s^{\pm} K^{\mp} \Rightarrow 2\beta_s + \gamma$

3. $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ (branching ratios)

$$\left.\begin{aligned} &Br\left(K^+ \to \pi^+ \nu \bar{\nu}\right) \\ &Br\left(K_L \to \pi^0 \nu \bar{\nu}\right) \end{aligned}\right\} \Rightarrow \beta \text{ and } \gamma$$

5.

$$K_L \to \pi^0 \nu \bar{\nu}$$

$$K^+ \to \pi^+ \nu \bar{\nu}$$

Master Formula for Weak Decays

AJB (2001)
hep-ph/0101336
hep-ph/0109197

Non-Perturbative Factors in the SM	QCD RG Factors	Short Distance Loop Functions (Penguins, Boxes)	New Flavour-Changing Parameters

Represent different Dirac and Colour Stuctures

$$A\left(\text{Decay}\right) = B_i \; \eta^i_{QCD} \; V^i_{CKM} \left[F^i_{SM} + F^i_{New}\right] + B^{New}_i \left[\eta^i_{QCD}\right]^{New} V^i_{New} \left[G^i_{New}\right]$$

(Summation over i)

Master Formula for Weak Decays

AJB (2001)
hep-ph/0101336
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Non-Perturbative Factors in the SM	QCD RG Factors	Short Distance Loop Functions (Penguins, Boxes)	New Flavour-Changing Parameters

Represent different Dirac and Colour Stuctures

$$A(\text{Decay}) = B_i \ \eta^i_{QCD} \ V^i_{CKM} \left[F^i_{SM} + F^i_{New}\right] + B^{New}_i \left[\eta^i_{QCD}\right]^{New} V^i_{New} \left[G^i_{New}\right]$$

(Summation over i)

$$A(K^+ \to \pi^+ \nu\bar{\nu}) = B_+ \left[\lambda_c \tilde{P}_c + \lambda_t X(v)\right]$$
$$A(K_L \to \pi^0 \nu\bar{\nu}) = B_L \ \mathrm{Im}(\lambda_t X(v))$$

$$\lambda_c = V^*_{cs} V_{cd}$$
$$\lambda_t = V^*_{ts} V_{td}$$

$$B_+, B_L \ \text{ from } \ K^+ \to \pi^0 e^+ \nu$$

$$X(v) = |X(v)| e^{i\theta_x}$$

v = parameters (m_t, ...)

Pure Short Distance Dynamics

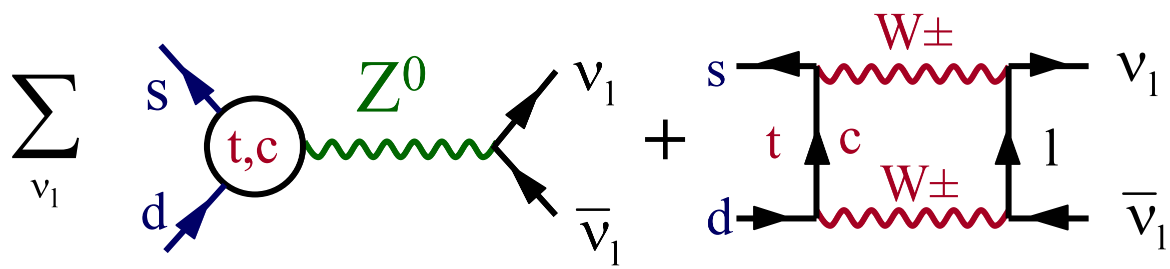
AJB, Romanino, Silvestrini (98)

$$\theta_x = \begin{cases} 0 & \text{SM} \\ 0, \pi & \text{MFV} \end{cases}$$

(AJB, Fleischer)

$$\boxed{K^+ \to \pi^+ \nu \bar{\nu}}$$

$$\boxed{V_{td}}$$



+ QCD Corrections:

Dib, Dunietz, Gilman LO (91)

Buchalla + AJB NLO (94)

$x_t = \overline{m}_t^2 / M_W^2$

$$H_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \sum_{\nu_l} \left(\lambda_c X_{NL}^l + \lambda_t X(x_t) \right) Q$$

$$\lambda_c = V_{cs}^* V_{cd} \qquad \lambda_t = V_{ts}^* V_{td} \qquad Q = (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}$$

$$X(x_t) = X_0(x_t) + \frac{\alpha_s}{4\pi} X_1(x_t) \equiv \eta_x X_0(x_t)$$

$$X_1(x_t) = \tilde{X}_1(x_t) + \underbrace{8x_t \frac{\partial X_0(t)}{\partial x_t} \ln \frac{\mu_t^2}{M_W^2}}$$

Cancels μ_t-dependence in $X_0(x_t(\mu_t))$

$$\boxed{\overline{m}_t(\mu_t)}$$

For $\mu_t \cong m_t$

$\eta_x = 0.995$

$$\boxed{X_{NL} \approx 10^{-3}}$$

$$\boxed{X(x_t) = 0.65 \cdot x_t^{0.575}}$$

$$\boxed{\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right)}$$

- **Take:** $$H_{eff}\left(K^+ \to \pi^0 e^+ \nu\right) = \frac{G_F}{\sqrt{2}} V_{us}^* (\bar{s}u)_{V-A} (\bar{\nu}e)_{V-A}$$
- **Use: Isospin Symmetry**

$$\left\langle \pi^+ \middle| (\bar{s}d)_{V-A} \middle| K^+ \right\rangle = \sqrt{2} \left\langle \pi^0 \middle| (\bar{s}u)_{V-A} \middle| K^+ \right\rangle$$

- **For single ν with $\mathbf{m_{\pi^+} = m_{\pi^0}}$**

$$\boxed{\frac{\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right)}{\mathrm{Br}\left(K^+ \to \pi^0 e^+ \nu\right)} = \frac{\alpha^2}{V_{us}^2\, 2\pi^2 \sin^4\theta_W} \left|\lambda_C X_{NL} + \lambda_t X(x_t)\right|^2}$$

- **Include Isospin breaking corrections** Marciano+ Parsa (95)

$$\left.\begin{array}{l} m_{\pi^+} \neq m_{\pi^0} \\ \text{Isospin violation in } K\to\pi \text{ formfactors} \\ \text{Electromagnetic corrections affecting} \\ \bar{s} \to \bar{u} e^+ \nu \ \text{ but not } \bar{s} \to \bar{d}\nu\bar{\nu} \end{array}\right\} \Rightarrow$$

Additional Factor

$$\boxed{r_{K^+} = 0.901}$$

Summing over 3 ν's

$$\mathrm{Br}(K^+) = \kappa_+\left[\left(\frac{\mathrm{Im}\,\lambda_t}{\lambda^5}X(x_t)\right)^2 + \left(\frac{\mathrm{Re}\,\lambda_c}{\lambda}P_c + \frac{\mathrm{Re}\,\lambda_t}{\lambda^5}X(x_t)\right)^2\right]$$

$$\kappa_+ = r_{K^+}\frac{3\alpha^2\mathrm{Br}\left(K^+\to\pi^0 e^+\nu\right)}{2\pi^2\sin^4\theta_W}\lambda^8 = 4.84\cdot 10^{-11}\left[\frac{\lambda}{0.224}\right]^8$$

$$\alpha = 1/128;\quad \sin^2\theta_W = 0.23;\quad \mathrm{Br}\left(K^+\to\pi^0 e^+\nu\right) = 4.87\cdot 10^{-2}$$

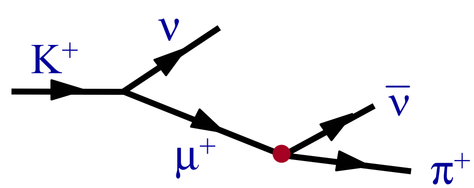
$$P_c = \frac{1}{\lambda^4}\left[\frac{2}{3}X^e_{NL} + \frac{1}{3}X^\tau_{NL}\right] = 0.39 \pm \overbrace{0.07}^{(m_c,\mu_c)}$$

			LO		NLO
μ_c, μ_t Uncertainty Br, $\lvert V_{td}\rvert$	:	Br	$\pm$ 22%	➡	$\pm$ 7%
		$\lvert V_{td}\rvert$	$\pm$ 14%		$\pm$ 4%

$$\left\{\overline{m}_t(\mu_t), \overline{m}_c(\mu_c):\ 100\ \mathrm{GeV}\le\mu_t\le 300\ \mathrm{GeV};\ 1\ \mathrm{GeV}\le\mu_c\le 3\ \mathrm{GeV}\right\}$$

LD Effects < 5%
Rein, Sehgal
Hagelin, Littenberg
Lu, Wise; Fajfer

Smallness of LD related to absence of internal γ contributions (present in $K_L\to\pi^0 e^+ e^-, K_L\to\mu\bar{\mu}$)



$$\boxed{Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)}$$

- **Consider one ν-flavour and denote:**

$$F \equiv \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W}\left(\lambda_c X_{NL} + \lambda_t X(x_t)\right)$$

$$\boxed{H_{eff} = F(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} + F^*(\bar{d}s)_{V-A}(\bar{\nu}\nu)_{V-A}}$$

- **Now:**

$$\boxed{K_L = \frac{1}{\sqrt{2}}\left((1+\bar{\varepsilon})\left|K^0\right\rangle + (1-\bar{\varepsilon})\left|\bar{K}^0\right\rangle\right)}$$

$$CP\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle; \qquad C\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle$$

- $$\boxed{\begin{aligned} A\left(K_L \to \pi^0 \nu\bar{\nu}\right) = \frac{1}{\sqrt{2}}\Big(&F(1+\bar{\varepsilon})\left\langle\pi^0\right|(\bar{s}d)_{V-A}\left|K^0\right\rangle \\ &+F^*(1-\bar{\varepsilon})\left\langle\pi^0\right|(\bar{d}s)_{V-A}\left|\bar{K}^0\right\rangle\Big)(\bar{\nu}\nu)_{V-A}\end{aligned}}$$

- $$\left\langle \pi^0 \middle| \left(\bar{d}s\right)_{V-A} \middle| \overline{K}^0 \right\rangle = -\left\langle \pi^0 \middle| \left(\bar{s}d\right)_{V-A} \middle| K^0 \right\rangle$$

$$\boxed{A\left(K_L \to \pi^0 \nu\bar{\nu}\right) = \frac{1}{\sqrt{2}} \underbrace{\left[F\left(1+\bar{\varepsilon}\right) - F^*\left(1-\bar{\varepsilon}\right)\right]}_{\approx 2\,\mathrm{Im}\,\lambda_t X(x_t)} \left\langle \pi^0 \middle| \left(\bar{s}d\right)_{V-A} \middle| K^0 \right\rangle \cdot \left(\bar{\nu}\nu\right)_{V-A}}$$

$$\left(\mathrm{Im}\,\lambda_c = -\mathrm{Im}\,\lambda_t\right)$$
$$\left(X_{NL} \ll X\left(x_t\right)\right)$$

- $$\left\langle \pi^0 \middle| \left(\bar{s}d\right)_{V-A} \middle| K^0 \right\rangle = \left\langle \pi^0 \middle| \left(\bar{s}u\right)_{V-A} \middle| K^+ \right\rangle$$

- {Isospin Breaking (Marciano, Parsa)} ⇨ $\left\{ r_{K_L} = 0.944 \right\}$

- **Summing over ν**

$$\boxed{Br\left(K_L \to \pi^0 \nu\bar{\nu}\right) = \kappa_{K_L} \left[\frac{\mathrm{Im}\,\lambda_t}{\lambda^5} X\left(x_t\right)\right]^2}$$ ★

$$\kappa_{K_L} = \frac{\tau_{K_L}}{\tau_{K^+}} \frac{r_{K_L}}{r_{K^+}} \kappa_{K^+} = 2.12 \cdot 10^{-10} \left[\frac{\lambda}{0.224}\right]^8$$

Waiting for Precise Measurements of $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

AJB, Schwab, Uhlig (04)

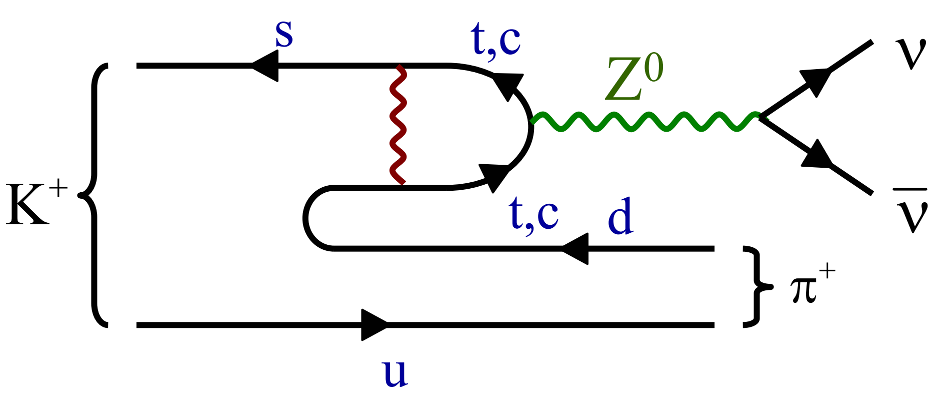
1. Present Status within SM (TH and Parametric Uncertainties)

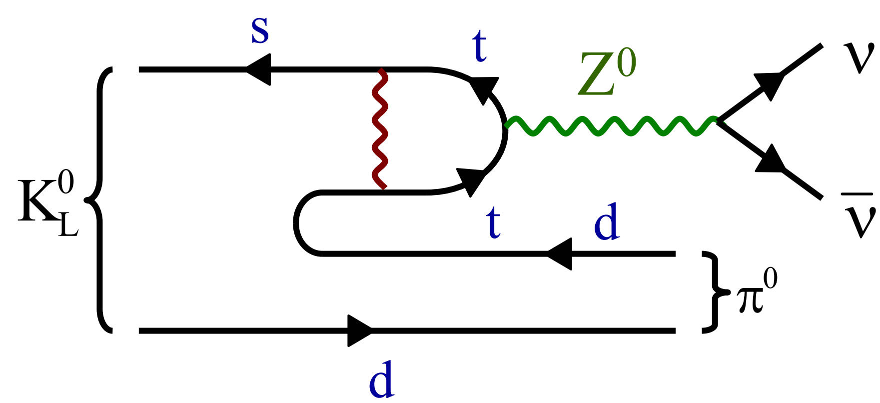
2. Impact of Present and Future Measurements of $K \to \pi\nu\bar{\nu}$ on CKM

3. $K \to \pi\nu\bar{\nu}$ in Scenarios with New Complex Phases in EWP and $B_d^0 - \bar{B}_d^0$ Mixing

4. Interplay of $K \to \pi\nu\bar{\nu}$ with
   - ~~CP~~ in B Decays
   - $\Delta M_d / \Delta M_s$, Rare Decays

Decays $K \to \pi\nu\bar{\nu}$





Isospin Symmetry

$$\left\langle \pi^+ \middle| (\bar{s}d)_{V-A} \middle| K^+ \right\rangle = \sqrt{2} \left\langle \pi^0 \middle| (\bar{s}u)_{V-A} \middle| K^+ \right\rangle$$

$$\left\langle \pi^0 \middle| (\bar{s}d)_{V-A} \middle| K^0 \right\rangle = \left\langle \pi^0 \middle| (\bar{s}u)_{V-A} \middle| K^+ \right\rangle$$

Leading Decay: $\boxed{K^+ \to \pi^0 e^+ \nu}$

<u>Isospin Breaking:</u>

Marciano, Parsa (Suppression)
K^+(10%) K_L(5%)

<u>Long Distance:</u>

K^+: +(6±2)% Isidori, Mescia, Smith (2005)
K_L: ≤1% Buchalla, Isidori

Express Review of $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

AJB
Schwab
Uhlig
hep-ph/0405132

NLO: Buchalla + AJB (94); NNLO: AJB, Gorbahn, Haisch, Nierste (05)

SM: $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (8.0 \pm 1.1) \cdot 10^{-11}$ $Br(K_L \to \pi^0 \nu\bar{\nu}) = (2.8 \pm 0.6) \cdot 10^{-11}$

Exp: $Br(K^+ \to \pi^+ \nu\bar{\nu}) = \left(14.7 {}^{+13.0}_{-8.9}\right) \cdot 10^{-11}$ $Br(K_L \to \pi^0 \nu\bar{\nu}) < 5.9 \cdot 10^{-7}$ (KTeV)

Brookhaven: E787, E949
(CKM, NA48, JPARC, ..)

Soon improved by E391a !!!
(J-PARC, ...)

$2.9 \cdot 10^{-7}$
90% C.L.

TH very clean : $\left(\begin{matrix}\text{With improved} \\ \text{CKM parameters} \\ \sim 2008\end{matrix}\right)$ $\Rightarrow$

$\sigma\left(Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right)\right) < 5\%$

$\sigma\left(Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)\right) < 5\%$

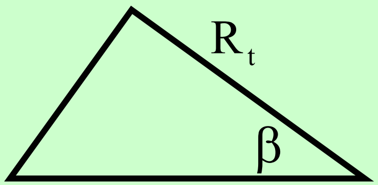
Very clean determination of Unitarity Triangle

$\sigma(Br) \cong 10\%$	$\sigma(\sin 2\beta \cong 0.04)$	$\sigma(\gamma) = 9^\circ$	$\sigma(\lvert V_{td}\rvert) = 7\%$
$\sigma(Br) \cong 5\%$	$\sigma(\sin 2\beta \cong 0.025)$	$\sigma(\gamma) = 5^\circ$	$\sigma(\lvert V_{td}\rvert) = 4\%$

Basic Formulae for $K^+ \to \pi^+ \nu\bar{\nu}$ (SM)

$$Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = 4.8 \cdot 10^{-11}\left[A^4 R_t^2 X^2 + 2 P_c A^2 R_t X \cos\beta + P_c^2\right]$$

$$X \equiv X(m_t)$$



$$= 10^{-11}\left[\underset{\text{(top)}}{4.2} + \underset{\text{(top-charm)}}{3.1} + \underset{\text{(charm)}}{0.7}\right]$$

$$A = \frac{|V_{cb}|}{\lambda^2} \cong 0.83$$

Buchalla AJB (94) NLO)

$$P_c = 0.367 \pm \underbrace{0.033}_{\Delta m_c = 50\,\mathrm{MeV}} \pm \underbrace{0.037}_{\text{theory}} \pm \underbrace{0.009}_{\alpha_s} \cong 0.37 \pm 0.07$$

BGHN NNLO (05)

$$P_c = 0.371 \pm 0.031_{m_c} \pm \underbrace{0.009}_{\text{theory}} \pm \underbrace{0.009}_{\alpha_s} = 0.37 \pm 0.04$$

$$Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right)_{SM} = \left[8.0 \pm \underbrace{0.5}_{P_c} \pm \underbrace{0.8}_{\text{CKM}}\right] 10^{-11} \cong (8.0 \pm 1.1) 10^{-11}$$

$$Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = \left[14.7\,{}^{+13.0}_{-8.9}\right] 10^{-11}$$

E787 (2)
E949 (1)
<u>3 Events</u>

QCD Corrections to $K \to \pi\nu\bar{\nu}$

LO | NLO | NNLO

Charm Part

$$P_c = \frac{4\pi}{\alpha_s(\mu_c)} P_c^{(0)} + P_c^{(1)} + \frac{\alpha_s(\mu_c)}{4\pi} P_c^{(3)}$$

$$\mu_c = 0(m_c)$$

Vainshtein, Zakharov, Novikov
Shifman (1977)
Ellis, Hagelin (1983)
Dib, Dunietz, Gilman (1991)

Buchalla
AJB
(1994)

AJB
Gorbahn
Haisch
Nierste
(2005)

$$x_t = \frac{m_t^2(\mu_t)}{M_W^2}$$

Top Part

$$X^{SM}(x_t) = X_0(x_t) + \frac{\alpha_s(\mu_t)}{4\pi} X_1(x_t)$$

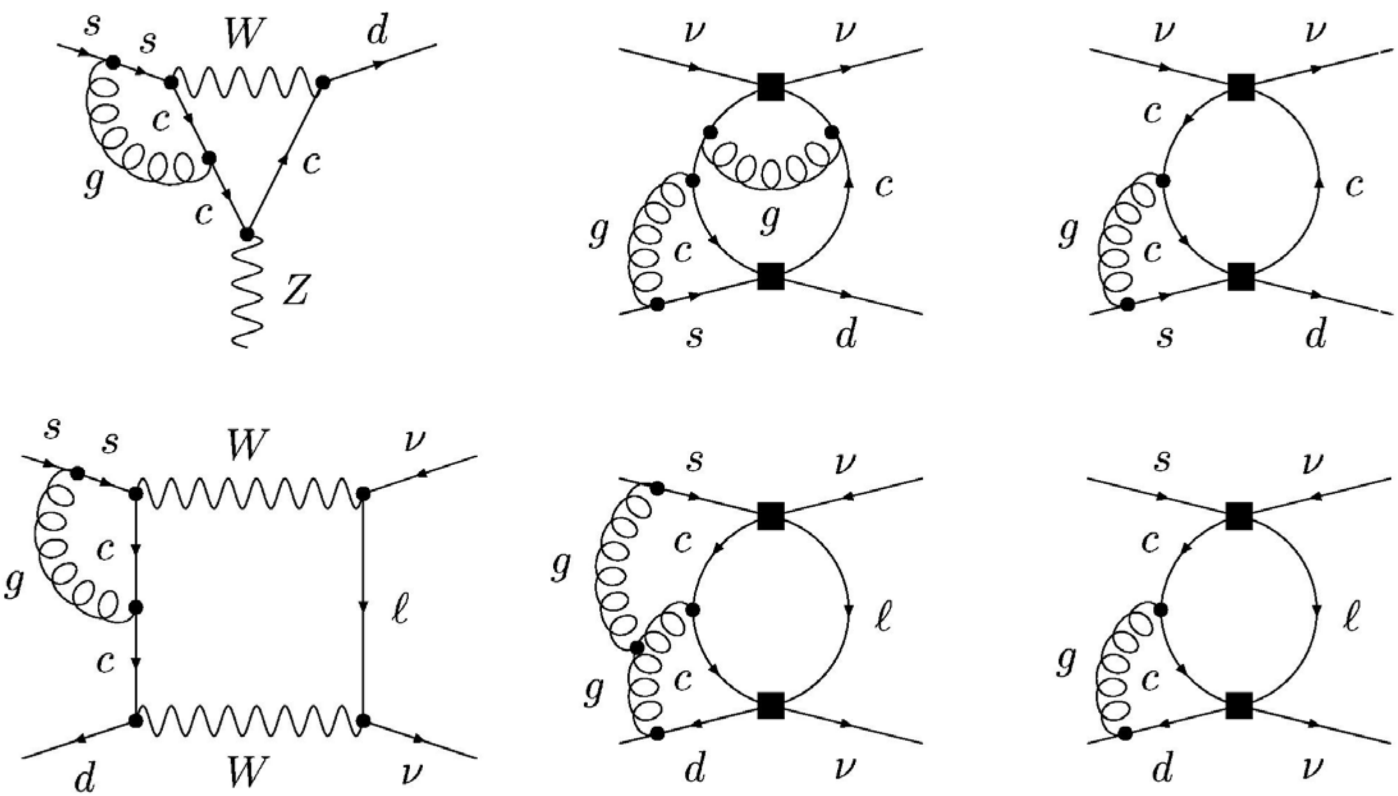
Inami, Lim (81)
AJB (81)

Buchalla, AJB (93)
Misiak, Urban (98)

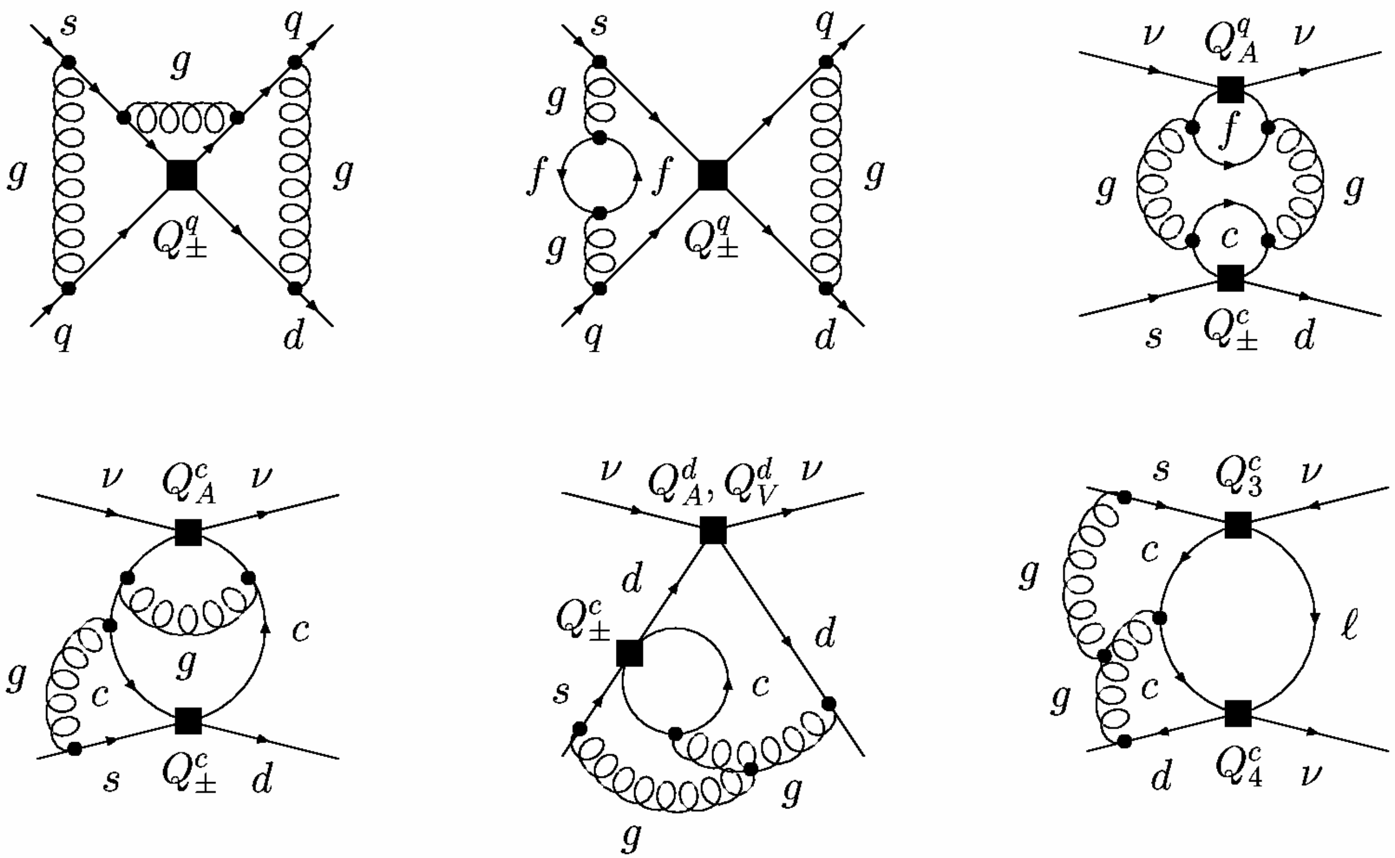
$C_i\ (\mu_w)$

3-Loop Anomalous Dimensions

Matrix Elements

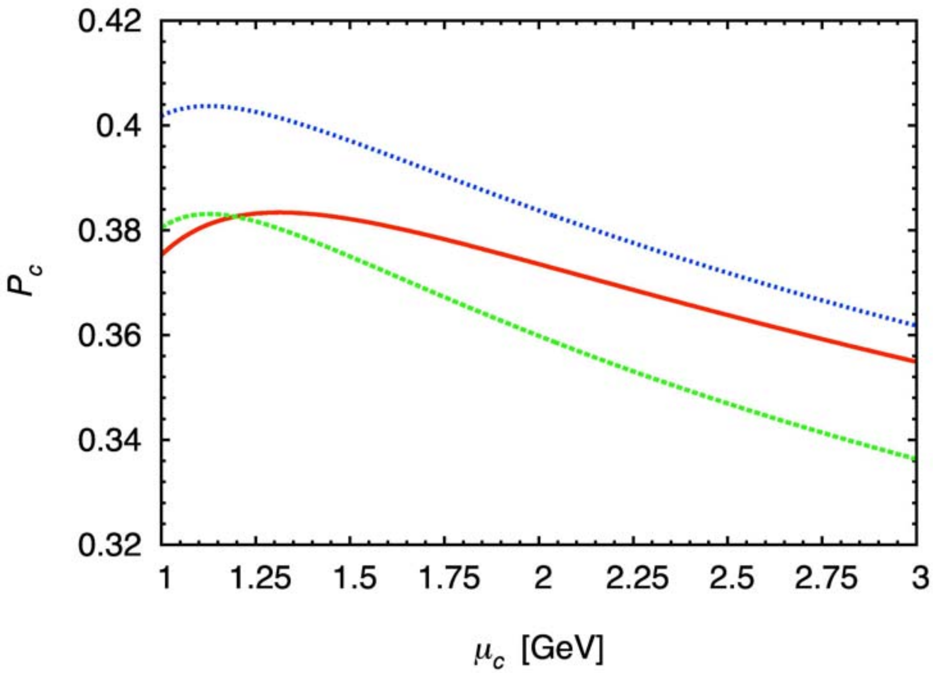


No Comments

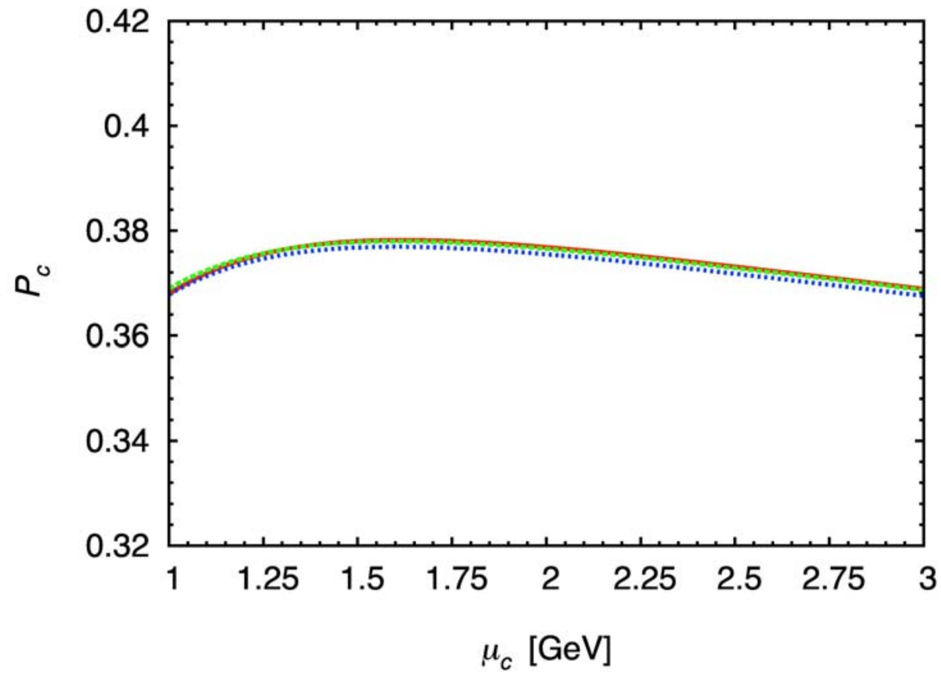


s q g g g $Q^q_\pm$ q d
s q g f f $Q^q_\pm$ g g q d
ν Q^q_A ν f g g c s $Q^c_\pm$ d
ν Q^c_A ν c g g c s $Q^c_\pm$ d
ν Q^d_A, Q^d_V ν d $Q^c_\pm$ d s c g g
s Q^c_3 ν c g ℓ g c d Q^c_4 ν

$P_c(\mu_c)$ for various calculations of $\alpha_s(\mu_c)$ from $\alpha_s(M_Z)$

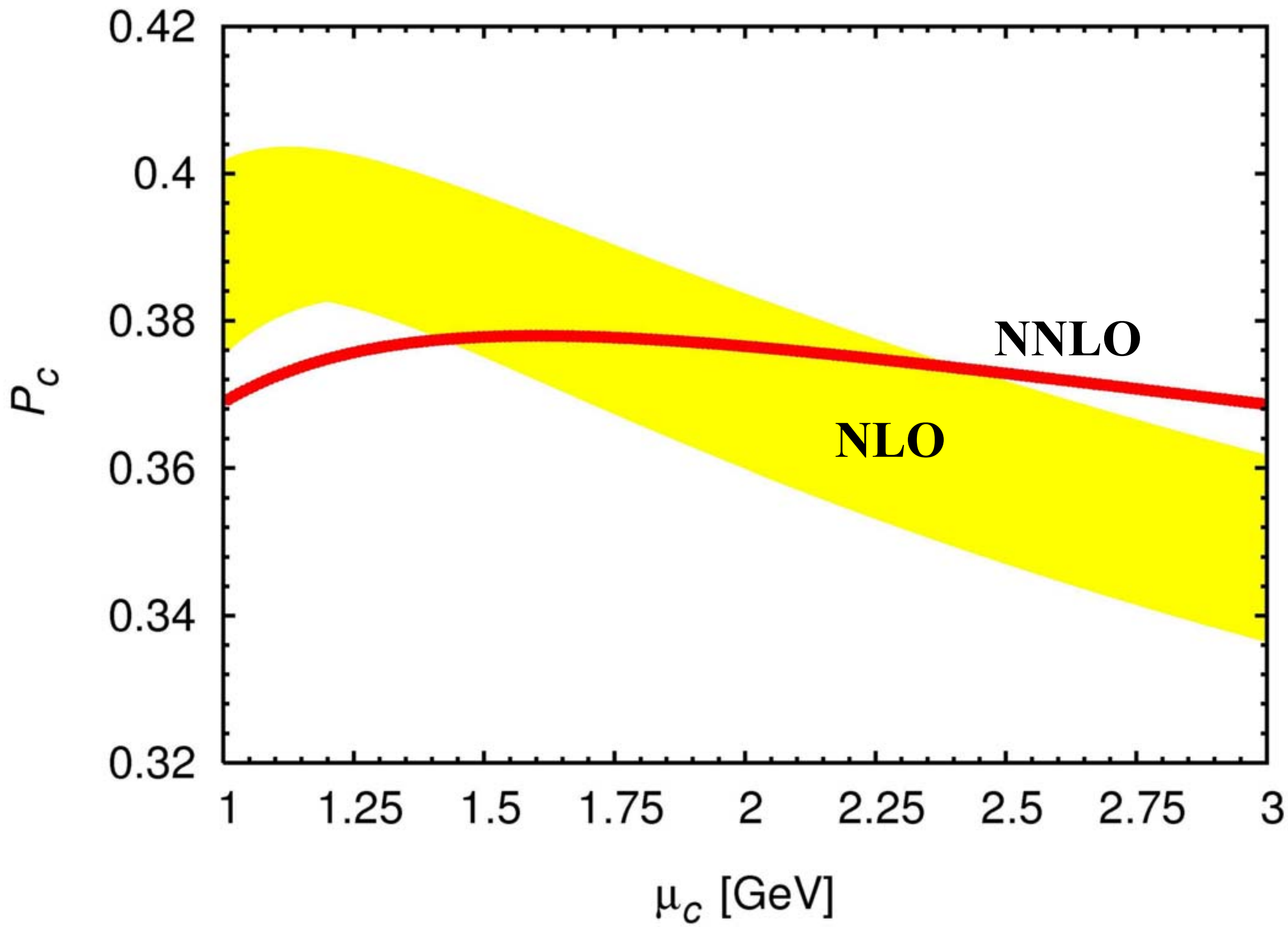


NLO



NNLO

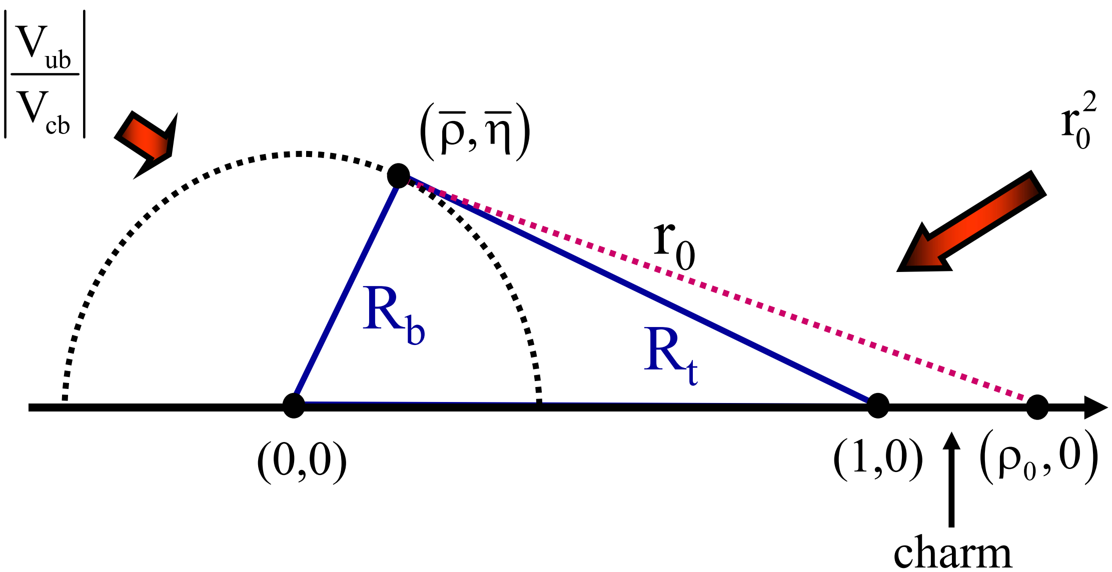
Reduction of TH Error in P_c



$K^+ \to \pi^+ \nu\bar{\nu}$ in the $(\bar{\rho}, \bar{\eta})$ Plane

$$\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = 4.31 \cdot 10^{-11} A^4 X^2(m_t) \frac{1}{\sigma}\left[\left(\sigma\bar{\eta}\right)^2 + \left(\rho_0 - \bar{\rho}\right)^2\right]$$

$$\sigma = \frac{1}{\left(1 - \lambda^2/2\right)^2} \qquad \rho_0 = 1 + \frac{P_c}{A^2 X(m_t)} \approx 1.4$$



$$r_0^2 = \frac{1}{A^4 X^2(m_t)}\left[\frac{\sigma \mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right)}{4.31 \cdot 10^{-11}}\right]$$

$$R_t = 1 + R_b^2 - 2\bar{\rho}$$

$$V_{td} = A\lambda^3\left(1 - \bar{\rho} - i\bar{\eta}\right)$$

$$|V_{td}| = \lambda |V_{cb}| R_t$$

Anatomy of $|V_{td}|$ from $K^+ \to \pi^+ \nu\bar{\nu}$

AJB
Schwab
Uhlig

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = 0.39\,\frac{\sigma(P_c)}{P_c} + 0.70\,\frac{\sigma(Br(K^+))}{Br(K^+)} + \frac{\sigma(|V_{cb}|)}{|V_{cb}|}$$

Present:	$\pm 4\%$	$\pm$ (Very Large)	$\pm 2\%$	
$\left.\begin{array}{l}\sigma(Br(K^+)) = 10\% \\ \sigma(P_c) = 0.03\end{array}\right\}$	$\pm 3\%$	$\pm 7\%$	$\pm 1.4\%$	(Scenario I)
$\left.\begin{array}{l}\sigma(Br(K^+)) = 5\% \\ \sigma(P_c) = 0.02\end{array}\right\}$	$\pm 2\%$	$\pm 3.5\%$	$\pm 1\%$	(Scenario II)

Determination
at 4-5% possible

Theoretically clean Relations

D´Ambrosio + Isidori (02)

$$\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = \bar{\kappa}_+ \left|V_{cb}\right|^4 X^2 \left[R_t^2 \sin^2\beta + \left(R_t \cos^2\beta + \frac{\lambda^4 P_c}{\left|V_{cb}\right|^2 X} \right)^2 \right]$$

$$R_t \sim \xi \frac{\sqrt{\Delta M_d}}{\sqrt{\Delta M_s}} \qquad \bar{\kappa}_+ = 7.64 \cdot 10^{-6} \qquad P_c = 0.37 \pm 0.04$$

AJB, Schwab, Uhlig (04)

$$\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = \bar{\kappa}_+ \left|V_{cb}\right|^4 X^2 \left[T_1^2 + \left(T_2 + \frac{\lambda^4 P_c}{\left|V_{cb}\right|^2 X} \right)^2 \right]$$

$$T_1 = \frac{\sin\beta \sin\gamma}{\sin(\beta+\gamma)} \qquad T_2 = \frac{\cos\beta \sin\gamma}{\sin(\beta+\gamma)}$$

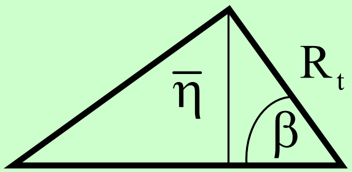
(Direct $\cancel{C}P$)

Basic Formulae for $K_L \to \pi^o \nu\bar{\nu}$

(SM)

Buchalla
AJB (NLO)

$$Br\left(K_L \to \pi^o \nu\bar{\nu}\right) = 2.8 \cdot 10^{-11} \left[\frac{\bar{\eta}}{0.35}\right]^2 \left[\frac{|V_{cb}|}{41.5 \cdot 10^{-3}}\right]^4 \left[\frac{X}{1.48}\right]^2$$



$$= \left(2.8 \pm \overbrace{0.6}^{\text{CKM}}\right) \cdot 10^{-11}$$

(AJB
Schwab
Uhlig)

E391a : $Br\left(K_L \to \pi^o \nu\bar{\nu}\right) < 2.9 \cdot 10^{-7}$

Future: E391a, JHF

Model independent bound (Grossman, Nir)

$$Br\left(K_L \to \pi^o \nu\bar{\nu}\right) \le 4.4\ Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \le 1.4 \cdot 10^{-9}\ (90\%\,C.L.)$$

E391a could get the first non-trivial upper bound.

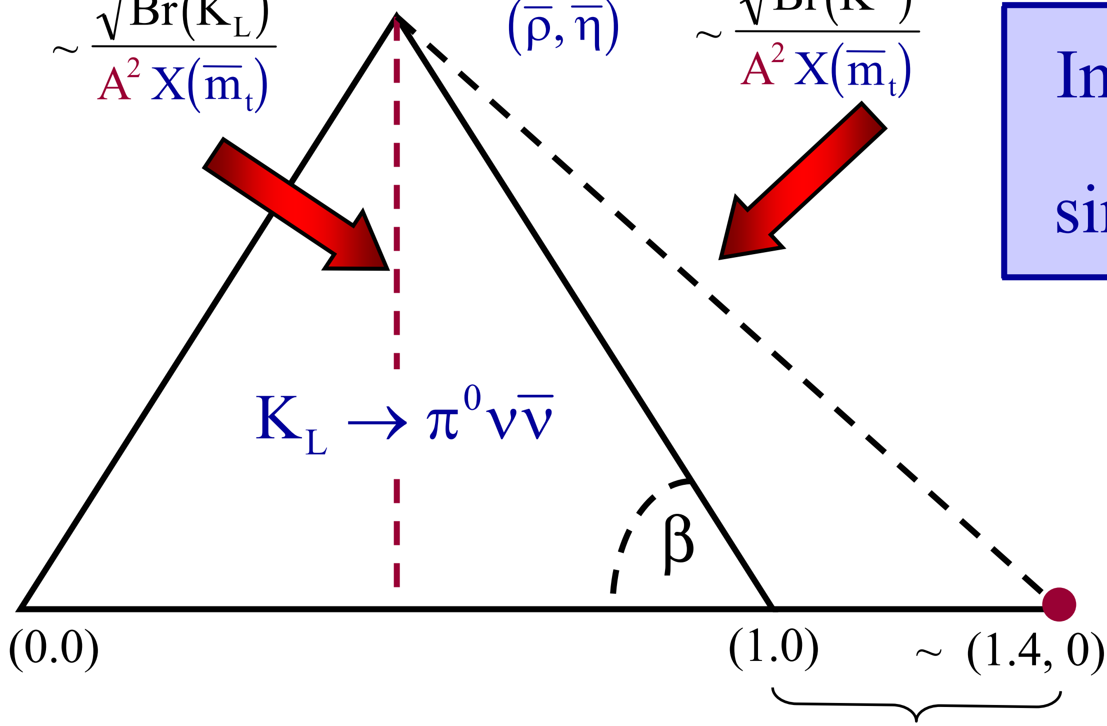
$$\boxed{\bar{\eta} = R_t \sin\beta}$$

E391 (JHF): ~ 1000 Events

UT from $K \to \pi\nu\bar{\nu}$

Buchalla
AJB

$\sim \dfrac{\sqrt{Br(K_L)}}{A^2 X(\overline{m}_t)}$ $(\bar{\rho}, \bar{\eta})$ $\sim \dfrac{\sqrt{Br(K^+)}}{A^2 X(\overline{m}_t)}$



$$\mathrm{Im}\,\lambda_t = F_1\left(\overline{m}_t, Br(K_L)\right)$$

$$\sin 2\beta = F_2\left(P_c, Br(K_L), Br(K^+)\right)$$

$$\lambda_t = V_{ts}^* V_{td}$$

$\sin 2\beta$ $(K \to \pi\nu\bar{\nu})$ $\Longleftrightarrow$ $\sin 2\beta$ $(B \to J/\psi, K_s)$ $\to \varphi K_s$

K-Physics $\Longleftrightarrow$ B - Physics

$[\, X(\overline{m}_t) \sim \overline{m}_t^{\,1.15} \,]$

$(0.37 \pm 0.04) \Rightarrow P_c$

$\sim \dfrac{P_c}{A^2 X(\overline{m}_t)}$

Test of SM and Beyond

The Angle β from $K \to \pi\nu\bar{\nu}$

Buchalla, AJB (94)
AJB, Schwab, Uhlig (04)

BSU:

$$\frac{\sigma(\sin 2\beta)}{\sin 2\beta} = 0.31\frac{\sigma(P_c)}{P_c} + 0.55\frac{\sigma(Br(K^+))}{Br(K^+)} \pm 0.39\frac{\sigma(Br(K_L))}{Br(K_L)}$$

$\sigma(\sin 2\beta) =$	± 0.041	$\pm ?$	$\pm ?$	(Present)
$\sigma(\sin 2\beta) =$	0.017	± 0.039	± 0.028	(Scenario I) Br´s at 10%
$\sigma(\sin 2\beta) =$	0.011	± 0.020	± 0.014	(Scenario II) Br´s at 5%

TH very clean

$\sigma(\sin 2\beta) \approx 0.02 - 0.03$ requires $\sigma(Br'\mathrm{s}) \leq 5\%$

The Angle γ from $K \to \pi\nu\bar{\nu}$

AJB, Schwab, Uhlig (04)

$$\frac{\sigma(\gamma)}{\gamma} = 0.75\frac{\sigma(P_c)}{P_c} + 1.32\frac{\sigma\left(Br\left(K^+\right)\right)}{Br\left(K^+\right)} + 0.07\frac{\sigma\left(Br\left(K_L\right)\right)}{Br\left(K_L\right)} + 4.1\frac{\sigma\left(\left|V_{cb}\right|\right)}{\left|V_{cb}\right|}$$

$\sigma(\gamma) =$	$\pm 8.3^\circ$	$\pm\,?$	$\pm\,?$	$\pm 4.9^\circ$	(Present)
$\sigma(\gamma) =$	$\pm 3.7^\circ$	$\pm 8.5^\circ$	$\pm 0.4^\circ$	$\pm 3.8^\circ$	(Scenario I) Br´s at 10%
$\sigma(\gamma) =$	$\pm 2.5^\circ$	$\pm 4.2^\circ$	$\pm 0.2^\circ$	$\pm 2.5^\circ$	(Scenario II) Br´s at 5%

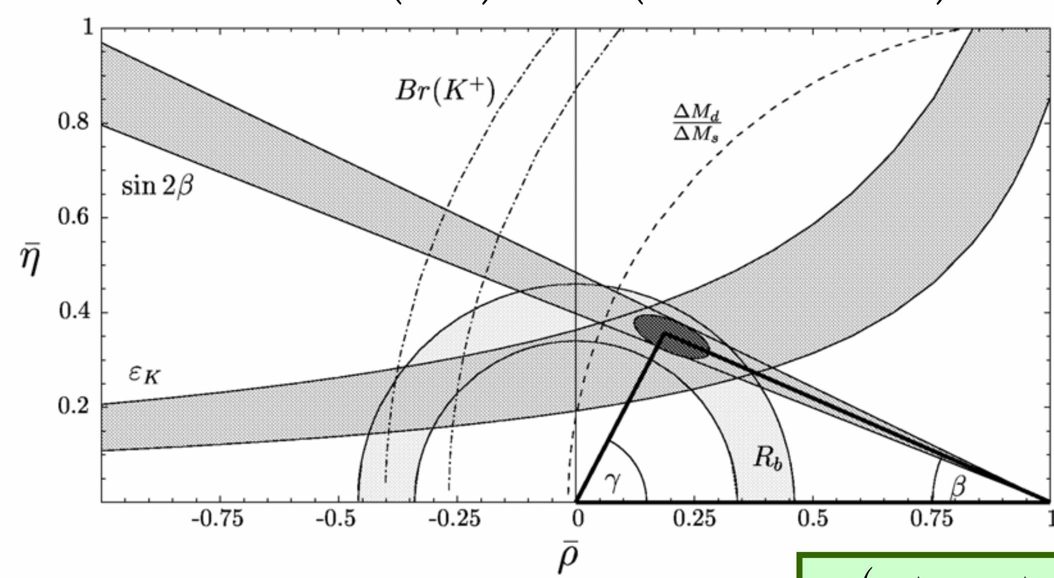
TH very clean

$\sigma(\gamma) \approx \pm 5^\circ$ requires $\sigma\left(Br\left(K^+\right)\right) \leq 5\%$

Unitarity Triangle 2004

(AJB, Schwab, Uhlig)

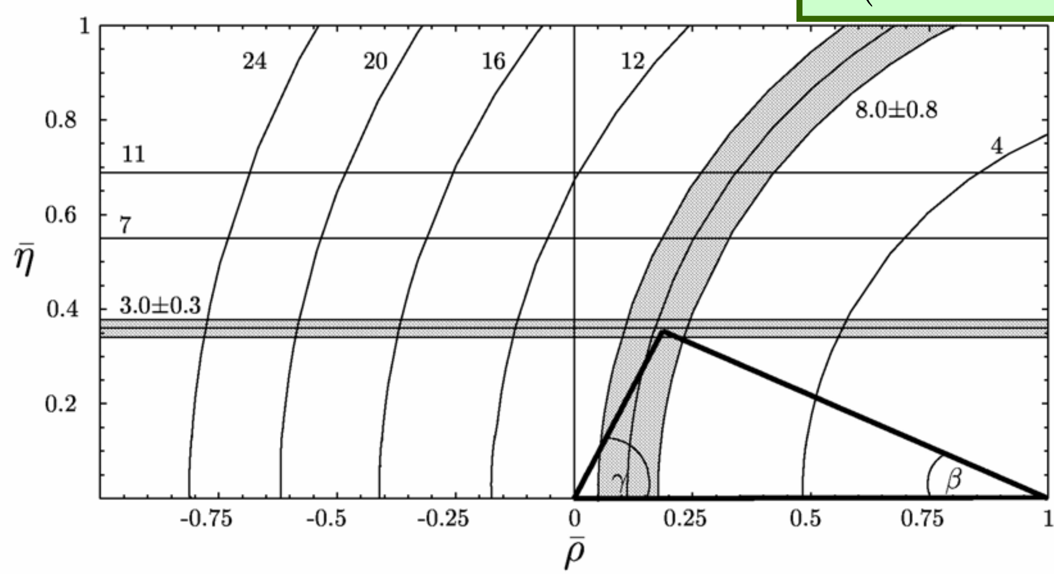
$$Br(K^+) \equiv Br(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = 14.7 \cdot 10^{-11}$$



$$P_c = \underline{\underline{0.37}} \pm 0.04$$

m_c, V_{cb}, μ_c

$Br(K^+ \rightarrow \pi^+ \nu\bar{\nu})$



$Br(K_L \rightarrow \pi^0 \nu\bar{\nu})$

Unitarity Triangle from $K \rightarrow \pi\nu\bar{\nu}$

(2012)

6.

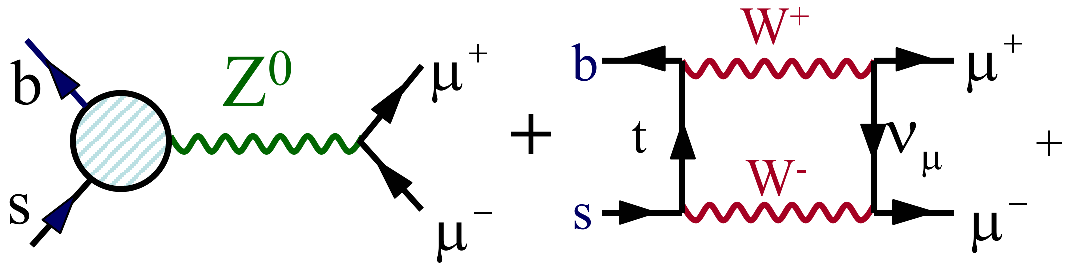
Rare B and K Decays

$$B_{s,d} \to \mu^+\mu^- \qquad B \to X_{s,d}\,\nu\bar{\nu}$$

$$B \to X_s\gamma \qquad B \to X_s l^+ l^-$$

$$K_L \to \pi^0 l^+ l^-$$

$$B_s \to \mu^+\mu^-$$



+ QCD Corrections

Buchalla, AJB (93)
Misiak, Urban (98)

$$Br(B_s \to \mu^+\mu^-) = 3.8 \cdot 10^{-9} \left[\frac{\tau(B_s)}{1.46\text{ps}}\right]\left[\frac{F_{B_s}}{230\text{MeV}}\right]^2 \left[\frac{|V_{ts}|}{0.040}\right]^2 [Y(x_t)]^2$$

$$Y(x_t) = 1.02 \left[\frac{m_t(m_t)}{170\text{GeV}}\right]^{1.56} \approx 1 \qquad F_{B_s} = (230 \pm 30)\text{MeV}$$

$$\tau(B_s) = (1.46 \pm 0.05)\text{ps}$$

(Dominant uncertainty)

SM: $Br(B_s \to \mu^+\mu^-) = (3.7 \pm 1.0) \cdot 10^{-9}$

$Br(B_s \to \mu^+\mu^-) < 5 \cdot 10^{-7}$

DØ, CDF 95% C.L.

$$\boxed{B_d \to \mu^+\mu^-}$$ (just replace s→d)

$$\mathrm{Br}\left(B_d \to \mu^+\mu^-\right) = 1.0\cdot 10^{-10}\left[\frac{\tau(B_d)}{1.54\,\mathrm{ps}}\right]\left[\frac{F_{B_d}}{190\,\mathrm{MeV}}\right]^2\left[\frac{|V_{td}|}{0.008}\right]^2\left[Y(x_t)\right]^2$$

$$\tau(B_d) = (1.540 \pm 0.014)\,\mathrm{ps}$$

$$F_{B_d} = (189 \pm 27)\,\mathrm{MeV}$$

$$|V_{td}| = (8.24 \pm 0.54)\cdot 10^{-3}$$

SM: $\mathrm{Br}\left(B_d \to \mu^+\mu^-\right) = (1.04 \pm 0.34)\cdot 10^{-10}$

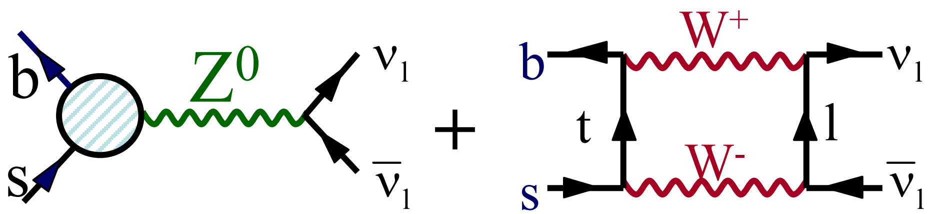
Belle: $\mathrm{Br}\left(B_d \to \mu^+\mu^-\right) < 1.6\cdot 10^{-7} \quad (95\%\ \mathrm{C.L.})$

$$\frac{F_{B_s}}{F_{B_d}} = 1.22 \pm 0.06$$

$$\frac{\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)}{\mathrm{Br}\left(B_d \to \mu^+\mu^-\right)} = \frac{\tau(B_s)}{\tau(B_d)}\frac{m_{B_s}}{m_{B_d}}\left[\frac{F_{B_s}}{F_{B_d}}\right]^2\left[\frac{|V_{ts}|}{|V_{td}|}\right]^2$$

Useful measurement of $|V_{td}|$

$B \to X_s \nu\bar{\nu}$

$\sum_{\nu_l}$ 

+ QCD Corrections

Buchalla, AJB (93)
Misiak, Urban (98)

$$Br\left(B \to X_s \nu\bar{\nu}\right) = 1.58 \cdot 10^{-5} \left[\frac{|V_{ts}|}{0.040}\right]^2 \left[X(x_t)\right]^2$$

$$\left[X(x_t)\right] = 1.57 \left(\frac{m_t(m_t)}{170\,\mathrm{GeV}}\right)^{1.15}$$

SM: $Br\left(B \to X_s \nu\bar{\nu}\right) = \left(3.66 \pm 0.21\right) \cdot 10^{-5}$

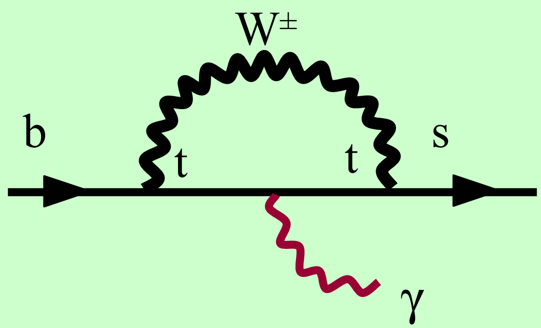
ALEPH: $Br\left(B \to X_s \nu\bar{\nu}\right) < 6.4 \cdot 10^{-4}$

$$\frac{Br\left(B \to X_d \nu\bar{\nu}\right)}{Br\left(B \to X_s \nu\bar{\nu}\right)} = \frac{|V_{td}|^2}{|V_{ts}|^2}$$

Theoretically cleanest measurement of $|V_{td}| / |V_{ts}|$ ★

Long Distance Effects neglegible: Buchalla, Isidori, Rey

$B \to X_s \gamma$



+ Large SD QCD Corrections

Bertolini, Borzumati, Masiero (1987)
Deshpande, Lo, Trampetic, Eilam, Singer (1987)

γ = on shell

Dominant Operator : $Q_7 = m_b \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$

(Magnetic Penguins)

QCD Enhancement (~ 3) governed by the Mixing of Q_7 with $Q_2 = (\bar{b}c)_{V-A}(\bar{c}s)_{V-A}$

$$Br(B \to X_s\gamma) = \begin{cases} (3.52 \pm 0.30)\cdot 10^{-4} & \text{CLEO, BaBar, Belle} \\ (3.70 \pm 0.30)\cdot 10^{-4} & \text{SM} \end{cases}$$

Sensitive to New Physics! Important for constraining Supersymmetry !!

NLO-QCD Corrections Saga 1994-2002

1993 : Identification of strong μ_b dependence (~ 60%): Ali, Greub; AJB, Misiak, Münz, Pokorski

Initial Conditions : Adel, Yao (93); Greub, Hurth (97); AJB, Kwiatkowski, Pott (97)
Ciuchini, Degrassi, Gambino, Giudice (97)

Two and Three-Loop Anomalous Dimensions : AJB, Jamin, Lautenbacher, Weisz (92); Ciuchini, Franco, Martinelli, Reina (93)
Misiak, Münz (95) (Two-Loop Mixing of Magnetic Operators)
Chetyrkin, Misiak, Münz (97) (Three-Loop Mixing between Q_7 and Q_2)

Operator Matrix Elements : Greub, Hurth, Wyler (1996)
AJB, Czarnecki, Misiak, Urban (2001)

Review:
AJB, Misiak (2003)

Gluon Bremstrahlung : Ali, Greub (91)
Pott (95)

$B \to X_s\gamma$ beyond NLO $\to$ NNLO

2001: Gambino, Misiak (significant uncertainty due to m_c)

⇒ Go beyond NLO

Considerable Progress made :

Misiak, Steinhauser (2004) — Initial Conditions

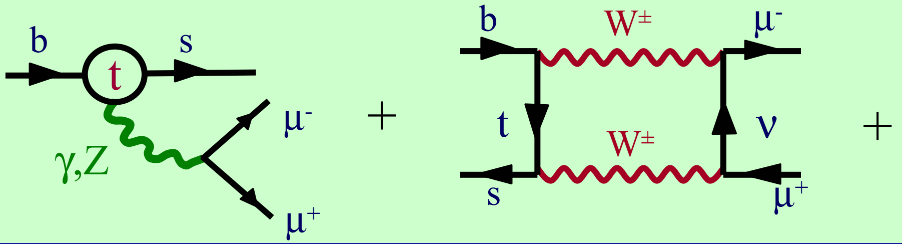
Bieri, Greub, Steinhauser (2003) — Matrix Elements (first steps)

Gorbahn, Haisch (2005)
Gorbahn, Haisch, Misiak (2005) — Three-Loop Mixing of Magnetic Penguins

4-Loop Mixing $Q_2 \leftrightarrow Q_7$: Czakon, ...

$\mathbf{B \to X_s \mu^+ \mu^-}$

Hou, Willey, Soni (87)



Smaller Magnetic Penguins

QCD (LO): Grinstein, Savage, Wise (89)

QCD (NLO): Misiak (94)
AJB + Münz (94)

Operators:

$$Q_{9V} = (\bar{s}b)_{V-A}(\bar{\mu}\mu)_V$$
$$Q_{10V} = (\bar{s}b)_{V-A}(\bar{\mu}\mu)_A$$

$+$

$$Q_7 = m_b \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$$

Known from $B \to X_s\gamma$

$$\frac{d\sigma}{ds} \approx \frac{\alpha^2}{4\pi}(1-s)^2 |V_{ts}|^2 \left[(1+2s)\left(|C_9(s)|^2 + |C_{10}|^2\right) + 4\left(1+\frac{2}{s}\right)|C_7|^2 + 12 C_7 C_9 \right]$$

$$s = \frac{\left(P_{\mu^+} + P_{\mu^+}\right)^2}{m_b^2}$$

$$C_9(s) = P_0^{NLO}(s) + \frac{Y(x_t)}{\sin^2\theta_w} - 4Z(x_t)$$

$$C_{10} = -\frac{Y(x_t)}{\sin^2\theta_w}$$

$[\;\;] = U(s)$

Recent Developments on $B \to X_s \mu^+ \mu^-$

NNLO QCD

Bobeth, Misiak, Urban (2000)
Ghinculov, Hurth, Isidori, Yao (2002-2004)
Asatryan, Asatrian, Greub, Walker (2002-2004)
Asatrian, Asatryan, Hovhannisyan, Poghosyan (2004)
Bobeth, Gambino, Gorbahn, Haisch (2004)

$$\mathrm{Br}\left(B \to X_s l^+ l^-\right) = \begin{cases} (4.5 \pm 1.0) \cdot 10^{-6} & \text{Exp} \\ (4.4 \pm 0.7) \cdot 10^{-6} & \text{SM} \end{cases}$$

(low s region)

(below $c\bar{c}$ resonance)

In order to test better look at $A_{FB}(s) \equiv$ Forward – Backward Asymmetry

(see Section 6)

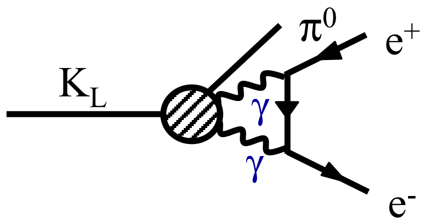
$$A_{FB}(s) = -3C_{10} \frac{\left[sC_9(s) + 2C_7\right]}{U(s)}$$

Vanishes at s_0: $s_0 C_9(s_0) + 2C_7 = 0$

Theoretically clean; sensitive to New Physics.

$$\boxed{K_L \to \pi^0 e^+ e^-}$$ (3 contributions)

① $K_L \to \pi^0\gamma\gamma \to \pi^0 e^+ e^-$ ⟸ **CP conserving**



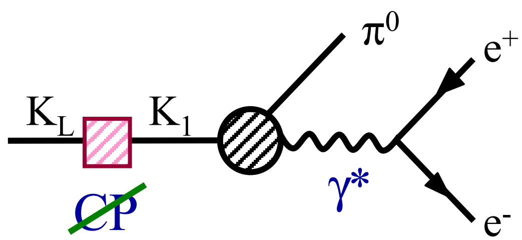
Donoghue, Holstein, Valencia, Ecker, Pich, de Rafael, Flynn, Randall, Seghal, Heiliger, Fajfer (95)
Cohen, Ecker, Pich (93)
Donoghue, Gabbiani (95)
Ambrosio, Portoles (97)

[Using KTeV (99) $K_L \to \pi\gamma\gamma$]

② $K_L \xrightarrow{K_1} \pi^0\gamma^* \to \pi^0 e^+ e^-$ ⟸ indirect ~~CP~~

$$\boxed{K_L \cong K_2 + \varepsilon K_1}$$

★ ($K_S \to \pi^0 e^+ e^-$ helps here!)

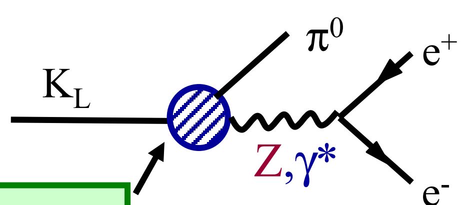


Ecker, Pich, de Rafael (91)
Heiliger, Seghal (93)
Donoghue, Gabbiani (95)
Fajfer (95)

③ $K_L \xrightarrow{K_2} \pi^0\gamma^* \to \pi^0 e^+ e^-$ ⟸ direct ~~CP~~

★ (TH very clean!)

New Physics can enter here



The action of Z^0, γ Penguins

LO: Dib, Dunietz, Gilman; Flynn, Randall; Buchalla, AJB, Harlander

NLO: AJB, Lautenbacher, Misiak, Münz (94)

Present Status on $K_L \to \pi^0 e^+ e^-$, $K_L \to \pi^0 \mu^+ \mu^-$

Buchalla, D´Ambrosio, Isidori; Isidori, Smith, Unterdorfer

$l = e, \mu$

$$\mathrm{Br}\left(K_L \to \pi^0 l^+ l^-\right) = \left[\underbrace{C^l_{mix}}_{\text{indirect } \cancel{CP}} + \underbrace{C^l_{int}\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)}_{\text{Interference of direct and indirect}} + \underbrace{C^l_{dir}\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2}_{\text{direct}} + \underbrace{C^l_{CPC}}_{\text{CP conserving}}\right] \cdot 10^{-12}$$

$$C^e_{mix} \cong 22.6 \pm 7.0$$
$$C^e_{int} \cong 7.4 \pm 1.5$$
$$C^e_{dir} \cong 2.4 \pm 0.2$$
$$C^e_{CPC} \cong 0$$

$$C^\mu_{mix} \cong 5.3 \pm 1.6$$
$$C^\mu_{int} \cong 1.9 \pm 0.4$$
$$C^\mu_{dir} \cong 1.0 \pm 0.1$$
$$C^\mu_{CPC} \cong 5.2 \pm 1.6$$

$$\mathrm{Br}\left(K_L \to \pi^0 e^+ e^-\right) = \left(3.7\,{}^{+1.1}_{-0.9}\right) \cdot 10^{-11}$$

$$\mathrm{Br}\left(K_L \to \pi^0 \mu^+ \mu^-\right) = (1.5 \pm 0.3) \cdot 10^{-11}$$

7.

Minimal Flavour Violation (MFV)

a) Generalities

b) Model with one Universal Extra Dimension

c) Littlest Higgs Model

d) MSSM at low tanβ

Review: AJB hep-ph/0310208

Generalities

$$A(\text{Decay}) = \sum_i B_i \eta^i_{QCD} V^i_{CKM} \overbrace{\underbrace{\left[F^i_{SM} + F^i_{New}\right]}_{\text{real}}}^{F_i(v)}$$

AJB, Gambino, Gorbahn, Jäger, Silvestrini
D'Ambrosio, Giudice, Isidori, Strumia

K and B Physics related to each other

1. All flavour changing processes governed by V^i_{CKM} .

2. Only SM Operators are relevant.

3. New Physics enters only through 7 Master Functions

$$F_i(v) = S(v), X(v), Y(v), Z(v), D'(v), E'(v), E(v)$$

v = collects parameters specific to a given MFV model.

SM: $\boxed{v = x_t}$

Universal Unitarity Triangle

AJB, Gambino, Gorbahn, Jäger, Silvestrini (00)

In the full class of MFV-models it is possible to construct quantities that depend on CKM parameters but in which the dependence on new physics parameters cancels out

↓

CKM Matrix determined without "New Physics Pollution"

↓

Universal Unitarity Triangle

Examples

$$R_t = 0.90\sqrt{\frac{\Delta M_d}{0.50/\mathrm{ps}}}\sqrt{\frac{18.4/\mathrm{ps}}{\Delta M_s}}\left[\frac{\xi}{1.22}\right]$$

$$a_{\psi K_s} = \sin 2\beta$$

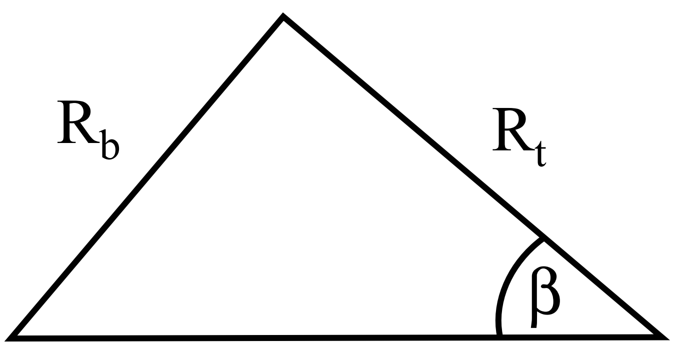
Universal Unitarity Triangle 2004

AJB, Schwab, Uhlig

Use only quantities that are independent of parameters specific to a given Minimal Flavour Violation model

$$\left|\frac{V_{ub}}{V_{cb}}\right| \Rightarrow R_b = \frac{\left(1-\lambda^2/2\right)}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

$$\frac{\Delta M_d}{\Delta M_s} \Rightarrow R_t = \frac{\xi_{th}}{\lambda}\sqrt{\frac{\Delta M_d}{\Delta M_s}} \qquad a_{\psi K_s} \Rightarrow \sin 2\beta$$



$$\xi_{th} = \frac{\sqrt{\hat{B}_s}\, F_{B_s}}{\sqrt{\hat{B}_d}\, F_{B_d}}$$

SM UT versus UUT of MFV

BSU (04)

SM

$$\bar{\eta} = 0.354 \pm 0.027$$
$$\bar{\rho} = 0.187 \pm 0.059$$

$$\gamma = (62.2 \pm 8.2)$$
$$R_t = 0.887 \pm 0.059$$
$$R_b = 0.400 \pm 0.039$$
$$|V_{td}| = (8.24 \pm 0.54) \cdot 10^{-3}$$

$$\mathrm{Im}\lambda_t = (1.40 \pm 0.12) \cdot 10^{-4}$$
$$\lambda_t = V_{ts}^* V_{td}$$

MFV

$$\bar{\eta} = 0.360 \pm 0.031$$
$$\bar{\rho} = 0.174 \pm 0.068$$

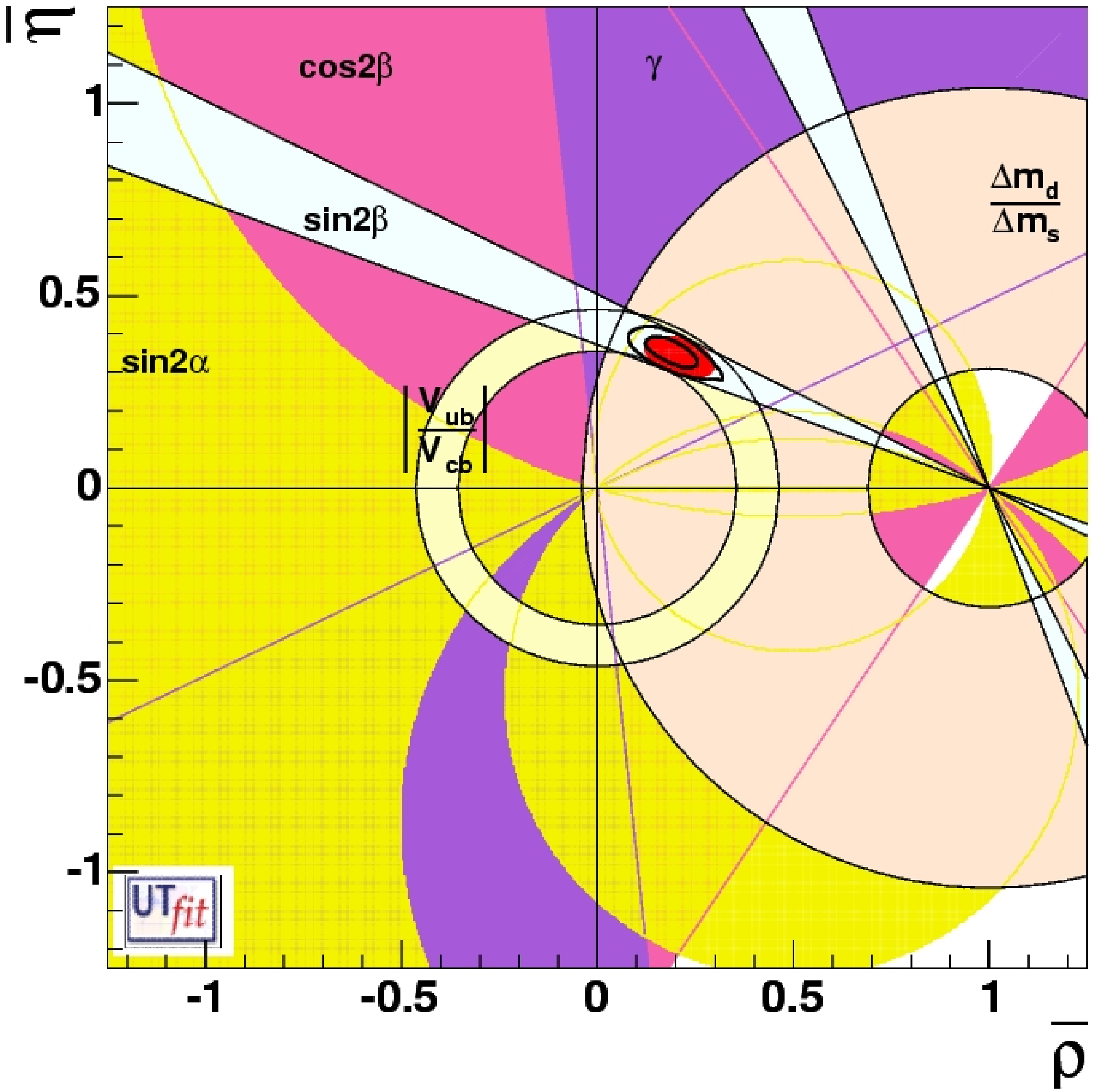
$$\gamma = (64.2 \pm 9.6)$$
$$R_t = 0.901 \pm 0.064$$
$$R_b = 0.400 \pm 0.044$$
$$|V_{td}| = (8.38 \pm 0.62) \cdot 10^{-3}$$

$$\mathrm{Im}\lambda_t = (1.43 \pm 0.14) \cdot 10^{-4}$$

UUT of MFV rather close to SM UT

Universal Unitarity Triangle (MFV)

UTfit Collaboration : Bona et al.



MFV "Sum Rules"

Relations that do not involve the Master Functions X, Y, Z, S, etc.

Violation of these relations signals new flavour (CP) violating interactions beyond CKM or new operators that are strongly suppressed in SM

Examples

$$\left(\sin 2\beta\right)_{\pi\nu\bar{\nu}} = \left(\sin 2\beta\right)_{\psi K_s}$$

$$\frac{Br\left(B_s \to \mu^+\mu^-\right)}{Br\left(B_d \to \mu^+\mu^-\right)} = \frac{\tau(B_s)}{\tau(B_d)}\frac{m_{B_s}}{m_{B_d}}\left[\frac{F_{B_s}}{F_{B_d}}\right]^2\left[\frac{|V_{ts}|}{|V_{td}|}\right]^2$$

$$\frac{\Delta M_d}{\Delta M_s} = \frac{m_{B_d}}{m_{B_s}}\frac{\hat{B}_d}{\hat{B}_s}\frac{F^2_{B_d}}{F^2_{B_s}}\frac{|V_{td}|^2}{|V_{ts}|^2}$$

$$\frac{Br\left(B \to X_d\nu\bar{\nu}\right)}{Br\left(B \to X_s\nu\bar{\nu}\right)} = \frac{|V_{td}|^2}{|V_{ts}|^2}$$

Impact of a Modified $|V_{td}|$

$$\left\{\Delta M_d \approx |V_{td}|^2 \cdot [S_{SM}(m_t) + \Delta s]\right\} \Rightarrow \boxed{|V_{td}|^2 \approx \frac{\Delta M_d}{[S(m_t) + \Delta s]}}$$

$$\boxed{Br(B_d \to \mu^+\mu^-) \approx |V_{td}|^2 \cdot [Y_{SM}(m_t) + \Delta Y]^2 \approx \Delta M_d \frac{[Y_{SM}(m_t) + \Delta Y]^2}{[S(m_t) + \Delta s]}}$$

But :

$$\boxed{Br(B_s \to \mu^+\mu^-) \approx |V_{ts}|^2 \cdot [Y_{SM}(m_t) + \Delta Y]^2}$$

$|V_{ts}| \approx |V_{cb}|$ (CKM Unitarity)

$$\left\{\begin{array}{l}\Delta S > 0 \\ \text{in most} \\ \text{extensions}\end{array}\right\} \Rightarrow \boxed{\left(\frac{Br(B_s \to \mu^+\mu^-)}{Br(B_d \to \mu^+\mu^-)}\right)_{NP} > \left(\frac{Br(B_s \to \mu^+\mu^-)}{Br(B_d \to \mu^+\mu^-)}\right)_{SM}}$$

Intriguing Property of Models with Minimal Flavour Violation

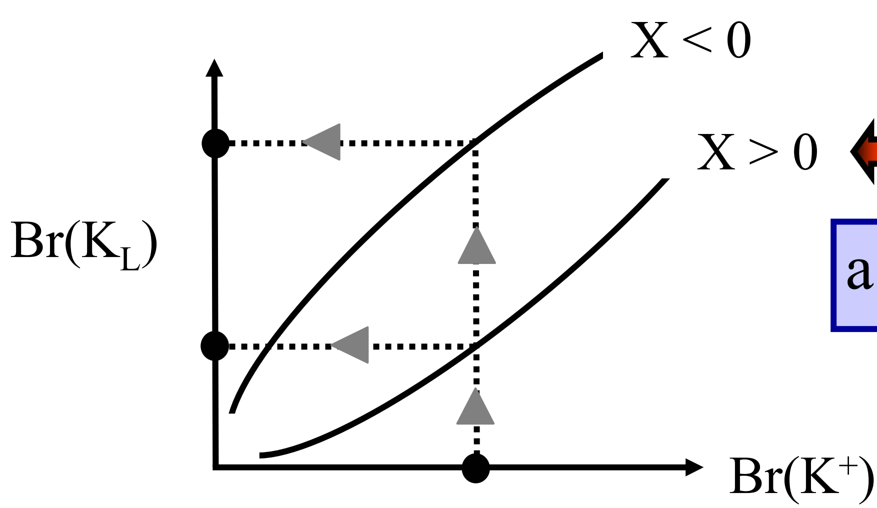
AJB, Fleischer (01)

$$Br\left(K_L\right) = F\left(Br\left(K^+\right),\ a_{\psi K_S},\ \mathrm{sgn}\left(X\right)\right)$$

TH very clean

Independently of any parameters, for given $Br(K^+)$ and $a_{\psi K_S}$ only <u>two values</u> of $Br(K_L)$ possible.

X < 0 very unlikely



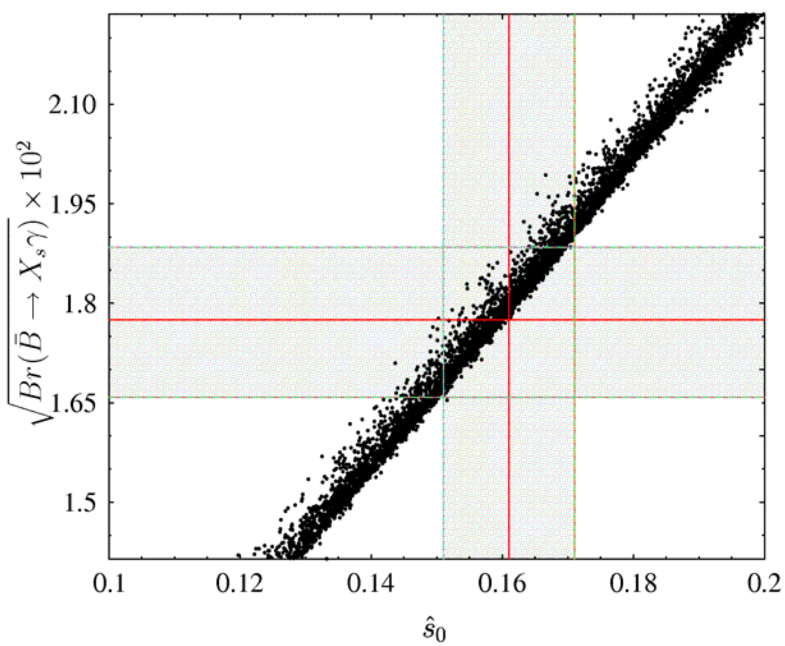
(SM, MSSM)

$$a_{\psi K_S} = \text{fixed}$$

Correlation: $Br(B \to X_s\gamma) \leftrightarrow \hat{s}_0$ in $A_{FB}(B \to X_s l^+ l^-)$

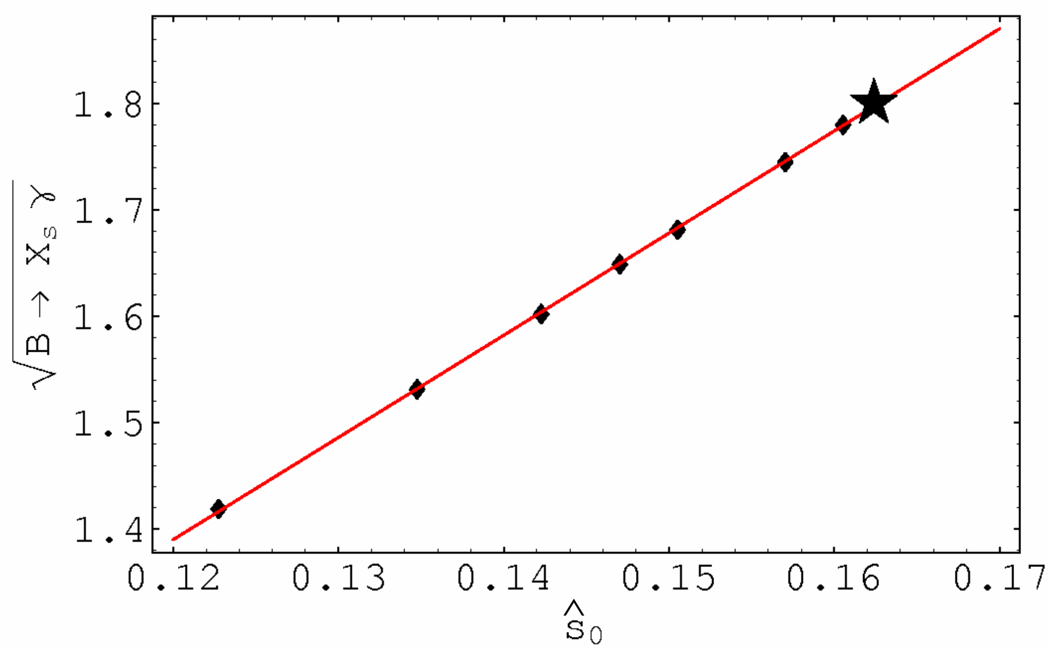
MSSM (MFV)

(Bobeth, AJB, Ewerth)



Universal Extra Dimensions

(AJB, Poschenrieder, Spranger, Weiler)



Relations between $\Delta M_{s,d}$ and $B_{s,d} \to \mu\bar{\mu}$ in Models with Minimal Flavour Violation

(AJB, hep-ph/0303060)

$$\Delta M_q \sim \hat{B}_q F_{B_q}^2 \left|V_{tq}\right|^2 S\left(x_t, x_{new}\right)$$

$$Br\left(B_q \to \mu\bar{\mu}\right) \sim F_{B_q}^2 \left|V_{tq}\right|^2 Y^2\left(x_t, \bar{x}_{new}\right)$$

Large hadronic uncertainties due to $F_{B_q}^2$

$$F_{B_d}\sqrt{\hat{B}_d} = \left(235 \pm 33 \begin{array}{c} +0 \\ -24 \end{array}\right) \mathrm{MeV} \qquad F_{B_d} = (189 \pm 27)\ \mathrm{MeV}$$

$$F_{B_s}\sqrt{\hat{B}_d} = \left(276 \pm 38\right) \mathrm{MeV} \qquad F_{B_s} = (230 \pm 30)\ \mathrm{MeV}$$

$$\hat{B}_d = 1.34 \pm 0.12$$

$$\hat{B}_s = 1.34 \pm 0.12 \qquad \frac{\hat{B}_s}{\hat{B}_d} = 1.00 \pm 0.03$$

(No problems with chiral logs and quenching)

$\mathrm{Br}\left(B_{s,d} \to \mu\bar{\mu}\right)$ from $\Delta M_{s,d}$

$$\mathrm{Br}\left(B_q \to \mu\bar{\mu}\right) = 4.36\cdot 10^{-10}\,\frac{\tau(B_q)}{\hat{B}_q}\,\frac{Y^2(x_t,\bar{x}_{new})}{S(x_t,x_{new})}\,\Delta M_q$$

No dependence on $F^2_{B_q}$

SM:

$$\mathrm{Br}\left(B_s \to \mu\bar{\mu}\right) = 3.42\cdot 10^{-9}\left[\frac{\tau(B_s)}{1.46\mathrm{ps}}\right]\left[\frac{1.34}{\hat{B}_s}\right]\left[\frac{\bar{m}_t(m_t)}{167\ \mathrm{GeV}}\right]^{1.6}\left[\frac{\Delta M_s}{18.0/\mathrm{ps}}\right]$$

$$\mathrm{Br}\left(B_d \to \mu\bar{\mu}\right) = 1.00\cdot 10^{-10}\left[\frac{\tau(B_d)}{1.54\mathrm{ps}}\right]\left[\frac{1.34}{\hat{B}_d}\right]\left[\frac{\bar{m}_t(m_t)}{167\ \mathrm{GeV}}\right]^{1.6}\left[\frac{\Delta M_d}{0.50/\mathrm{ps}}\right]$$

(Example)

$\Delta M_s = (18.0 \pm 0.5/\mathrm{ps})$ ⇒ $\mathrm{Br}\left(B_s \to \mu\bar{\mu}\right) = (3.42 \pm 0.54)\cdot 10^{-9}$

$\Delta M_d = (0.503 \pm 0.006/\mathrm{ps})$ ⇒ $\mathrm{Br}\left(B_d \to \mu\bar{\mu}\right) = (1.00 \pm 0.14)\cdot 10^{-10}$

Moreover new Physics Effects can be easier seen

Testing MFV through $B_{s,d} \to \mu\bar{\mu}$ and $\Delta M_{s,d}$

$$\frac{Br(B_s \to \mu\bar{\mu})}{Br(B_d \to \mu\bar{\mu})} = \underbrace{\frac{\hat{B}_d}{\hat{B}_s}}_{(1.00 \pm 0.03)} \underbrace{\frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d}}_{\text{Experiment}}$$

Valid in MFV models in which only SM operators relevant.

Violation of this relation would indicate the presence of new operators and generally of non-minimal flavour violation.

The Impact of Universal Extra Dimensions on FCNC Processes

Based on:

AJB, M. Spranger, A. Weiler $\equiv$ (BSW) (hep-ph/0212143)

AJB, A. Poschenrieder, M. Spranger, A. Weiler (hep-ph/0306158)

The Next Steps

1. ACD Model in D = 5
2. Impact of KK on Inami-Lim Functions
3. Impact on:

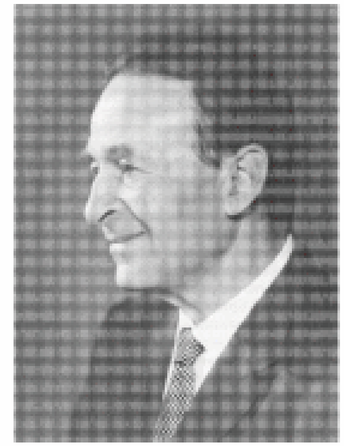
$$\Delta,\ \ \varepsilon_K,\ \ \Delta M_{d,s},\ \ K \to \pi\nu\bar{\nu}$$
$$K_L \to \mu\bar{\mu},\ B \to X_{d,s}\nu\bar{\nu},\ B_{s,d} \to \mu\bar{\mu}$$

$$B \to X_s\gamma,\ \ B \to X_s\mathrm{gluon},\ \ \varepsilon'/\varepsilon$$
$$B \to X_s\mu\bar{\mu},\ \ K_L \to \pi^0 e^+ e^-,$$

4. Conclusions

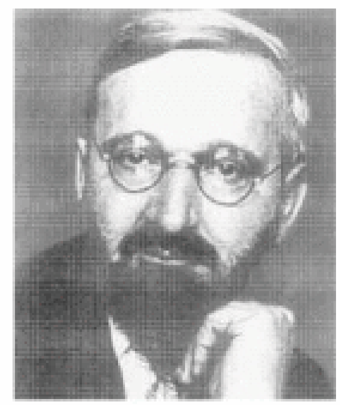
Introduction to the Model

Kaluza (1921) and Klein (1926)
Unification of gravity and electrodynamics in $D = 5$ compactified on S^1.



Some extra dimensional Models:

- brane world: SM on brane, gravity in the bulk, localization mechanism
- gravity and gauge bosons in bulk, fermions on brane $R^{-1} >$ few TeV, localization mechanism
- Universal extra dimensions (UED): everything in the bulk, no localization mechanism required, gravity not considered



ACD Model

Appelquist, Cheng, and Dobrescu (ACD)

hep-ph/0012100

- All SM fields live in the bulk $D = 4 + 1$, Gravity not considered.
- Orbifold: Replace S^1 by S^1/Z_2
- Simple extension of SM, 1 extra parameter (R, radius of ED), boundary terms set to zero
- provides excellent dark-matter candidate
  Servant,Tait '02; Cheng,Feng,Matchev '02
- bounds on $1/R$ are rather weak
  $1/R \gtrsim 250$ GeV, $M_H > 250$ GeV,
  $1/R \gtrsim 300$ GeV, $M_H < 250$ GeV. Appelquist,Yee '02

Appelquist, Cheng, Dobrescu Model (ACD)

$(D = 5)$

Universal Extra Dimensions:

All SM fields live in extra dimensions

Particle Content

in an Effective $D = 4$ Theory

SM Fields $(n = 0)$ (Zero Modes)

+

Corresponding KK Models $(n = 1, 2, \ldots)$ $W^{\pm}_{(n)}$, $Z^{0}_{(n)}$, etc.

+

Additional Physical Scalar Modes $a^{0}_{(n)}$, $a^{\pm}_{(n)}$; $n = 1, 2, \ldots$

Single New Parameter: Compactification Scale $1/R$

$$1/R \geq \begin{cases} 250 \text{ GeV} & (M_H > 250 \text{ GeV}) \\ 300 \text{ GeV} & (M_H < 250 \text{ GeV}) \end{cases}$$

(ACD, AY: Electroweak Precision Observables)

Mass Spectrum

$$M^2_{\gamma(n)} = \frac{n^2}{R^2}$$

$$M^2_{Z(n)} = \frac{n^2}{R^2} + M^2_Z$$

$$M^2_{W(n)} = \frac{n^2}{R^2} + M^2_W$$

$$m^2_{q(n)} = \frac{n^2}{R^2} + m^2_q$$

$$m^2_{l(n)} = \frac{n^2}{R^2} + m^2_l$$

$$m^2_{a^\pm(n)} = \frac{n^2}{R^2} + M^2_W \qquad n \geq 1$$

$$m^2_{a^0(n)} = \frac{n^2}{R^2} + M^2_Z \qquad n \geq 1$$

$(n = 0, 1, 2 \ldots)$

Interactions

1. Full Set of Feynman Rules in BSW
2. Vertices depend on n/R
3. Conservation of KK Parity $\Rightarrow$ Absence of tree level KK contributions
4. Minimal Flavour Violation (CKM Matrix; no new operators)

Properties Relevant for FCNC Processes

$\varepsilon_K, \Delta M_{d,s}$: $S(x_t, 1/R) = S_0(x_t) + \sum_{n=1}^{\infty} S_n\left(x_t, \frac{n}{R}\right)$ $\begin{pmatrix}\Delta F{=}2 \\ \text{Boxes}\end{pmatrix}$

$\left(x_t = \frac{m_t^2}{M_W^2}\right)$

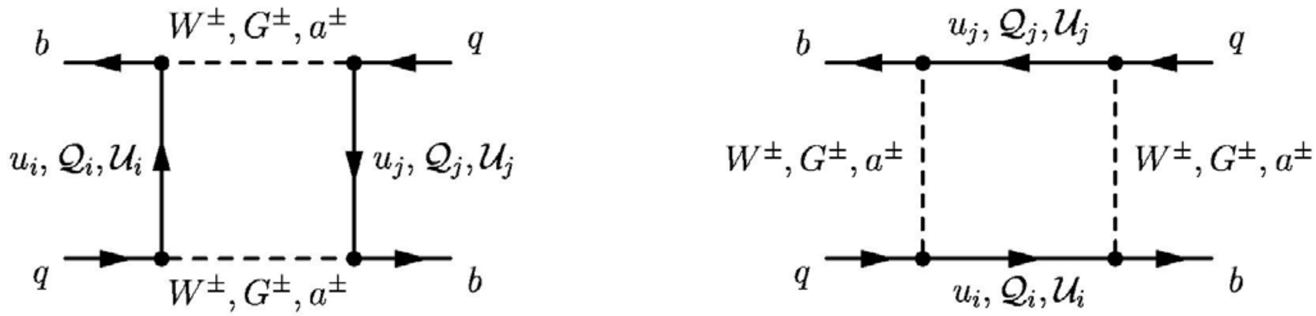
$K^+ \to \pi^+ \nu\bar{\nu}$
$K_L \to \pi^0 \nu\bar{\nu}$
$B \to X_{s,d} \nu\bar{\nu}$: $X(x_t, 1/R) = \underbrace{X_0(x_t)}_{\substack{(C_0\text{-}4B_0) \\ \text{SM}}} + \sum_{n=1}^{\infty} C_n\left(x_t, \frac{n}{R}\right)$

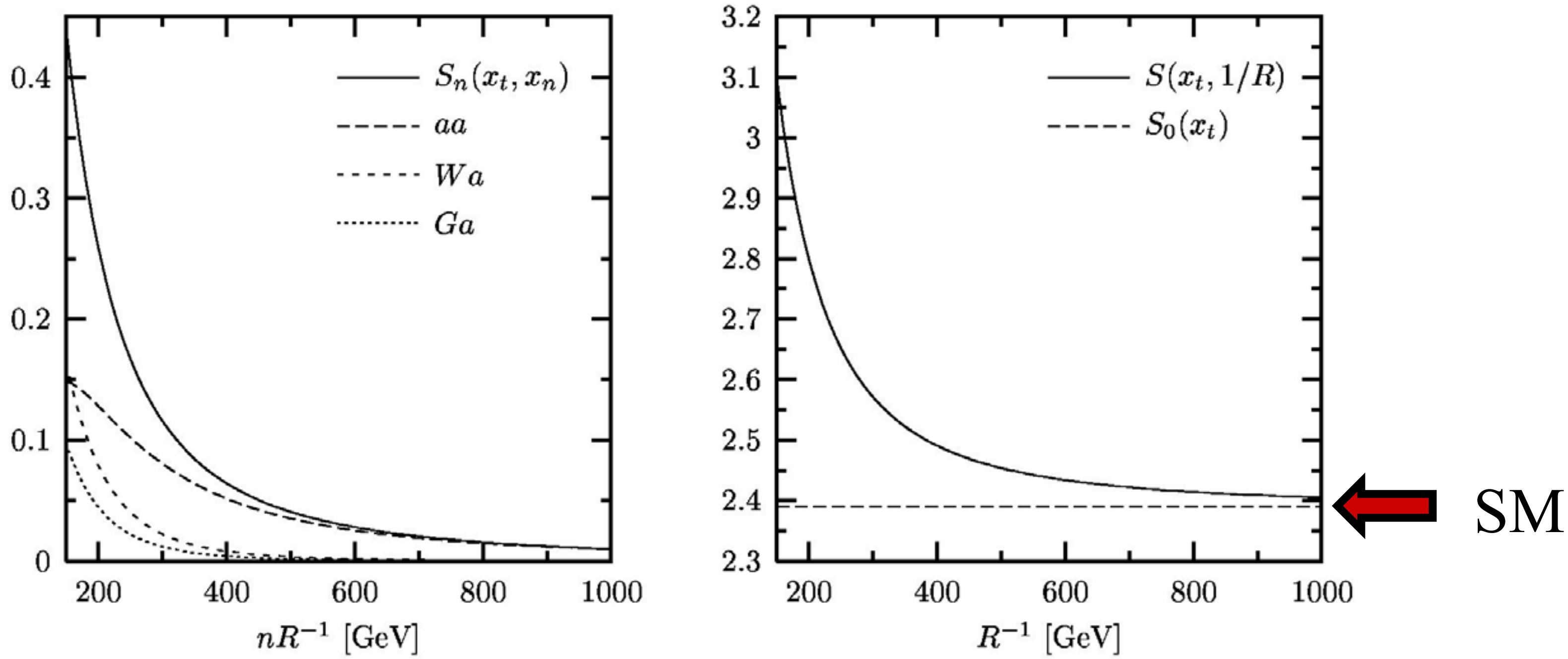
$B_{s,d} \to \mu^+\mu^-$
$K_L \to \mu^+\mu^-$: $Y(x_t, 1/R) = \overbrace{Y_0(x_t)}^{\text{SM}}_{(C_0\text{-}B_0)} + \sum_{n=1}^{\infty} C_n\left(x_t, \frac{n}{R}\right)$

Z^0 Penguins

GIM mechanism improves significantly the convergence of the sum over the $(KK)_t$ Modes and essentially removes the contributions of $(KK)_{u,c}$ in the first two generations.

Results for the Function $S(x_t, 1/R)$





(a) (b)

Basic Formulae for UT Analysis

1. ε_K - Hyperbola

$$\boxed{\bar{\eta}\left[\left(1-\bar{\rho}\right)A^2 F_{tt}\eta_{QCD}^{tt} + P_c\left(\varepsilon\right)\right]A^2\hat{B}_K = 0.213}$$

$$\boxed{\begin{aligned} F_{tt}^{SM} &= S_0\left(x_t\right) \\ F_{tt}^{ACD} &= S\left(x_t, 1/R\right) \end{aligned}}$$

$\eta_{QCD}^{tt} = 0.57 \pm 0.01;\quad P_C\left(\varepsilon\right) = 0.28 \pm 0.05;$

2. $B_d^0 - \bar{B}_d^0$ Mixing Constraint

$$\boxed{R_t = 0.86\left[\frac{0.041}{\left|V_{cb}\right|}\right]\sqrt{\frac{2.34}{F_{tt}}}\sqrt{\frac{\Delta M_d}{0.50/ps}}\left[\frac{230MeV}{\sqrt{\hat{B}_d}F_{B_d}}\right]\sqrt{\frac{0.55}{\eta_B^{QCD}}}}$$

$$\left\{F_{tt}^{ACD} > F_{tt}^{SM}\right\}$$

$$\Downarrow$$

$$\left\{R_t^{ACD} < R_t^{SM}\right\}$$

$\left|V_{cb}\right| = 0.041 \pm 0.001;\quad \Delta M_d = \left(0.503 \pm 0.006\right)/ps;\quad \eta_B^{QCD} = 0.55 \pm 0.01$

3. $B_s^0 - \bar{B}_s^0$ Mixing Constraint $(\Delta M_d/\Delta M_s)$

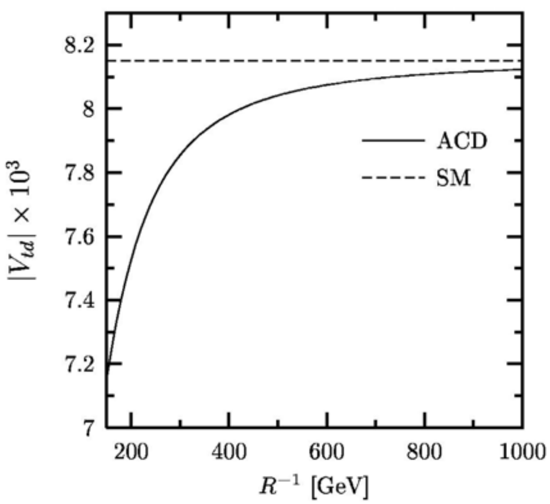
$$\boxed{R_t = 0.90\sqrt{\frac{\Delta M_d}{0.50/ps}}\sqrt{\frac{18.4/ps}{\Delta M_s}}\left[\frac{\xi}{1.22}\right]}$$

$$\boxed{\xi = \frac{\sqrt{\hat{B}_s}F_{B_s}}{\sqrt{\hat{B}_d}F_{B_d}}}$$

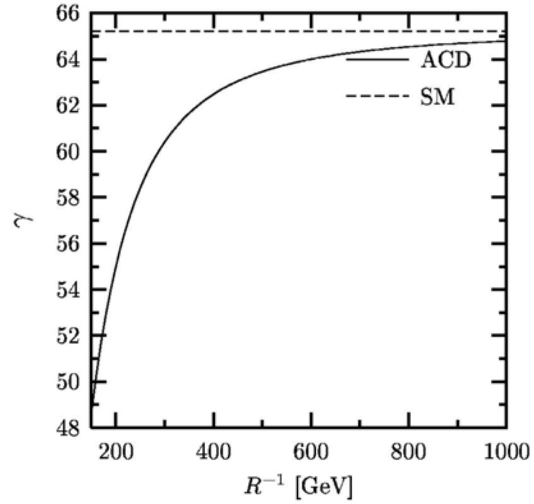
(No dependence on 1/R)

$\Delta M_s > 14.4\ /\ ps$ (95% C.L.) LEP (SLD)

Implications for Unitarity Triangle



(a)

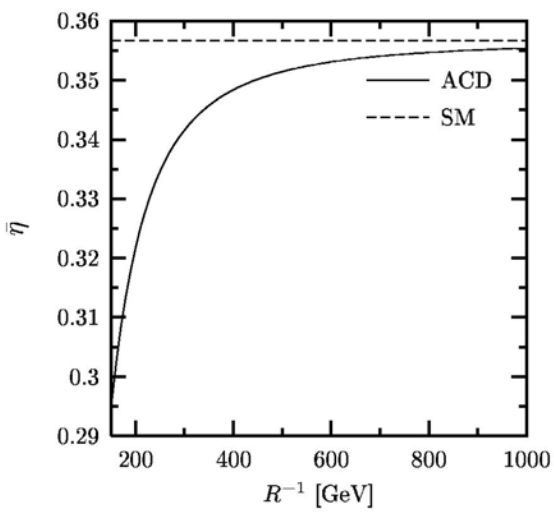


(b)

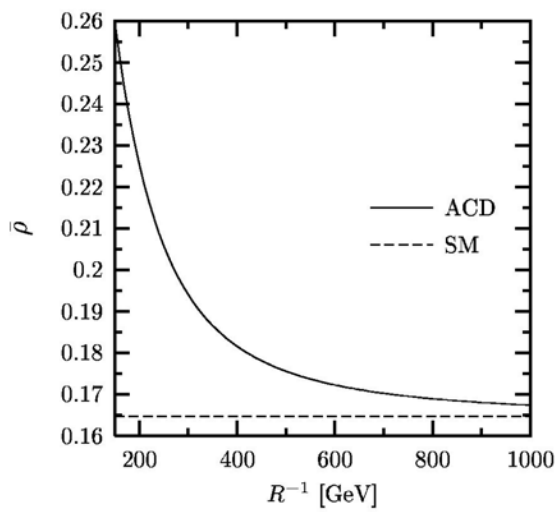
1/R = 200 (300) GeV

Suppressions :

$\lvert V_{td} \rvert$	:	8%	(4%)
$\bar{\eta}$	:	11%	(4.5%)
γ	:	10^o	(5^o)



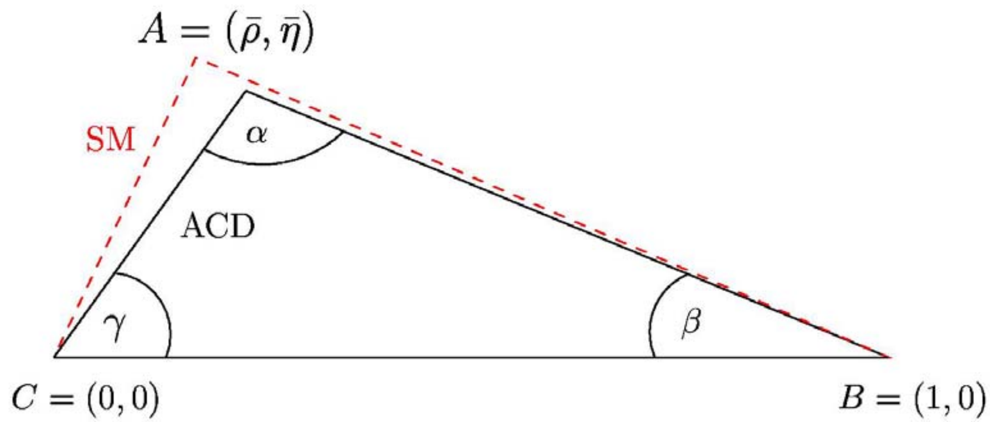
(c)



(d)

Unitarity Triangle in the ACD Model

1/R = 200 GeV

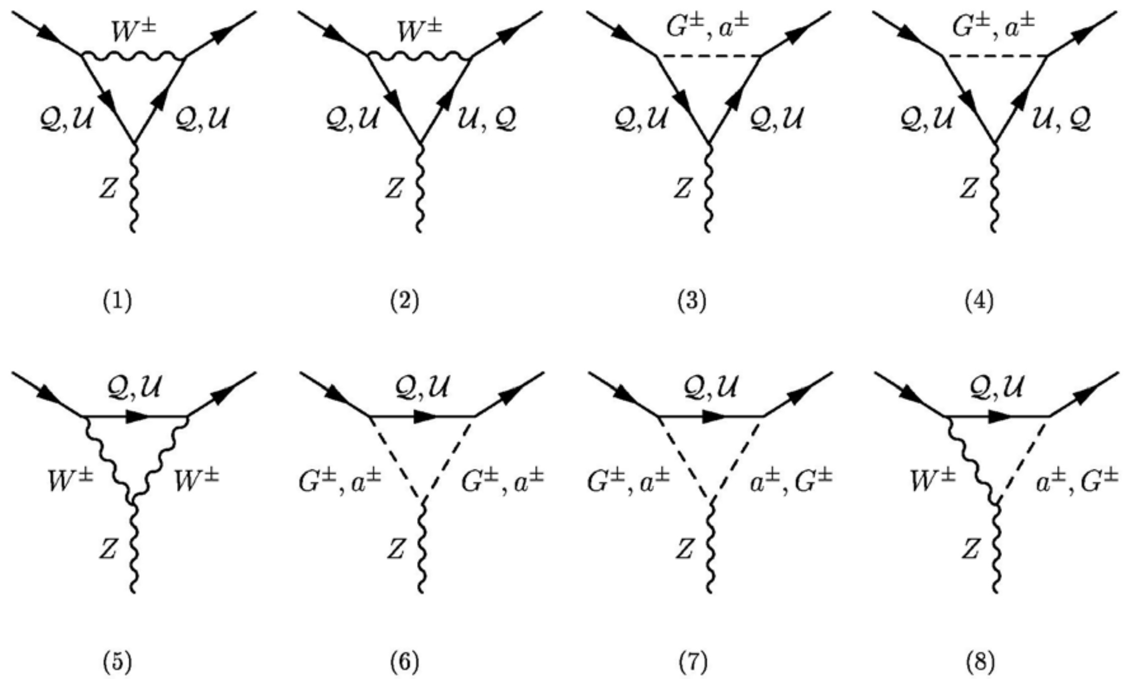


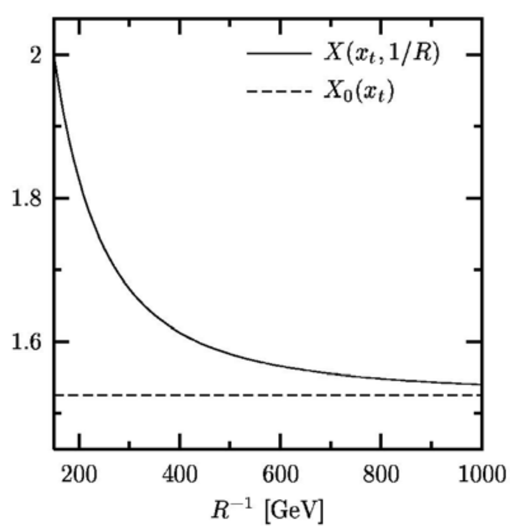
At $\quad 1/R = 200$ GeV $\qquad \gamma_{SM} = 65^\circ \rightarrow \gamma_{ACD} = 49^\circ$

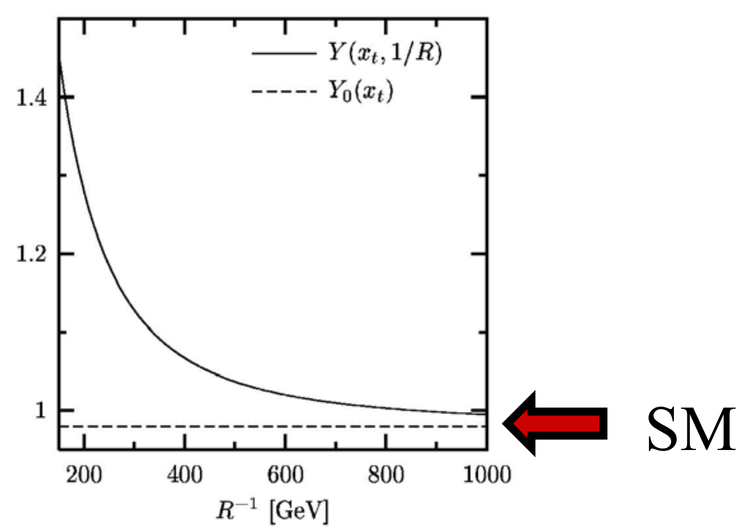
but at $1/R = 300\ (400)$ GeV $\quad \gamma_{ACD} = 60^\circ (63^\circ)$

Very difficult to see the difference in view of hadronic uncertainties.

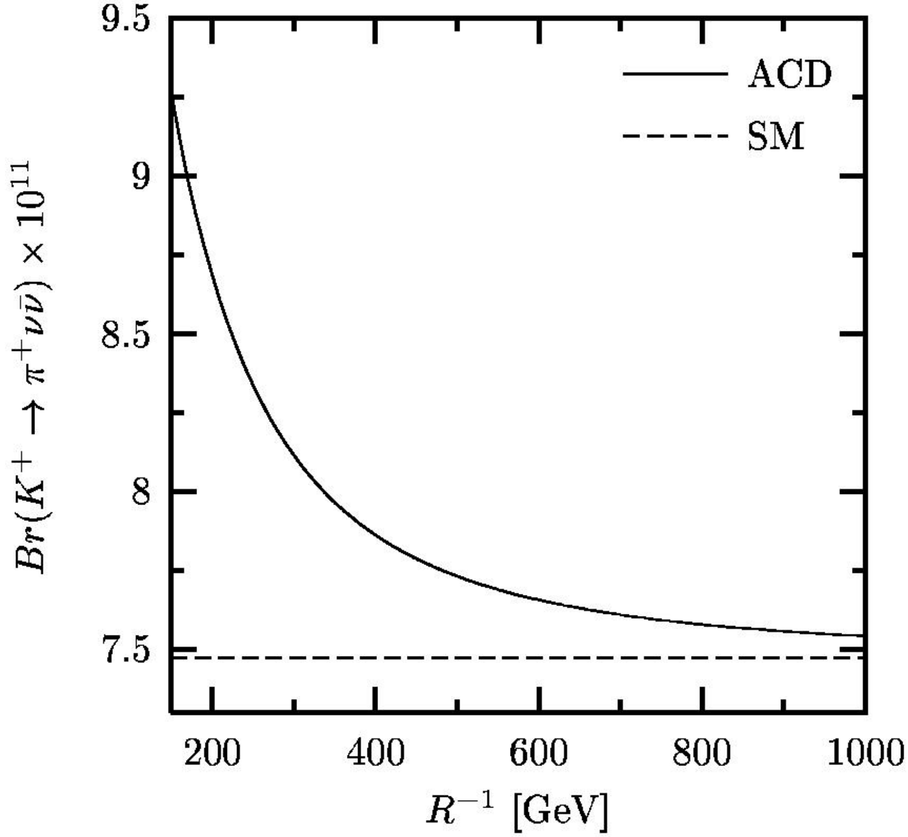
Results for the Functions $X(x_t, 1/R)$, $Y(x_t, 1/R)$

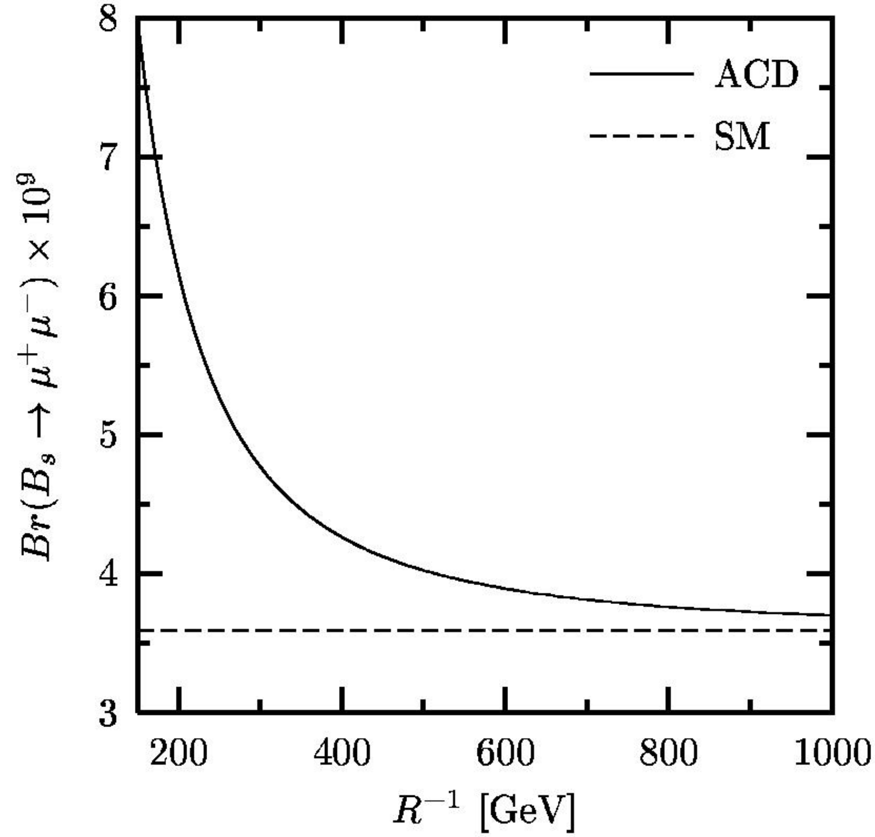






Implications for Rare K and B Decays

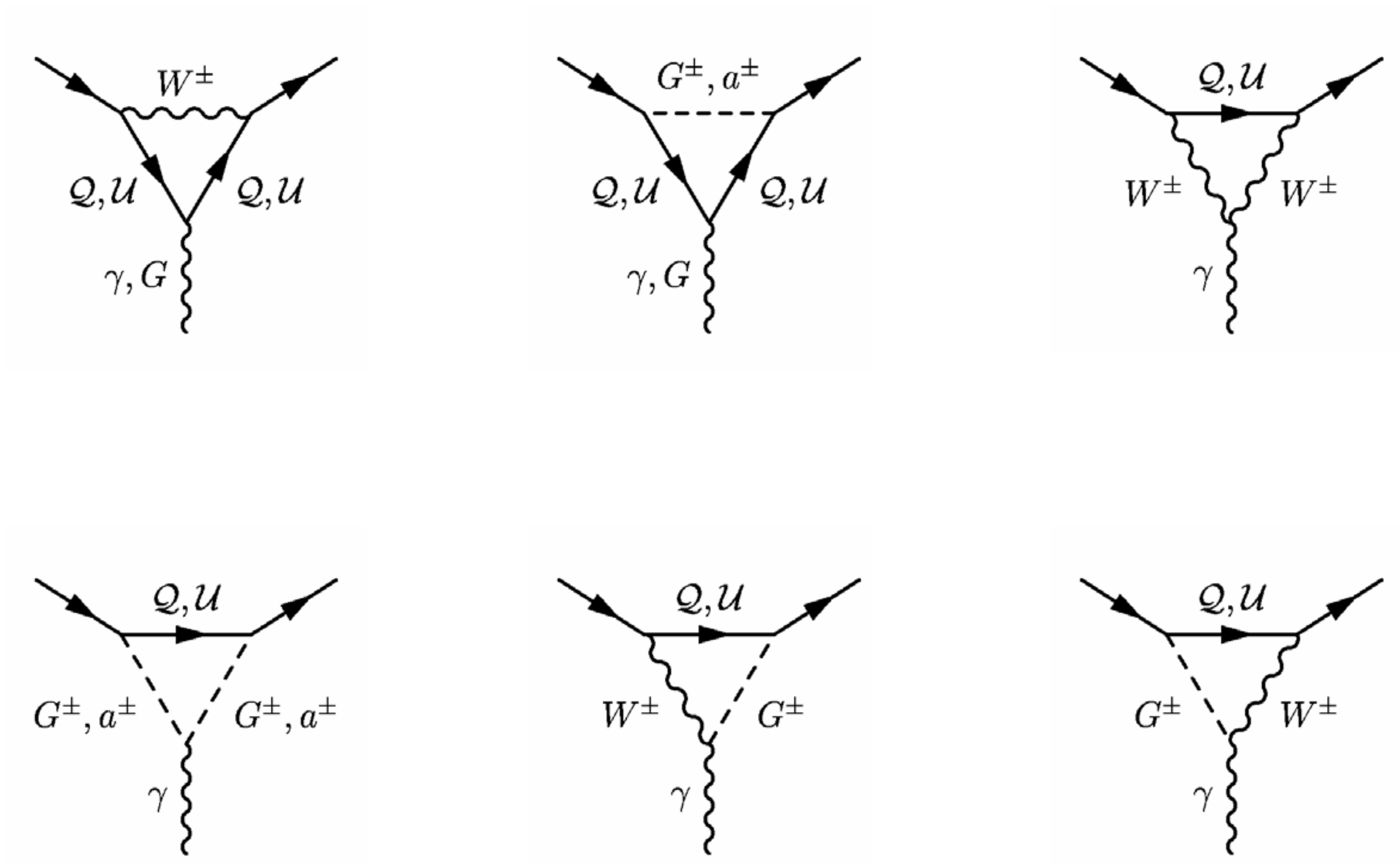




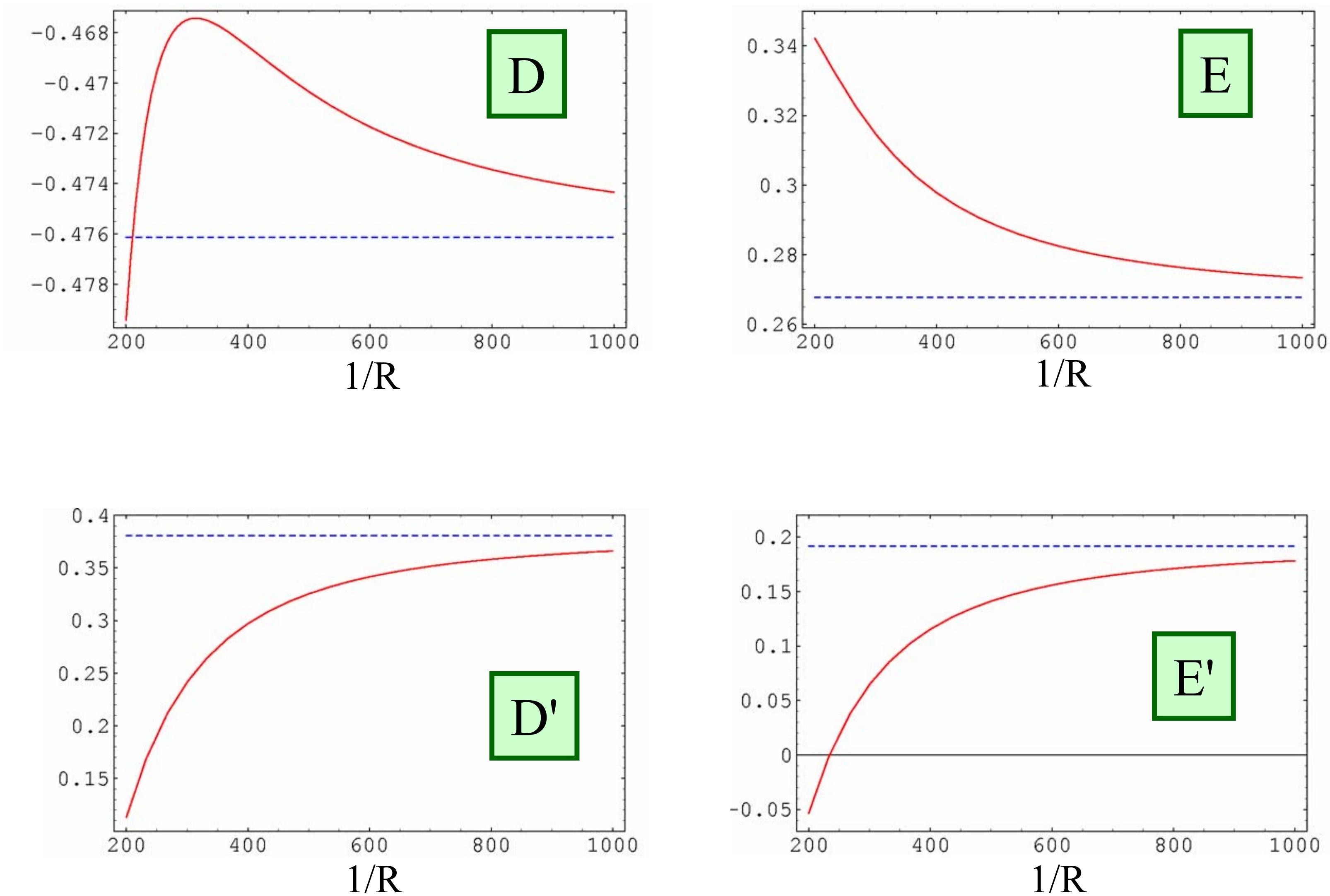
The Impact of Universal Extra Dimensions on $B \to X_s\gamma$, $B \to X_s\mu^+\mu^-$, $K_L \to \pi^0 e^+ e^-$

Andrzej J. Buras, Anton Poschenrieder, Michael Spranger, Andreas Weiler

Diagrams Contributing to D, E, D', E'



Results for D, E, D', E'



Impact of KK on $B \to X_s\gamma$

$$\left.\begin{array}{l}\text{CLEO}\\ \text{ALEPH}\\ \text{BaBar}\\ \text{Belle}\end{array}\right\} Br\left(B \to X_s\gamma\right) = \left(3.28\begin{array}{l}+0.41\\-0.36\end{array}\right)\cdot 10^{-4}$$

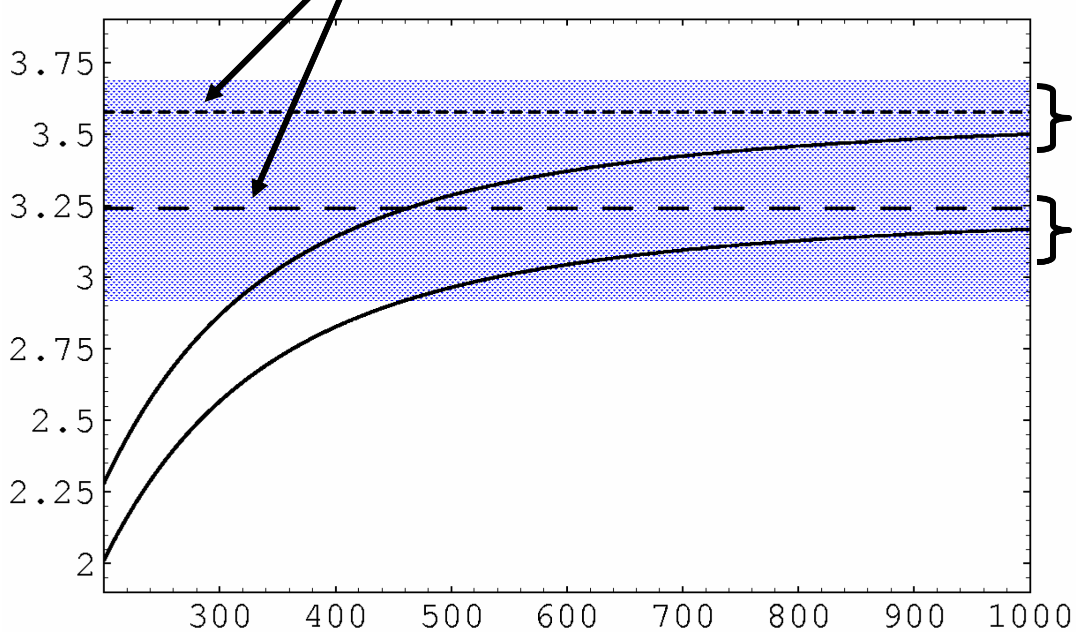
SM

$$\left(3.57 \pm 0.30\right)\cdot 10^{-4}$$

$$\text{Gambino} + \text{Misiak} \quad \left(\frac{m_c^2}{m_b^2} = 0.22\right)$$

SM

$Br\left(B \to X_s\gamma\right)$ $\left[10^{-4}\right]$



1/R

$m_c^2 / m_b^2 = 0.22$

$m_c^2 / m_b^2 = 0.29$

Impact of KK on $B \to X_s \mu\bar{\mu}$

$$\left\{\begin{array}{c}\text{Integration}\\ \text{over}\\ \text{full dileption}\\ \text{mass spectrum}\end{array}\right\}$$

$$\mathrm{Br}\left(B \to X_s \mu\bar{\mu}\right) = \left(7.9 \pm 2.1 \begin{array}{l}+2.0\\ -1.5\end{array}\right) \cdot 10^{-6} \quad (\text{Belle})$$

$$\text{SM}: \quad (4.1 \pm 0.7) \cdot 10^{-6} \quad (\text{Ali, Lunghi, Greub, Hiller})$$

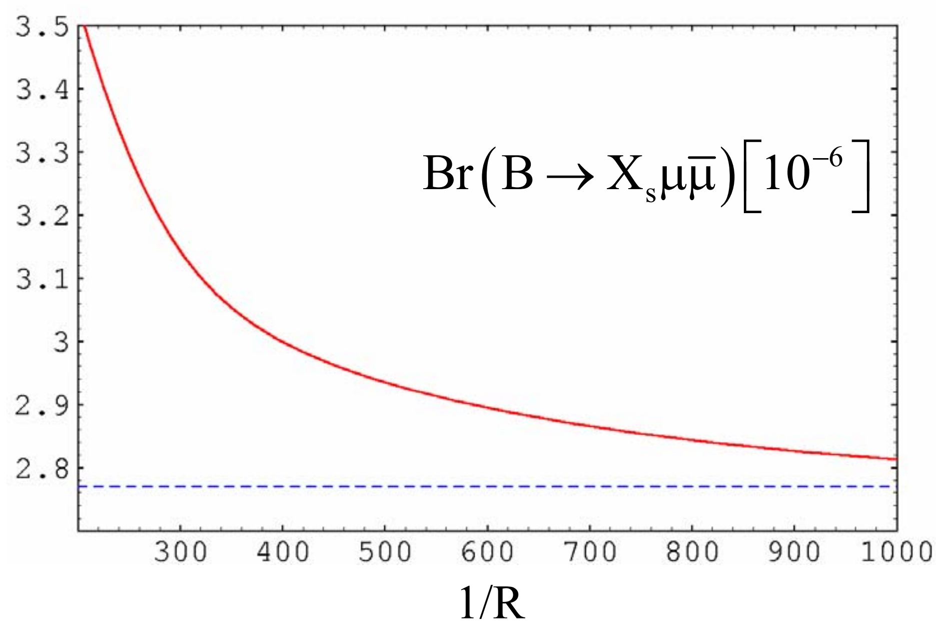
$$\text{ACD}: \quad (4.8 \pm 0.8) \cdot 10^{-6} \quad (\text{BPSW; } 1/R = 300\text{GeV})$$

$$\hat{s} = \frac{\left(P_{\mu_+} + P_{\mu_-}\right)^2}{m_b^2}$$

$$\left\{\begin{array}{c}\text{Integration}\\ \text{over}\\ \text{low dileption}\\ \text{mass spectrum}\end{array}\right\}$$

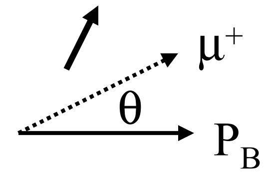
$$0.05 \lesssim \hat{s} \lesssim 0.25$$

$$\text{SM}: \quad (2.75 \pm 0.45) \cdot 10^{-6}$$



Forward-Backward Asymmetry in $B \to X_s \mu^+ \mu^-$ (SM)

$$A_{FB}(\hat{s}) = \frac{1}{\Gamma(b \to ce\bar{\nu})} \int_{-1}^{+1} d\cos\theta_L \, \frac{d^2\Gamma(b \to s\mu^+\mu^-)}{d\hat{s}\cos\theta_L} \, \mathrm{sgn}(\cos\theta_L)$$



$$A_{FB}(\hat{s}) = -3\tilde{C}_{10} \frac{\left[\hat{s}\,\mathrm{Re}\,\tilde{C}_9^{\mathrm{eff}}(\hat{s}) + 2C_{7\gamma}^{(0)\mathrm{eff}}\right]}{U(\hat{s})}$$

$$\hat{s}_0 \cong -\frac{2C_{7\gamma}^{(0)\mathrm{eff}}}{\mathrm{Re}\,\tilde{C}_9^{\mathrm{eff}}(\hat{s}_0)}$$

$$C_9 \leftrightarrow (\bar{s}b)_{V-A}(\bar{\mu}\mu)_V$$
$$C_{10} \leftrightarrow (\bar{s}b)_{V-A}(\bar{\mu}\mu)_A$$
$$C_{7\gamma} \leftrightarrow B \to X_s\gamma$$

SM :

NLO: $\hat{s}_0 \cong 0.14 \pm 0.02$ **Ali Mannel Morozumi**

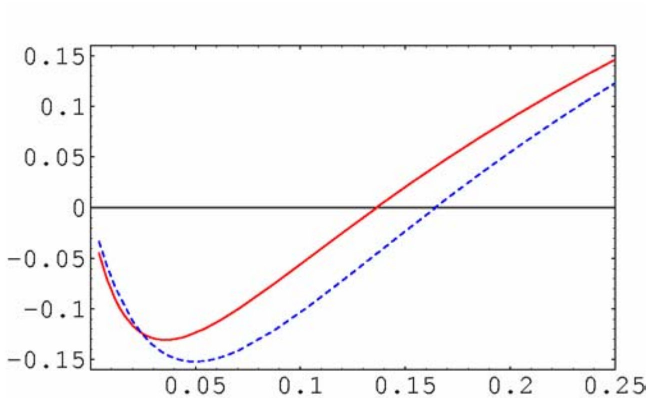
NNLO: $\hat{s}_0 \cong 0.162 \pm 0.008$

Asatrian, Asatrian, Greub, Walker, Bieri Hovhannisyan

Ghinculov, Hurth, Isidori, Yao

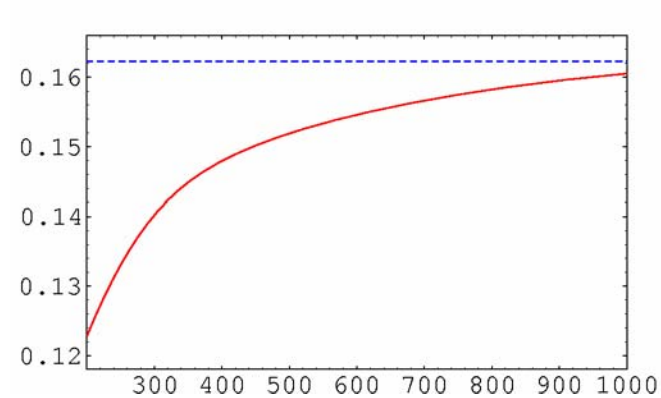
Impact of KK on $A_{FB}(\hat{s})$

SM
ACD



$A_{FB}(\hat{s})$

$\hat{s}$



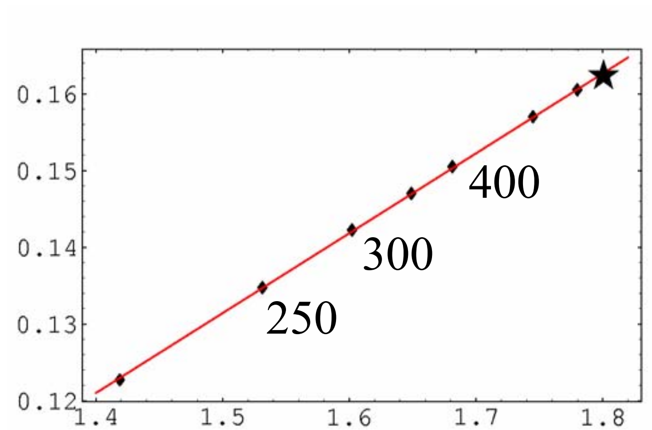
$\hat{s}_0$

$1/R$

$$\left\{ \begin{matrix} \widetilde{C}_9^{\text{eff}} \text{ very} \\ \text{weakly affected} \end{matrix} \right\}$$

$$\Downarrow$$

$$\hat{s}_0 \sim \sqrt{Br(B \to X_s \gamma)}$$



$\hat{s}_0$

$\sqrt{Br(B \to X_s \gamma)}$

Summary

1. ACD Model consistent with the data on FCNC Processes with $1/R \cong 300$ GeV

2. Only small impact on UT relative to the SM

3. Enhancements : $K^+ \to \pi^+ \nu\bar{\nu}\,(9\%)$, $K_L \to \pi^0 \nu\bar{\nu}\,(10\%)$

   $1/R = 300$ GeV

   $B \to X_s \mu\bar{\mu}\,(12\%)$, $B \to X_d \nu\bar{\nu}\,(12\%)$

   $B \to X_s \nu\bar{\nu}\,(21\%)$, $K_L \to \mu^+\mu^-\,(20\%)$

   $B_d \to \mu^+\mu^-\,(23\%)$, $B_s \to \mu^+\mu^-\,(33\%)$

4. Suppressions : $B \to X_s \gamma\,(20\%)$, $B \to X_s\,\text{gluon}\,(40\%)$

   $\hat{s}_0 : 0.162 \to 0.142$; ε'/ε

5. With improved Exp+Th for $B \to X_s \gamma$ and $\hat{s}_0$ strong lower bound on $1/R$ could be obtained.

FCNC Processes in the "Littlest" Higgs Model

LH Model : Arkani-Hamed, Cohen, Katz, Nelson (2002)

FCNC Processes : AJB, Poschenrieder, Uhlig : hep-ph/0410309 $B^0_{d,s} - \overline{B}^0_{d,s}$ Mixing
0501230 Rare Decays
05xxyyy Technical Details

New Effect generally below 30%

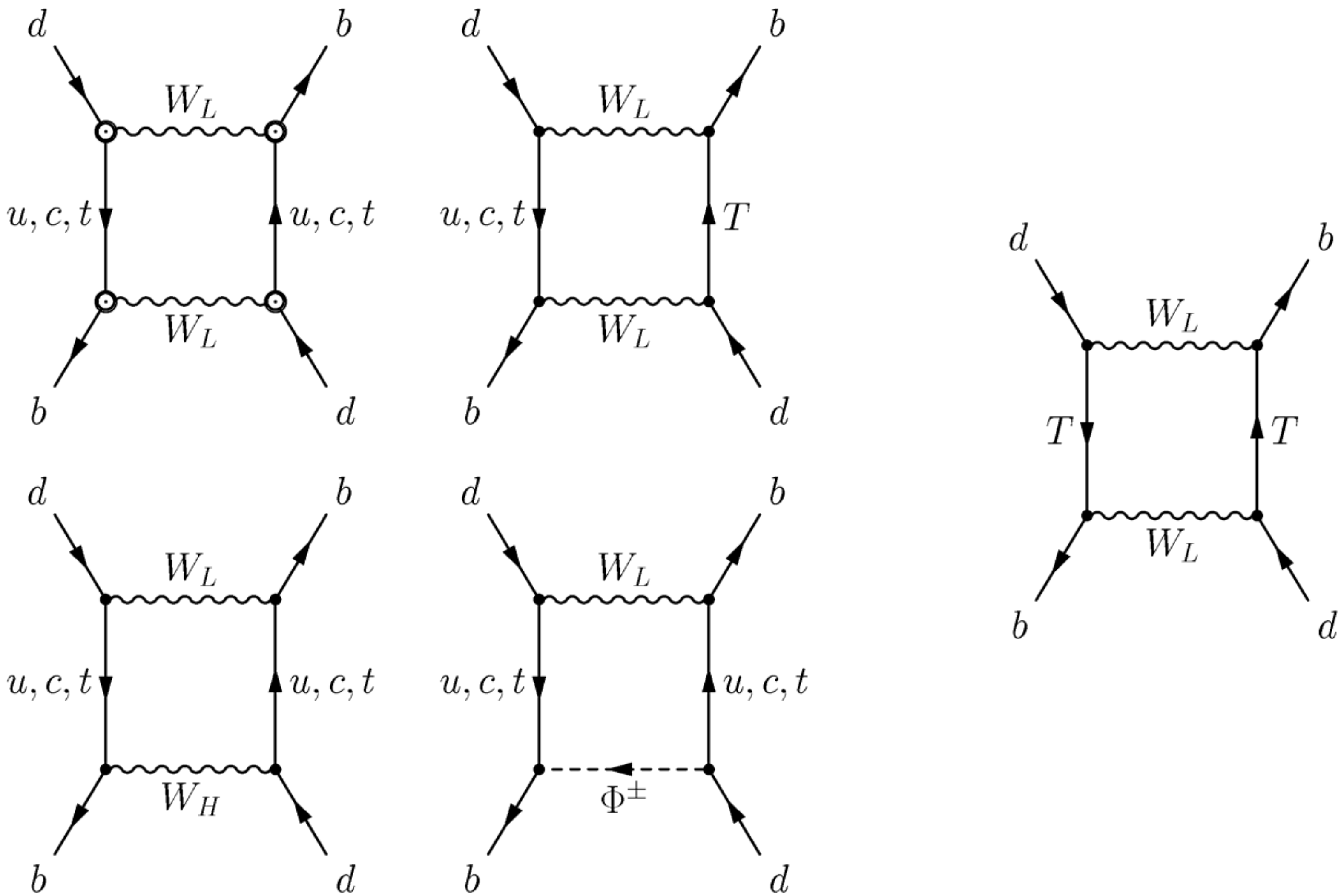
A symphony of penguin and box diagrams with

SM	New Particles
$W^\pm_L,\ Z^0_L$	$W^\pm_H,\ Z^0_H,\ A^0_H$ (new Gauge Bosons)
t	$T,\ \Phi^\pm$ (new heavy top, charged scalar)

New Particles $m_i > 1\ \text{TeV}$

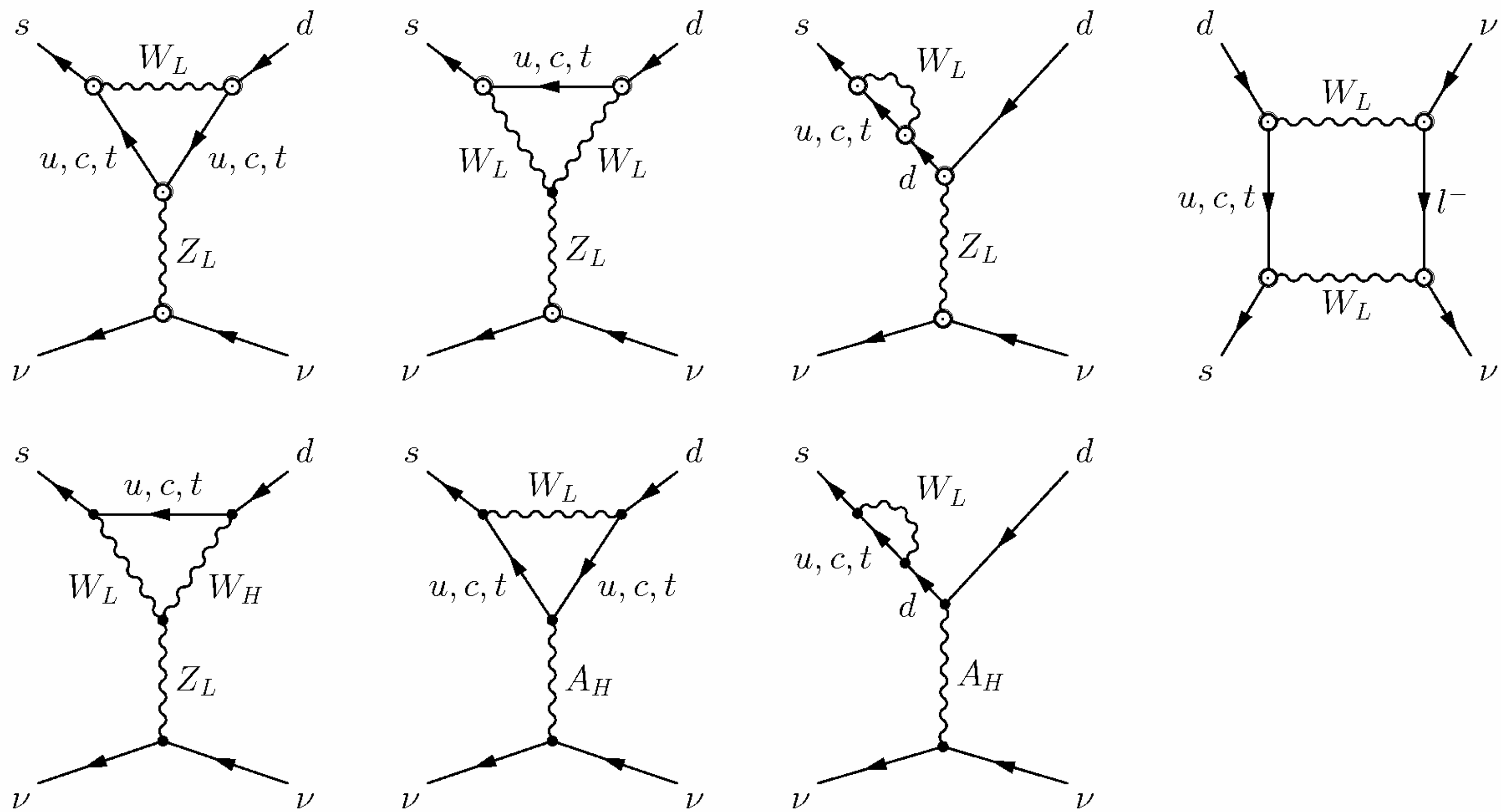
Related work: Choudhury, Gaur, Goyal, Mahajan, 0407050 (confirmed our results)
Choudhury, Gaur, Joshi, McKellar, 0408125 (?)

Movement 1



d
b
W_L
u, c, t
u, c, t
W_L
b
d
d
b
W_L
u, c, t
T
W_L
b
d
d
b
W_L
u, c, t
u, c, t
W_H
b
d
d
b
W_L
u, c, t
u, c, t
$\Phi^{\pm}$
b
d
d
b
W_L
T
T
W_L
b
d

Movement 2



s
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u, c, t
u, c, t
Z_L
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ν
s
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W_L
W_L
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s
ν
s
d
u, c, t
W_L
W_H
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ν
s
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W_L
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ν
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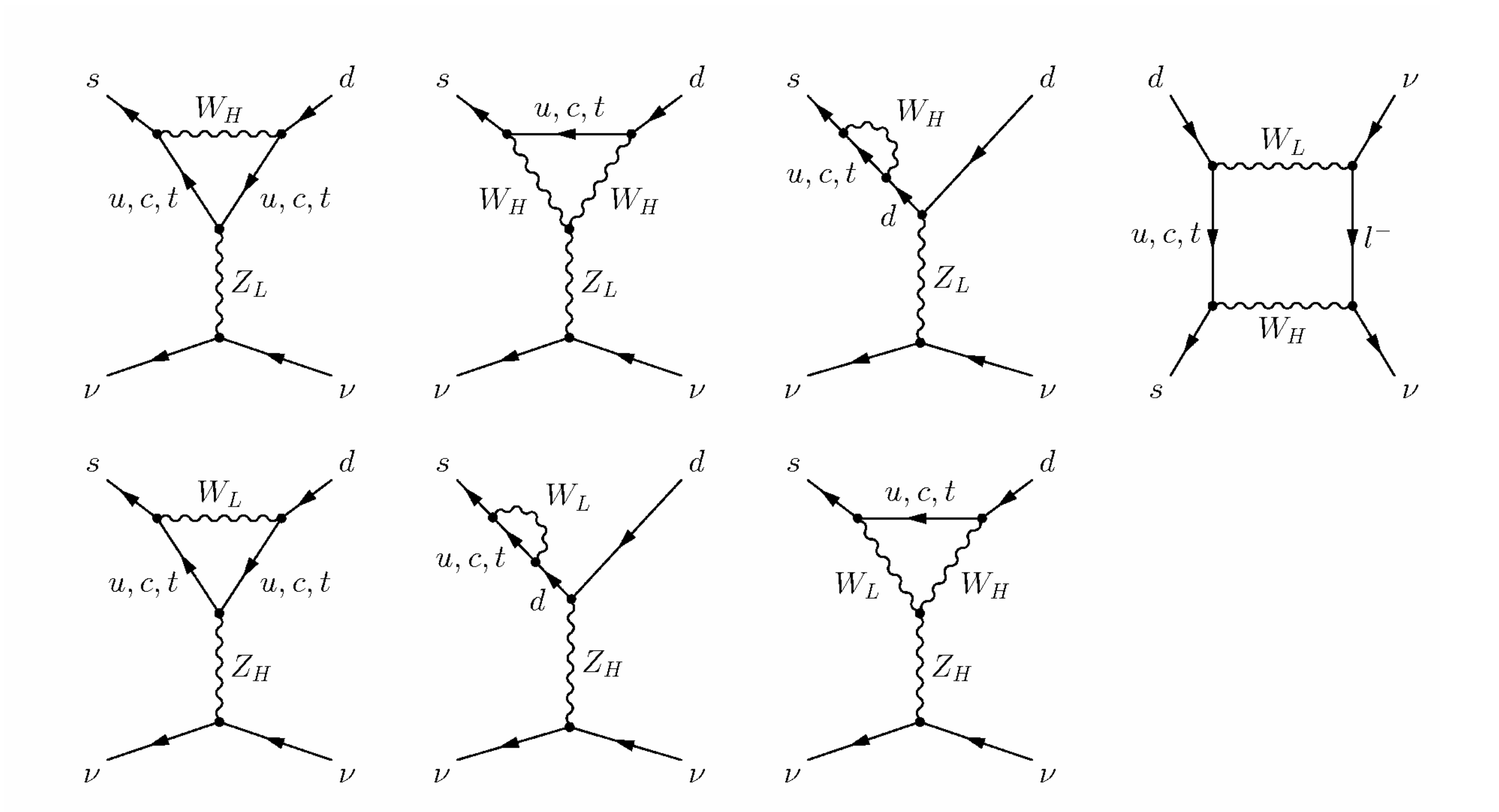
Movement 3



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Z_H
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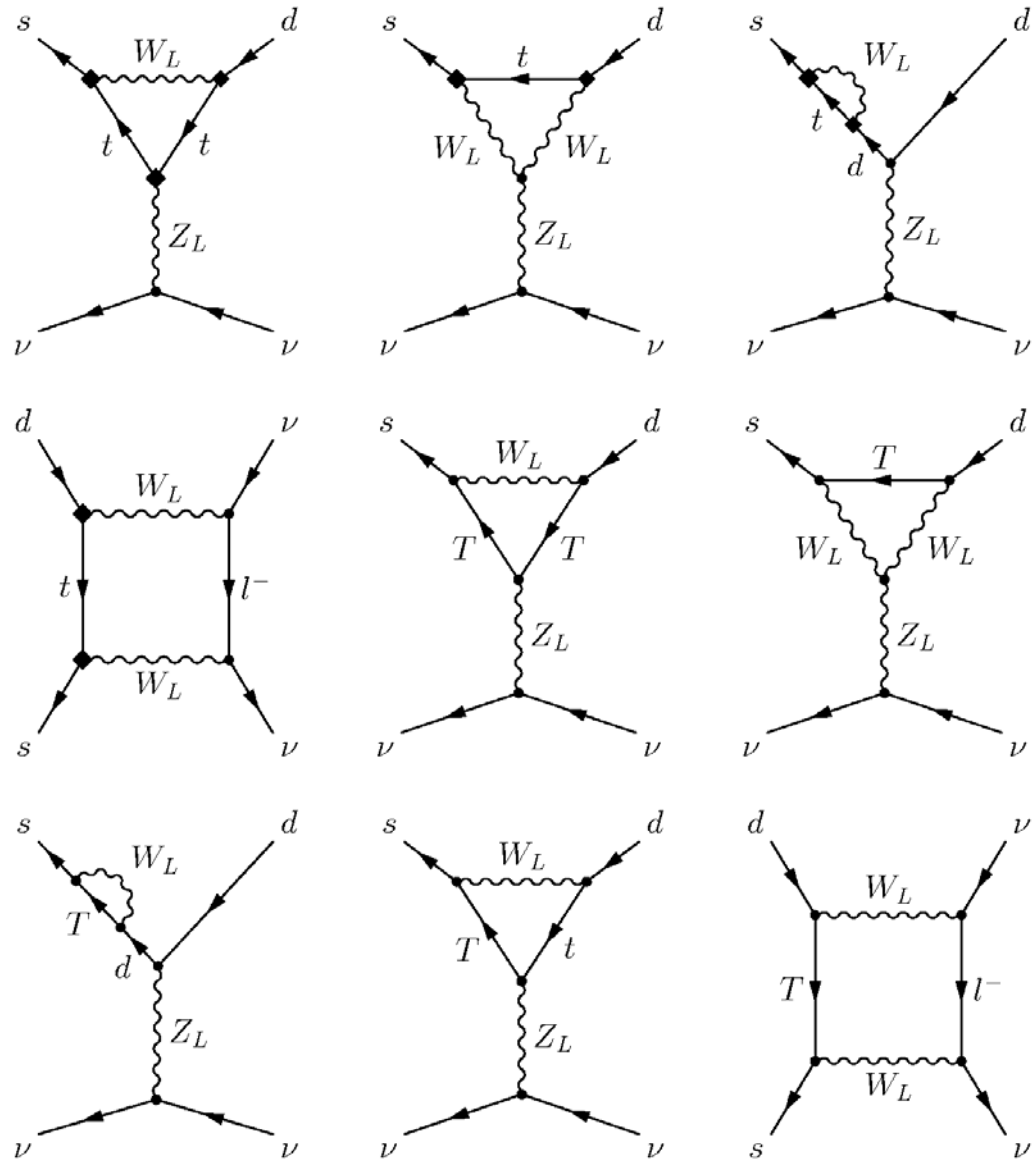
Z_H
ν
ν

Movement 4


s
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FCNC Processes in MSSM (MFV)

Classic Paper : Bertolini, Borzumati, Masiero, Ridolfi (1991)

Last Analysis : AJB, Gambino, Gorbahn, Jäger, Silvestrini (2000)

$$T(Q) \equiv \frac{Q_{\text{MSSM}}}{Q_{\text{SM}}}$$

$$0.65 \le T\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \le 1$$

$$0.41 \le T\left(K_L \to \pi^0 \nu\bar{\nu}\right) \le 1$$

$$0.73 \le T\left(B \to X_s \nu\bar{\nu}\right) \le 1.34$$

$$0.68 \le T\left(B_s \to \mu\bar{\mu}\right) \le 1.53$$

Governed by the modification of $\underbrace{X(v)}_{\text{enhanced or suppressed}}$ and $V_{td} \Downarrow$

Governed by the modification of the functions $\underbrace{X(v)\,,\,Y(v)}_{\text{enhanced or suppressed}}$

V_{ts} not modified

MFVfit Collaboration

(BBBEPSW)
hep-ph/0505110

Use the existing results for

1. UUTfit
2. $B \to X_s \gamma$
3. $B \to X_s l^+ l^-$
4. $K^+ \to \pi^+ \nu \bar{\nu}$

$\Rightarrow$

$X_{max}(v) = 1.95 \qquad Y_{max}(v) = 1.43$

$(X_{SM} \cong 1.53) \qquad (Y_{SM} \cong 0.99)$

$\Downarrow$

Model Independent Upper Bounds within MFV Scenario

Conclusion : Large Departures from SM within MFV not possible

Upper Bounds on Rare K and B Decays from MFV

Bobeth, Bona, AJB, Ewerth, Pierini, Silvestrini, Weiler hep-ph/0505110

Branching Ratios	MFV (95%)	SM (95%)	SM (68%)	Exp
$Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right)\cdot 10^{11}$	<11.9	<10.9	8.3±1.2	$14.7^{+13.0}_{-8.9}$
$Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)\cdot 10^{11}$	<4.6	<4.2	3.1±0.6	$<5.9\cdot 10^4$
$Br\left(B \to X_s \nu\bar{\nu}\right)\cdot 10^{5}$	<5.2	<4.1	3.7±0.2	<64
$Br\left(B_s \to \mu^+\mu^-\right)\cdot 10^{9}$	<7.4	<5.9	3.7±1.0	$<5.0\cdot 10^2$
$Br\left(B_d \to \mu^+\mu^-\right)\cdot 10^{10}$	<2.2	<1.8	1.1±0.4	$<1.6\cdot 10^3$