Electroweak loop corrections at TeV energies

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(1) **Introduction**

- high-energy behaviour of electroweak loop corrections

(2) **1 loop level**

- general formula for leading electroweak corrections

(3) **Phenomenological impact**

- pp $\to$ Z+jet at the LHC

(4) **2 loop level**

- recent results, open problems, phenomenological impact

1. Introduction

LHC and **ILC** will explore interactions of the SM constituents at

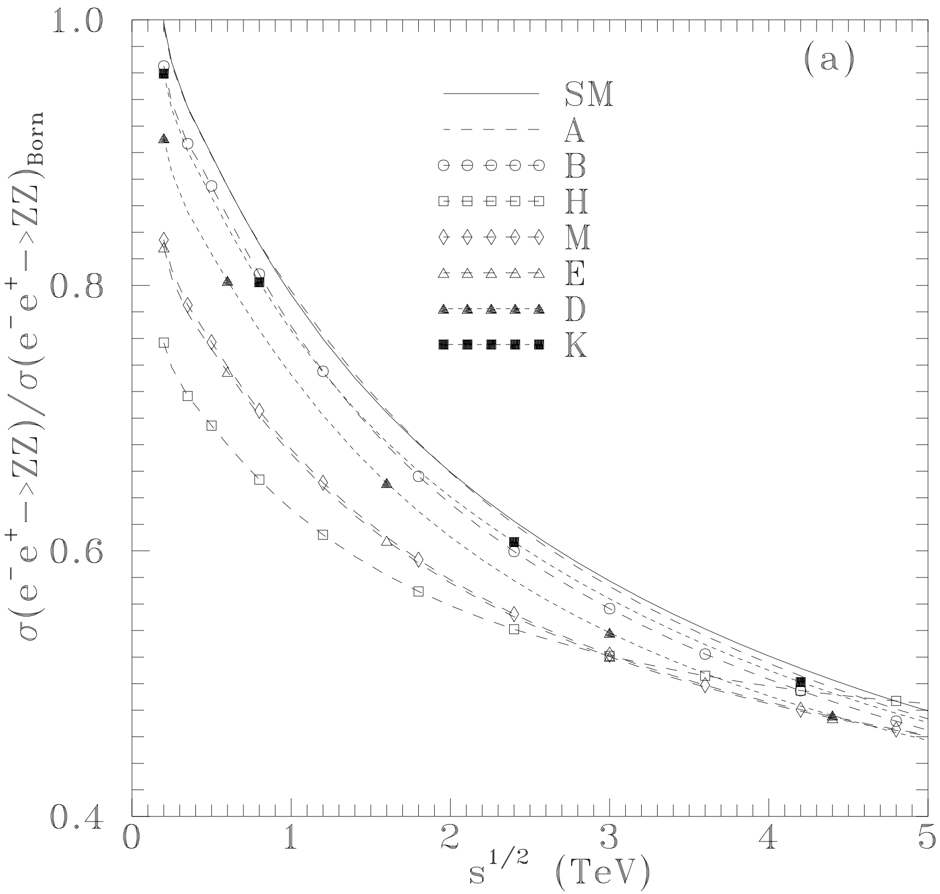
- center-of-mass energies in the TeV range
- high luminosity, high precision

Interpretation of precision measurements requires loop corrections (signal and background processes)

Loop effects needed to reach 1% precision (naive estimte):

$$\underbrace{\alpha_S \sim 10\%, \quad \alpha_S^2 \sim 1\%,}_{\mathrm{QCD}\,:\;1+2\,\mathrm{loop!}} \qquad \underbrace{\alpha \sim 1\%}_{\mathrm{EW}\,:\;1\,\mathrm{loop\,sufficient}}$$

One loop electroweak corrections to $e^+e^- \to ZZ$

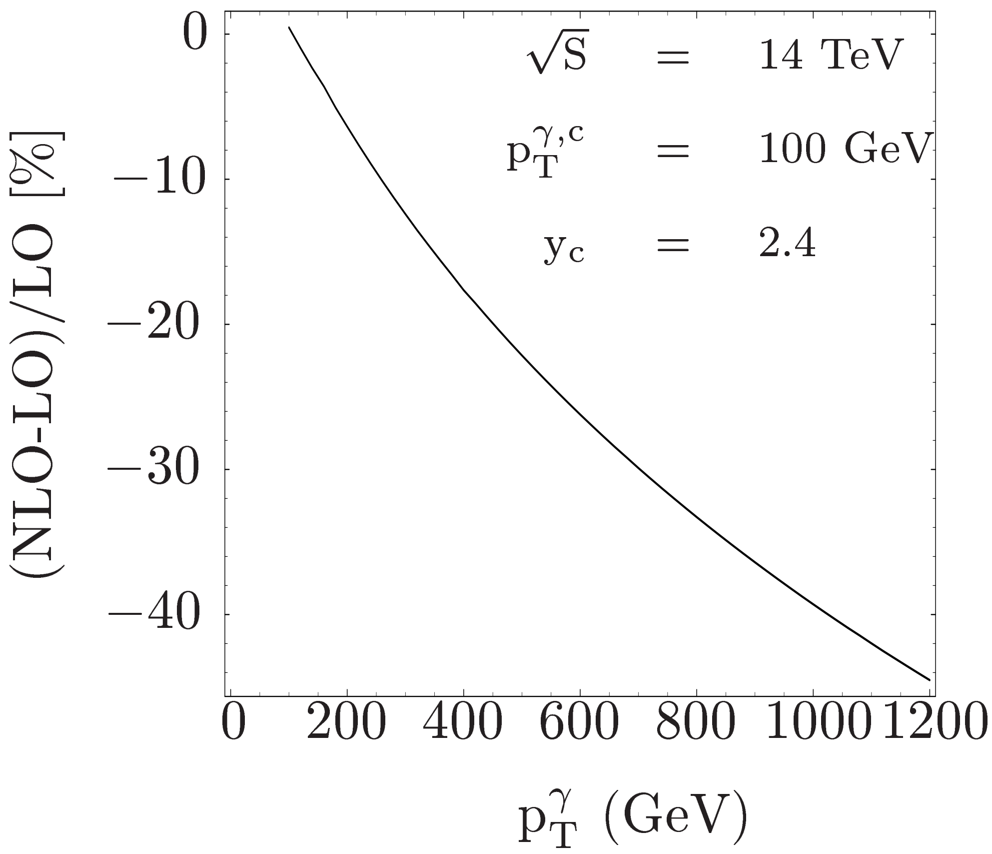


Gounaris, Layssac, Renard (2002)

Large negative corrections at TeV energies

- start at $\sqrt{s} \sim 200$ GeV
- increase with energy
- -20% at $\sqrt{s} \sim 1$TeV !

One loop EW corrections to $pp \to Z\gamma$ at the LHC



Hollik, Meier (2004)

Large negative corrections

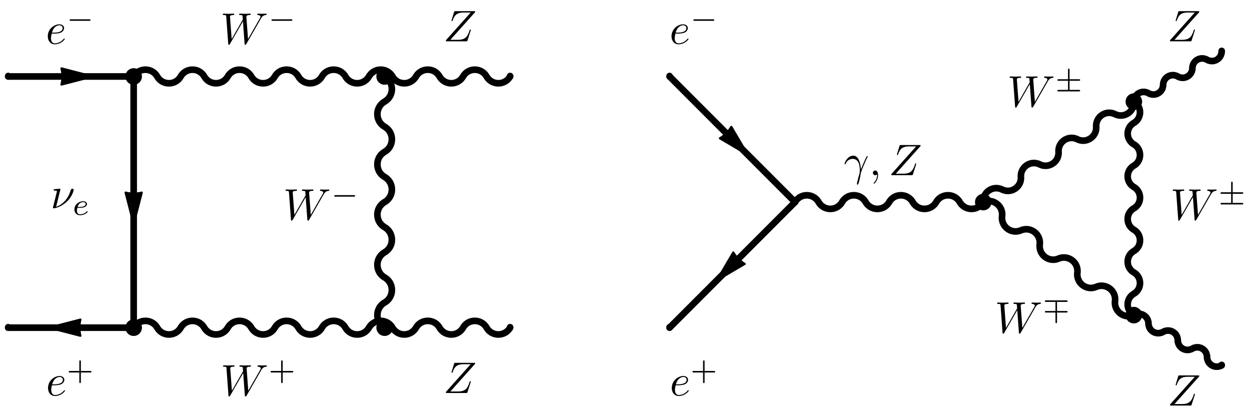
- similar behaviour
- -40% at $p_T \sim 1$ TeV !

Questions

- process independent feature?
- origin?

Origin of large corrections: $s \gg M_\mathrm{W}^2$

Asymptotic expansion of 1-loop Feynman diagrams



$\Rightarrow \alpha \ln^2\left(\frac{s}{M_\mathrm{W}^2}\right)$

Leading EW corrections at $\sqrt{s} \sim 1$ TeV

- from bosonic vertex and box diagrams
- double logarithms: $\ln^2\left(\frac{s}{M_\mathrm{W}^2}\right) \sim 25$
- general feature of hard scattering processes

General form of 1 loop EW corrections for $s \gg M_W^2$

$$\alpha \left[C_2 \underbrace{\ln^2\left(\frac{s}{M_W^2}\right)}_{\text{LL}} + C_1 \underbrace{\ln^1\left(\frac{s}{M_W^2}\right)}_{\text{NLL}} + C_0 \right] + \mathcal{O}\left(\frac{M_W^2}{s}\right)$$

How to predict C_2, C_1, C_0?

Process-by-process

- expansion of exact result
- many processes
- applicable only at 1 loop

Process-independent approach

- general predictions for $C_{2,1}$
- limited to logarithms
- applicable beyond one loop

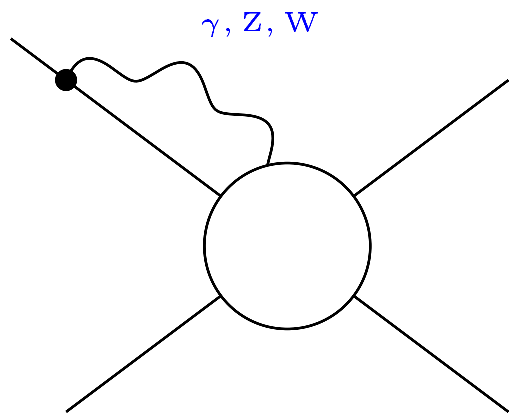
2. General results at 1 loop

The $\log(s/M^2)$ terms represent **mass singularities**!

<u>Origin of mass singularities</u>

- virtual gauge bosons $(\gamma, Z, W^{\pm})$ coupling to **on-shell external legs**

- **soft** $(q^\mu \to 0)$ and **collinear** $(q^\mu \to xp^\mu)$ regions



Process-independent treatment
(Ward identities, eikonal/collinear couplings, ...)

Universality and factorization of 1-loop EW logarithms

- arbitrary polarized processes $(e, \nu, u, d, t, b, \gamma, Z, W^{\pm}, H, g)$
- Born $\times$ external-leg factors

$$\mathcal{M}_1 = \Big(1 + \underbrace{\frac{\alpha}{4\pi} \sum_{\textbf{legs } k} \delta^{1}_{\mathrm{EW}}(k)}_{\text{soft,coll.}}\Big) \underbrace{\mathcal{M}_0}_{g \to g(s)}$$

Universal external-leg factors (process independent)

$$\delta^{1}_{\mathrm{EW}}(k) = -\frac{1}{2} C^{\mathrm{ew}}(k) \log^2 \frac{s}{M^2} + \sum_{l \neq k} \sum_{a=\gamma,\mathrm{Z},\mathrm{W}^{\pm}} I^{a}(k) I^{\bar{a}}(l) \log \frac{r_{kl}}{s} \log \frac{s}{M^2} + \gamma^{\mathrm{ew}}(k) \log \frac{s}{M^2}$$

$$-\frac{1}{2} Q^2(k) \left[2 \log \frac{s}{m_k^2} \log \frac{M^2}{\lambda^2} - \log^2 \frac{M^2}{m_k^2} - 2 \log \frac{M^2}{\lambda^2} - \log \frac{M^2}{m_k^2} \right] + \sum_{l \neq k} Q(k) Q(l) \log \frac{r_{kl}}{s} \log \frac{M^2}{\lambda^2}$$

Denner and P. (2001)

Typical size at $\sqrt{s} \simeq 1\,\mathrm{TeV}$

For $2 \to 2$ processes:

$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\mathrm{LL}} \simeq -\frac{\alpha}{\pi s_{\mathrm{W}}^2}\log^2\frac{s}{M^2} \simeq -26\%$$

$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\mathrm{NLL}} \simeq +\frac{3\alpha}{\pi s_{\mathrm{W}}^2}\log\frac{s}{M^2} \simeq +16\%$$

One-loop corrections

- $\mathcal{O}(10\%)$
- important impact on **LHC** and **ILC** observables!

3. pp $\rightarrow$ Z+jet at the LHC

Kühn, Kulesza, P., Schulze (2005)

Motivation to study this process

- high statistics and clean experimental signature (Z$\rightarrow l\bar{l}$)
- extract gluon PDF with precision at 1% level

Study of weak corrections at large p_T

- complete one loop calculation
- high-p_T expansion and comparison with NLL approximation
- dominant logarithmic effects a 2 loops

Analytic results: 1 loop weak corrections to $\bar{q}q \to Zg$

$$\overline{\sum}|\mathcal{M}_1|^2 = 8\pi^2\alpha\alpha_{\mathrm{S}}(N_{\mathrm{c}}^2 - 1) \sum_{\lambda=\mathrm{R,L}} \Bigg\{ \left(I^Z_{q_\lambda}\right)^2 \Bigg[H_0 \left(1 + 2\delta C^{\mathrm{A}}_{q_\lambda}\right)$$

$$+ \frac{\alpha}{2\pi} \sum_{V=\mathrm{Z,W}^\pm} \left(I^V I^{\bar{V}}\right)_{q_\lambda} H_1^{\mathrm{A}}(M_V^2) \Bigg] + \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}^3} T^3_{q_\lambda} I^Z_{q_\lambda} \Bigg[2H_0 \delta C^{\mathrm{N}}_{q_\lambda} + \frac{\alpha}{2\pi} \frac{1}{s_{\mathrm{W}}^2} H_1^{\mathrm{N}}(M_W^2) \Bigg] \Bigg\}$$

Main ingredients: $H_1^{\mathrm{A/N}}$ (loop diagrams)

- 14 Passarino-Veltman scalar integrals
- nontrivial analytic structure
- coefficients: rational functions of $\hat{s}, \hat{t}, \hat{u}, M_Z, M_W$

$\Rightarrow$ long expressions!

Asymptotic expansion: $\hat{s}, \hat{t}, \hat{u} \gg M_W^2$

$$H_1^{\mathrm{A/N}}(M_V^2) = \mathrm{Re}\left[g_0^{\mathrm{A/N}}(M_V^2)\,\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} + g_1^{\mathrm{A/N}}(M_V^2)\,\frac{\hat{t}^2-\hat{u}^2}{\hat{t}\hat{u}} + g_2^{\mathrm{A/N}}(M_V^2)\right],$$

$$g_0^{\mathrm{N}}(M_W^2) = 2\left[\frac{2}{4-D} - \gamma_{\mathrm{E}} + \log\left(\frac{4\pi\mu^2}{M_Z^2}\right)\right] + \log^2\left(\frac{-\hat{s}}{M_W^2}\right) - \log^2\left(\frac{-\hat{t}}{M_W^2}\right) - \log^2\left(\frac{-\hat{u}}{M_W^2}\right)$$
$$\log^2\left(\frac{\hat{t}}{\hat{u}}\right) - \frac{3}{2}[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right)] - \frac{20\pi^2}{9} - \frac{\pi}{\sqrt{3}} + 2,$$

$$g_0^{\mathrm{A}}(M_V^2) = -\log^2\left(\frac{-\hat{s}}{M_V^2}\right) + 3\log\left(\frac{-\hat{s}}{M_V^2}\right) + \frac{3}{2}[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right)$$
$$+\log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right)] + \frac{7\pi^2}{3} - \frac{5}{2},$$

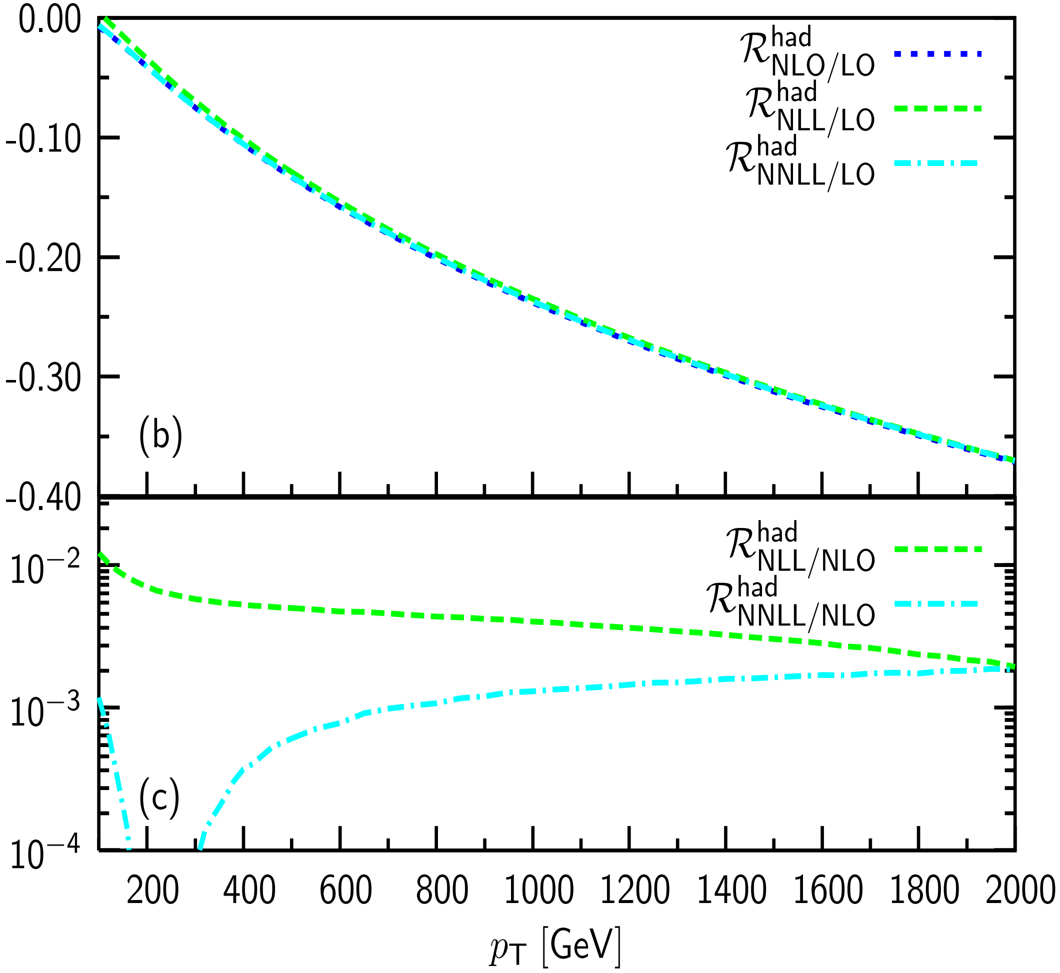
$$g_1^{\mathrm{N}}(M_V^2) = -g_1^{\mathrm{A}}(M_V^2) + \frac{3}{2}[\log\left(\frac{\hat{u}}{\hat{s}}\right) - \log\left(\frac{\hat{t}}{\hat{s}}\right)] = \frac{1}{2}[\log^2\left(\frac{\hat{u}}{\hat{s}}\right) - \log^2\left(\frac{\hat{t}}{\hat{s}}\right)],$$

$$g_2^{\mathrm{N}}(M_V^2) = -g_2^{\mathrm{A}}(M_V^2) = -2[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right) + \log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right)] - 4\pi^2.$$

Simple and compact expressions (easy to implement!)

- NLL predicted by process-independent formula
- remainder: $\pi^2, \log(\hat{t}/\hat{u}), \ldots$ not growing with energy

Corrections to the p_T distribution for $pp \to Z + \text{jet}$ at LHC



Large negative corrections

- increase with p_T
- -25% at $p_T \sim 1$ TeV

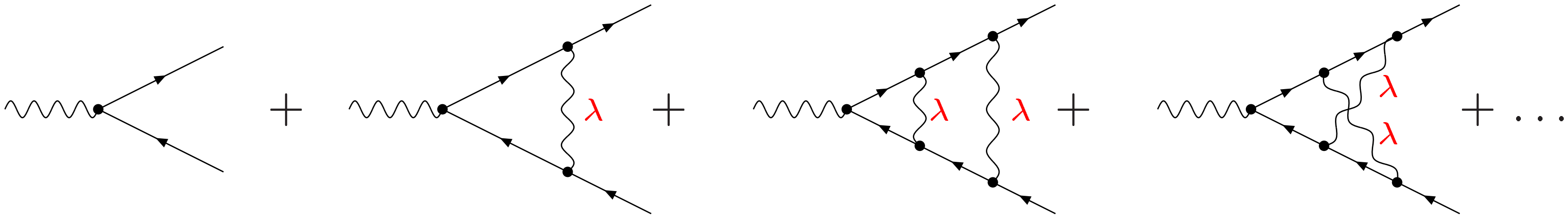
Quality of high-energy approximations

- $2 \times 10^{-3} \leq \text{NLL/NLO} - 1 \leq 10^{-2}$
- $\text{NNLL/NLO} - 1 \leq 2 \times 10^{-3}$

$\Rightarrow$ very precise!

4. Status and open problems at 2 loops

Higher-order leading logarithms (LL) in QED vertex ($Q^2 \gg \lambda^2$)



$$= \mathrm{i}e\,\bar{u}(p_1)\gamma^\mu v(p_2) \underbrace{\left\{1 - \frac{\alpha}{4\pi}\ln^2\left(\frac{Q^2}{\lambda^2}\right) + \frac{1}{2!}\left[\frac{\alpha}{4\pi}\ln^2\left(\frac{Q^2}{\lambda^2}\right)\right]^2 + \ldots\right\}}_{\exp\left[-\frac{\alpha}{4\pi}\ln^2\left(\frac{Q^2}{\lambda^2}\right)\right]}$$

Exponentiation!

Sudakov (1956)

Typical size of 2 loop EW logarithms at $\sqrt{s} \simeq 1\,\text{TeV}$

Assuming exponentiation

$$\exp\left(\frac{\delta\sigma_1}{\sigma_0}\right) = 1 + \left(\frac{\delta\sigma_1}{\sigma_0}\right) + \frac{1}{2}\left(\frac{\delta\sigma_1}{\sigma_0}\right)^2 + \dots$$

1 loop: $\mathcal{O}(10\%)$ corrections

$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\text{LL}} \simeq -\frac{\alpha}{\pi s_{\text{W}}^2}\log^2\frac{s}{M_W^2} \simeq -26\%$$

$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\text{NLL}} \simeq +\frac{3\alpha}{\pi s_{\text{W}}^2}\log\frac{s}{M_W^2} \simeq +16\%$$

2 loop: $\mathcal{O}(1\%)$ corrections

$$\left(\frac{\delta\sigma_2}{\sigma_0}\right)_{\text{LL}} \simeq +\frac{\alpha^2}{2\pi^2 s_{\text{W}}^4}\log^4\frac{s}{M_W^2} \simeq 3.5\%$$

$$\left(\frac{\delta\sigma_2}{\sigma_0}\right)_{\text{NLL}} \simeq -\frac{3\alpha^2}{\pi^2 s_{\text{W}}^4}\log^3\frac{s}{M_W^2} \simeq -4.1\%$$

$\Rightarrow$ important for precision measurements at **ILC** (!) and **LHC** (?)

Asymptotic expansion of 2-loop EW corrections

General form for $s/M_W^2 \gg 1$

$$\alpha^2 \left[C_4 \underbrace{\ln^4\left(\frac{s}{M_W^2}\right)}_{\text{LL}} + C_3 \underbrace{\ln^3\left(\frac{s}{M_W^2}\right)}_{\text{NLL}} + \ldots \right]$$

Hierarchy at $\sqrt{s} \sim 1\,\text{TeV}$

- $\ln^4\left(\frac{s}{M_W^2}\right) \gg \ln^3\left(\frac{s}{M_W^2}\right) \gg \ldots$
- start to investigate the leding terms $C_4, C_3, \ldots$

Resummations of electroweak logarithms

arbitrary processes

- **LL**

  Fadin, Lipatov, Martin, Melles (2000)

- **LL + NLL**

  Melles (2001,2002,2003)

$f\bar{f} \to f'\bar{f}'$ (massless)

- **LL + NLL + NNLL**

  Kühn, Moch, Penin, Smirnov (2000,2001)

Prescriptions based on **QCD/QED resummation techniques**

- in the high-energy limit of the EW theory **spontaneous symmetry breaking neglected** (mixing, mass-gaps, couplings with mass dimension, . . .)
- **need to be proven!**

Diagrammatic calculations at 2 loops

arbitrary processes

- **LL**

  Melles (2000); Hori et. al. (2000);

  Beenakker and Werthenbach (2000,2002)

- **LL + ang-dep NLL**

  Denner, Melles and P. (2003)

$gf\bar{f}$ vertex

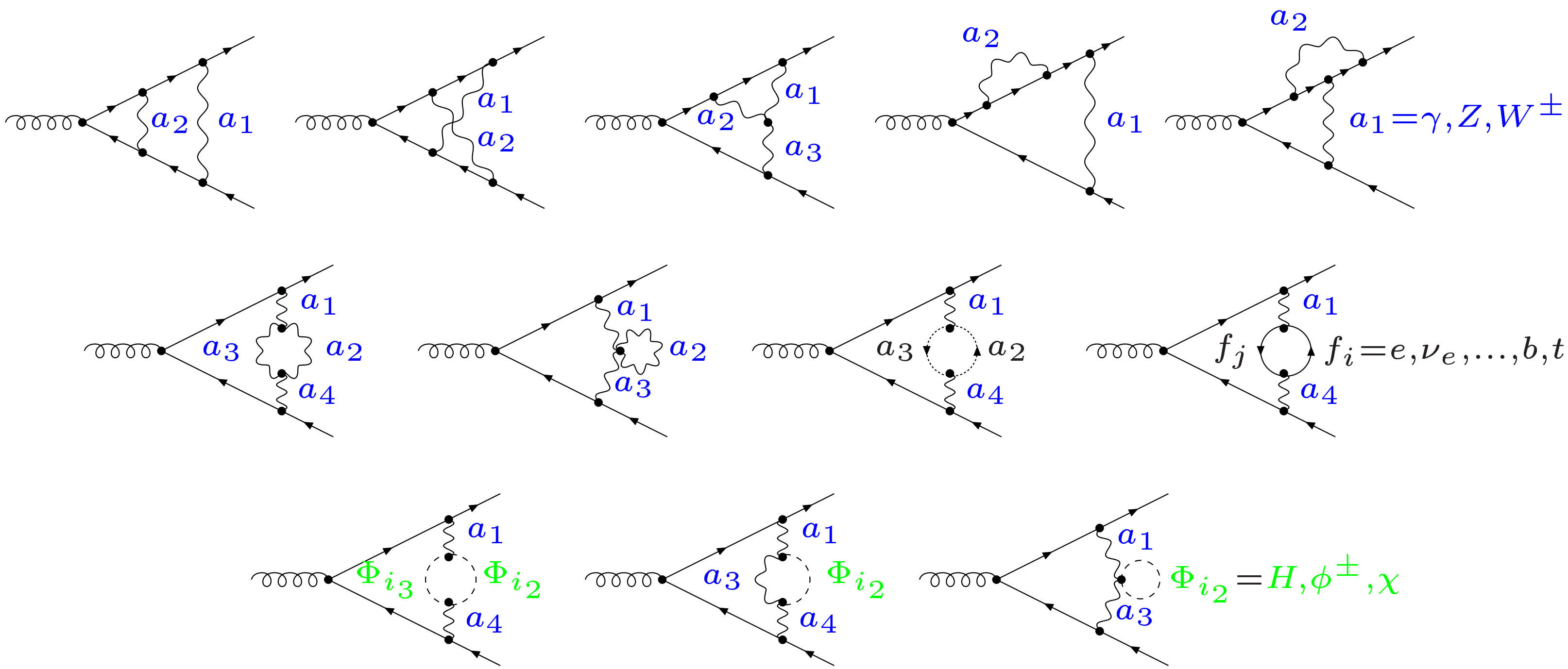
- **LL + NLL**

  P. (2004)

Based on the **electroweak Feynman rules** (symmetry breaking!)

- Present results: **resummation prescriptions confirmed**
- Next important goal: **complete set of NLL** for arbitrary processes

Next-to-leading logarithms in the $gf\bar{f}$ vertex at 2 loops



Calculation of 2-loop integrals

- in the logarithmic approximation
- automatized algorithm

Algorithm for the high-energy expansion of multi-loop diagrams

- based on Feynman parametrization, sector decomposition
- UV and mass singularities (γ, f) in $D = 4 - 2\epsilon$
- diagrams with various energy and mass parameters in the limit

$$s \sim t \sim \cdots \sim u \gg M_W^2 \sim M_Z^2 \sim M_t^2 \sim M_H^2$$

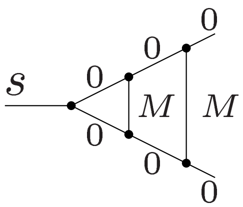
- for arbitrary multi-loop topologies extract

$$\left(\frac{1}{\epsilon}\right)^{r_0} \ln^{r_1}\left(\frac{s}{M_W^2}\right)$$

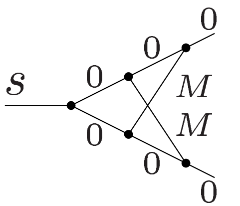
- automatized up to next-to-leading level
  ($r_0 + r_1 = 4, 3$ at two loops)
- subleading terms in the future ($r_0 + r_1 \leq 2$ at two loops)

Denner and P. (2005)

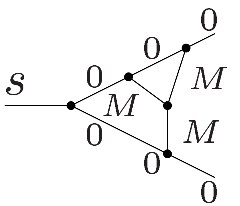
Mathematica implementation

$(q_2 p_1)(q_2 p_2) \times$ 

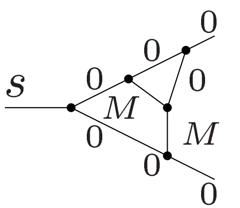
$$\overset{(16+0.1)}{=} -\left(\frac{\mu^2}{s}\right)^{2\epsilon}\left[\frac{1}{16\epsilon}\log^2\left(\frac{s}{M^2}\right)+\frac{1}{24}\log^3\left(\frac{s}{M^2}\right)\right]$$



$$\overset{(7+32)}{=} -\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{7}{12 s^2}\log^4\left(\frac{s}{M^2}\right)$$

$(q_2 p_1) \times$ 

$$\overset{(4+20)}{=} \left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{1}{s}\left[\frac{1}{24}\log^4\left(\frac{s}{M^2}\right)-\frac{1}{12}\log^3\left(\frac{s}{M^2}\right)\right]$$

$(q_2 p_1) \times$ 

$$\overset{(3+17)}{=} -\left(\frac{\mu^2}{s}\right)^{2\epsilon}\frac{1}{s}\left[\frac{5}{48}\log^4\left(\frac{s}{M^2}\right)+\frac{1}{12\epsilon}\log^3\left(\frac{s}{M^2}\right)+\frac{1-2\gamma_E}{12}\log^3\left(\frac{s}{M^2}\right)\right]$$

$(t_{sd}+t_{int})=$ **computing time in seconds**

5. Two-loop effects for pp $\to$ Z+jet at the LHC

Two-loop contributions for $\bar{q}q \to Zg$

$$\overline{\sum}|\mathcal{M}_2|^2 = \overline{\sum}|\mathcal{M}_1|^2 + 2\alpha^3\alpha_{\mathrm{S}}(N_c^2-1)\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\sum_{\lambda=\mathrm{L,R}}\left\{\frac{1}{2}\left(I^Z_{q_\lambda}C^{\mathrm{ew}}_{q_\lambda}+\frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}}T^3_{q_\lambda}\right)\right.$$
$$\times\left[I^Z_{q_\lambda}C^{\mathrm{ew}}_{q_\lambda}X_1+\frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}}T^3_{q_\lambda}X_2\right]-\frac{T^3_{q_\lambda}Y_{q_\lambda}}{8s^4_{\mathrm{W}}}X_2$$
$$\left.+\frac{1}{6}I^V_{q_\lambda}\left[I^Z_{q_\lambda}\left(\frac{b_1}{c^2_{\mathrm{W}}}\left(\frac{Y_{q_\lambda}}{2}\right)^2+\frac{b_2}{s^2_{\mathrm{W}}}C_{q_\lambda}\right)+\frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}}T^3_{q_\lambda}b_2\right]X_3\right\}$$

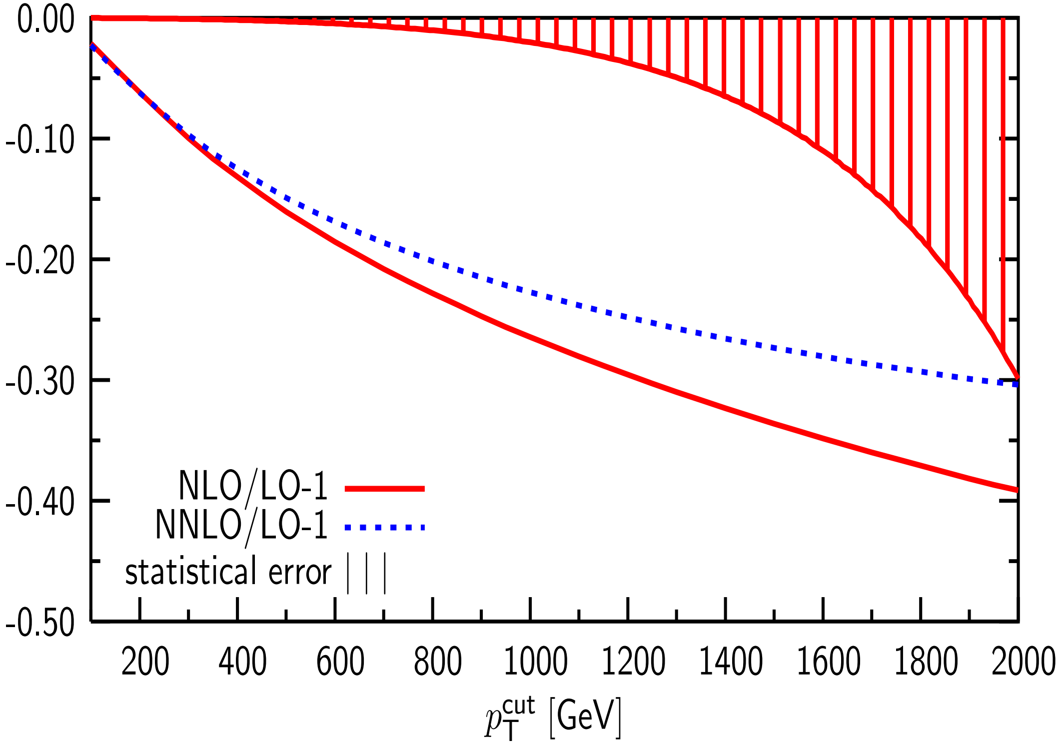
with LL and NLL terms

Kühn, Kulesza, P., Schulze (2005)

$$X_1 = \ln^4\left(\frac{\hat{s}}{M_W^2}\right)-6\ln^3\left(\frac{\hat{s}}{M_W^2}\right), \quad X_2 = \ln^4\left(\frac{\hat{t}}{M_W^2}\right)+\ln^4\left(\frac{\hat{u}}{M_W^2}\right)-\ln^4\left(\frac{\hat{s}}{M_W^2}\right),$$

$$X_3 = \ln^3\left(\frac{\hat{s}}{M_W^2}\right)$$

Predictions for pp → Z+jet at the LHC



Size of corrections at $p_{\mathrm{T}} \sim 1\,\mathrm{TeV}$

- 1-loop: -26%
- 1+2-loop: -26% + 4% = -22%

Comparison with statistical error

- $\mathcal{L} = 300\,\mathrm{fb}^{-1}$, Z → leptons

⇒ $(\Delta\sigma/\sigma)_{\mathrm{stat}} \sim 2\%$ at 1 TeV

Summary

At TeV energies EW corrections are enhanced by large logarithms

At 1 loop

- $\mathcal{O}(10\%)$ corrections
- well understood
- good approximation

At 2 loops

- $\mathcal{O}(1\%)$ corrections
- LL well understood
- ansatz for NLL resummation

New tool to study 2-loop logarithms

- algorithm to extract logarithms from multi-loop diagrams

Important for interpretation of precision measurements of many reactions at LHC and ILC