Supersymmetry and electroweak corrections at high energy

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e.g. $\begin{cases} \text{mSUGRA 5 parameters} \\ \text{cMSSM 4 parameters} \end{cases}$ but these are *ad hoc* hypotheses. . .

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iii. $\begin{pmatrix}\sim \text{helicity conservation} \\ \text{at high energy (kinematic} \\ \text{and dynamic reasons)}\end{pmatrix} \Rightarrow \begin{pmatrix}\text{Many amplitudes are} \\ \text{negligible (at tree} \\ \text{level and beyond} \ldots \text{)}\end{pmatrix}$

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◇ It is not unreasonable in some MSSM scenarios ($\sim$SPS 4)

Sudakov logarithms: general structure

At L loops electroweak corrections of many processes have an asymptotic limit of the form ($s = E^2_{c.m.},\ \alpha_W = \frac{e^2}{4\pi \sin^2 \theta_W}$):

$$A = A^{\mathrm{Born}} \left(1 + \sum_{\ell=1}^{L} \alpha_W^{\ell} \sum_{k=1}^{2\ell} c_{\ell,k} \log^k \left(\frac{s}{M^2} \right) \right) + \dots$$

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- A lot of relevant processes have such a simple asymptotic expansion:
  - ◇ ILC processes: $e^+e^- \to f\overline{f},\ \chi^0\chi^0,\ \chi^+\chi^-,\ H^+H^-, W^+W^- \ldots$

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  - ◇ LHC (partonic) processes:
    $$\begin{cases} t\bar{t} \text{ production: } gg \to t\bar{t},\ q\bar{q} \to t\bar{t} \\ \text{single top production: } bq \to tq',\ q\bar{q} \to t\bar{b},\ bg \to tW^-,\ bg \to H^-t \end{cases}$$

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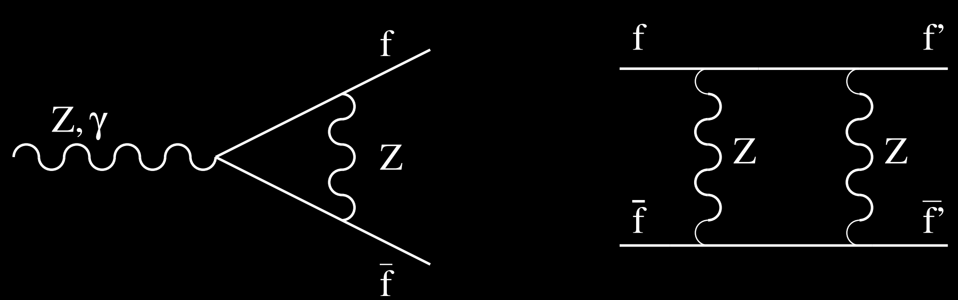
- Diagrammatic origin:
  - ◇ For each coefficient c, gauge invariant combinations of vertex's corrections, self-energies and boxes

    for example:



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  - ◇ At $\sqrt{s} \geq 1$ TeV these terms:

$$\left(\begin{array}{l} \text{Give corrections to cross} \\ \text{sections at the level of } 10-30\% \end{array} \right) \Rightarrow \left(\begin{array}{l} \text{These contributions} \\ \text{are detectable at LHC} \end{array} \right)$$

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$\Downarrow$

(At ILC resummation issues)

[Available resummations at log NLO $i.e.\ \log^{2L}\frac{s}{M_W^2}$, $\log^{2L-1}\frac{s}{M_W^2}$]

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Detailed discussion is available:

- M. Beccaria, S. Bentvelsen, M. Cobal, F.M. Renard and C. Verzegnassi, *"Special SUSY features of large invariant mass unpolarized and polarized top-antitop production at the CERN LHC".* Phys. Rev. D71, 073003, (2005).

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So it needs a pragmatic attitude:
in a light SUSY scenario expansion is a first step which can reveal the presence of large corrections and can be used to direct the efforts towards a full 1 loop treatment

Deeper inside Yukawa contributions

At L loops (purely) Yukawa contributions are of the form:

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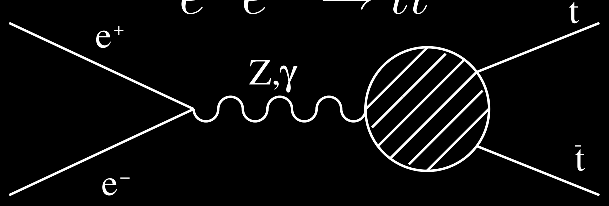

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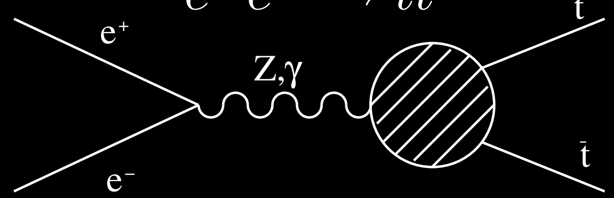

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  - ◇ Can be computed in a <u>gaugeless</u> limit of MSSM:

$$\mathcal{W} = \frac{g}{\sqrt{2}}\frac{m_t}{M_W}\frac{1}{\sin\beta} t_R(t_L H_u^0 - b_L H_u^+) + \frac{g}{\sqrt{2}}\frac{m_b}{M_W}\frac{1}{\cos\beta} b_R(t_L H_d^- - b_L H_d^0).$$

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◇ M. Beccaria and E. Mirabella,
"Yukawa enhanced electroweak corrections at high energy in the MSSM"
Phys. Rev. D72, 055004, (2005).

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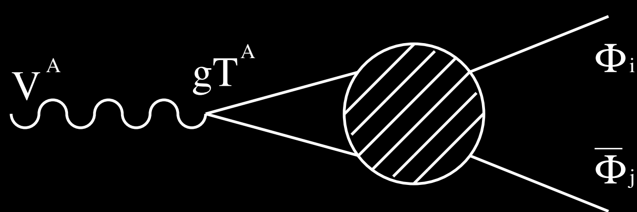
- Due to the U.V. origin of Yukawa terms, and exploiting Supersymmetry:

$\overline{\psi}\sigma^\mu\psi A_\mu \leftrightarrow$



$$\equiv \Gamma_{(V)ij} \begin{cases} \overline{\psi}\sigma^\mu\psi\ A_\mu, \\ i(\varphi\partial^\mu\varphi^* - \partial^\mu\varphi\varphi^*)A_\mu, \\ \overline{\lambda}\ \overline{\psi}\varphi + \text{h.c.} \end{cases}$$

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We consider leading Yukawa contributions to $\overline{\psi}\sigma^\mu\psi A_\mu$ *i.e.* at L loops:

$$\alpha^L \left(\frac{m_t^2}{\sin^2\beta M_W^2}\right)^p \left(\frac{m_b^2}{\cos^2\beta M_W^2}\right)^q \log^{L}\frac{s}{M^2}, \quad p+q=L.$$

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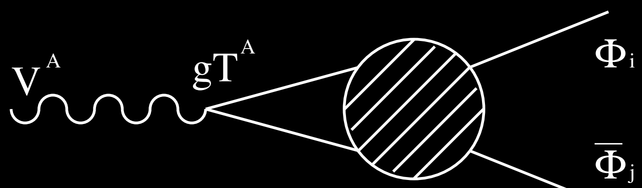

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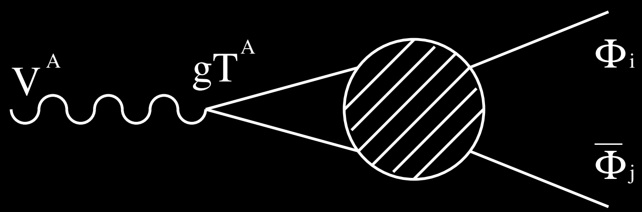

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$$\left(\mu\frac{\partial}{\partial\mu}+\beta_t(y_t,y_b)\frac{\partial}{\partial y_t}+\beta_b(y_t,y_b)\frac{\partial}{\partial y_b}-\gamma_{ij}(y_t,y_b)\right)\Gamma_{(V)ij}\left(\frac{Q}{\mu},y_t,y_b\right)=0.$$

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$$\Gamma_{(V)ij}\left(\frac{Q}{\mu}\right) \stackrel{LO}{=} c_{ij}\left(\frac{Q}{\mu}\right)\,\Gamma^{\mathrm{Born}}_{(V)ij}, \qquad (\triangle)$$

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Applications and discussions

Study of relevance of Yukawa corrections in many processes:

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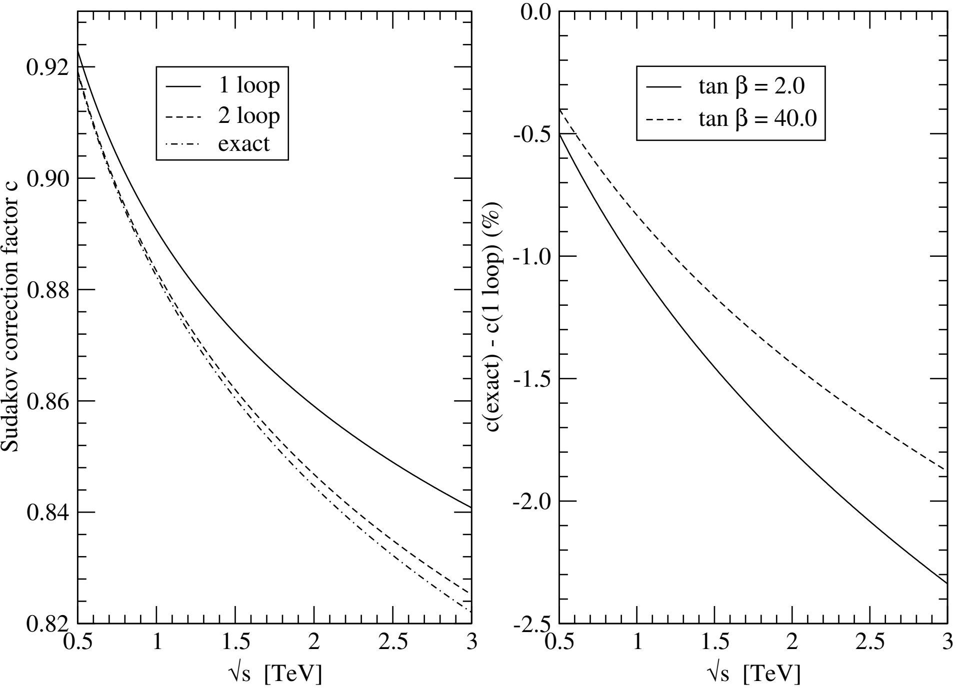
- ◇ LHC process: $u\overline{d} \to t_L \overline{b}_L$

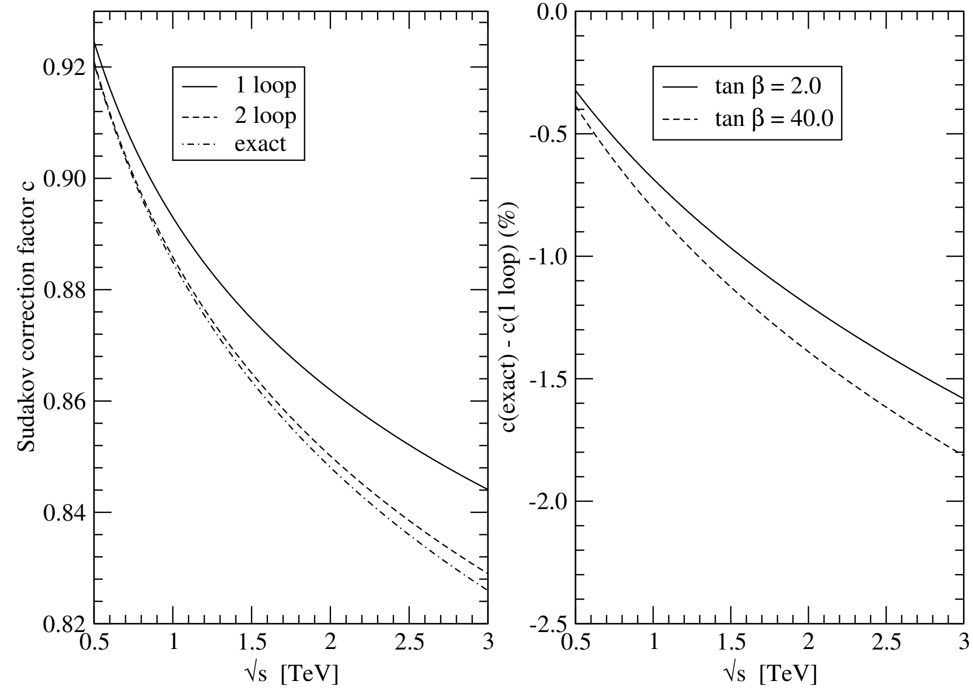
In other words we have compared:

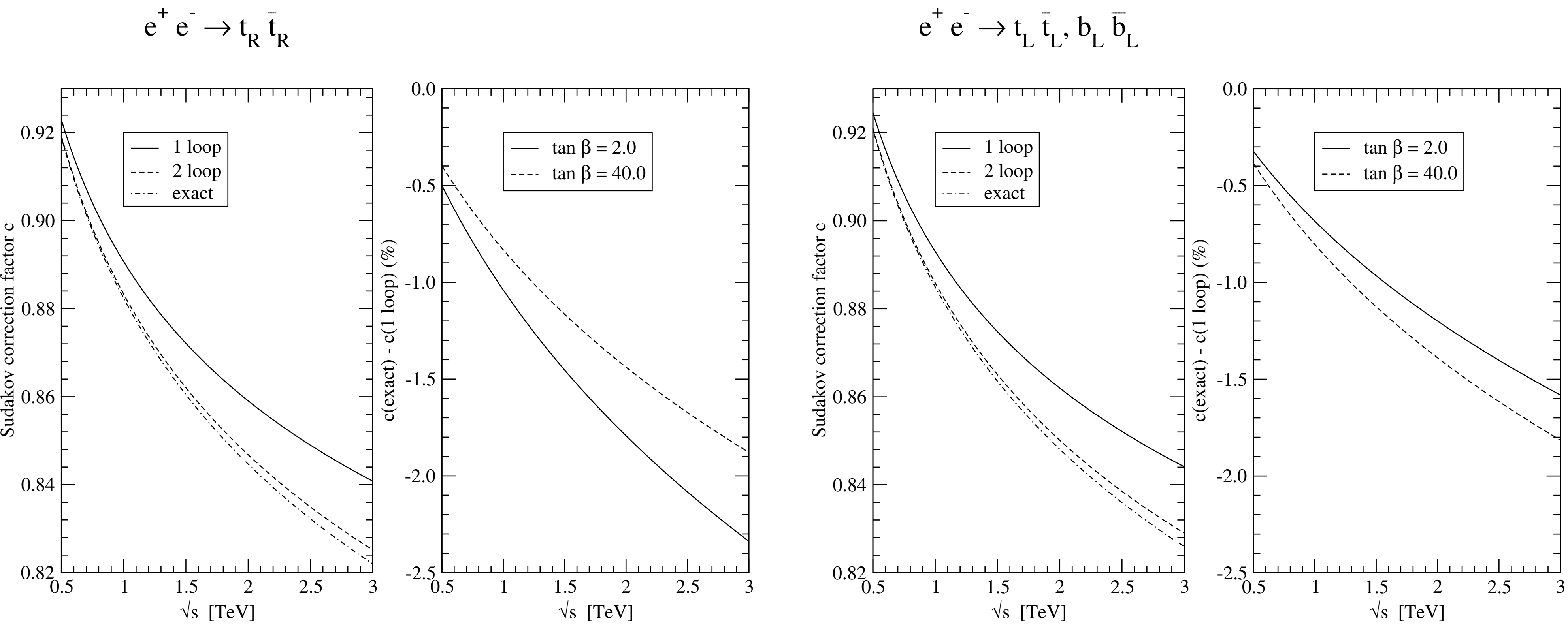
$$c_{\text{exact}} - c_{1\text{ loop}} - c_{2\text{ loops}}$$

$e^+ e^- \to t_R \bar{t}_R$

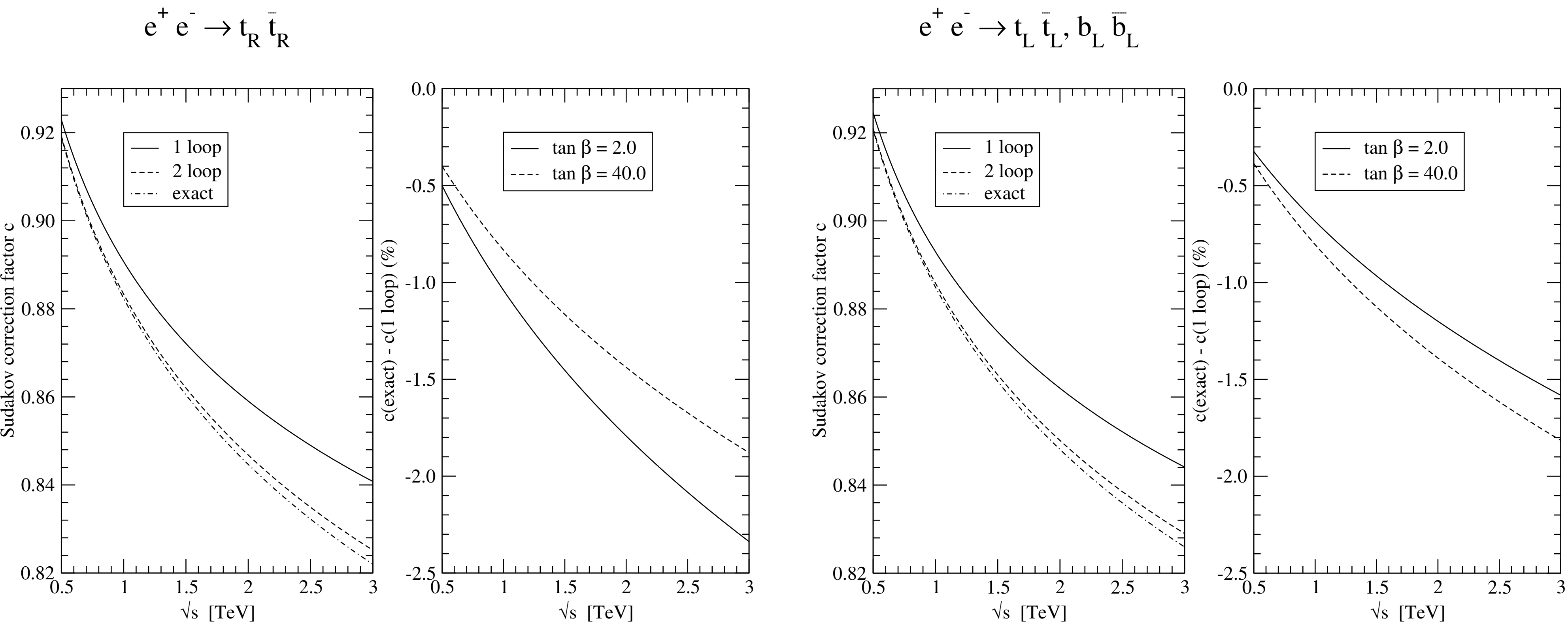


$e^+ e^- \to t_L \bar{t}_L, b_L \bar{b}_L$





- Second order corrections are significant



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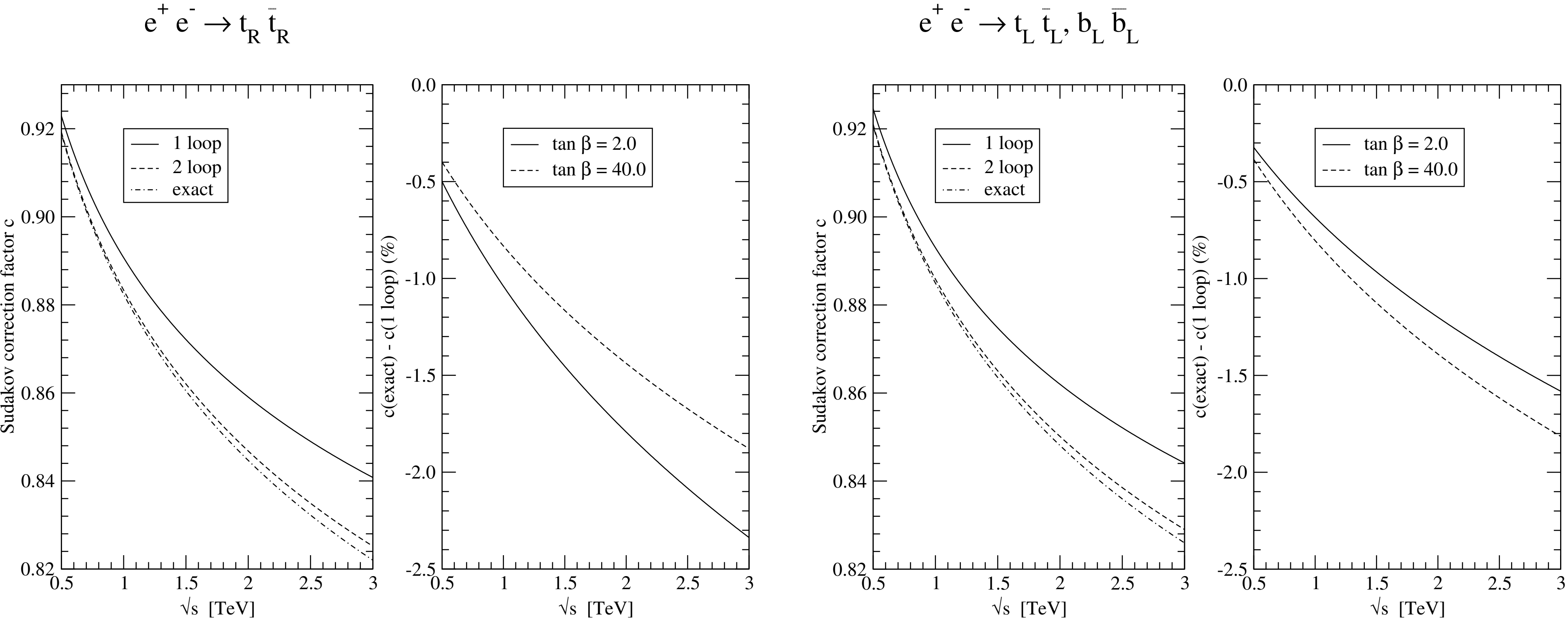

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- Relative differences between exact and one loop calculations are visible at ILC

Sudakov Supersymmetric Relations

A fact:
At 1 loop and at log NLO there are some relations (SSR) between c coefficients of Sudakov corrections of the amplitudes of different processes involving particles of the same SUSY multiplet, *e.g.*

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◇ M. Beccaria and E. Mirabella, *"Supersymmetric structure of electroweak Sudakov corrections"*. Phys. Rev. D **71**, 115016,(2005).

Study of SSR: strategy

Hints for a SUSY origin of SSR

- (If SUSY exact) $\Rightarrow$ (W.I. ensure that SSR are valid at all orders)
- At 1 loop and at log NLO: $\exists$ GBHC rules,
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So SSR may have a SUSY origin

One can demonstrate it neglecting MSSM's soft terms

and computing in the asymptotic limit

the various 1 loop corrections

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$$e^+_\alpha e^-_\alpha \longrightarrow f_\beta \overline{f}_\beta, \qquad\qquad e^+_\alpha e^-_\alpha \longrightarrow \widetilde{f}_\beta \widetilde{f}^*_\beta,$$

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in both cases

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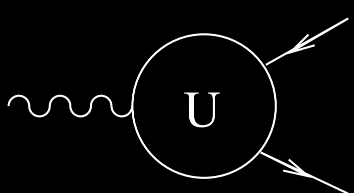
$$\Downarrow$$



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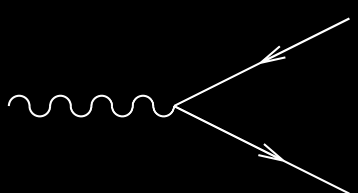

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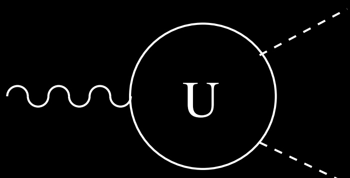

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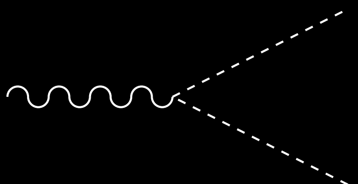

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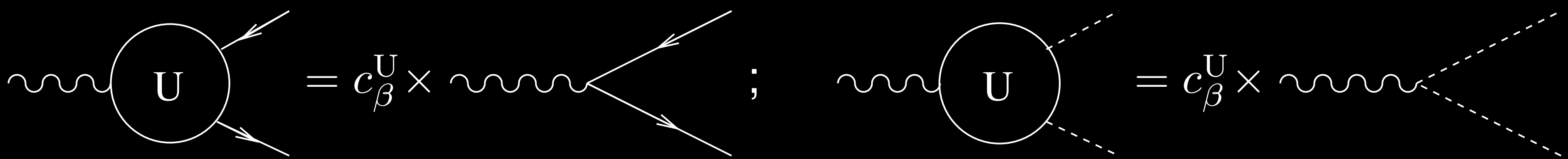

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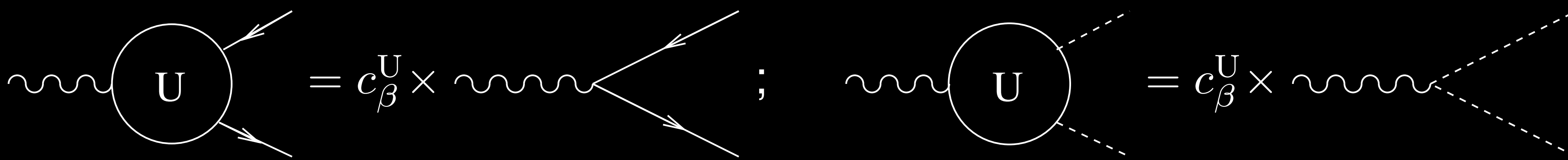

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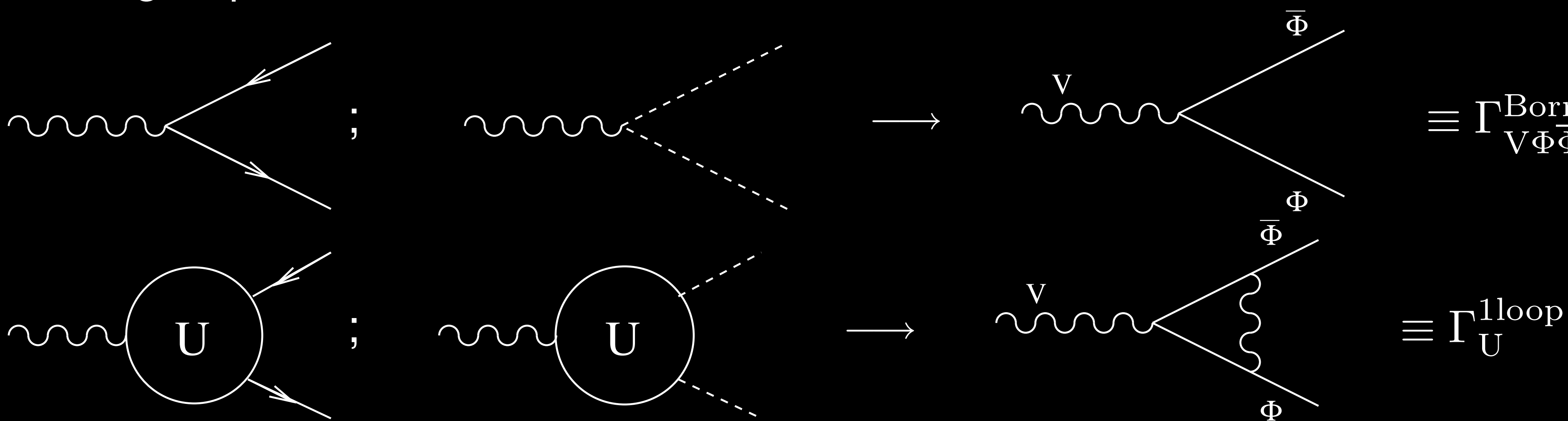

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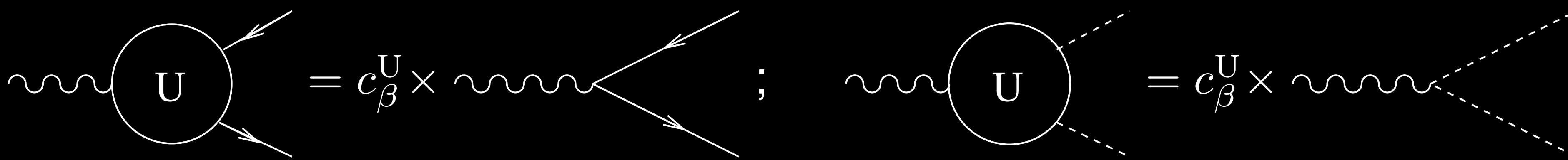

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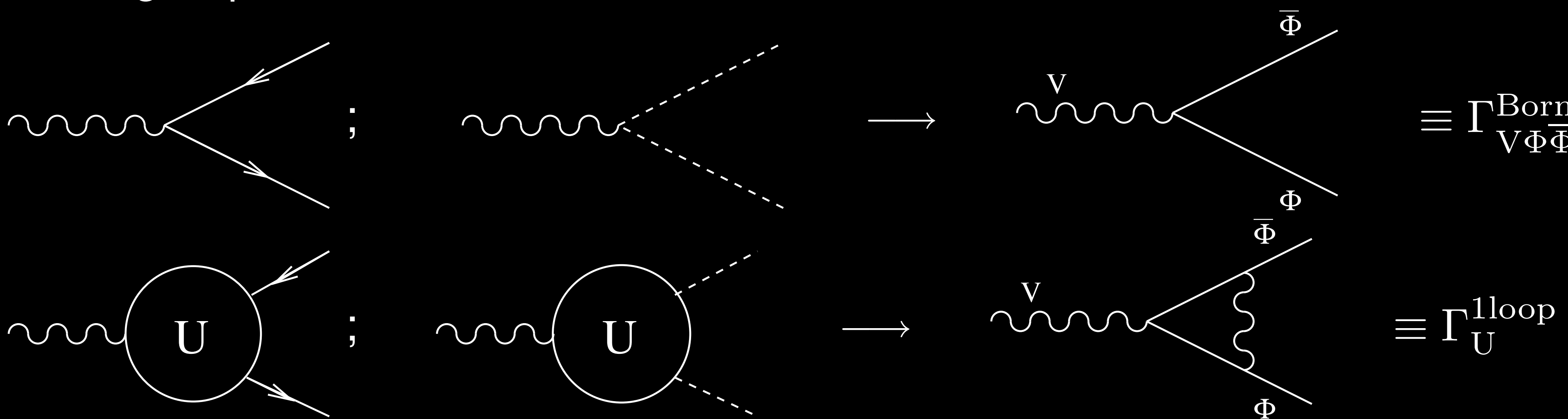

So SUSY origin of SSR is demonstrated if:

$$\Gamma^{\mathrm{1loop}}_{\mathrm{U}} = c_\beta^{\mathrm{U}} \times \Gamma^{\mathrm{Born}}_{\mathrm{V}\Phi\overline{\Phi}}$$

Study of SSR: results

Coming back at the general case if we show:

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- Convenient $\left\{ \begin{array}{l} \text{Diagrams with precise physical origin} \\ \text{Independence from renormalization scheme} \end{array} \right.$

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These SSR have been confirmed by a calculation in components.

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- Original results of my research activity:
  ◇ Resummation of leading Yukawa logarithms at all orders in perturbative expansion for many important processes.
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- Possible estensions in a "wider" perspective:

  ◇ Exploitation of Sudakov expansions as an analitical information useful in a full 1 loop calculation