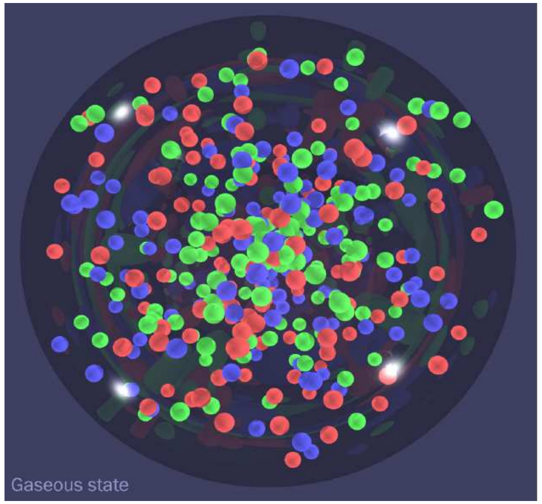
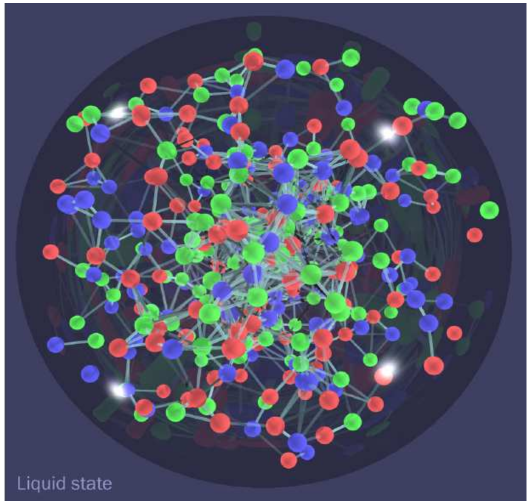
Quark Gluon Plasma and the AdS/CFT-correspondence

by Matthias Kaminski

IMPRS Particle Physics School Munich Seminar, 12th of May 2006





[Berkley Lab webpages: http://www.lbl.gov/Science-Articles/Archive/sabl/2005/May/02-RHIC.html]

Collision movie

Overview

1. Quark-Gluon-Plasma at the RHIC collider
2. Strings and the AdS/CFT-correspondence
3. Kubo formulae
4. Application to different theories
5. Summary and Outlook

2. Quark-Gluon-Plasma at RHIC

[BNL 73847-2005 Formal Report]

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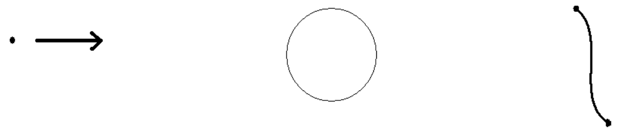
☞ PHENIX collaboration claims that they will be able to ' . . . *address color and thermal transport properties* . . . '

3. Strings and AdS/CFT

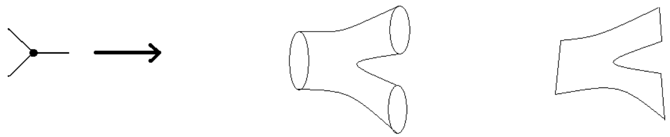
Question: How is String Theory related to Quantum Field Theory?

3.1 – String Theory

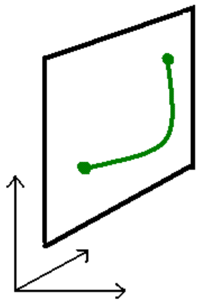
point particle | extended strings



pointlike vertex | worldsheets



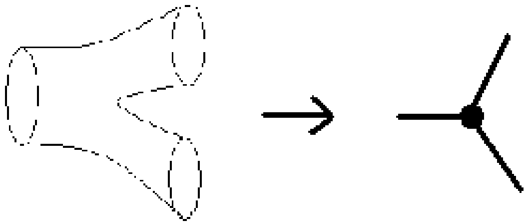
Also contains D branes e.g. D3-brane



- open strings end on D-branes (determines dynamics)
- hypersurfaces embedded in 10-dimensional spacetime
- field theory ’lives’ on branes (at low-energy *string* $\rightarrow\ \cdot$)

3.2 – SUGRA-limit of String Theory

$$\lambda = g_S N \rightarrow \infty$$



(later $N \rightarrow \infty$)

string length: $l_{\text{string}}^4 = \frac{L^4}{4\pi(g_s N)} \rightarrow 0$

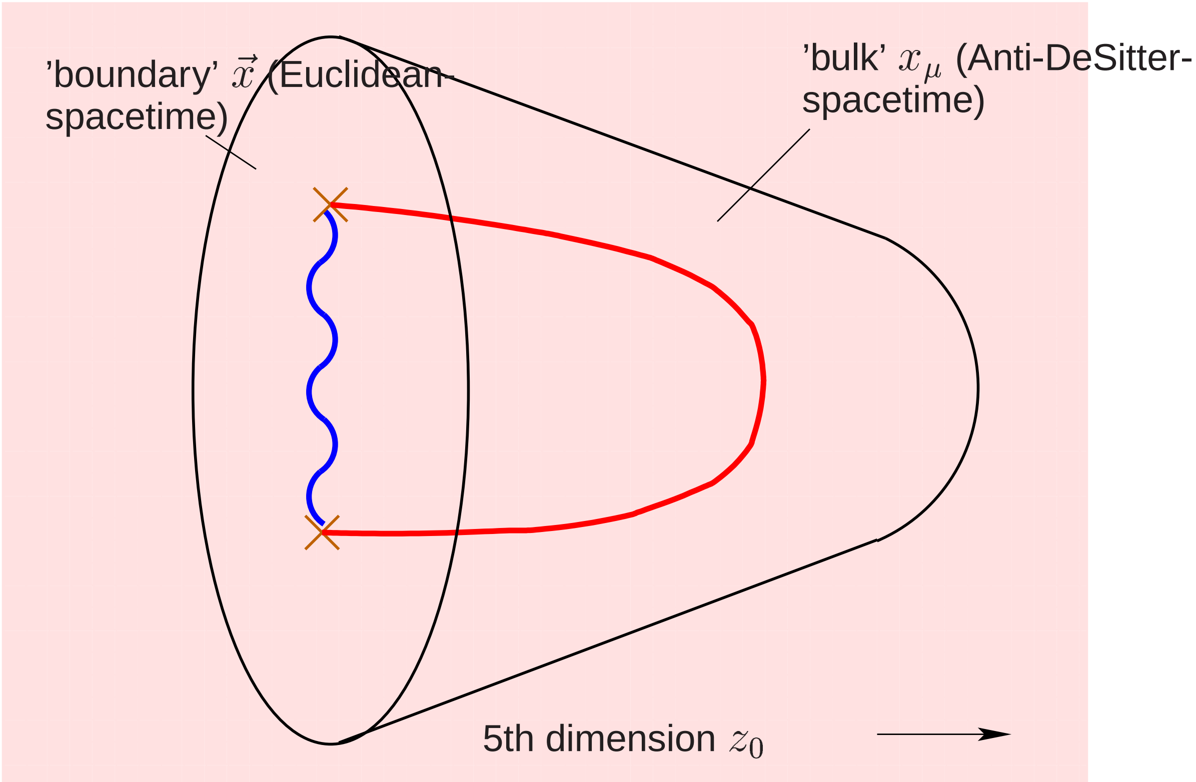
$\Longrightarrow$ SUGRA is a classical perturbative field theory expanded in $\alpha' = l_{\text{string}}^2$.

Possible solutions to SUGRA: solitons$\equiv$ D-branes

3.3 – AdS/CFT-conjecture

[Maldacena, '97]

Using the two 'faces' of D-branes



Dictionary (by limit $\lambda \to \infty$):

operator $\mathcal{O}$ in Conformal Field Theory (CFT)$\longleftrightarrow$ SUGRA-field ϕ in Anti-deSitter space $AdS_5 \times S^5$

3.4 – Correlators from AdS/CFT

Formally stated for Euclidean metric:

$$\langle e^{\int d^4x \phi_0(\vec{x}) \mathcal{O}(\vec{x})} \rangle_{CFT} = Z_{\mathrm{SUGRA}}[\phi(z, \vec{x})] \Big|_{\phi(0,\vec{x})=\phi_0(\vec{x})} \quad (1)$$

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and by functional differentiation on both sides:

$$\langle \mathcal{O}(\vec{x}) \mathcal{O}(\vec{y}) \rangle_{CFT} = \frac{\delta}{\delta\phi_0(\vec{x})} \frac{\delta}{\delta\phi_0(\vec{y})} Z_{\mathrm{SUGRA}} \Big|_{\phi=\phi_0} \quad (2)$$

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Conjectured prescription for Minkowskian metric: [Son & Starinets, '02]

$$\langle \mathcal{O}\mathcal{O} \rangle_{CFT} = -2\mathcal{F}(k,z) \Big|_{z_B}$$

where $\mathcal{F}$ comes from the quadratic part of the SUGRA on-shell action

$$S_{\text{SUGRA}}^{\text{on-shell}} = \int \frac{d^4k}{(2\pi)^4} \, \phi_0(-k) \mathcal{F}(k,z) \phi_0(k) \Big|_{z=z_B}^{z=z_H} \tag{3}$$

4. Kubo formulae

[Hosoya et al., ’83]

Question: How are 2-point correlators connected to physical observables?

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For the operator $\mathcal{O} \equiv T_{\mu\nu}$: [e.g. Zubarev]

$$\varrho_{\mathrm{non-eq}} = \exp\{\underbrace{-\int d^3x F^\nu T_{0\nu}}_{=\beta H \text{ equilibrium}} + \int d^3x \int_\infty^t dt_1 e^{\epsilon(t_1 - t)} \underbrace{T_{\mu\nu}}_{\text{current}} \underbrace{\partial^\mu F^\nu \}}_{\text{thermo force/gradient}} \tag{6}$$

where

$$F^\nu = \beta u^\nu \tag{7}$$

$\beta \equiv$ inverese temperature, $u^\nu \equiv$ four-velocity component of fluid

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Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

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To find coefficient, seperate tensor, vector and scalar processes [Curie's Theorem, 1894]:

$$T_{\mu\nu} = \pi_{\mu\nu} + \epsilon u_\mu u_\nu - p g_{\mu\nu} + p u_\mu u_\nu + P_\mu u_\nu + P_\nu u_\mu \tag{10}$$

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Seperating e.g. the tensor process from others:

$$\langle \pi_{\mu\nu}\rangle_{\mathrm{non-eq}} \approx \underbrace{\langle \pi_{\mu\nu}\rangle_{\mathrm{eq}}}_{\equiv 0} + \int d^3x' \int_{-\infty}^{t} dt' \mathrm{e}^{\epsilon(t'-t)} \underbrace{\left(\pi_{\mu\nu}(\vec{x},t), \pi_{\rho\sigma}(\vec{x}',t')\right)}_{\propto(\text{tensor structures}_{\mu\nu\rho\sigma})\times(\pi_{\alpha\beta},\pi^{\alpha\beta})} \beta\partial^\rho u^\sigma \tag{12}$$

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  $(\pi_{\mu\nu}, \pi_{\rho\sigma}) = (\text{ tensor structures }_{\mu\nu\rho\sigma}) \times (\pi_{\alpha\beta}, \pi^{\alpha\beta})$
- interprete coefficient as shear viscosity (by definition):

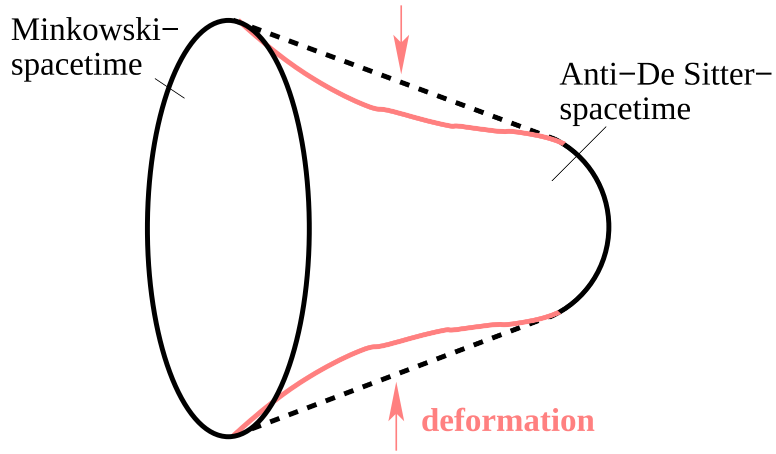
$$\eta \equiv \frac{\beta}{5} \int d^3x' \int_{-\infty}^{t} dt' e^{\epsilon(t'-t)} \underbrace{(\pi_{\alpha\beta}(\vec{x}, t), \pi^{\alpha\beta}(\vec{x}', t'))}_{\langle \pi_{\alpha\beta}, \pi^{\alpha\beta} \rangle \text{retarded}}$$

Kubo Formula

5. Applications

Using different SUGRA-theories (Actions and metrics) we can learn much about the strongly coupled dual field theories.

5.1 – Towards QCD



☞ for a R-charged $\mathcal{N} = 4$ SUSY $SU(N)$ Yang-Mills plasma

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Son, Starinets, '06]

5.2 – Deriving a coupling

[M.K., Erdmenger, '06]

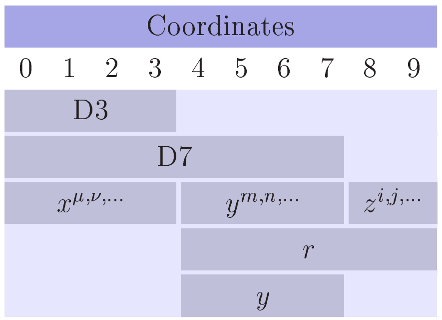
D7-brane worldvolume action [Erdmenger,Große,Guralnik,'05]

$$S_{D7} = \frac{(2\pi\alpha')^2 T_7}{4} \int d^4x\, d^4y\, \mathrm{tr}\Big[H(r) F_{\mu\nu} F_{\mu\nu} + 2 F_{m\nu} F_{m\nu} + H^{-1}(r)\, \tfrac{1}{2} (F_{mn} -^* F_{mn})(F_{mn} -^* F_{mn}) \Big] , \tag{13}$$

and the metric

$$ds^2_{\mathrm{D7}} = H^{-1/2}(r) \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/2}(r) d\vec{y}^{\,2} \tag{14}$$

with $r^2 = y^2 + (2\pi\alpha')^2 m^2, \quad H(r) = \frac{L^4}{r^4}$



Two-point correlators from this action are proportional to $g_s N$.

Summary and Outlook

☞ back to 'roadmap'

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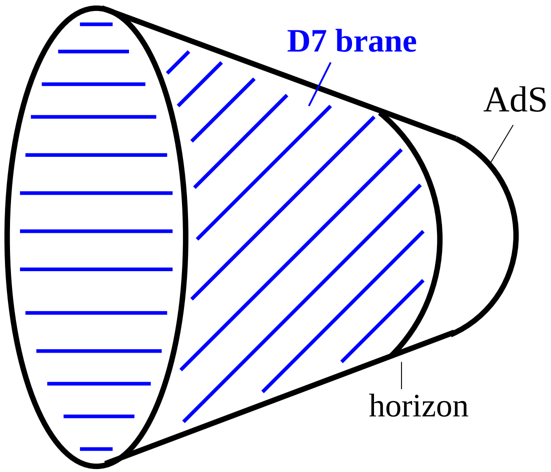
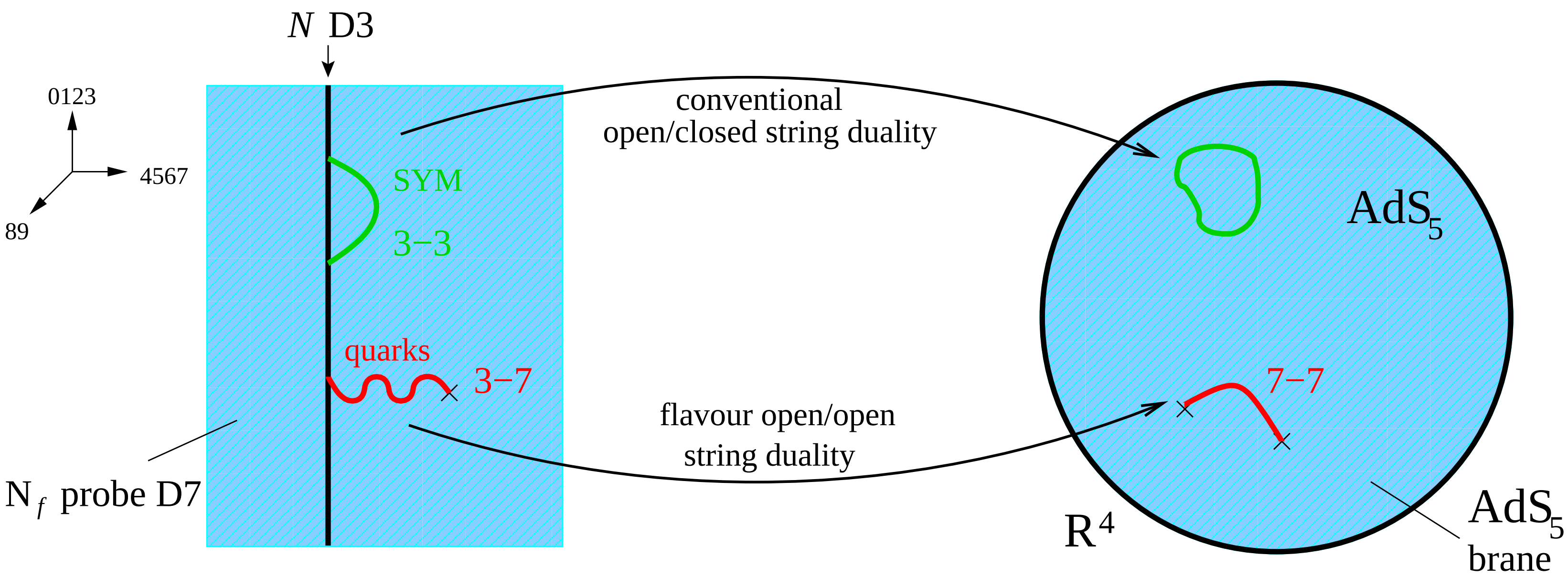


☞ incorporate massive fields by embedding D7-branes

☞ include finite chemical potential ($\equiv$ R-charge)

☞ transport coefficients for this theory

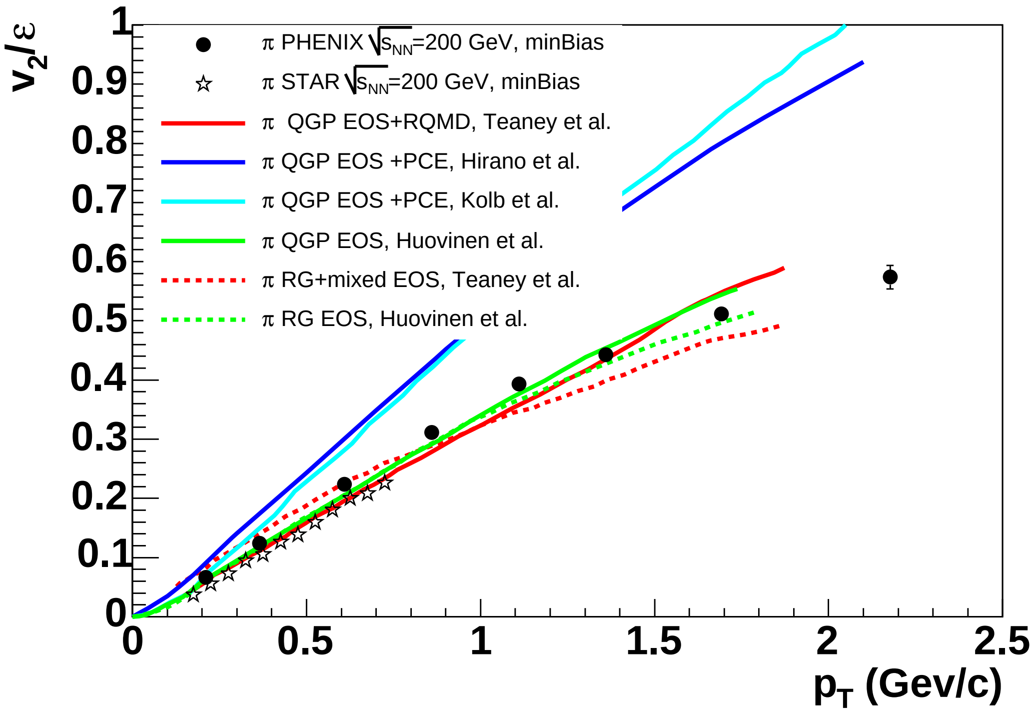
APPENDIX



N D3
0123
4567
89
SYM
3−3
quarks
3−7
N f probe D7
conventional
open/closed string duality
flavour open/open
string duality
AdS5
7−7
R 4
AdS5
brane

Comparision: pure hydrodynamics calculations $\leftrightarrow$ RHIC data

[PHENIX coll.]



Eccentricity defined with Cartesian x and y: $\varepsilon = \frac{\langle y^2\rangle - \langle x^2\rangle}{\langle y^2\rangle + \langle x^2\rangle}$.

Measure for 'elliptic flow':

Second Fourier coefficient v_2 where ϕ is the azimuthal angle: $\frac{d^2N}{d\phi dp_T} = N_0(1 + 2v_2(p_T)\cos(2\phi))$.