Entropy maximization

in the presence of higher-curvature interactions

Jan Perz
MPI/LMU Munich

JHEP05(2006)028 [hep-th/0603211] (G. L. Cardoso, D. Lüst, J. Perz)

Vast landscape

$$10^{500}$$

Flux compactifications and black holes

- IIB compactification on $M \times S^2$
  with RR 5-form $\mathbf{F}_{[5]} = \mathbf{F}_{[3]} \wedge \omega$
- fluxes $p^I = \int_{A^I} \mathbf{F}_{[3]} \qquad q_I = \int_{B_I} \mathbf{F}_{[3]}$
- conditions for susy vacua

$$Y^I - \bar{Y}^I = ip^I \qquad F_I - \bar{F}_I = iq_I \qquad AdS_2$$

$$Y^I \sim \int_{A^I} \Omega \qquad F_I \sim \int_{B_I} \Omega$$

$$z^A = \frac{Y^A}{Y^0}$$

Flux compactifications and black holes

- $N = 2$, ½-BPS, 4D $\qquad M \times S^2 \times AdS_2$
  black hole near-horizon geometry
- charges $\quad p^I = \int_{S^2_\infty} dA^I \qquad q_I = \int_{S^2_\infty} *dA_I$
- attractor equations

$$Y^I - \bar{Y}^I = ip^I \qquad F_I - \bar{F}_I = iq_I$$

$$F_I(Y) = \frac{\partial F(Y)}{\partial Y^I}$$

Entropic principle

- probability density for a flux compactification proportional to exponentiated black hole entropy

$$\mathcal{S}(p^I, q_J) = \mathcal{S}(Y^0, \bar{Y}^0, z^A, \bar{z}^A) \qquad z^A = Y^A / Y^0$$

- charge rescaling ambiguity removed by fixing Y^0 $\quad$ ($\mathcal{S} = \mathcal{S}(z, \bar{z})$)
- charges quantized
  – attractor equations Diophantine

At the two-derivative level

- entropy $\mathcal{S} = \pi i \left(\bar{Y}^I F_I^{(0)} - Y^I \bar{F}_I^{(0)} \right) = \pi \left| Y^0 \right|^2 e^{-G(z,\bar{z})}$
- one-modulus example, singularity at $V = 0$

$$F^{(0)}(Y) = -i (Y^0)^2 \left(\frac{\beta}{2\pi} V^2 \log V + a \right), \qquad V = -i z^1$$

(conifold of the IIB mirror quintic, *ST* model...)

- extremization in z independent of Y^0

$$e^{-G} = 4\,\mathrm{Re}\,a - \frac{\beta}{2\pi}(V + \bar{V})^2 - \frac{2\beta}{\pi}|V|^2 \log|V|$$

- $\mathrm{Re}\,a > 0$ (large black holes)

...entropy has a maximum at $V = 0$

- $\beta < 0$ from the positivity of the Kähler metric

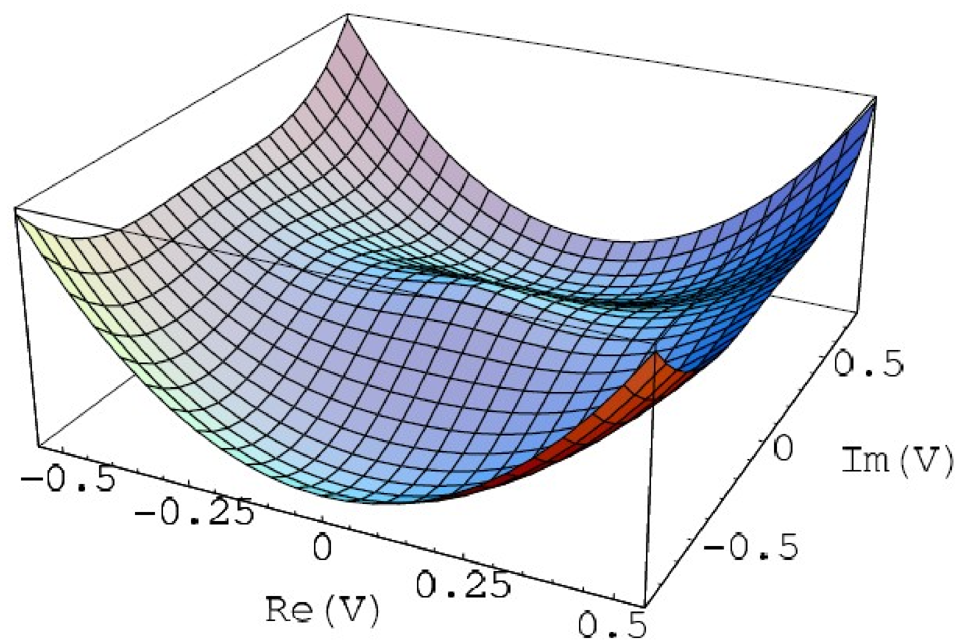
$$g_{V\bar{V}} \approx \frac{\beta}{4\pi \,\mathrm{Re}\, a} \log|V|^2$$

- extra massless charged hypermultiplets

$$\beta = \frac{n_{\mathrm{v}} - n_{\mathrm{h}}}{2}$$

- theory IR free

$$\tilde{g}^{-2} \approx \frac{\beta}{4\pi} \log|V|^2$$



With higher-curvature interactions

- Wald's formula

$$\mathcal{S} = \pi\Big(i\Big(\bar{Y}^I F_I(Y,\Upsilon) - Y^I \bar{F}_I(\bar{Y},\bar{\Upsilon})\Big) + 4\,\mathrm{Im}(\Upsilon F_\Upsilon)\Big)$$

- perturbative expansion

$$F(Y,\Upsilon) = \sum_{g=0}^{\infty} F^{(g)}(Y)\Upsilon^g$$

- one vanishing modulus

$$F^{(1)}(Y) \approx -\frac{i}{64\cdot 12\pi}\beta\log V$$

- enhancement by Wald's term: $4\,\mathrm{Im}(\Upsilon F_\Upsilon) \to \infty$

Higher $F^{(g)}$

- for the conifold of the mirror quintic — divergent with alternating coefficients

$$F^{(g)}(Y) = i\,\frac{A_g}{(Y^0)^{2g-2}(iV)^{2g-2}} \qquad g \geq 2$$

- indeterminate contribution to the entropy
- non-holomorphic corrections less singular (at least for $F^{(1)}$)

Non-perturbative F

- resolved conifold in type IIA (from $F_{\rm top}$)

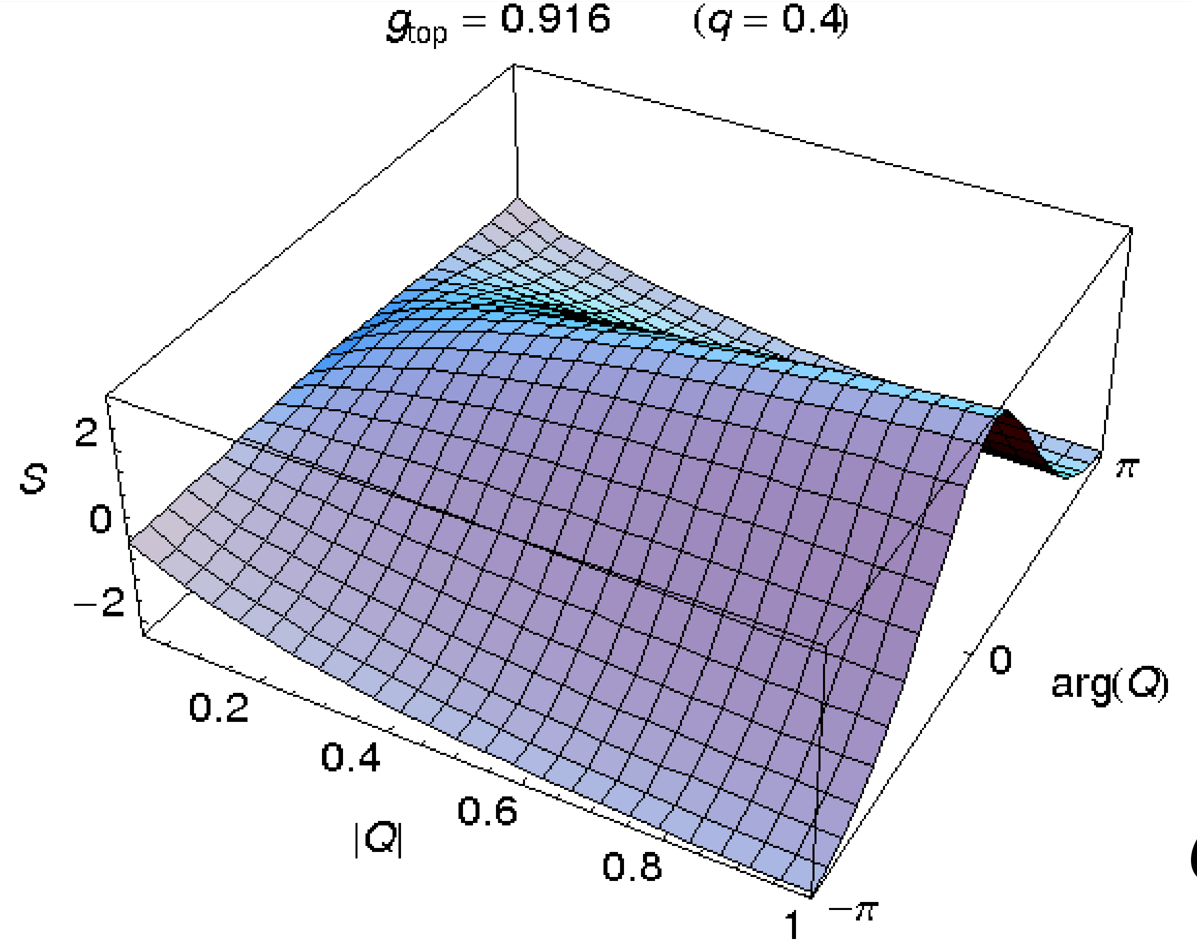
$$F = \frac{i}{2\pi}\sum_{n=1}^{\infty} n \log(1 - q^n Q)$$

$$q = e^{-g_{\rm top}} \qquad Q = e^{-2\pi V} \qquad g_{\rm top}^2 = \frac{4\pi^2}{(Y^0)^2}$$

- Q-independent terms suppressed
- $\mathrm{Re}\, g_{\rm top} > 0 \qquad\qquad \mathrm{Re}\, V \geq 0$
- numerical approximation

Real g_{top}

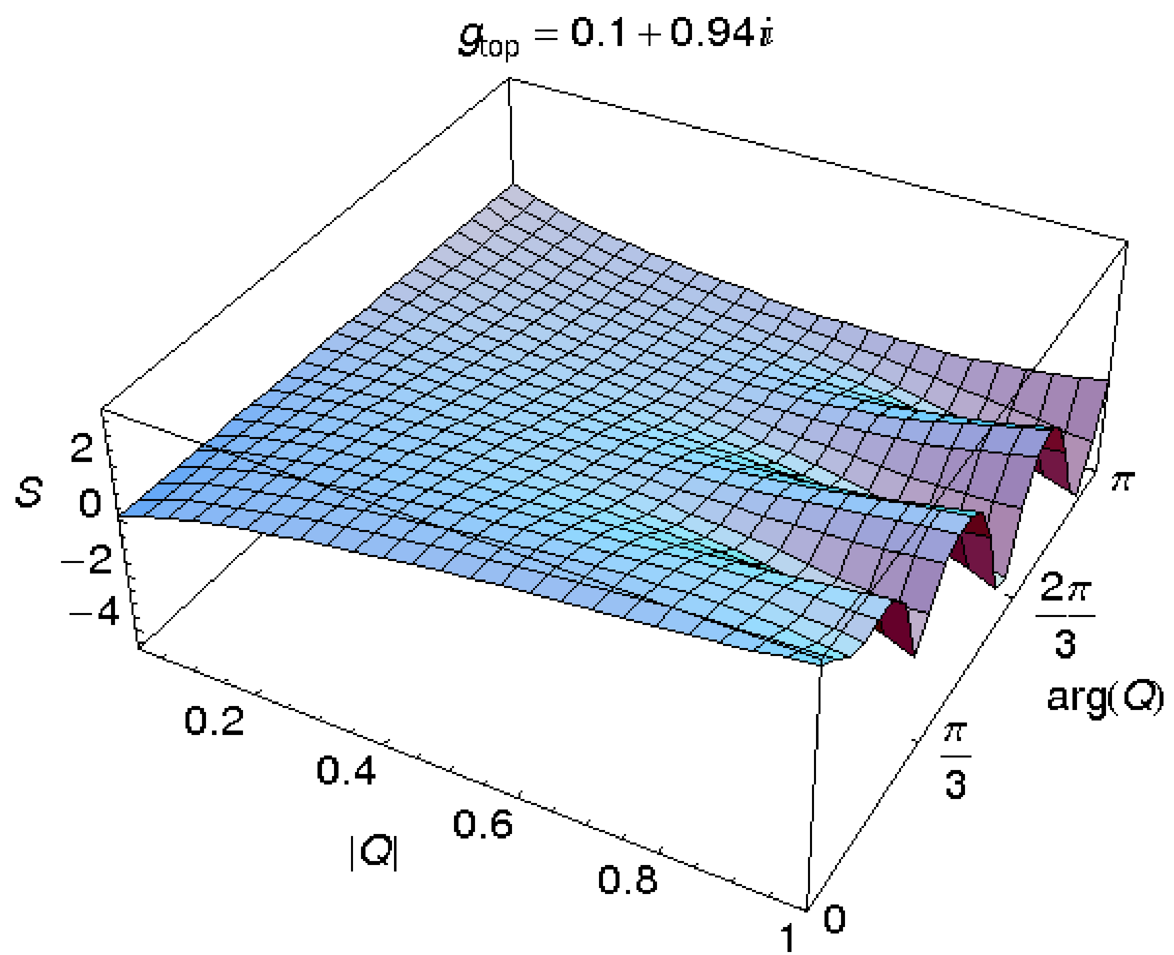
- entropy has a maximum at $V = 0$ (boundary)



$$Q = e^{-2\pi V}$$

Complex g_{top}

- conifold point generically no longer extremum, new local extrema



OSV free energy

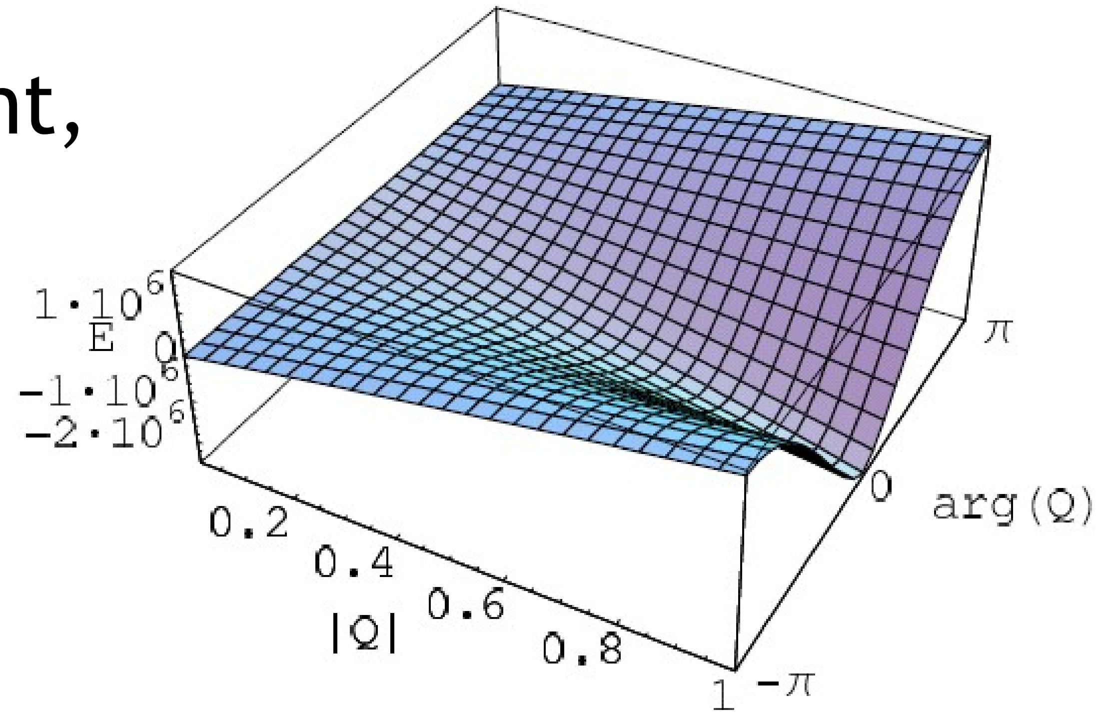
- entropy as a Legendre transform

$$\mathcal{S} = \underbrace{4\pi \operatorname{Im} F}_{E} - \underbrace{4\pi\phi^I \,\partial \operatorname{Im} F / \partial\phi^I}_{L}, \qquad \phi^I = 2 \operatorname{Re} Y^I$$

$$e^{\mathcal{S}} = \left|Z_{\text{top}}\right|^2 e^{-L}$$

- at the conifold point, for real g_{top}, E minimized

$$e^{\mathcal{S}} = \left|Z_{\text{top}}\right|^{-2}$$



Conclusions

- with Y^0 fixed and for real $g_{\rm top}$ the entropy attains a *maximum* at the conifold point
- following the entropic principle IR-free theories would be preferred
- difference with respect to GSV
  $e^{\mathcal{S}}$ vs. e^{E}