# Hamiltonian Analysis of the Arbitrary-Dimensional Hilbert-Palatini Theory

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Idea: try and consider a theory with a natural connection variable

$\Rightarrow$ Hilbert-Palatini theory (in arbitrary dimensions?)

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Action of the gravitational field (Einstein-Hilbert action):

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Hilbert-Palatini action:

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$\Rightarrow$ **Matter-free** GR and HP theory are equivalent **on-shell**

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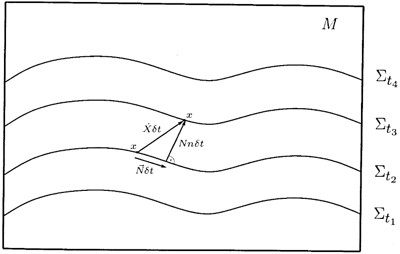
# Hilbert-Palatini theory: Spacetime split




Source: T. Thiemann. *Modern Canonical Quantum General Relativity* (2007)

← Spacetime split

Assumption: Spacetime topology $M \cong \mathbb{R} \times \sigma$

Foliation of $M$ into hypersurfaces $\Sigma_t \cong \sigma$ of constant time $t$

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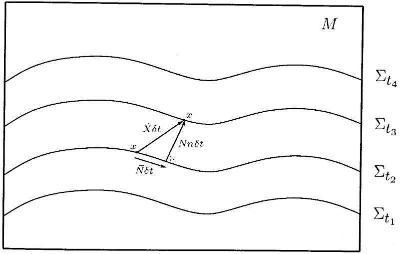


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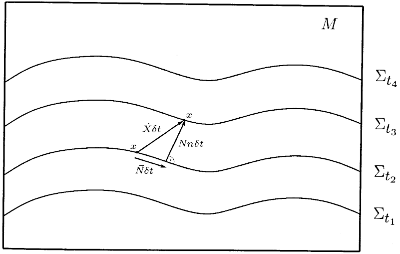


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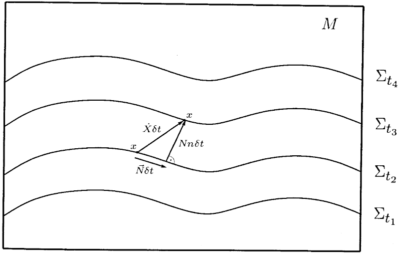


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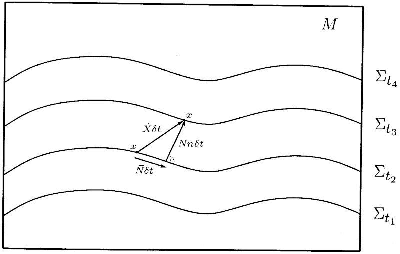


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$N^\mu \longrightarrow N^a$, $e_\mu{}^I \longrightarrow e_t{}^I, e_a{}^I$,

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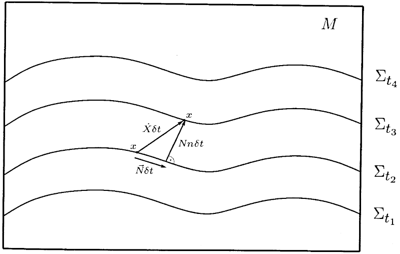


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$N = -\frac{1}{e e^t{}_I e^{tI}}$, $N^a = -\frac{e^t{}_I e^{aI}}{e^t{}_J e^{tJ}}$

# Hilbert-Palatini theory: Legendre transform

Canonical variables:

| Coordinates | ${e_t}^I$ | ${e_a}^I$ | ${\omega_t}^{IJ}$ | ${\omega_a}^{IJ}$ |
|---|---|---|---|---|
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Get rid of additional freedom by introducing simplicity constraint:

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HP-Hamiltonian:

$$H = \int d^D\mathbf{x} \cdot \left(\mathcal{N}\mathcal{H} + \mathcal{N}^a\mathcal{H}_a + \Lambda^{IJ}\mathcal{G}_{IJ} + c_{ab}{}^{IJKL}\mathcal{S}^{ab}{}_{IJKL}\right)$$

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Still, $\mathcal{S}^{ab}{}_{IJKL}$ and $\mathcal{B}^{ab}{}_{IJKL}$ are second-class contraints

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No further constraints needed to obtain ADM

# Outline

# Conclusions & Outlook

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## Outlook

1. Investigations of other GR reformulations
2. Maybe LQG "predicts" $3+1$-dimensional spacetime?

# Literature on gravity and quantum gravity


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