

The Cosmic Background Radiation (CMB)

Freeze out of CMB photons:

For $T \gg 2m_e$: reaction $\gamma \leftrightarrow e^+ + e^-$ in thermal equilibrium.

→ as many $e^+ + e^-$ pairs created as annihilated.

Subsequent expansion → cooling down of universe → reaction leaves thermal equilibrium.

Photons, γ , still in thermal equilibrium with matter through **Thomson/Compton scattering**

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

And through ionization and recombination of Hydrogen (chemical potential):

$$H + \gamma \leftrightarrow e^- + p$$

With these reactions in equilibrium: no free propagation of photons!

Once temperature below ionization energy of Hydrogen:

→ electrons not available as scattering partners as they were bound in neutral hydrogen

→ Free propagation of photons through the universe.

a) “Re”combination, i.e. the disappearance of the reaction partners of the photons.

It can be shown that mean free path becomes larger than observable universe for redshifts down to $z \sim 1100$.

→ From freeze out lecture: particle density as function of temperature for *non-relativistic* particles ($B \sim$ binding energy of Hydrogen where $B = 13.6$ eV) in thermal equilibrium:

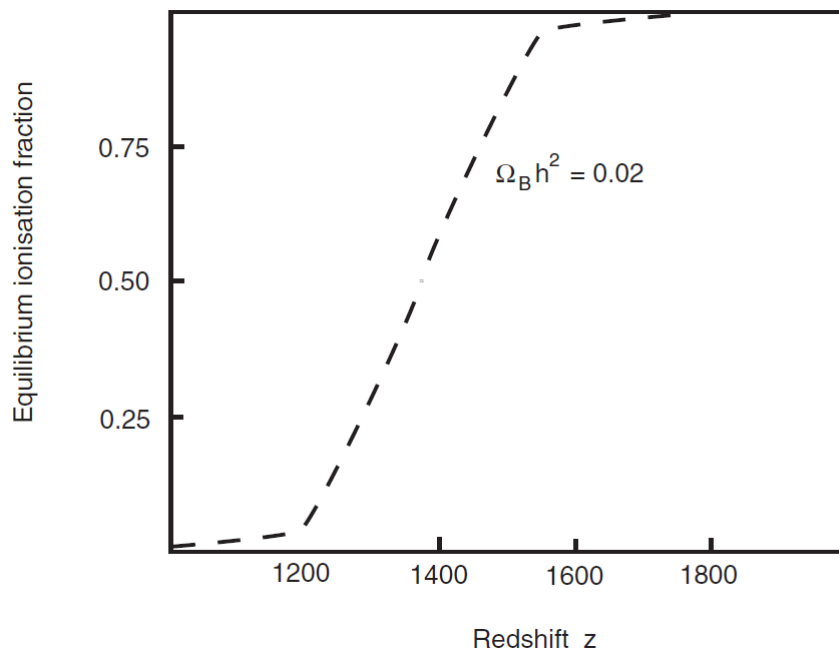
With $\eta_B = \frac{n_B}{n_\gamma} = 2.7 \cdot 10^{-8} \Omega_b h^2$ and $m_H = m_p + m_e - B$ (the)

→ Saha equation: fractional ionization at equilibrium $X_e = \frac{n_p}{n_B} = \frac{n_p}{n_p + n_H} = \frac{n_e}{n_e + n_H}$ (number of scattering partners) as a function of temperature for processes in thermal equilibrium.

$$\frac{1 - X_e}{X_e} \propto \eta_B \left(\frac{T}{m_e} \right)^{\frac{3}{2}} e^{\frac{B}{T}}$$

This determines mean free path of photons as function of T or corresponding z.

“Re”combination occurred ($X_e \sim 0.1$) at a redshift z in the range of **1200 to 1300**, while it can be calculated that the mean free path for photons is larger than the radius of the observable universe around **$z \sim 1050$** .



Where $\eta_b = \frac{n_b}{n_\gamma}$ is the Baryon to Photon ratio, m_e the electron (positron) mass and $B = 13.6 \text{ eV}$ the binding energy of the hydrogen atom.

b) Dilution of the reaction partners (Hydrogen, electrons and protons) by Hubble expansion.

For thermal equilibrium to be maintained (validity of Saha equation) we have to demand that the reaction rate $H + \gamma \leftrightarrow e^- + p$ is faster than the Hubble expansion rate.

Remember from thermal freeze out simplified Boltzmann equation:

$$\dot{n}_e + 3Hn_e = -\langle\sigma_{rev}|v|\rangle [n_e^2 - (n_e^{eq})^2],$$

$$\text{With } \langle\sigma_{rev}|v|\rangle = 4.7 \cdot 10^{-24} \left(\frac{1 \text{ eV}}{T}\right)^{\frac{1}{2}} \text{ cm}^2$$

→ Numerical analysis: $T_f \sim 0.25 \text{ eV}$. At this time the ionization fraction X_e (Saha equation) is frozen at $X_e = 2.7 \cdot 10^{-5} (\Omega_b h)^{-1}$. Also the number of free streaming photons per co-moving volume was fixed at this time.

→ **Surface of last scattering of cosmic microwave background photons.** Since then these photons moved on geodesics and did not participate in any scattering reactions.

Freeze out of photons at $T_e \sim 0.25 \text{ eV}$ not at temperature $B = 13.6 \text{ eV}$ for Hydrogen as one could naively assume as the photon to baryon ratio is $\sim 10^9$, i.e. far from 1. Many photons from the tail of the distribution can still lead to ionization!

Energy spectrum of these photons: black body spectrum.

Temperature measured today is red-shifted to $T_0 = \frac{T_e}{1+z}$.

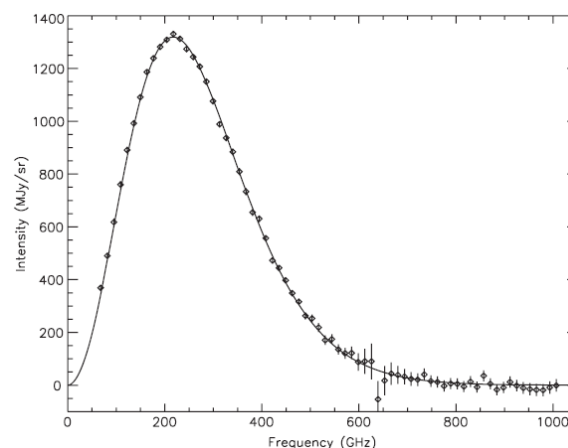
CMB was predicted by G. Gamow in 1946. His main motivation: explain why not all protons ended up in heavy elements (he believed that all isotopes were created in Big Bang nucleosynthesis). Peebles (Nobel Prize 2019) and Dicke came to same conclusion – apparently independently in 1965.

CMB discovered in 1965 by Penzias and Wilson accidentally (Nobel Prize 1978): they tried to measure galactic 21cm line from H_2 with a radio antenna → discovered isotropic noise with temperature $T_{CMB} = (3.5 \pm 1) K$, which they first interpreted as the black body radiation from pigeon feces on the antenna.

Nowadays CMB measured to extremely high accuracy. The spectrum fits a perfect black body on the range of three orders of magnitudes! The exact value is given with $T_{CMB} = (2.72548 \pm 0.00057) K$ (taken from The Astrophysical Journal, 707:916–920, 2009).

Measurement of 30 – 500 GHz radiation *not trivial*. Many other sources are present: atmosphere, black body radiation of detector, galaxy, etc.

→ Satellite experiments with ultra-cold detectors are necessary to remove most of the background noise.



CMB spectrum measured with the WMAP satellite. The shape fits a perfect black body with temperature $(2.72548 \pm 0.00057) K$.

Fluctuations in the CMB:

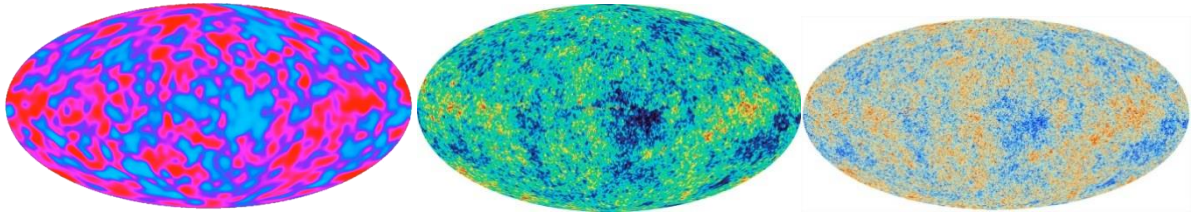
We see structures in our universe today

→ there must have been small fluctuations in the early universe from which structures could condense.

1st detection of fluctuations in the CMB: COBE satellite in 1992 (Nobel prize G. Smoot, 2006).

Since: high precision measurements of the CMB:

WMAP (released anisotropy data in 2006) and PLANCK (released data in 2013) with ever finer resolution.



Anisotropies as measured with the COBE (left), WMAP (center) and PLANCK (right) satellites. The different colors display fluctuations of the temperature of the order $\Delta T \sim 0.00001K$.

The observed temperature fluctuations result from differences in the gravitational potential from which photons at last scattering surface have to escape: **Sachs-Wolfe Effect**:

We know from energy conservation:

$$\left(\frac{\Delta T}{T}\right)_0 = \left(\frac{\Delta T}{T}\right)_e + \Phi_e$$

observed emitted potential

where the 1st term on r.h.s. describes intrinsic temperature fluctuation at a given point wrt average temperature at **surface of last scattering**. The 2nd term describes the gravitational potential at the point of observation. The higher the potential from which photons need to escape, the more energy they lose on the way to observer, the cooler the area will appear (redshift).

But also: the higher the potential, the higher the energy density from which photons have to escape, hence the higher the temperature in these areas.

These are two competing processes → quantitative treatment needed.

Remember: $a \propto t^{\frac{2}{3(1+\alpha)}}$ and $aT = \text{const}$ (unless phase transition)

$$\rightarrow \Delta(aT) = 0 \quad \rightarrow \frac{\Delta T}{T} = -\frac{\Delta a}{a}$$

and

$$\frac{\Delta a}{a} = \frac{2}{3(1+\alpha)} \frac{\delta t}{t}$$

From general relativity we know (time dilation):

$$\frac{\delta t}{t} \approx \Phi.$$

Combining the expressions yields:

$$\left(\frac{\Delta T}{T}\right)_e = -\frac{\Delta a}{a} = -\frac{2}{3(1+\alpha)} \frac{\delta t}{t} = -\frac{2}{3(1+\alpha)} \Phi_e$$

or

$$\left(\frac{\Delta T}{T}\right)_0 \cong \frac{1+3\alpha}{3+3\alpha} \Phi_e$$

Hence, for a matter dominated universe with $\alpha = 0$ we can write

$$\left(\frac{\Delta T}{T}\right)_0 \cong \frac{1}{3} \Phi_e$$

→ Energy loss due to escape from gravitational potential Φ_e is dominating over additional energy due to higher temperature in denser areas.

→ CMB allows us to measure the gravitational potential at time of “re”combination/photon decoupling.

The fluctuations are visible over regions that are far bigger than the event horizon at decoupling. This means that the fluctuations are intrinsic!

During journey of CMB photons from emission to observation: trajectory through gravitational potentials of large scale structure. Energy gain when falling into the potential, energy loss when “climbing out”.

During propagation through potential: universe is expanding

→ distortion of the potential

→ reduction of the potential depth

→ CMB photons use less energy to leave the potential well.

Integrated Sachs Wolfe (ISW) effect is observable in the CMB power spectrum.

The CMB is **polarized** by **Thomson scattering** of photons off electrons during decoupling. The polarization pattern induced by gravitational wells has *no curl* (E-mode polarization). Another mechanism to induce polarization is due to **gravitational waves** emitted in the early universe (inflation). This component of the polarization pattern is *predicted to have a curl* (B-mode polarization). There is no other known mechanism that could produce B-mode polarization.

→ Observation of B-mode polarization would, hence, be direct evidence for gravitational waves produced during inflation.

BICEPS (BICEPS2)

Quantification of CMB fluctuations as a function of angular separation on the sky: expansion in spherical harmonics:

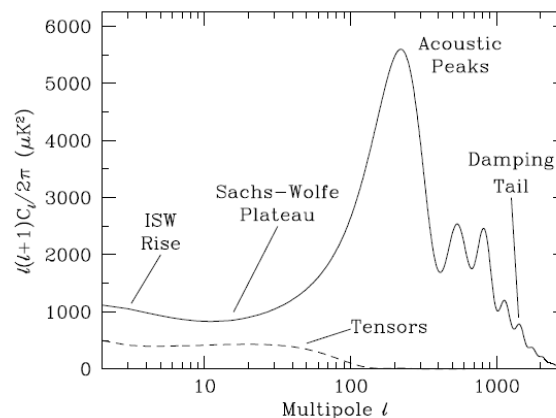
$$\frac{\Delta T(\vec{n})}{T} = \sum_{l=2}^{\infty} \sum_{m=-l}^{m=l} a_{lm} Y_{lm}(\theta, \varphi)$$

Dipole term $l = 1$ is omitted: governed by peculiar velocity of earth/our galaxy in the coordinate system of co-moving coordinates.

Measurement of the temperature map (COBE/WMAP/PLANCK): all a_{lm} can be determined and an average can be given by

$$C_l = \langle a_{lm} a_{lm}^* \rangle = \frac{1}{2l+2} \sum_{m=-l}^l a_{lm} a_{lm}^*$$

C_l contains the information on the average temperature fluctuation at an angle corresponding to multipole l .



Theoretical prediction for CMB fluctuations for a Λ CDM model. The contribution expected from gravitational waves is also shown (Tensors).

The precise form of the power spectrum of the CMB fluctuations is closely linked to the content and geometry of the universe at time of decoupling.

This can be understood by considering the content that is in thermodynamic equilibrium as a fluid in a given geometry. Oscillations inside the universe will appear between dense and dilute regions: particles fall into gravitational potential, will be stopped and eventually even reversed by radiation pressure inside the dense regions.

→ baryon photon fluid starts to perform “acoustic oscillations”.

Isotropy spectrum will have peaks at characteristic angles corresponding to different modes:

1. peak: mode that reached maximum compression at surface of last scattering
2. peak: mode that went through compression and reached maximum dilution (rerefaction)
3. peak: mode that reached second maximum of compression

...

The frequency and amplitude of these oscillations depends on content and structure of universe. Angle and amplitude of 1st mode: determined by the Hubble radius and the geometry of the universe at time of decoupling

→ It is sensitive to the cosmological parameter $\Omega_{tot} = \Omega_m + \Omega_\Lambda - \frac{k}{aH} = \Omega_m + \Omega_\Lambda + \Omega_k$.

It is the baryon-photon fluid that is oscillating in the gravitational potential. Latter has a contribution from dark matter that is not in thermal equilibrium with the baryon-photon fluid → power spectrum is also sensitive to Ω_{DM} and Ω_B .

The baryon to photon ratio η_b determines the amplitude of the 1st mode peak. The ratio of Ω_B and Ω_{DM} determines the relative heights of 1st and 2nd peaks. Many other cosmological parameters can be determined from the acoustic peaks of the CMB anisotropy.

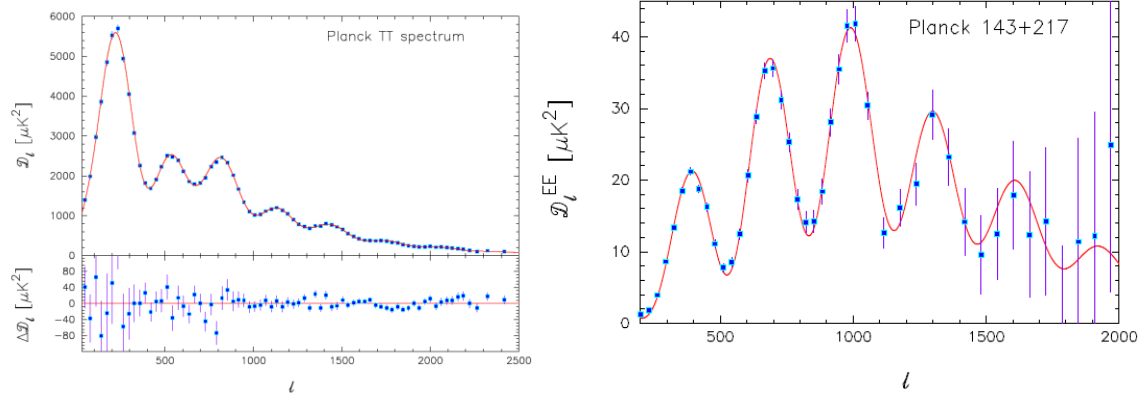
Oscillations in the CMB at small scales are suppressed, as the photons escape from a finite shell thickness: Variations corresponding to the thickness of the shell are washed out: Silk damping.

The ESA PLANCK satellite was launched in 2008. The PLANCK collaboration has released their data in 2013. Cosmological parameters could be determined with unprecedented precision.

With the PLANCK satellite also fluctuations in the polarization of the CMB could be measured for the first time.

Ω_{tot}	$1.0010^{+0.0033}_{-0.0031}$	
$\Omega_m h^2$	0.1423 ± 0.0029	$\Omega_m = 0.315^{+0.016}_{-0.017}$
Ω_Λ	0.685 ± 0.016	
$\Omega_B h^2$	0.02207 ± 0.00027	$\Omega_B = 0.0485 \pm 0.0012$
$\Omega_{DM} h^2$	0.1196 ± 0.0031	$\Omega_{DM} = 0.2633 \pm 0.0095$
η_b	$(6.047 \pm 0.074) \cdot 10^{-10}$	
Age of the universe	$(13.813 \pm 0.058) \cdot 10^9 \text{ yr}$	
H_0	$(67.3 \pm 1.2) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$	
z_* at decoupling	1090.37 ± 0.65	

Set of parameters as derived from the PLANCK CMB anisotropies assuming a Λ CDM cosmology.



Power spectrum of temperature fluctuations of the CMB as measured with the PLANCK satellite compared to the best fit Λ CDM model (left). With PLANCK also the anisotropies in polarization of the CMB could be measured with high precision (right). The red line represents the prediction of the CMB polarization as computed from the best fit Λ CDM model (taken from A&A 571, A16, 2014).

Oscillations of the baryon-photon fluid in the early universe should also lead to a characteristic density fluctuation of galaxies in the observable universe (known as Baryon Acoustic oscillations, BAO). These fluctuations could be found in large scale galaxy surveys like SDSS (Sloan Digital Sky Survey).