The most pressing problems with the cosmological concordance model and with the standard model of particle physics:

- Flatness ←
- Horizon ←
- What is Dark Energy?
- What is Dark Matter? (←)
- Baryon/Lepton Asymmetry of the Universe ←
- Hierarchy problem
- Strong CP problem ←
- Smallness of the neutrino mass

Flatness problem:

Remember that the Friedman equation

$$(\frac{\dot{a}}{a})^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho_{tot}$$

can be written as

$$\frac{8\pi G}{3}\rho(t) \cdot a^2(t) - \dot{a}^2(t) = k$$

The right hand side of this equation is independent of time, thus can be written by quantities determined at current time:

$$\frac{8\pi G}{3}\rho(t) - H^2(t) = (\frac{8\pi G}{3}\rho_0 - H_0^2)\frac{a_0^2}{a^2(t)}$$

or with $\rho_{krit}(t) = \frac{3H^2}{8\pi G}$

$$\rho(t) - \rho_{krit}(t) = (\rho_0 - \rho_{krit,0})\frac{a_0^2}{a^2(t)}$$

And after some rearrangement:

$$\rightarrow \Omega(t) = \frac{\Omega_0}{\Omega_0 + (1-\Omega_0)\frac{\rho_0}{\rho(t)} \cdot \frac{a_0^2}{a^2(t)}}$$

Remember that $\rho \propto a^{-n}$

$$\rightarrow \Omega(t) = \frac{\Omega_0}{\Omega_0 + (1-\Omega_0)(\frac{a}{a_0})^{n-2}}$$

with n=4 for radiation dominated n=3 for matter dominated universe

→ $\Omega_{rad}(t) = \frac{\Omega_0}{\Omega_0+(1-\Omega_0)(\frac{a}{a_0})^2}$ $\Omega_{mat}(t) = \frac{\Omega_0}{\Omega_0+(1-\Omega_0)\frac{a}{a_0}}$

Because $\lim_{t\to 0} a(t) = 0$ this means that $\lim_{t\to 0} \Omega(t) = 1$ which in turn means that at very early epochs of the universe the density must have been extremely close to the critical density. Today we observe a density of close to unity.

If there is a "slight" deviation from critical density observed now this means that the initial conditions must have been extremely fine tuned

- ➔ **Flatness problem**

Either there is a principle that universe has critical density or we find a scenario in which our universe looks flat.

Horizon problem:

How far can we look back in time?
Light is moving on geodesic $ds^2 = 0$ → $c^2 dt = a(t)\frac{dr}{\sqrt{1-kr^2}}$
Let our co-moving coordinate be $r = 0$.
Light that was emitted at r_e at time t_e will reach us at time $t = \int_{t_e}^{t_0} \frac{dt}{a(t)}$ and has to travel a distance (radial, ϑ=ϕ=0):

$$D = a_0 \cdot \int_{t_e}^{t_0} \frac{c \cdot dt}{a(t)}$$

We cannot look back further in time then to $t_e \to 0$.

We have seen earlier that scale factor is evolving in time as
$a(t) \propto t^{\frac{2}{3(1+\alpha)}}$
$\alpha = 0$ for matter domination → $a_{mat}(t) \propto t^{\frac{2}{3}}$
$\alpha = \frac{1}{3}$ for radiation domination → $a_{rad}(t) \propto t^{\frac{1}{2}}$

→ The size of the horizon is growing like

Matter dominated universe $d^h_{mat}(t) = 3 \cdot c \cdot t$

Radiation dominated universe $d^h_{rad}(t) = 2 \cdot c \cdot t$

This means that the **horizon is growing faster than the universe itself**. In turn this means that today we see a much bigger part of the universe than could be in causal connection at early times.

Remember: There is a homogeneous cosmic microwave background (CMB). How can this be so homogeneous if parts observed could not have been in causal connection? The horizon at time of emission of the CMB corresponds to a patch on the sky of roughly 1^0!

→ **Horizon Problem**

→ **Inflationary universe: The De Sitter model**
(universe with vanishing Energy-momentum tensor**):**

→ $a(t) = e^{Ht}$ with $H = \sqrt{\frac{\Lambda}{3}}$ → Inflation!

This corresponds to negative pressure in energy momentum tensor in co-moving coordinates:

$$T_{\mu\nu}^{\Lambda} = \frac{1}{c^2}\begin{pmatrix} \rho_\Lambda & 0 & 0 & 0 \\ 0 & -\rho_\Lambda & 0 & 0 \\ 0 & 0 & -\rho_\Lambda & 0 \\ 0 & 0 & 0 & -\rho_\Lambda \end{pmatrix} \quad \text{where} \quad \rho_\Lambda = \frac{\Lambda}{8\pi G}$$

Flatness and Horizon problems could both be solved by Inflation:

We have seen earlier that in universe dominated by vacuum energy the scale factor grow exponentially:

$$\Omega_\Lambda \gg \Omega_{rad}, \Omega_m \rightarrow a \propto e^{Ht}$$

It is imaginable that at very high temperatures in the early universe, say at the GUT scale, $T_{GUT} \sim 10^{15} GeV$, phase transitions of fields could occur during which energy from the field was transferred into the vacuum. If this was the case over a cosmologically relevant time scale this corresponds to above situation.

→ Inflationary universe (like in economy)!

The Hubble time at T_{GUT} was around $10^{-34}s$, i.e. if above condition is fulfilled for $10^{-32}s$ this corresponds to 100 Hubble times, i.e. is cosmologically relevant!

→ During this time the expansion of space (scale factor) is exponential, while the expansion of the event horizon is linear (see above). This means that patches of the universe that were in causal connection can be outside the event horizon after the phase transition (horizon problem).
→ As space is stretched exponentially, any curvature before inflation is stretched out and hence appears flat after the phase transition (flatness problem).

→ Quantum fluctuations are magnified from quantum level to macroscopic scales: First seeds for structure formation, consistent with anisotropy observed in CMB!

Baryon Asymmetry of the universe? Accordingly Lepton asymmetry of the universe?

Nucleons have baryon number +1 → quarks carry baryon number 1/3, while anti-quarks have baryon number -1/3.
So far no baryon number violation could be experimentally observed.

Proton Decay

The nearly conserved symmetry leads to a quasi-stable proton half-life: $T_{1/2}(p)>2.9\cdot10^{29}$ yr.
Grand Unification (GUT): Above certain energy Baryons and Leptons have the same couplings, i.e. are (nearly) indistinguishable
→ Baryons can be transformed into Leptons and vice versa
→ Proton is NOT stable

There are many possible decay channels like (limits from Superkamiokande):

p	→	$e^+\pi^0$	$T_{1/2} > 8.2\cdot10^{33}$ yr
p	→	$\mu^+\pi^0$	$T_{1/2} > 6.6\cdot10^{33}$ yr
p	→	νK^+	$T_{1/2} > 5.9\cdot10^{33}$ yr

Every good extension of the SM (GUT/TOE) must involve proton decay!

The search for proton decay has led to important progress in neutrino research: Originally the big neutrino detectors were built for search of proton decay. Neutrinos turned out to be a background for these detectors!

From **CMB**, its power spectrum & **BBN** we know:

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.2 \pm 0.4)\cdot 10^{-10}$$

→In early universe there must have been a slight asymmetry: per 10^9 antiparticles 10^9+1 particles

This creates the problem of the Baryon Asymmetry of the Universe (BAU): how this asymmetry was created???.

→Sakharov conditions in order to make possible Baryon asymmetry:

→*Baryon number must be violated in nature:* If Baryon number would be conserved, no Baryon asymmetry whatsoever could be produced.

→*CP must be violated: :*

If C would be conserved, no asymmetry between particles and antiparticles could be created CP-symmetry violation is necessary otherwise equal numbers of left-handed baryons and right-handed anti-baryons would be produced, as well as equal numbers of left-handed anti-baryons and right-handed baryons.

→Responsible process must *deviate from thermal equilibrium*: If all processes are always in equilibrium, processes that create asymmetry would be in equilibrium with processes that creates symmetry → No creation of asymmetry possible.

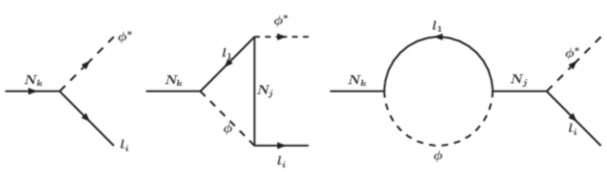
In the standard model baryon number is conserved in all interactions.
But: there are B-number violating "transitions": Sphalerons, Instantons:
between degenerate vacua with different B+L number.

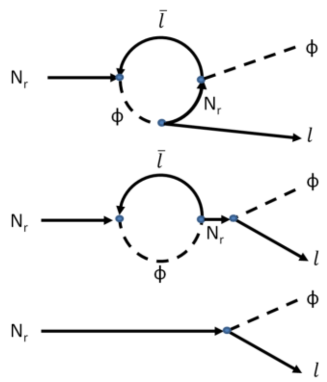
B-L conserved!

Weak interaction → Neutrinos → Sphalerons ?

Baryogenesis via Leptogenesis

Idea:
CP violation exists in weak interaction (→ neutrino mixing)!





Decay of heavy right handed neutrino has CP violating term through interference between one loop diagram and self-absorptive term

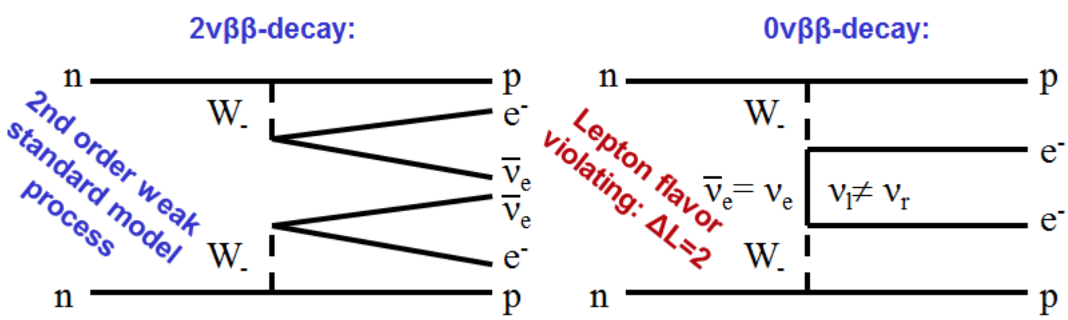
P(N$_r$→ l H)=1/2(1+ε)P

P(N$_r$→ $\bar{l}$ H)=1/2(1-ε)P

→ Creation of more Leptons than anti Leptons by decay of heavy Majorana neutrinos

Lepton asymmetry is transformed into Baryon asymmetry by "Sphalerons"

- ➔ This is an extension of SM: Right handed neutrinos (Majorana Singlet) are required for consistency of the theory! They could also explain low effective neutrino mass through see-saw mechanism!
- ➔ Prediction: Neutrinos are Majorana particles (i.e. their own antiparticles
- ➔ Search for **neutrinoless double-beta-decay** $2\mathrm{n} \rightarrow 2\mathrm{p} + 2\mathrm{e}^-$ only possible if neutrinos have Majorana of nature!



$2\nu\beta\beta -$ decay can be observed if single $\beta -$ decay is energetically forbidden, while coherent decay of two neutrons into two protons is allowed as 2nd order weak process
→ $2\nu\beta\beta$ Half-lives of isotopes are of the order $10^{18} - 10^{21} yr$.

$0\nu\beta\beta$ –decay can only occur if:

- Neutrino is a Majorana particle
- Helicity flip occurs in the vertex Standard assumption: if decay exists: dominated by exchange of Majorana neutrino. In general: all Lepton flavor violating processes leading to helicity flip can induce $0\nu\beta\beta$ –decay.

Observation is possible for even even nuclei due to pair binding term in Weizsäcker formula: Two parabolas of binding energy vs. Z:

Half-life limits: $T_{1/2} > 10^{26}$ yr!

The strong CP problem

Remember: CKM matrix contains CP violating phases!

QCD vacuum very complicated: So called θ-vacuum of QCD also contains CP violating phase θ.

→ $\overline{\boldsymbol{\Theta}}\frac{\alpha_s}{8\pi}G_{\mu\nu a}\widetilde{G}_a^{\mu\nu}\epsilon\mathcal{L}_{QCD}$ with $\overline{\boldsymbol{\Theta}} = \boldsymbol{\Theta} - \mathbf{arg\,det}\,\boldsymbol{M_q}$

Observable CP violation given by $\overline{\boldsymbol{\Theta}}$

$\overline{\boldsymbol{\Theta}}$ leads to non-vanishing electric dipole moment

$d_n \sim \overline{\boldsymbol{\Theta}} \cdot 10^{-16}\mathrm{e\ cm}$

Experimtets:

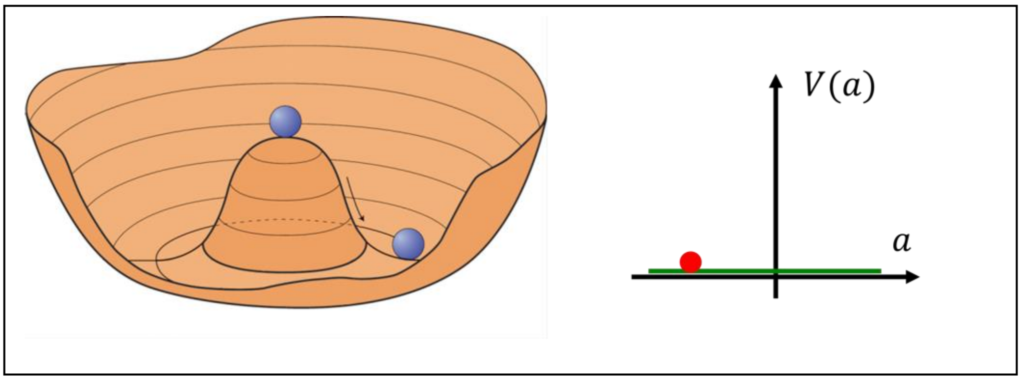
$d_n < 10^{-26}\mathrm{e\ cm}$ → $\overline{\boldsymbol{\Theta}} < \mathbf{10^{-10}}$ **!!**

Why do two CP violating arbitrary phases from QCD vacuum and from CKM matrix cancel each other out to this high degree?

➔ **Strong CP problem!**

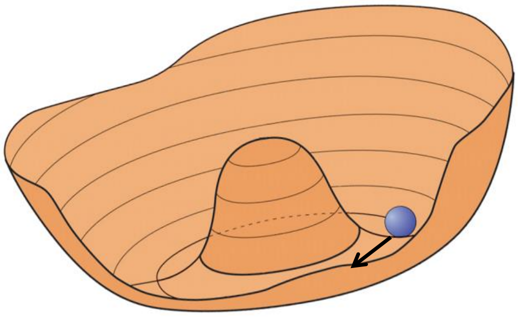
Idea for solving this:

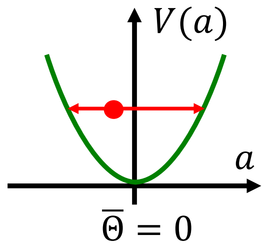
→Introduce Peccei Quinn symmetry based on spontaneously broken $U(1)_{PQ}$ Complex scalar field: (remember Higgs?). $\overline{\boldsymbol{\Theta}}$ corresponds to phase oft the field.



„Topological susceptibility“ of QCD vacuum leads to dependence of field potential on $\overline{\boldsymbol{\Theta}}$**:**

$\boldsymbol{V}(\overline{\boldsymbol{\Theta}} = \mathbf{0})$ is minimal





→ generation of mass (second derivative of potential at minimum)

Mass suppressed by distance between origin of field and minimum of trough: Energy scale of symmetry breaking!

→ Remember**: NON-THERMAL Production of local field oscillations, i.e. particle population without initial momentum: NON RELATIVISTIC!**

➔ **Ideal Dark Matter candidate: the Axion**

Number density depends on initial alignment of θ after symmetry breaking.