Renormalons and Top Quark Mass with Jet Substructure

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Top Quark Mass Determination and Renormalons

The heaviest elementary particle in the Standard Model, important for electroweak precision fits and vacuum stability.

[M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)]

t-Quark Mass (Direct Measurements)

The following measurements extract a t-quark mass from the kinematics of $t\bar{t}$ events. They are sensitive to the top quark mass used in the MC generator that is usually interpreted as the pole mass, but the theoretical uncertainty in this interpretation is hard to quantify. See the review "The Top Quark" and references therein for more information.

VALUE (GeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
$\boxed{\mathbf{172.9 \pm 0.4}}$ **OUR AVERAGE**	Error includes scale factor of 1.3. See the ideogram		

t-Quark Mass from Cross-Section Measurements

The top quark $\overline{\text{MS}}$ or pole mass can be extracted from a measurement of $\sigma(t\bar{t})$ by using theory calculations. We quote below the $\overline{\text{MS}}$ mass. See the review "The Top Quark" and references therein for more information.

VALUE (GeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
$\mathbf{160.0^{+4.8}_{-4.3}}$	[1] ABAZOV 11S	D0	$\sigma(t\bar{t})$ + theory

t-Quark Pole Mass from Cross-Section Measurements

VALUE (GeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
173.1±0.9 OUR AVERAGE			

Need to quantify the non-perturbative contributions in the observables used in top quark mass determination which might have large IR-sensitivity.

- IR renormalons in the heavy quark observables.

Renormalons in Heavy Quark Pole Mass

· Perturbative expansion as an asymptotic series:

$$f(\alpha) = \sum f_n \alpha^n , \qquad f_n \overset{n\to\infty}{\sim} a^n n^b n!$$

Analytic continuation with Borel transformation, ambiguous for fixed-sign factorials,

$$B[f](t) = \sum \frac{f_n}{n!} t^n \quad \Longrightarrow \quad f^B(\alpha) = \int_0^\infty e^{-t/\alpha} B[f](t)$$

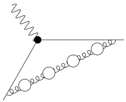
IR renormalons arise from the logarithms from renormalization procedure:

$$\int_0^q \frac{dk^2}{k^{2-m}} \left[|\beta_0| \ln\left(\frac{q^2}{k^2}\right)\right]^n \sim \left(\frac{2|\beta_0|}{m}\right)^n n! \quad \overset{\text{QCD}}{\Longrightarrow} \quad \text{power corrections } \delta \sim \left(\Lambda_{\text{QCD}}/Q\right)^{a/2|\beta_0|}$$

Leading IR renormalon in the heavy quark pole mass:

$$m_{\text{pole}} = m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}}) \left[1 + \sum_n r_n \alpha_s^n (m_{\overline{\text{MS}}})\right], \quad r_n \sim (2|\beta_0|)^n \Gamma\left[n + 1 - |\beta_1|/(2\beta_0)^2\right]$$

· IR renormalons in large-N_f limit and effective gluon propagator with small mass λ.
[Beneke, Braun, hep-ph/9411229; Ball, Beneke, Braun, hep-ph/9502300]

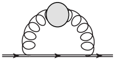


· Mellin representation $\frac{1}{k^2-\lambda^2} = \frac{1}{2\pi i}\frac{1}{k^2} \int_{-1/2-i\infty}^{-1/2+i\infty} dt\, \Gamma(-t)\Gamma(1+t) \left(-\frac{\lambda^2}{k^2}\right)^t$.

· residuals of renormalon poles given by non-analytic terms in λ^2.

· leading IR renormalon ($\delta \sim \Lambda_{\text{QCD}}$) $\Longleftrightarrow$ linear term in λ.

Heavy quark self-energy diagram with an effective gluon propagator:

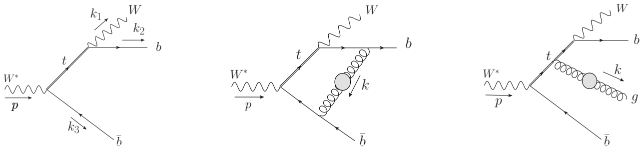


$$\Longrightarrow \quad m_{\text{pole}}^{(\lambda)} \sim \bar{m} \left\{ 1 - \frac{g_s^2 C_F}{(4\pi)^2} \left[2\pi \frac{\lambda}{\bar{m}} + \frac{\lambda^4}{4\bar{m}^4} \ln \frac{\lambda^2}{\bar{m}^2} + \ldots \right] \right\}$$

Infrared Renormalons in Inclusive Top Cross Section I

- Cross section intrinsically less sensitive to long-distance effects than the pole mass.

→ Leading IR renormalon cancels when changing the pole mass to a short-distance mass.
[Ravasio, Nason, Oleari, 1810.10931]



Gluon mass term λ^2 comes from resumming the bubble chain and acts as an IR regulator.

Expansion by regions for the soft gluon with $k^\mu \sim \lambda \ll m_t$, [Beneke, Smirnov, hep-ph/9711391]
e.g. in the heavy quark pole mass:

$$\int \frac{d^dk}{(2\pi)^d} \frac{a + b\lambda}{(k^2 - \lambda^2 + i\epsilon)[(p+k)^2 - m_t^2 + i\epsilon]} \underset{p^2 = m_t^2}{\xrightarrow{\text{NLP in } \lambda}} \int \frac{d^dk}{(2\pi)^d} \frac{a}{(k^2 - \lambda^2)\, 2p \cdot k} = -\frac{i}{(4\pi)^2} \frac{\pi}{m_t} a\lambda$$

IR renormalons not affected by top decay width, while $\Gamma_t > \Lambda_{\mathrm{QCD}}$. [Smith, Willenbrock, hep-ph/9612329]

Narrow width approximation ($\Gamma_t \to 0$), for inclusive cross section,

$$\sigma(W^* \to Wb\,\bar{b}) \longmapsto \sigma(W^* \to t\,\bar{b}) \cdot \mathrm{BR}(t \to Wb) + \mathcal{O}\left(\frac{\Gamma_t}{m_t}\right)$$

Infrared Renormalons in Inclusive Top Cross Section II

Focus on $\sigma(W^* \to t\bar{b})$, **soft expansion** for both the amplitude and the phase space, e.g.

$$\sigma_{\mathrm{real}}(W^* \to t\bar{b}) = \int d\Phi_t d\Phi_{\bar{b}} d\Phi_k (2\pi)^d \delta^{(d)}(p - q_1 - q_2 - k)\, |\mathcal{M}|^2_{\mathrm{real}}$$

Expand the on-shell δ-functions with reverse unitarity, after integrating over the energy-momentum δ-function, e.g.

$$\begin{aligned}
(-2\pi i)\delta^+((P-k)^2 - m_t^2) &= (-2\pi i)\delta^+(P^2 - m_t^2) + (-2\pi i)\frac{\partial \delta^+(P^2 - m_t^2)}{\partial P^\mu}(-k^\mu) + \mathcal{O}(\lambda^2) \\
&= \left[\frac{1}{P^2 - m_t^2}\right]_{\mathrm{cut}} - k\cdot\partial_P \left[\frac{1}{P^2 - m_t^2}\right]_{\mathrm{cut}} + \mathcal{O}(\lambda^2) \\
&= \left[\frac{1}{P^2 - m_t^2}\right]_{\mathrm{cut}} + 2k\cdot P \left[\frac{1}{(P^2 - m_t^2)^2}\right]_{\mathrm{cut}} + \mathcal{O}(\lambda^2)
\end{aligned}$$

Perform IBP reduction for both the virtual and real contributions to few master integrals.

The cancellation of linear in λ contribution in the inclusive cross section,

$$\left[\sigma(W^* \to t\bar{b})\right]^{(\lambda)} = \underbrace{\frac{3C_F g_s^2 g_w^2 \lambda m_t \left(p^2 - m_t^2\right)\left(m_t^2 + p^2\right)}{16\pi^2 p^2}}_{\text{change scheme from tree level}} + \underbrace{\frac{-3C_F g_s^2 g_w^2 \lambda m_t \left(p^2 - m_t^2\right)\left(m_t^2 + p^2\right)}{16\pi^2 p^2}}_{\text{virtual and real and on-shell wave-function renorm.}} = 0$$

expected by only inspecting IR sensitivity. (no $\ln\lambda$ by Bloch-Nordsieck cancellation)

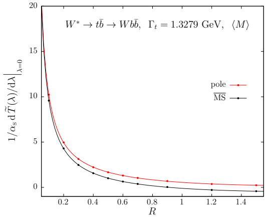
Infrared Sensitivity and Renormalons in Jet Observables with Grooming

IRC safety for sufficiently inclusive observables $\Longrightarrow$ IR finiteness, but *IR insensitivity* ?
[Beneke, hep-ph/9807443]

Leading IR renormalon $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ in top quark observables due to jet requirements, bringing in uncertainty in the top-mass determination.
[Ravasio, Nason, Oleari, 1810.10931]

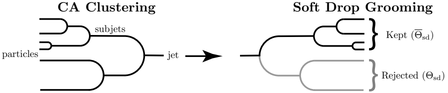
Radius dependent effects in jet observables:
[Dasgupta, Magnea, Salam, 0712.3014]

- Underlying event + Pile-up $\propto R^2 + \mathcal{O}(R^4)$.
- Hadronization $\propto -1/R + \mathcal{O}(R)$.
- Perturbative radiation $\propto \ln R + \mathcal{O}(1)$.
- IR renormalons $\propto 1/R + \mathcal{O}(1)$.
[Korchemsky, Sterman, hep-ph/9411211]



[Ravasio, Nason, Oleari, 1810.10931]

Soft drop jet declustering and grooming: remove the soft wide-angle radiations.
[Larkoski, Marzani, Soyez, Thaler, 1402.2657]



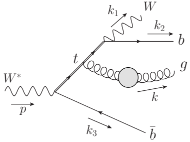
Soft Drop Condition (e^+e^-) :

$$\frac{\min[E_i, E_j]}{(E_i + E_j)} > z_{\mathrm{cut}} \left(\frac{1 - \cos\theta_{ij}}{1 - \cos R_0} \right)^{\beta/2}$$

Grooming the soft contaminations in the jet observables to reduce IR sensitivity.

Infrared Renormalons in Reconstructed Top Mass and Grooming I

Reconstructing the top mass from its decay products, i.e. the W and b jet,



} soft gluon might be clustered into the b jet

Measurement function in this case,

$$\mathcal{O}[gb,\bar{b}] \overset{\mathrm{C/A}}{=} \Theta(\cos\theta_{b,g} - \cos R)\Theta(\cos\theta_{b,g} - \cos\theta_{\bar{b},g})\Theta(\cos R - \cos\theta_{bg,\bar{b}}) \times (k^\mu + k_1^\mu + k_2^\mu)^2$$

Cannot perform IBP reduction in the real contribution due to phase space constraints.

Soft expansion for amplitudes and phase space (including measurement function), e.g.

$$\langle\mathcal{O}\rangle_{\mathrm{real}}^{\mathrm{cut}} = \int \mathrm{d}\Phi_b \mathrm{d}\Phi_{\bar{b}} \mathrm{d}\Phi_W \mathrm{d}\Phi_g (2\pi)^d \delta^{(d)}(p - k_1 - k_2 - k_3 - k)\,|\mathcal{M}_g|^2\,\mathcal{O}$$

Expand the energy-momentum conservation condition for $k^\mu \sim \lambda$,

$$\delta^{(d)}(p - k_1 - k_2 - k_3 - k) = \delta^{(d)}(p - k_1 - k_2 - k_3) - \frac{\partial\delta^{(d)}(p - k_1 - k_2 - k_3)}{\partial p^\mu} k^\mu + \mathcal{O}(\lambda^2)$$

Basis integrals of the soft gluon in the NWA have the following form:

$$I = \int \frac{d^d k}{(2\pi)^{d-1}} \delta^+(k^2 - \lambda^2) \frac{1}{(k\cdot k_i)^\alpha\,[k\cdot(p - k_3)]^\beta} \Theta(\cos\theta_{bg} - \cos R)\Theta(\cos\theta_{bg} - \cos\theta_{\bar{b}g})$$

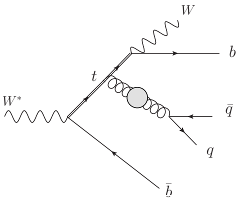
Infrared Renormalons in Reconstructed Top Mass and Grooming II

[Calculations are still ongoing]

Next step is to consider applying the grooming procedure,

$$\mathcal{O}_{\rm SD}[gb,\bar{b}] \overset{\rm C/A}{=} \Theta(\cos\theta_{b,g}-\cos R)\Theta(\cos\theta_{b,g}-\cos\theta_{\bar{b},g})\Theta(\cos R-\cos\theta_{bg,\bar{b}})$$
$$\times\left[\underbrace{\Theta\left(k^0-\tilde{z}_{\rm cut}\left(\frac{1-\cos\theta_{b,g}}{1-\cos R}\right)^{\beta/2}\right)}_{\text{soft drop passed}}(k^\mu+k_1^\mu+k_2^\mu)^2+\underbrace{\Theta\left(\tilde{z}_{\rm cut}\left(\frac{1-\cos\theta_{b,g}}{1-\cos R}\right)^{\beta/2}-k^0\right)}_{\text{soft drop failed}}(k_1^\mu+k_2^\mu)^2\right]$$

Need to consider also the case with $q\bar{q}$ in the fermion bubbles resolved $\Longrightarrow$ 5 particles in the final states with non-trivial phase space constraints.



Summary and Outlook

- Non-perturbative effects persist in jet observables, obstructions for precision of collider phenomenology.
- Renormalon analysis combined with analytic calculation techniques such as expansion by regions can help to understand IR sensitivity and non-perturbative effects.
- Understanding the jet-related renormalons in observables used in top-mass determination with jet substructure techniques promising for reducing the top-mass uncertainty.
- More interesting results expected.

Thank you for your attention.

Back Up

Infrared Renormalons I

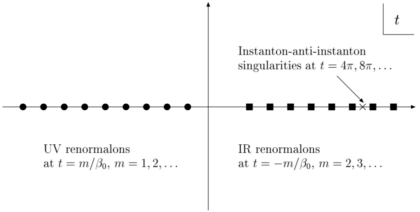
Given a formal power series in α

$$f(\alpha) = \sum_{n=0}^{\infty} f_n \alpha^{n+1}$$

the Borel transformation and the Borel sum are given as

$$B[f](t) = \sum_{n=0}^{\infty} \frac{f_n}{n!} t^n \qquad \Longrightarrow \qquad f^B(\alpha) = \int_0^{\infty} e^{-t/\alpha} B[f](t)$$

Instanton and renormalon poles in the Borel plane: [Beneke, hep-ph/9807443]



Infrared Renormalons II

Each non-analytic term proportional to $(\sqrt{\lambda^2})^{2n+1}$ corresponds to a single-pole singularity of $B[R](u)$ at positive half-integers $u = n + 1/2$. Each non-analytic term proportional to $\lambda^{2n} \ln \lambda^2$ corresponds to a single pole at positive integer $u = n$.

The dispersive method of evaluating the bubble diagrams.

$$\begin{aligned} \frac{1}{1+\Pi(k^2)} &= \frac{1}{\pi}\int_0^\infty d\lambda^2 \, \frac{1}{k^2-\lambda^2} \, \frac{\mathrm{Im}\Pi(\lambda^2)}{|1+\Pi(\lambda^2)|^2} \\ &+ \int_{-\infty}^\infty d\lambda^2 \, \frac{1}{k^2-\lambda^2} \, \frac{\lambda_L^2}{(-\beta_{0f}\alpha_s)} \, \delta(\lambda^2-\lambda_L^2) \end{aligned}$$

Effective gluon propagator

$$B[\alpha_s D_{\mu\nu}^{AB}(k)](u) = i\delta^{AB}\left(\frac{e^C}{\mu^2}\right)^{-u} \frac{k_\mu k_\nu - k^2 g_{\mu\nu}}{(-k^2)^{2+u}}$$

Mellin representation of massive propagator

$$\frac{1}{k^2-\lambda^2} = \frac{1}{2\pi i}\,\frac{1}{k^2}\int_{-1/2-i\infty}^{-1/2+i\infty} \mathrm{d}u \, \Gamma(-u)\Gamma(1+u)\left(-\frac{\lambda^2}{k^2}\right)^u$$

Infrared Renormalons III

From large N_f limit and the fermion bubble chain to large b_0 approximation.

$$n_f \to -\frac{11C_A}{4T_R} + n_l$$

where $C_A = 3$, $T_R = 1/2$ and n_l is the number of light flavors.

Renormalon ambiguity and QCD scale setting uncertainty. [Ball, Beneke, Braun, hep-ph/9502300]

Conversion relation between the pole mass and $\overline{\text{MS}}$ mass. [Marquard, Smirnov, Smirnov, Steinhauser, 1502.01030]
Assuming the top-quark $\overline{\text{MS}}$ mass $\overline{m}_t = m_t(\overline{m_t}) = 163.643$ GeV, and $\alpha_s^{(6)}(m_t) = 0.1088$, we have

$$m_{pole} = 163.643 + 7.557 + 1.617 + 0.501 + (0.195 \pm 0.005)\,\text{GeV}$$

Ultimate uncertainty of the Top quark pole mass. [Beneke, Marquard, Nason, Steinhauser, 1605.03609]

Fixed-order convergence and all-order resummation of $\ln R$ for small-radius jets.
[Dasgupta, Dreyer, Salam, Soyez, 1411.5182, 1602.01110]

Resummations carry nontrivial information of non-perturbative corrections.

Finite-Width Effects and Collinear Anomaly

· Collinear anomaly / analytic regularization problem might arise in Master Integrals when assuming scaling for the finite width, but absent in the NWA.
$\Longrightarrow$ account for non-resonance effects / non-factorizable effects with Unstable Particle Effective Theory. [Beneke, Chapovsky, Signer, Zanderighi, hep-ph/0312331]

$$F \equiv \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2-\lambda^2)(k^2+2k\cdot k_i)^{1-\alpha}\left[k^2+\Delta+2k\cdot(p-k_3)\right]^{1+\alpha}}$$

Sudakov decompose the loop momentum as : $k = k_- k_i + k_+ k_j + k_\perp$, regions for NLP in λ:
Soft region, $k^\mu \sim \lambda$,

$$F^{(s)} = \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2-\lambda^2)(2k\cdot k_i)^{1-\alpha}\left[\Delta+2k\cdot(p-k_3)\right]^{1+\alpha}}$$

2-collinear region, $(k_-, k_+, k_\perp) \sim (\lambda^0, \lambda^2, \lambda)$,

$$F^{(2c)} = \mu^{4-d} \int \frac{dk_- dk_+ d^{d-2}k_\perp}{(2\pi)^d (k^2-\lambda^2)(k^2+2k\cdot k_i)^{1-\alpha}\left[2k_- k_i\cdot(p-k_3)\right]^{1+\alpha}}$$
$$= \mu^{4-d} \int \frac{d^d k}{(2\pi)^d (k^2-\lambda^2)(k^2+2k\cdot k_i)^{1-\alpha}(2k\cdot k_j)^{1+\alpha}} \left[\frac{k_i\cdot k_j}{k_i\cdot(p-k_3)}\right]^{1+\alpha}$$

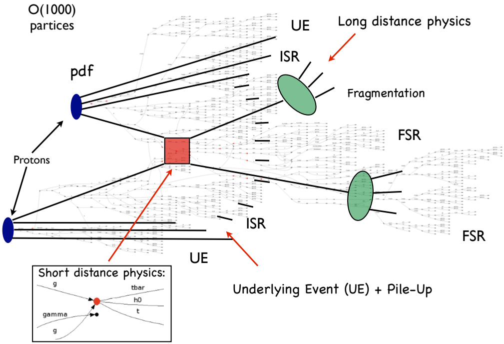
Singular terms in analytic regulator cancel between the 2-collinear region and the soft region,

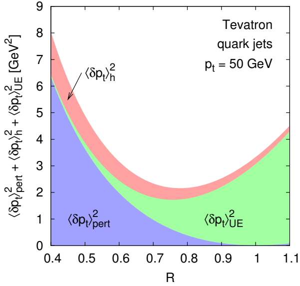
$$F^{(s)(1/\alpha)} + F^{(2c)(1/\alpha)} = \frac{i\left[\epsilon\left(-\log\left(4\pi\mu^2\right)\right)+\gamma\epsilon-1\right]}{64\pi^2\alpha b\epsilon} - \frac{i\left[\epsilon\left(-\log\left(4\pi\mu^2\right)\right)+\gamma\epsilon-1\right]}{64\pi^2\alpha b\epsilon} = 0$$

with $b = k_i\cdot(p-k_3)$.

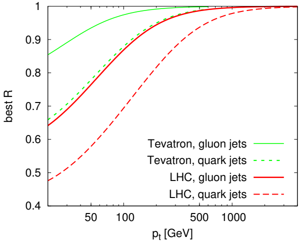
Non-Perturbative Effects in Jet Observables

- Underlying event + Pile-up $\propto R^2 + \mathcal{O}(R^4)$.
- Hadronization $\propto -1/R + \mathcal{O}(R)$.
- Perturbative radiation $\propto \ln R + \mathcal{O}(1)$.
- IR renormalons $\propto 1/R + \mathcal{O}(1)$.





[M. Dasgupta et al., arXiv:0712.3014]



[M. Dasgupta et al., arXiv:0712.3014]

Generalized k_t algorithms.

Define distances between entities (particles, pseudojets) i and j and d_{iB} between entity i and the beam (B).

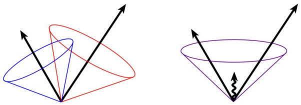
$$d_{ij} = \min(p_{T,i}^{2p}, p_{T,j}^{2p})\frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = p_{T,i}^2 \quad \text{(pp collision)}$$

$$d_{ij} = \min(E_i^{2p}, E_j^{2p})\frac{(1-\cos\theta_{ij})}{(1-\cos R)}, \quad d_{iB} = E_i^{2p} \quad (e^+e^- \text{ collision})$$

The recombination algorithm now works as follows:

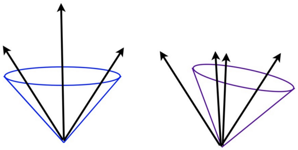
- Compute d_{ij} and d_{iB} for all particles in the final state, and find the minimum value.
- If the minimum is a d_{iB}, declare particle i a jet, remove it from the list, and go back to step one.
- If the minimum is a d_{ij}, combine particles i and j, and go back to step one.
- Iterate until all particles have been declared jets.

IRC Safety and Sudakov Safety



infrared safe:
configuration must not change when adding a further soft particle

infrared unsafe:
after emission of soft gluon jets are merged:
2 jets $\rightarrow$ 1 jet



collinear safe:
configuration does not change when substituting one particle with two collinear particles

Sudakov safe observables: observables do not have a valid α_s expansion but are nevertheless calculable in pQCD using all-orders resummation. These observables are called "Sudakov safe" since singularities at each α_s order are regulated by an all-orders Sudakov form factor.

(Modified) Mass Drop Tagger

The mass-drop tagger was designed to be used with jets found by the Cambridge/Aachen algorithm. It involves two parameters y_{cut} and μ and, for an initial jet labelled j, proceeds as follows:

1. Break the jet j into two subjets by undoing its last stage of clustering. Label the two subjets j_1, j_2 such that $m_{j_1} > m_{j_2}$.
2. If there was a significant mass drop, $m_{j_1} < \mu m_j$, and the splitting is not too asymmetric, $y = \min(p_{tj_1}^2, p_{tj_2}^2)\Delta R_{j_1 j_2}^2 / m_j^2 > y_{\text{cut}}$, then deem j to be the tagged jet.
3. Otherwise redefine j to be equal to j_1 and go back to step 1 (unless j consists of just a single particle, in which case the original jet is deemed untagged).

Typical parameter choices are for example $\mu = 2/3$ and $y_{\text{cut}} \in 0.09 - 0.15$.

For modified mass-drop tagger, change the above last step with

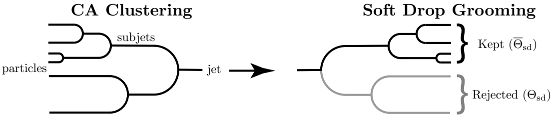
3. Otherwise redefine j to be that of j_1 and j_2 with the larger transverse mass $(m^2 + p_t^2)$ and go back to step 1 (unless j consists of just a single particle, in which case the original jet is deemed untagged).

At leading order, since there is no recursion, this modified MDT (mMDT) behaves identically to the original MDT.

Soft Drop Declustering Algorithm

[A. Larkoski et al., arXiv:1402.2657]

Given a jet with radius R_0 using Cambridge-Aachen (CA) clustering [1]



Undo the last step of jet clustering and examine the sub-jets with

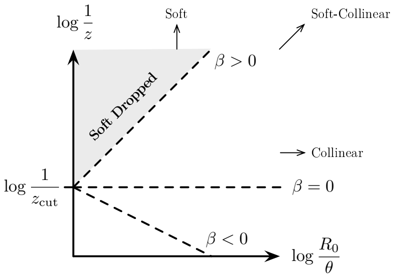
$$\text{Soft Drop Condition:} \quad \begin{cases} \dfrac{\min[p_{Ti}, p_{Tj}]}{(p_{Ti}+p_{Tj})} > z_{\text{cut}} \left(\dfrac{R_{ij}}{R_0}\right)^{\beta} & p\,p \;\; \text{collision} \\[2ex] \dfrac{\min[E_i, E_j]}{E_i+E_j} > z_{\text{cut}} \left(\dfrac{1-\cos\theta_{ij}}{1-\cos R_0}\right)^{\beta/2} & e^+e^- \; \text{collision} \end{cases}$$

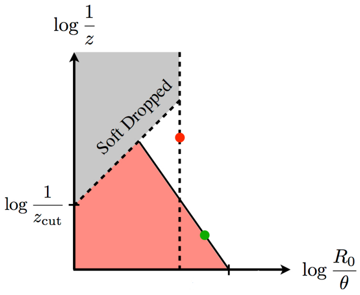
recursively until the condition is satisfied, with z_{cut} the soft threshold and β the angular exponent controlling the grooming.

- Other k_t-family algorithms might be preferred.
- **CA algorithm** most suitable for substructure.

IRC Safety and β-dependence of Soft Drop

[A. Larkoski et al., arXiv:1402.2657]





- $\beta < 0$: removes soft & collinear radiation.
- $\beta = 0$: corresponds roughly to the mMDT.

} IRC safe for tagging mode

- $\beta > 0$: removes large-angle soft radiation. $\rightarrow$ IRC safe for grooming mode

$\rightarrow$ Remanent collinear soft modes after grooming, controlled by $\beta > 0$.

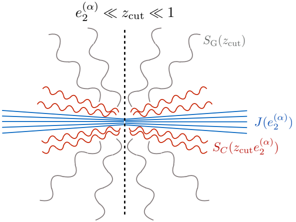
$\rightarrow$ Remanent radiations with $\theta_{ij} < R_g$ as soft drop grooming terminates.

Factorization for Soft Drop Jet Observables

[C. Frye et al., arXiv:1603.09338]

$S_G(z_{cut})$: Global soft mode.
$S_C(z_{cut} e_{2,R}^{(\alpha)})$: Collinear-soft mode.
$J(e_{2,L}^{(\alpha)})$: Jet mode.

EEC: $e_2^{(\alpha)} = \frac{1}{E_J^2} \sum_{i<j\in J} E_i E_j \left(\frac{2p_i\cdot p_j}{E_i E_j}\right)^{\alpha/2}$



In $e^+e^- \to$ dijets events, the factorization formula for soft-drop groomed left and right hemisphere jets:

$$\frac{d^2\sigma}{de_{2,L}^{(\alpha)}\, de_{2,R}^{(\alpha)}} = H(Q^2) S_G(z_{cut}) \left[S_C(z_{cut} e_{2,L}^{(\alpha)}) \otimes J(e_{2,L}^{(\alpha)})\right] \left[S_C(z_{cut} e_{2,R}^{(\alpha)}) \otimes J(e_{2,R}^{(\alpha)})\right]$$

for $e_{2,L}^{(\alpha)}, e_{2,R}^{(\alpha)} \ll z_{cut} \ll 1$.

- NGLs absent to all orders.
- Process independent.
- Non-perturbative corrections reduced.