Møller scattering of massless fermions in a parity preserving Maxwell-Chern-Simons QED$_3$

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The action for a parity preserved Maxwell-Chern-Simons QED_3 with a local $U(1)_A \times U(1)_a$ symmetry is:

$$S = \int d^3x \Big\{ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}f^{\mu\nu}f_{\mu\nu} + \mu\epsilon^{\mu\rho\nu}A_\mu\partial_\rho a_\nu + i\overline{\psi}_+ \not{D}\psi_+ + i\overline{\psi}_- \not{D}\psi_- + - \frac{1}{2\alpha}(\partial^\mu A_\mu)^2 - \frac{1}{2\beta}(\partial^\mu a_\mu)^2 \Big\}, \quad (1)$$

where the parameters g and e are dimensionful, with mass dimension $\frac{1}{2}$, and, μ with mass dimension 1.

The terms $F_{\mu\nu}=\partial_\mu A_\nu - \partial_\nu A_\mu$ and $f_{\mu\nu}=\partial_\mu a_\nu - \partial_\nu a_\mu$ are the usual field strength. The covariant derivative is defined by $\not{D}\psi_\pm \equiv (\not{\partial} + ie\not{A} \pm ig\not{a})\psi_\pm$ and the gamma matrices represented in terms of Pauli matrices $\gamma^\mu=(\sigma_z, -i\sigma_x, i\sigma_y)$.

The CPT-even action (1) is invariant under the parity symmetry in D=1+2[††], P, defined as:

$$\psi_\pm \stackrel{P}{\longrightarrow} \psi^P_\pm = -i\gamma^1\psi_\mp \; , \; \overline{\psi}_\pm \stackrel{P}{\longrightarrow} \overline{\psi}^P_\pm = i\overline{\psi}_\mp\gamma^1 \; , \tag{2}$$

$$A_\mu \stackrel{P}{\longrightarrow} A^P_\mu = (A_0, -A_1, A_2) \; , \tag{3}$$

$$a_\mu \stackrel{P}{\longrightarrow} a^P_\mu = (-a_0, a_1, -a_2) \; . \tag{4}$$

The gauge $U(1)_A \times U(1)_a$ symmetry ($\delta_{\rm g}$) reads:

$$\delta_{\rm g}\psi_\pm(x) = i[\theta(x) \pm \omega(x)]\psi_\pm(x) \quad \text{and} \quad \delta_{\rm g}A_\mu(x) = -\frac{1}{e}\,\partial_\mu\theta(x) \; , \tag{5}$$

$$\delta_{\rm g}\overline{\psi}_\pm(x) = -i[\theta(x) \pm \omega(x)]\overline{\psi}_\pm(x) \quad \text{and} \quad \delta_{\rm g}a_\mu(x) = -\frac{1}{g}\,\partial_\mu\omega(x) \; . \tag{6}$$

Reference

†† M. A. De Andrade, O. M. Del Cima and J. A. Helayël-Neto, Il Nuovo Cimento A111 (1998) 1145.

The Maxwell-Chern-Simon action is:

$$\Sigma_{MCS} = \int d^3x \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} f^{\mu\nu} f_{\mu\nu} + \mu \epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho \right\}$$

The equations of motion for the fields are:

$$\frac{\delta \Sigma_{MCS}}{\delta A_\mu} = 0 \Rightarrow \partial_\nu F^{\mu\nu} - \mu \omega^\mu = 0 \quad (7)$$

$$\frac{\delta \Sigma_{MCS}}{\delta a_\mu} = 0 \Rightarrow \partial_\nu f^{\mu\nu} - \mu W^\mu = 0 \quad (8)$$

Where, $\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho} f_{\nu\rho}$ e $W^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho} F_{\nu\rho}$.

Defining the analogues of the electric and magnetic fields[1] $E_i = F_{0i}$, $e_i = f_{0i}$, $B = -F_{12}$ and $b = -f_{12}$. In terms of these fields, the equations (7) and (8) and the homogeneous equations $\partial_\mu \omega^\mu = \partial_\mu W^\mu = 0$ are written:

$$\begin{cases} \vec{\nabla}.\vec{E} - \mu b = 0, & \vec{\nabla}.\vec{e} - \mu B = 0, \\ \vec{\tilde{\nabla}} B - \dfrac{\partial \vec{E}}{\partial t} - \mu \vec{\tilde{e}} = 0, & \vec{\tilde{\nabla}} b - \dfrac{\partial \vec{e}}{\partial t} - \mu \vec{\tilde{E}} = 0, \\ \vec{\nabla}.\vec{\tilde{E}} + \dfrac{\partial B}{\partial t} = 0, & \vec{\nabla}.\vec{\tilde{e}} + \dfrac{\partial b}{\partial t} = 0 \end{cases} \tag{9}$$

Using the notation $\vec{V} = (V_1, V_2) = (V_x, V_y)$; $\vec{\tilde{V}} = (V_y, -V_x)$ e $\vec{\nabla} = \left(\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}\right)$.

It follows from (9) that all fields satisfy the Proca equation

$$\left(\Box + \mu^2\right) \phi_j = 0, \tag{10}$$

with $\phi_j = \vec{E}, B, \vec{e}$ or b

[1] i=1,2

The free Dirac equations associated to ψ_+ and ψ_-, which stem from the action (1), read:

$$i\partial\!\!\!/\psi_\pm(x) = 0 \, , \tag{11}$$

The field operators ψ_+ and ψ_- in terms of the c-number plane wave solutions, with operator-valued amplitudes, a_+, b_+, a_- and b_- (annihilation operators), and $a_+^\dagger$, $b_+^\dagger$, $a_-^\dagger$ and $b_-^\dagger$ (creation operators) are given by:

$$\psi_\pm(x) = \int \frac{d^2\vec{k}}{(2\pi)\sqrt{2k^0}}\{a_\pm(k)u_\pm(k)e^{-ikx} + b_\pm^\dagger(k)v_\pm(k)e^{ikx}\} \, . \tag{12}$$

Assuming $p^\mu{=}(E, p_x, p_y)$, the wave functions, u_+, v_+, u_- and v_-, follow as bellow:

$$u_\pm(p) = \frac{p\!\!\!/}{\sqrt{E}}\begin{bmatrix}1\\0\end{bmatrix} \, , \quad \text{and} \quad v_\pm(p) = \frac{-p\!\!\!/}{\sqrt{E}}\begin{bmatrix}0\\1\end{bmatrix} . \tag{13}$$

The spinors being such that:
$\overline{u}_\pm(p)u_\pm(p) = \overline{v}_\pm(p)v_\pm(p) = \overline{u}_\pm(p)v_\pm(p) = \overline{v}_\pm(p)v_\pm(p) = 0$ and
$u_\pm^\dagger(p)u_\pm(p) = v_\pm^\dagger(p)v_\pm(p) = 2E$

The quantum operators associated to the electric charge ($Q_\pm$) and Néel (chiral) charge ($q_\pm$), both stemming from the ($U(1)_A \times U(1)_a$) symmetry, read:

$$Q_\pm = -e \int d^2\vec{x} :\psi_\pm^\dagger(x)\psi_\pm(x) := -e \int d^2\vec{k} \left\{a_\pm^\dagger(k)a_\pm(k) - b_\pm^\dagger(k)b_\pm(k)\right\} \ ,$$

$$q_\pm = \mp g \int d^2\vec{x} :\psi_\pm^\dagger(x)\psi_\pm(x) : = \mp g \int d^2\vec{k} \left\{a_\pm^\dagger(k)a_\pm(k) - b_\pm^\dagger(k)b_\pm(k)\right\} \ ,$$

where the results by acting them upon the wave functions, u_+, v_+, u_- and v_-, can be summarized by the table bellow:

Exploring the physics of the model

The massless fermions: charges

Wave function	Eletric charge	Chiral charge	Particles
$u_+(p)$	$-e$	$-g$	$e^-_{(+)}$ (fermion)
$u_-(p)$	$-e$	$+g$	$e^-_{(-)}$ (fermion)
$v_+(p)$	$+e$	$+g$	$e^+_{(+)}$ (antifermion)
$v_-(p)$	$+e$	$-g$	$e^+_{(-)}$ (antifermion)

When dealing with massless particles in 3 spacetime dimensions, there is no rest frame upon which one can define spin and neither one can talk about helicity. Nonetheless, spin/helicity (in the generalized sense of a quantum number labelling the representation of the little group) is still an important kinematic physical property and this may be seen in a number of different ways. In our present case of massless fermions, we have that

$$\left[H_D, L + \frac{1}{2}\sigma_z\right] = 0, \tag{14}$$

where $H_D = \vec{\alpha}.\vec{p}$ and $L = xp_y - yp_x$. So one can see that $\frac{1}{2}\sigma_z$ is the spin operator.

Exploring the physics of the model

The massless fermions: spin

Another way of seeing it is coupling these fermions to external magnetic and magnetic-chiral fields. Remembering that we have two types of fermions, $\psi_\pm$, we have the hamiltonians

$$H_\pm = \vec{\alpha}.(\vec{p} - e\vec{A} \mp g\vec{a}) \Rightarrow E_{i,n,s} = \pm\omega_i\sqrt{n + \frac{1}{2} + s};\ n = 0, 1, 2, \ldots$$

Where $\omega_\mp = \omega_0\sqrt{1 \pm \frac{gb}{eB}}$, $\omega_0 = \sqrt{2eB}$, $B = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$ and $b = \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}$. The i label refers to the type of fermion, $\psi_\pm$, and s is a splitting due to the existence of spin, being $s = \pm\frac{1}{2}$.

It is interesting to notice the emergence of another splitting on the Landau levels due to the other interaction. The existence of such a splitting is an experimental fact for graphene monolayers in high magnetic fields[†].

References

† Y. Zhang et al, PRL 96 136806 (2006)

The propagators play a key role on the analysis of the physical spectrum consistency, which includes the unitarity at the tree-level. From the classical action (1) we find the propagators in momenta space:

$$\langle A^\mu(k)A^\nu(k)\rangle = -i\left\{\frac{1}{k^2-\mu^2}\left(\eta^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)+\frac{\alpha}{k^2}\frac{k^\mu k^\nu}{k^2}\right\}, \tag{15}$$

$$\langle a^\mu(k)A^\nu(k)\rangle = \frac{\mu}{k^2(k^2-\mu^2)}\epsilon^{\mu\lambda\nu}k_\lambda , \tag{16}$$

$$\langle a^\mu(k)a^\nu(k)\rangle = -i\left\{\frac{1}{k^2-\mu^2}\left(\eta^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)+\frac{\beta}{k^2}\frac{k^\mu k^\nu}{k^2}\right\}, \tag{17}$$

$$\langle\overline{\psi}_+(k)\psi_+(k)\rangle = i\frac{\not{k}}{k^2} , \tag{18}$$

$$\langle\overline{\psi}_-(k)\psi_-(k)\rangle = i\frac{\not{k}}{k^2} . \tag{19}$$

The spectrum analysis by using the current-propagator-current coupling ($\mathcal{A}_{\Phi_i\Phi_j} = \mathcal{J}^*_{\Phi_i}(k)\langle\Phi_i(k)\Phi_j(k)\rangle\mathcal{J}_{\Phi_j}(k)$) may verify the causality ($k^2 \geq 0$) and taking the imaginary part of the residue at the poles ($\Im\{\mathcal{R}es(\mathcal{A}_{\phi_i\phi_j}|_{poles})\} \geq 0$) the unitarity at *tree-level*.

$$\Im\ \mathcal{R}es\ \mathcal{A}_{AA}|_{k^2=\mu^2} = |Z_A|^2 > 0\ , \tag{20}$$

$$\Im\ \mathcal{R}es\ \mathcal{A}_{aa}|_{k^2=\mu^2} = |Z_a|^2 > 0\ , \tag{21}$$

$$\Im\ \mathcal{R}es\ \mathcal{A}_{Aa}|_{k^2=0,\mu^2} = 0\ . \tag{22}$$

$$\Im\ \mathcal{R}es\ \mathcal{A}_{\overline{\psi}_+\psi_+}|_{k^2=0} = k_0|\theta_{1+} + i\theta_{2+}|^2 > 0 \tag{23}$$

$$\Im\ \mathcal{R}es\ \mathcal{A}_{\overline{\psi}_-\psi_-}|_{k^2=0} = k_0|\theta_{1-} + i\theta_{2-}|^2 > 0\ . \tag{24}$$

Comment

It can be concluded that the model is free from tachyons (all poles have $k^2 \geq 0$) and unitary at tree-level (free from ghosts). All the propagators ($\langle A^\mu(k)A^\nu(k)\rangle$, $\langle a^\mu(k)a^\nu(k)\rangle$, $\langle\overline{\psi}_+(k)\psi_+(k)\rangle$ and $\langle\overline{\psi}_-(k)\psi_-(k)\rangle$ propagate two degrees of freedom and the propagator $\langle a^\mu(k)A^\nu(k)\rangle$ is not dynamic.

From the previous results, we conclude that the MCSQED_3 parity invariant model is free from tachyons and ghosts at tree-level. However, to have full control of the unitarity at tree-level, it's necessary to study the behavior of the cross section at the limit of high energy in the center of momentum frame ($E_{\mathrm{CM}} = \sqrt{s} \to \infty$), by checking the Froissart-Martin[†] limit.

$$\lim_{\sqrt{s}\to\infty} \sigma \leq C\, s^{\frac{1}{2}} \ln s\ .$$

It shall be stressed that due to the presence of massless fermions, infrared singularities show up in S-matrix jeopardizing its unitarity, therefore it has been also to be verified the infrared lower bound limit of the cross section.

References

[†] M. Chaichian, J. Fischer e Yu.S. Vernov, Nucl.Phys.B383 (1992) 152;
O.M. Del Cima, Mod.Phys.Lett.A9 (1994) 1695.

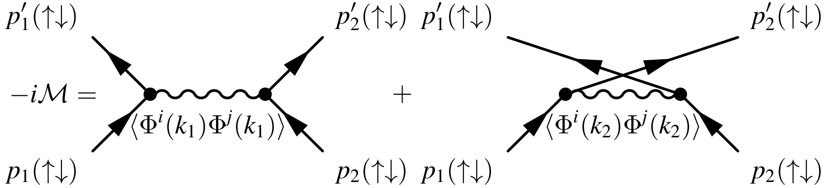
Møller scattering

Scattering amplitudes

The fermion-fermion scattering amplitudes[†] mediated by A_μ and a_μ read as bellow

$$
\begin{aligned}
-i\mathcal{M} &= \bar{u}_m(p_1')[\mathcal{V}^{\mu}_{m\,\Phi^i m}]u_m(p_1)\,[\langle\Phi^i_\mu(p_1-p_1')\Phi^j_\nu(p_1-p_1')\rangle]\,\bar{u}_n(p_2')[\mathcal{V}^{\nu}_{n\,\Phi^j n}]u_n(p_2)\,+ \\
&\quad -\bar{u}_m(p_2')[\mathcal{V}^{\mu}_{m\,\Phi^i m}]u_m(p_1)\,[\langle\Phi^i_\mu(p_1-p_2')\Phi^j_\nu(p_1-p_2')\rangle]\,\bar{u}_n(p_1')[\mathcal{V}^{\nu}_{n\,\Phi^j n}]u_n(p_2)
\end{aligned}
\tag{25}
$$

where m and n may assume $\{+,-\}$ representing the negatively charged $(-g)$ and positively charged $(+g)$ with respect to the chiral charge,respectively, and $\Phi^i_\mu = (A_\mu, a_\mu)$.



[†] J.J. Sakurai, *Advanced Quantum Mechanics*, Addison Wesley (1967).

Working in the center of momentum (CM) reference frame, the momenta of the interacting particles and the momentum transfer are fixed as

$$
\begin{aligned}
p_1 &= (E, p, 0) \quad \text{and} \quad p_1' = (E, p\cos\varphi, p\sin\varphi)\ , \\
p_2 &= (E, -p, 0) \ \text{and} \quad p_2' = (E, -p\cos\varphi, -p\sin\varphi)\ ,
\end{aligned} \tag{26}
$$

and

$$
\begin{aligned}
k_1 &= p_1 - p_1' = p_2' - p_2 = (0, p(1-\cos\varphi), -p\sin\varphi) = (0, \vec{\mathbf{k}})\ , \\
k_2 &= p_1 - p_2' = p_1' - p_2 = (0,\ p(1+\cos\varphi),\ p\sin\varphi) = (0, \vec{\mathbf{k}'})\ .
\end{aligned} \tag{27}
$$

and making use of the Lorentz invariant Mandelstam variables evaluated at the CM frame for massless particles:

$$
\begin{aligned}
s &= (p_1 + p_2)^2 = 4E^2\ , \\
t &= (p_1 - p_1')^2 = -2p^2(1-\cos\varphi) = -4p^2\sin^2\left(\frac{\varphi}{2}\right), \\
u &= (p_1 - p_2')^2 = -2p^2(1+\cos\varphi) = -4p^2\cos^2\left(\frac{\varphi}{2}\right).
\end{aligned}
$$

The amplitudes are, thus, given by:

$$\mathcal{M}_{\pm\pm} = (e^2 + g^2)e^{\pm i\varphi}\left(\frac{s-u}{t-\mu^2} + (u \leftrightarrow t)\right)$$

$$\mathcal{M}_{\pm\mp} = -(e^2 - g^2)\left(\frac{s-u}{t-\mu^2} - (u \leftrightarrow t)\right)$$

The scattering potentials are given by[†]:

$$V(\vec{r}) = \frac{1}{(2\pi)^2}\int d^2\vec{k}e^{i\vec{k}.\vec{r}}\beta_1\beta_2 F^{CM}(\vec{k}) \tag{28}$$

where

$$\mathcal{M} = \overline{u}_1'\overline{u}_2'\ F\ u_1 u_2 \tag{29}$$

taking into account only the t-channel part of the scattering amplitude evaluated at the CM frame.

References

† J. Sucher, "*Potentials from field theory: Non-uniqueness, Gauge dependence and all that*", AIP Conference Proceedings (1989), 337-370;

Accordingly to the prescription (28) the scattering potentials of interaction between the massless fermions, mediated by the photon and the Néel quasiparticle, read:

$$V_{\pm\mp}(r) = (1 - \vec{\alpha}_1.\vec{\alpha}_2)\frac{(e^2 - g^2)}{2\pi}K_0(\mu r) \tag{30}$$

$$V_{\pm\pm}(r) = (1 - \vec{\alpha}_1.\vec{\alpha}_2)\frac{(e^2 + g^2)}{2\pi}K_0(\mu r) \tag{31}$$

Due to the presence of the relativistic term $\vec{\alpha}_1.\vec{\alpha}_2$ it is not yet obvious that this potential may lead to bound states. One can see that this term is of order $\frac{v^2}{c^2}$ and, therefore, can be neglected in the low energy (low speed) approximation in the massive case †.

References

† O. M. Del Cima and E. S. Miranda, "*Electron-polaron electron-polaron bound states in mass-gap graphene-like planar quantum electrodynamics: s-wave bipolarons*", The European Physical Journal B, vol. 91, p. 212, Oct 2018.

It remains to be investigated if it is possible for massless fermions interacting through this potential to form bound states. However the Dirac equation for this system needs to be handled with care in order to avoid the problem of continuum dissolution † and the correct equation to be investigated is:

$$\vec{\alpha}_1.\vec{p}\psi - \vec{\alpha}_2.\vec{p}\psi + \Lambda(1)\Lambda(2)V(r)\Lambda(2)\Lambda(1)\psi = E\psi \tag{32}$$

Where $\Lambda(i) = \frac{1}{2}\left(1 + \frac{H_{Di}}{E}\right)$ are the projectors onto the positive energy space, being $H_{Di} = \vec{\alpha}_i.\vec{p}_i$ and $E = |\vec{p}|$. And we have used the fact that the potential is evaluated at the CM frame where $\vec{p}_1 = -\vec{p}_2 \equiv \vec{p}$

References

† J. Sucher, "*Continuum Dissolution and the Relativistic Many-Body Problem: A Solvable Model*" ,Phys. Rev. Lett., vol. 55, pp. 1033 -1035, Sep 1985.

Conclusions and Perspectives

Conclusions:

* We obtained the free Maxwell-Chern-Simons equations and showed that all fields satisfy the Proca equation;
* We observed the existence of two types of "spin" $\frac{1}{2}$ fermions, and their respective anti-fermions, each with different chiral charges;
* A splitting of the Landau levels was observed. It might be in accord with known experimental results for graphene monolayers when submitted to high magnetic fields;
* The model is physically consistent at tree level in the sense that it is causal and unitary;
* The amplitudes for the Møller scattering were obtained for the massless fermions. They are Lorentz invariant and change sign when the two fermions are swapped, as expected for fermions;
* From the amplitudes, the potentials were obtained and seen to be non-confining, contrary to what happens in massive QED_3 † when fermions are scattered by massless gauge field, which results in completely confining potential – logarithm potential.

Reference

† P. Maris, Phys. Rev. D52, (1982) 1511.

Perspectives:

* Verify the Froissart-Martin bound of the model in the ultraviolet and infrared limits;
* The algebraic renormalization of the model by adopting the BPHZL (Bogoliubov-Parasiuk-Hepp-Zimmermmann-Lowenstein);
* Further investigation of the splitting of the Landau levels;
* Investigate the possibility of the formation of bound states of massless fermions interacting through the potential that we obtained. The possibility was already proved[†] for the massive case in the low energy limit.

References

[†] arXiv:1810.02008 and submitted to Reviews in Mathematical Physics

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