Halo-independent interpretation of dark matter searches with CRESST

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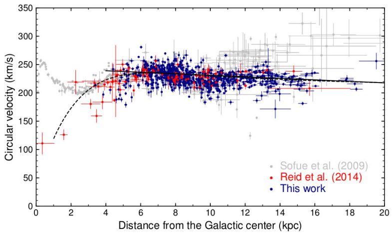
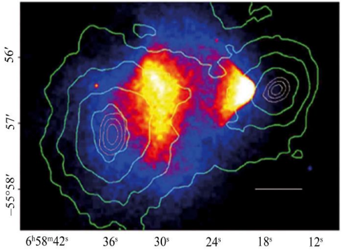
Evidence for dark matter

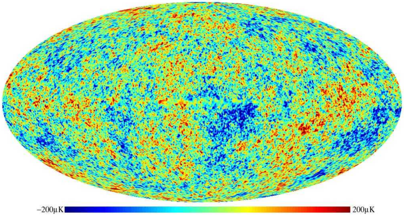
Gaxaxy rotation curve



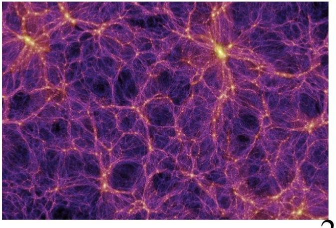
Colliding galaxy clusters



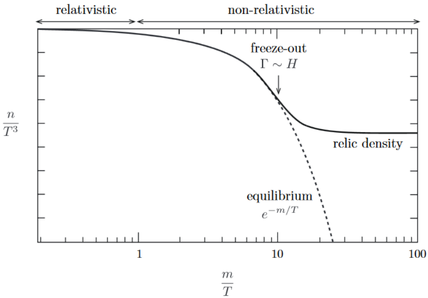
Cosmic Microwave Background

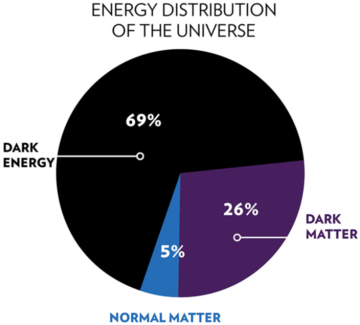


Large-scale structure



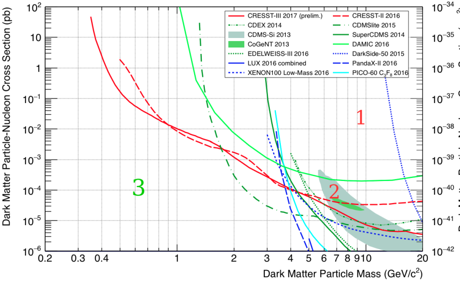
Cold Dark Matter : WIMPS

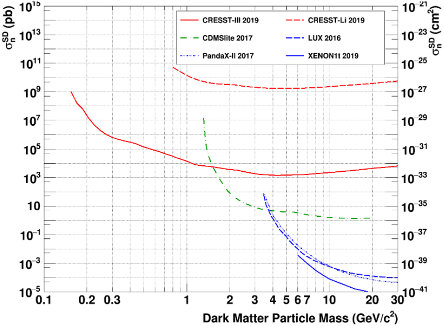




- **Dark Matter** → **WIMPS (Weakly Interacting Massive Particles)** produced thermally → Relic abundance determined by freeze-out
- **WIMP miracle** → A σ_{DM} of the weak interaction range leads to the correct DM relic abundance

Direct detection : CRESST





DD set **strong bounds** in the $\sigma_{DM} - m_{DM}$ parameter space

× 1: Ruled out by several experiments

× 2: "Islands" not compatible with other experiments results.

✓ 3: Unexplored region, and CRESST is the world leading experiment below $1.6 GeV$!

Direct Detection

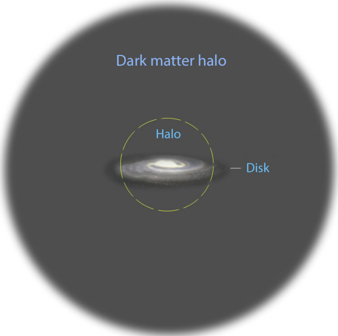
- Differential rate of DM-induced scattering :

$$\frac{dR}{dE_R} = \frac{\rho_{dm}}{m_A\, m_{DM}} \int_{v \geq v_{min}(E_R)} d^3v\, v\, f(\vec{v} + \vec{v}_{obs}(t)) \frac{d\sigma}{dE_R}$$

- **Astrophysical uncertainties**
- **Particle/nuclear physics uncertainties**

The Standard Halo Model : Isothermal Sphere

- The equilibrium distribution of a gas of self-gravitating particles is an **isothermal sphere with density profile** $\rho \propto r^{-2}$
- The velocity distribution $f(\vec{v})$ arises as the solution to the collisionless Boltzmann-equation
- The Maxwell Boltzmann distribution is truncated at the local escape velocity of the Milky Way $v_{esc} \approx 544\ km/s$



$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

$$f(v) \propto v^2 \exp(-v^2/2\sigma_v^2)$$

$$\sigma_v \approx 156\ km/s$$

Astrophysical uncertainties : The local Dark Matter density

- **Local measures** : Vertical kinematics of stars near the sun (tracers)

  $\rho_{dm} \approx 0 - 0.85 GeV/cm^3$

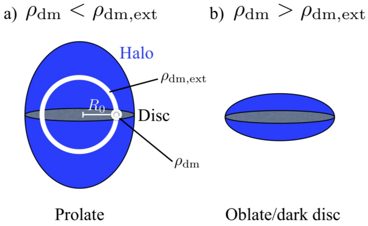
- **Global measures** : Extrapolate ρ_{dm} from the rotation curve

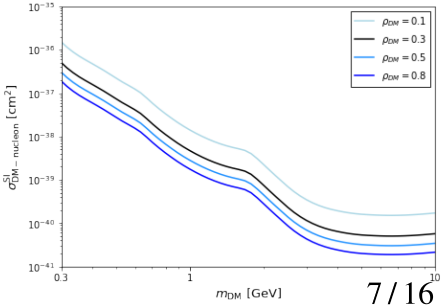
  $\rho_{dm} \approx 0.2 - 0.4 GeV/cm^3$

- Common choice :

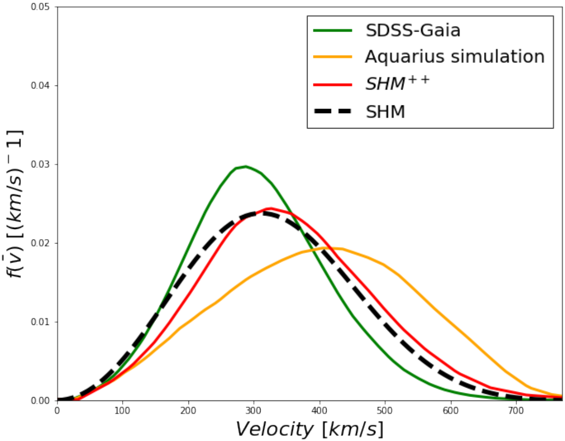
  $\rho_{dm} = 0.3 GeV/cm^3$

- ρ_{dm} linear in the DM scattering rate : **Uncertainties can be treated by rescaling the signal normalization**





- **Dark matter substructure** in the Milky Way → subhalos, streams, debris flow...
- **SHM** is neither a good fit to observations nor to simulations



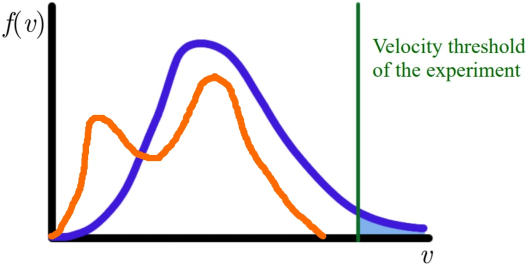
THE SHM MIGHT NOT BE A CORRECT DESCRIPTION OF THE DM HALO!

Halo-independent approach

- A certain dark matter mass and cross section (σ, m_{DM}) is ruled out independently of the velocity distribution if

$$min_{f(\vec{v})} \{R(\sigma, m_{DM})\} > R_{max}$$

- A single direct detection experiment is not sufficient to probe a dark matter model in a fully halo independent way, **Why?**



Some velocity distributions could escape detection by the experiment

How to parameterize the deviation from the SHM?

- The velocity distribution can be expressed as a superposition of streams $\rightarrow f(\vec{v}) = \sum_i c_{v_i} \delta(\vec{v} - v_0)$

✓ **Information divergences** : Rich and varied library of distance measures → Useful to parameterize different physics phenomena!

$$\text{Minimize } N_{expected}\,(\sigma, m_{DM})$$

$$\text{Subject to: } D(\, f_{MB} \,||\, f \,) \leq const.$$

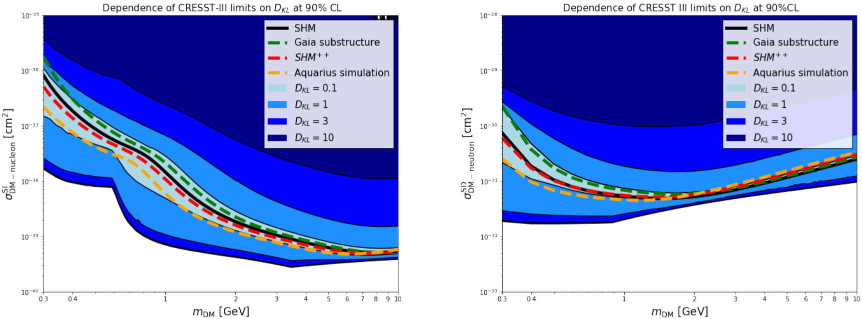
$$c_{v_i} \geq 0$$

$$\sum_i c_{v_i} = 1$$

- **Maxwell Boltzmann velocity distribution**
- **True velocity distribution**

- The **Kullback-Leibler divergence** is physically motivated and popular in Information theory → It represents the relative entropy between two distributions

$$D_{KL} = \int f_{MB}(v) \log\left(\frac{f_{MB}(v)}{f(v)}\right) dv$$



- Specific **dark matter models** could be rescued when **considering astrophysical uncertainties**

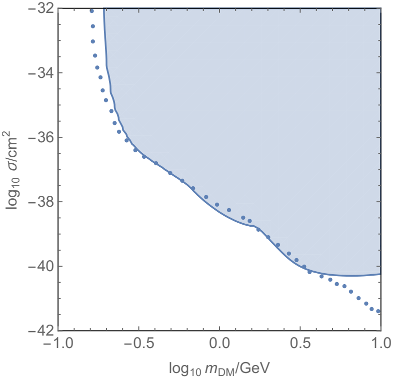
✓ Parameterized **model-independent** analysis!

DDCalc 2.2.0 : Implementing CRESST III

- DDCalc is the standard dark matter direct detection phenomenology software

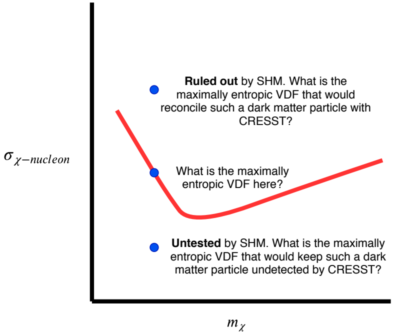
- I implemented with the help of Felix Kahlhoefer and Andreas Rappelt the CRESST III experiment, available in the latest version 2.2.0.

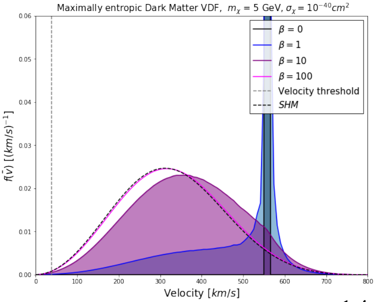
✓ Good agreement with the official CRESST collaboration limit



Bayesian estimate of the Dark Matter VDF

- The **principle of maximum entropy** constructs the maximally uninformative VDF given some constraints
- CRESST data can be used as a restriction in the set of possible VDF's $\rightarrow \beta D_{KL}(f, f_{MB}) - \frac{1}{2}\chi^2(f)$
- The parameter β **quantifies** our knowledge of the prior distribution





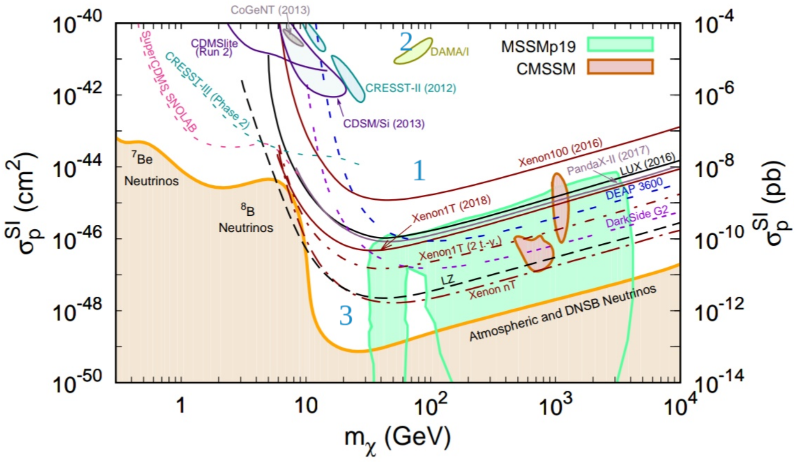
Conclusions

- The DM velocity distribution inside the Solar System is unknown. Hints from simulations and observations disfavor the Standard Halo Model.
- We have developed a method to quantify the impact of astrophysical uncertainties in a direct detection experiment, based on tools from information theory.
- We have derived upper limits on the DM scattering cross-section using CRESST data, including uncertainties from the velocity distribution.

Thanks for your attention

BACK-UP SLIDES

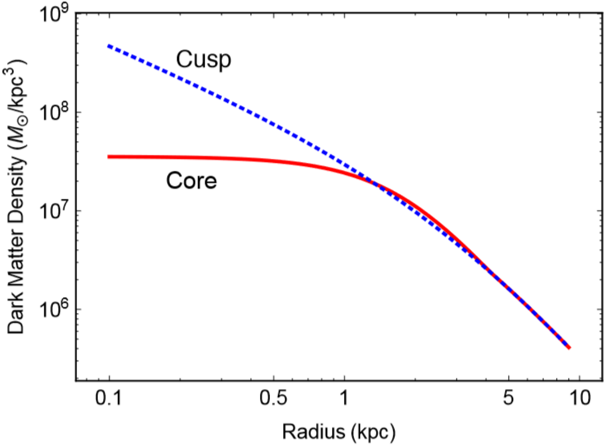
Direct detection limits: High masses



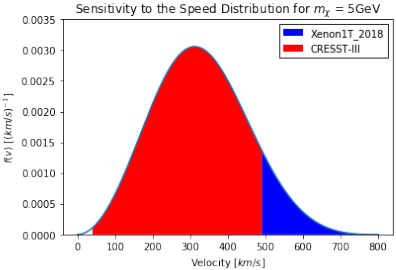
- What is the impact of astrophysical uncertainties?
- How do these conclusions depend on $f(\vec{v})$?

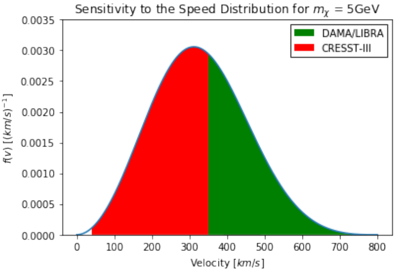
Density profile: Core-cusp problem

Discrepancy between inferred dark matter density profile $\rho(r)$ from rotation curves and cosmological N-body simulations predictions (core-cusp problem)



- Minimal velocity to induce a recoil: $v_{min}(E_R) = \sqrt{\frac{E_R m_N}{2\mu^2}}$
- Most experiments are only sensitive to the high velocity tail for $m_\chi < 10\ GeV$



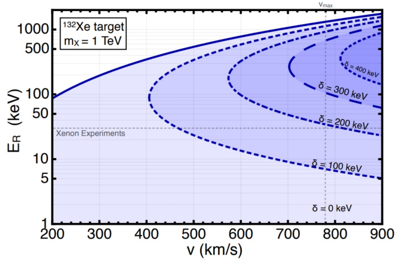


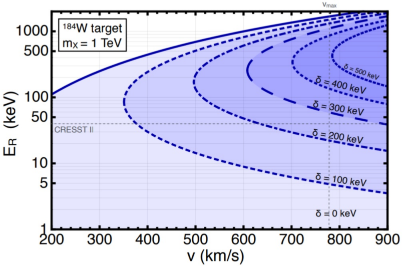
CRESST-III is (almost) able to probe the whole velocity spectrum for light WIMPS

Different kinematics : Inelastic DM

- In some models Dark Matter can interact inelastically with nucleai accesing an excited state with mass splitting δ
- Minimal velocity to induce a recoil: $v_{min} = \sqrt{\frac{2\delta}{\mu}}$

(arXiv:1608.02662)





$v_{min,Xe}(E_R) = 392 km/s$

$v_{min,W}(E_R) = 340 km/s$

Experiments are only sensitive to the high velocity tail for IDM

Information theory: Entropy

Proposed by Claude Shannon in 1948 in his article *"A Mathematical Theory of Communication"*

Suppose that you receive a message that consists of a string of symbols a or b, say

$$ababbabaabbabaaaab....$$

where a occurs with probability p and b with probability $1 - p$

How much information can one extract from a long message of this kind, say N letters?

How is this information *measured*?

For large N, the message will consist approximately of pN occurrences of a and $(1-p)N$ occurrences of b. The number of such messages is

$$\frac{N!}{(pN)!((1-p)N)!} \approx \frac{1}{(p)^{pN}(1-p)^{(1-p)N}} = 2^{NH} \tag{1}$$

where H is the **entropy** per letter

$$H = -plogp - (1-p)log(1-p) \tag{2}$$

More generally

$$\boxed{H(p) = -\sum_i p_i log(p_i)} \tag{3}$$

Information is measured in bits (base 2), nats (base e)...

The information gain in observing such a message is NH

KL divergence

Q predicts x_i with probability q_i. P predicts x_i with p_i. After observing N times x, how sure could we be that Q is wrong?

If P is the correct distribution, we'll observe outcome $i \approx p_i N$ times. We will judge the probability of what we have seen to be

$$\mathcal{P} = \prod_{i=1}^{N} q_i^{p_i N} \frac{N!}{\prod_{i=1}^{N} (p_j N)!} \tag{4}$$

for large N

$$\frac{N!}{\prod_{i=1}^{N} (p_j N)!} \approx 2^{-NH} \tag{5}$$

and

$$\mathcal{P} \approx 2^{-N \sum_i p_i log(\frac{p_i}{q_i})} \tag{6}$$

The relative entropy (per observation) or KL divergence is

$$\boxed{D_{KL}(p, q) = \sum_i p_i log\left(\frac{p_i}{q_i}\right)} \tag{7}$$

Shannon entropy vs Boltzmann entropy

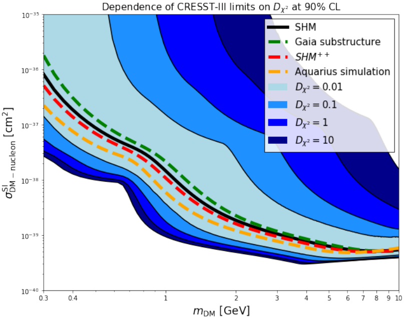
- **Information theory**
- Entropy : $H(p) = -\sum_i p_i log(p_i)$
- p_i : Probability of a discrete random variable
- Entropy quantifies the average amount of information conveyed by a message (event), when considering all possible outcomes.

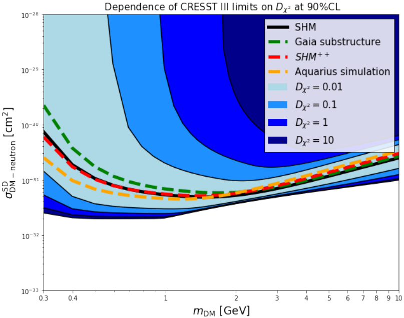
- **Statistical Mechanics**
- Entropy: $S = -k_B \sum p_i log(p_i)$
- p_i : Probability of the microstate i
- For a given set of macroscopic variables, the entropy measures the degree to which the probability of the system is spread out over different possible microstates

Useful connection? : The MB distribution can be obtained from both theories independently.

The MB distribution is the *Maximally uninformative distribution* under conservation of energy $< H >= E$

Results for χ^2 divergence

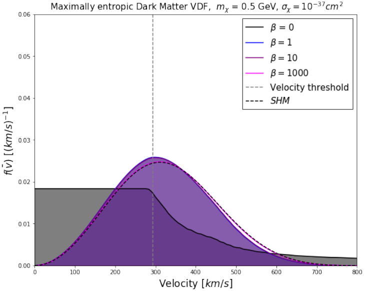


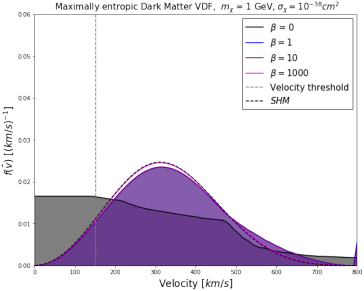


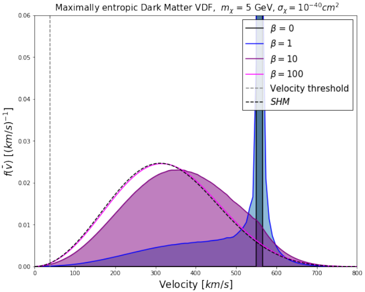
$$D_{\chi^2} = \int \frac{\left(f(v) - f_{MB}(v)\right)^2}{f_{MB}(v)} dv$$

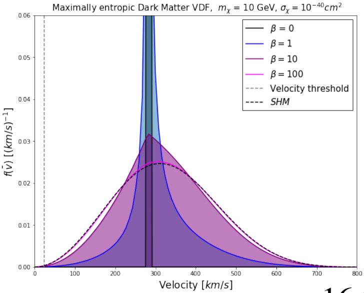
Different scaling than D_{KL}

Additional plots of bayesian approach









CRESST Limits: The Yellin Methods

- **Backgrounds are not known a priori**
- **Maximum gap**: A cross section σ is excluded as being too high if most random experiments would give smaller maximum gaps $\rightarrow$ Function $C_0(x)$ that equals the desired **confidence**
- Extension to **Optimum interval** method $\rightarrow$ consider all integrals with 1,2,..n_{obs} events, new $C_n(x_n)$ obtained via Monte Carlo
- **Frequentist method !**