# Superstring scattering from D-branes in the pure spinor formalism

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# Outline

# Why do we bother about the pure spinor formalism?

- ▶ Alternative organization scheme of worldsheet degrees of freedom

- ▶ There are already two different prescriptions: RNS and GS formalism

- ▶ Major motivation of the pure spinor formalism:
    - ▶ Manifest spacetime supersymmetry ($X^m \leftrightarrow \theta^\alpha$)
    - ▶ Manifest Lorentz invariance
    - ▶ PSF simplifies the calculation of scattering amplitudes, stunning example: massless four-point amplitudes at two-loops

- ▶ So far all results were shown to be equivalent

- ▶ For flat spacetime, it was even possible to calculate a closed string 3-loop amplitude

                                    Gomez & Mafra 2013

  and the complete open string n-pt massless tree level amplitude

                            Stieberger, Schlotterer, Mafra 2011

# The pure spinor formalism

Worldsheet matter action:

- ▶ Siegel's modification of GS superstring
  $S_{matter} = \frac{1}{4\pi} \int_\Sigma \mathrm{d}^2 z \left( \frac{1}{2} \partial X^m \bar{\partial} X_m + p_\alpha \bar{\partial} \theta^\alpha + \bar{p}_{\hat{\alpha}} \partial \bar{\theta}^{\hat{\alpha}} \right)$

- ▶ Spacetime supersymmetric variables
  $\Pi^m = \partial X^m + \frac{1}{2}(\theta \gamma^m \partial \theta)$
  $d_\alpha = p_\alpha - \frac{1}{2}(\partial X^m + \frac{1}{4}(\theta \gamma^m \partial \theta))(\gamma_m \theta)_\alpha$

Problems of this model:

- ▶ total central charge $c_{total} = -22 \rightarrow$ Theory can't be quantized consistently

- ▶ Level $k_{Siegel} = 4$ of the Lorentz current algebra for $S_{matter}$ in contrast to $k_{RNS} = 1$ in RNS
  $\rightarrow$ Level have to agree, to get equivalent scattering amplitudes

# The ghost sector

- ▶ Solution: Adding a new ghost sector
  $S_{PS} = S_{matter} + \frac{1}{4\pi} \int_{\Sigma} \mathrm{d}^2 z \left( w_\alpha \bar{\partial} \lambda^\alpha + \bar{w}_{\hat{\alpha}} \partial \bar{\lambda}^{\hat{\alpha}} \right)$

- ▶ $c_{matter} + c_{ghost} = 0 \Rightarrow$ if $w$ and $\lambda$ have 11 components each
  $\Rightarrow$ pure spinor constraint $(\lambda \gamma^m \lambda) = 0, \quad \forall m = 0, \dots, 9$
  eliminates five degrees of freedom for each $w$ and $\lambda$

- ▶ Ghost fields are spacetime spinors, they transform in a representation of the Lorentz group
  $\Rightarrow$ Contribution to the matter Lorentz current algebra
  $\Rightarrow$ Just the right correction of the level, such that RNS and PSF agree

# BRST symmetry

- $S_{PS}$ has a global fermionic symmetry generated by the BRST operator $Q = \oint \frac{\mathrm{d}z}{2\pi i} \lambda^\alpha(z) d_\alpha(z)$

- BRST operator was first only postulated

  Berkovits 2000

- Later on, derived from gauge fixing

  Berkovits 2015

- BRST charge is nilpotent, because of the pure spinor constraint

# Massless vertex operators

- Vertex operators are defined by BRST cohomology
- ghost number one $\leftrightarrow$ RNS analogue: fixed vertex operator $cV^{(q)}$
  $\Rightarrow h = 0$ vertex operator $V = \lambda^\alpha A_\alpha(X, \theta) \sim e^{ikX}$
  V is BRST closed $QV = 0$
- ghost number zero $\leftrightarrow$ RNS analogue: integrated vertex operator
  $\Rightarrow h = 1$ vertex operator (invariant upon integration)
  $U = \partial\theta^\alpha A_\alpha + \Pi^m A_m + d_\alpha W^\alpha + \frac{1}{2}N^{mn}F_{mn}$
  $QU = \partial V \Rightarrow \int \mathrm{d}z U(z)$   BRST invariant

# Scattering amplitudes in the pure spinor formalism
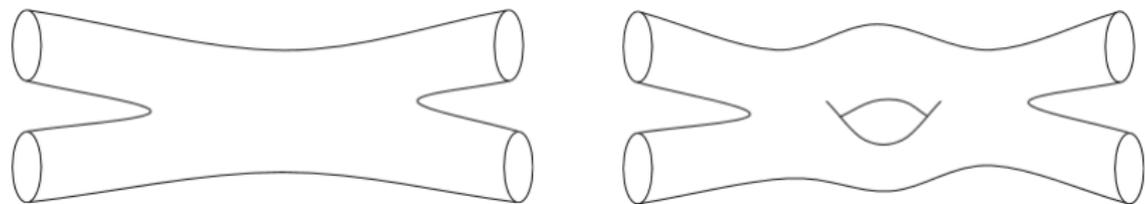


- ▶ Order in perturbation theory corresponds to a certain worldsheet topology
- ▶ Tree level scattering amplitudes refer to
    - ▶ Disk for open strings
    - ▶ Sphere for closed strings

  as worldsheet topology
- ▶ Conformal Killing vectors of worldsheet topology allow to fix position of vertex operators

  $\rightarrow$ Insertions of unintegrated vertex operators at fixed position

# Tree level scattering amplitude prescription

$$\mathcal{A}_{n\text{-}pt}^{tree} = \int \mathrm{d}z_2 \ldots \mathrm{d}z_{n-2} \, \langle V_1(0) U_2(z_2) \ldots U_{n-2}(z_{n-2}) V_{n-1}(1) V_n(\infty) \rangle$$

Evaluation of the correlator in two steps

- Integrate out the $h = 1$ fields by using the corresponding OPEs
  $\rightarrow$ Automatically eliminates the non-zero modes of the $h = 0$ fields

- Integrate over the zero modes of the $h = 0$ fields
  $\theta, \lambda, (X \rightarrow \delta(\sum_i k^i))$
  $\rightarrow$ Zero mode prescription:
  $\langle (\lambda \gamma^m \theta)(\lambda \gamma^n \theta)(\lambda \gamma^p \theta)(\theta \gamma_{mnp} \theta) \rangle = 2880$

# Closed string two-point function on the disk

**Motivation:**
Obtain the same result in PSF as in RNS for the bosonic closed string 2-point function on the disc

Garousi & Myers 1996

- ▶ Tree level prescription not clear for this amplitude
- ▶ Good guess would be $\int \mathrm{d}y \langle V_1(iy) \tilde{U}_1(-iy) V_2(i) \tilde{V}_2(-i) \rangle$ inspired from RNS with vertex operator position fixing $z_1 = iy, z_2 = i$
- ▶ Kinematic factor suggests that this is not sufficient, need an additional term
  $\rightarrow$ Logical choice to add $\int \mathrm{d}y \langle U_1(iy) \tilde{V}_1(-iy) V_2(i) \tilde{V}_2(-i) \rangle$
- ▶ Still strange, because we integrate over an unintegrated vertex operator
- ▶ Integration domain unclear, probably as in RNS from 0 to 1

# Gauge fixing the PSF action

- Gauge fixing can be done for the pure spinor superstring via BRST quantization

  Hoogeveen & Skenderis 2007

- Combines integrated and unintegrated vertex operator to make the description more natural
  $$\mathcal{V}(z) = V(z) + \zeta U(z)$$

- For tree level scattering amplitudes this leads to the prescription from before

**Main result:**

- Doing the gauge fixing for two-point function on the disk leads to
  $$\mathcal{A}^{disk}_{2\text{-}pt\ closed} =$$
  $$\int_0^1 \mathrm{d}y \left\langle i\Big( V_1(iy)\tilde{U}_1(-iy) + U_1(iy)\tilde{V}_1(-iy) \Big) V_2(i)\tilde{V}_2(-i) \right\rangle$$
  $\rightarrow$ After applying the steps from above, matches the result of RNS formalism

# Summary & Outlook

Summary:

- ▶ Pure spinor formalism is an efficient tool to calculate superstring scattering amplitudes
- ▶ All calculated scattering amplitudes in PSF match the results of RNS and GS

Outlook:

- ▶ one point function of a closed string on the disk
- ▶ vacuum amplitude on the torus