QUANTUM LEPTOGENESIS

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DESY

based on
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in collaboration with A.Anisimov, W. Buchmüller, M. Drewes

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Outline

- Introduction
- Non-equilibrium dynamics
  - Boltzmann equations
  - Kadanoff-Baym
- Solution to the Kadanoff-Baym
- Thermal leptogenesis
  - Boltzmann equations
  - Kadanoff-Baym
- Conclusions

Matter-antimater asymmetry

CMB : $\frac{n_B}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10}$

WMAP: $\frac{n_B}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}$

Sakharov's conditions

- Baryon number violation
- C, CP violation
- Departure from thermal equilibrium

Baryogeneis Models

- GUT baryogenesis
- Electroweak baryogenesis (SM, MSSM, NMSSM)
- Affleck-Dine baryogenesis (supersymmetry, flat directions, coherent oscillations)
- Baryogenesis via Leptogenesis (partial conversion of the lepton asymmetry into baryon asymmetry by sphaleron processes)

Thermal Leptogenesis

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \partial\!\!\!/ \nu_R - \bar{l}_L \tilde{\Phi} \lambda \nu_R - \frac{1}{2} \bar{\nu}_R^c M \nu_R + h.c.$$

- see-saw mechanism explains small neutrino masses
- mass term of $N \approx \nu_R + \nu_R^c$ violates lepton number
- mass and flavour eigenstates are not identical
- complex phases appear
- decay of N generally violates CP

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$\Rightarrow$ lepton and baryon asymmetry can be generated

CP-asymmetry

Leptogenesis is inherently a quantum effect

$$\epsilon_{CP} = \frac{\Gamma(N \to l\phi) - \Gamma(N \to \bar{l}\phi^c)}{\Gamma(N \to l\phi) + \Gamma(N \to \bar{l}\phi^c)}$$

In vacuum

- hierarchical spectrum

$$\epsilon = \frac{3}{16\pi} \frac{\mathrm{Im}(K_{12})^2}{K_{11}} \frac{M_1}{M_2}$$

Non-Equilibrium Dynamics of Quantum Systems

Methods

- Boltzmann equations (BE)
- quantum Boltzmann equations (QBE)
- kinetic equations for *matrices of density*
- Kadanoff-Baym equations (KBE)

Quantum Boltzmann Equations

$$\dot{n}(t) + 3H(t)n(t) + \Gamma(t)(n(t) - n_{eq}) = 0$$

- describe time evolution of classical particle numbers
- cross sections are importet from quantum field theory
- BE are know to work well in some examples, e.g.
  - photon decoupling,
  - big bang nucleosynthesis,
- but note that
  - BE cannot describe coherent oscillations ,
  - BE assume particles move freely between scatterings,
  - BE are Markovian ,
  - classical particle number is not well defined in interacting quantum field theory.

Is a Quantum Treatment possible?

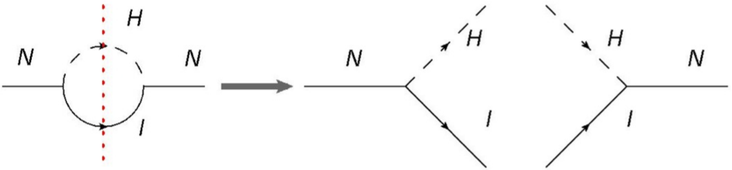
- spacial homogeneity
- weak coupling $\Rightarrow$ perturbative
- background plasma is in equilibrium
- backreaction can be neglected

'Weak coupling to a thermal bath'

Boltzmann vs Kadanoff-Baym Equations

- initial value problem for density matrix $\rho(t)\ldots$
- $\ldots$or for correlation functions $\langle\ldots\rangle = \mathrm{tr}(\rho\ldots)$
- KBE contain full quantum mechanics

$$\text{particle numbers} \Leftrightarrow \text{correlation functions}$$
$$\text{collision term} \Leftrightarrow \text{self energies}$$



Π, Π encode information about all decay and scattering processes

KBE Formalism

Statistical and Spectral Propagators

Scalars

$$\begin{aligned}\Delta^{+}(x_1,x_2) &= \frac{1}{2}\langle\{\Phi(x_1),\Phi(x_2)\}\rangle_c \\ \Delta^{-}(x_1,x_2) &= i\langle[\Phi(x_1),\Phi(x_2)]\rangle_c\end{aligned}$$

Fermions

$$\begin{aligned}S^{+}_{\alpha\beta}(x_1,x_2) &= \frac{1}{2}\langle[\Psi(x_1)_\alpha,\bar{\Psi}_\beta(x_2)]\rangle_c \\ S^{-}_{\alpha\beta}(x_1,x_2) &= i\langle\{\Psi(x_1)_\alpha,\bar{\Psi}_\beta(x_2)\}\rangle_c\end{aligned}$$

Majorana

$$\begin{aligned}G^{+}_{\alpha\beta}(x_1,x_2) &= \frac{1}{2}\langle[N_\alpha(x_1),N_\beta(x_2)]\rangle_c \\ G^{-}_{\alpha\beta}(x_1,x_2) &= i\langle\{N_\alpha(x_1),N_\beta(x_2)\}\rangle_c\end{aligned}$$

Kadanoff-Baym Equations

$$
\begin{aligned}
(\Box_1 + m^2)\Delta^-(x_1,x_2) &= -\int d^3\mathbf{x}' \int_{t_2}^{t_1} dt'\,\Pi^-(x_1,x')\Delta^-(x',x_2) \\
(\Box_1 + m^2)\Delta^+(x_1,x_2) &= -\int d^3\mathbf{x}' \int_{t_i}^{t_1} dt'\,\Pi^-(x_1,x')\Delta^+(x',x_2) \\
&\quad + \int d^3\mathbf{x}' \int_{t_i}^{t_2} dt'\,\Pi^+(x_1,x')\Delta^-(x',x_2)
\end{aligned}
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Weak Coupling to a thermal Bath

- consider fields that are weakly coupled to a large bath in equilibrium
- assume interaction mainly with bath fields $\mathcal{X}$
  - then self energies are computed with equilibrium propagators
  - in practice realised by using couplings that are linear in the field of interest, e.g. $g\phi\mathcal{O}[\mathcal{X}]$, $g\Psi\mathcal{O}[\mathcal{X}]$, at leading order in g

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For such systems Anisimov/Buchmülller/Drewes/SM

- spectral propagators Δ^-, S^-, G^- are time translation invariant
- KBE are equivalent to a stochastic Langevin equation
- KBE can be solved analytically up to a *memory integral*

Kadanoff-Baym Equations

$$(\partial_{t_1}^2 + \omega_{\mathbf{q}}^2)\Delta_{\mathbf{q}}^-(t_1 - t_2) = -\int_{t_2}^{t_1} dt' \Pi_{\mathbf{q}}^-(t_1 - t')\Delta_{\mathbf{q}}^-(t' - t_2)$$

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Solutions

Solutions for Scalars

$$\Delta^-_{\mathbf{q}}(t_1 - t_2) = i \int \frac{d\omega}{2\pi} e^{-i\omega(t_1 - t_2)} \rho_{\mathbf{q}}(\omega)$$

$$\rho_{\mathbf{q}}(\omega) = \frac{i}{\omega^2 - \omega_{\mathbf{q}}^2 - \Pi^A_{\mathbf{q}}(\omega) - i\omega\epsilon} - \frac{i}{\omega^2 - \omega_{\mathbf{q}}^2 - \Pi^R_{\mathbf{q}}(\omega) + i\omega\epsilon}$$

$$\begin{aligned} \Delta^+_{\mathbf{q}}(t_1, t_2) &= \Delta^+_{\mathbf{q},\mathrm{in}} \dot{\Delta}^-_{\mathbf{q}}(t_1) \dot{\Delta}^-_{\mathbf{q}}(t_2) + \ddot{\Delta}^+_{\mathbf{q},\mathrm{in}} \Delta^-_{\mathbf{q}}(t_1) \Delta^-_{\mathbf{q}}(t_2) \\ &+ \dot{\Delta}^+_{\mathbf{q};\mathrm{in}} \left(\dot{\Delta}^-_{\mathbf{q}}(t_1) \Delta^-_{\mathbf{q}}(t_2) + \Delta^-_{\mathbf{q}}(t_1) \dot{\Delta}^-_{\mathbf{q}}(t_2) \right) \\ &+ \int_0^{t_1} dt' \int_0^{t_2} dt'' \Delta^-_{\mathbf{q}}(t_1 - t') \Pi^+_{\mathbf{q}}(t' - t'') \Delta^-_{\mathbf{q}}(t'' - t_2) \end{aligned}$$

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Properties of the Solutions

- retarded self-energy $\Pi^R = \Pi^R|_{T=0} + \delta\Pi^R(T)$ is the decisive quantity
- $\mathrm{Re}\Pi^R$ gives thermal mass
- $\mathrm{Im}\Pi^R$ decay width Γ to resonance

Three regimes

1. $|\mathrm{Re}\Pi|, |\mathrm{Im}\Pi| \ll \omega_{\mathbf{q}}^2$
2. $|\mathrm{Re}\Pi| \approx \omega_{\mathbf{q}}^2, |\mathrm{Im}\Pi| \ll \omega_{\mathbf{q}}^2$
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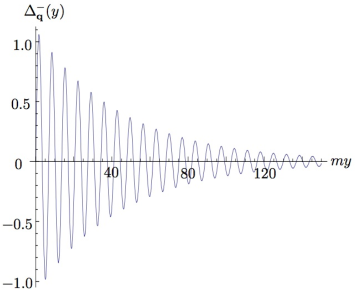
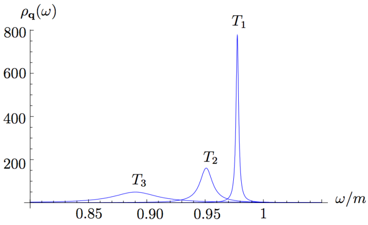
   single resonance kinematically behaves like quasiparticle
   but total energy receives vacuum contribution
3. $|\mathrm{Re}\Pi|, |\mathrm{Im}\Pi| \approx \omega_{\mathbf{q}}^2$
   particle interpretation and Boltzmann equations break down,
   at large T possibly even in a weakly coupled theory

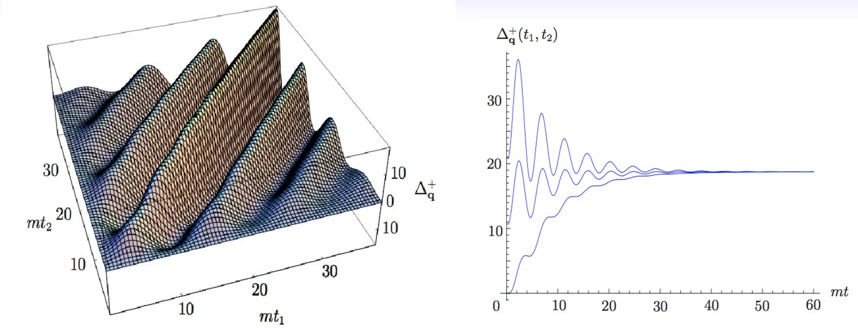
The Spectral Function





- Damped oscillatory behaviour
- Breitt-Wigner breaks down at high temperatures

The Statistical Propagator



- depends on two time arguments
- equilibrates independent of initial conditions after characteristic time $\tau \sim {}^1/_\Gamma$
- oscillates with plasma frequency

Non-equilibrium fermion

The width

$$\Pi_{\mathbf{q}}(\omega) = a_{\mathbf{q}}(\omega)\not{q} + b_{\mathbf{p}}(\omega)\not{u}$$

$$\Gamma = -2\mathrm{Im}\left(\mathrm{b}(\omega_{\mathbf{q}}) + \frac{\mathrm{a}(\omega_{\mathbf{q}})\mathrm{M}^2}{\omega_{\mathbf{q}}}\right)$$

Small width solution

$$S^-_{\mathbf{q}}(y) = \left(i\gamma_0 \cos[\omega_{\mathbf{q}} y] + \frac{M - \mathbf{q}\gamma}{\omega_{\mathbf{q}}}\sin[\omega_{\mathbf{q}} y]\right) e^{-\frac{\Gamma|y|}{2}}$$

$$S^+_{\mathbf{q}}(t,y) = -\left(i\gamma_0 \sin[\omega_{\mathbf{q}} y] - \frac{M - \mathbf{p}\gamma}{\omega_{\mathbf{q}}}\cos[\omega_{\mathbf{q}} y]\right)$$

$$\times \left[\frac{\tanh\left(\frac{\beta\omega}{2}\right)}{2} e^{-\frac{\Gamma|y|}{2}} + f_N^{eq}(\omega) e^{-\Gamma t}\right]$$

Application to Leptogenesis

Hierarchical Majorana Masses

- integrate out all but the lightest heavy neutrino

$$\mathcal{L} = \bar{l}_{Li}\tilde{\phi}\lambda^*_{i1}N + N^T\lambda_{i1}Cl_{Li}\phi - \frac{1}{2}MN^TCN + \frac{1}{2}\eta_{ij}l^T_{Li}\phi\, Cl_{Lj}\phi + \frac{1}{2}\eta^*_{ij}\bar{l}_{Li}\tilde{\phi}\, C\bar{l}^T_{Lj}\tilde{\phi}\,;$$

- effective vertex:

$$\eta_{ij} = \sum_{k>1}\lambda_{ik}\frac{1}{M_k}\lambda^T_{kj}$$

Boltzmann approach

Boltzmann equations are

- 1st order (Markovian, no memory effects)
- local in time (no oscillations)

The coupled differential equations are given by

$$\frac{\partial f_N}{\partial t} = C[f_N] \qquad \text{for Majorana}$$

$$\frac{\partial f_{l-\bar{l}}}{\partial t} = C[f_{l-\bar{l}}] \qquad \text{for lepton asymmetry}$$

$C[f]$: collision term

Boltzmann approach for Majorana

Solving for theMajorana neutrino

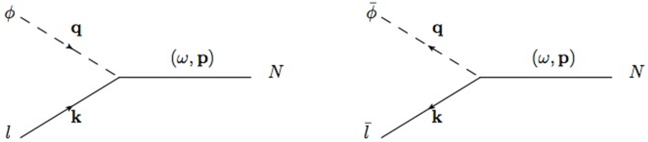
$$f_N(t) = f_N^{eq}(1 - e^{-\Gamma t})$$

The decay rate is given by

$$\Gamma = (\lambda^\dagger \lambda)_{11} \frac{2}{\omega} \int_{\mathbf{q},\mathbf{p}} (2\pi)^4 \delta^4(k+q-p) f_{l\phi}\, p \cdot k$$

with

$$f_{l\phi} = f_l f_\phi - (1-f_l)(1+f_\phi) = 1 - f_l + f_\phi$$



Boltzmann approach for asymmetry

Without wash-out terms

$$f_{l-\bar{l}} = -\epsilon_{CP}\frac{1}{k}\int_{\mathbf{q},\mathbf{p}}(2\pi)^4\delta^4(k+q-p)p\cdot k \times f_{l\phi}f_N^{eq}\frac{1}{\Gamma}(1-e^{-\Gamma t})$$

with the CP parameter

$$\epsilon_{CP} = \frac{3\mathrm{Im}(\lambda^\dagger\eta\lambda)\mathrm{M}}{16\pi(\lambda^\dagger\lambda)_{11}}$$

KB approach for the Asymmetry

How to the asymmetry without reference to particle number?

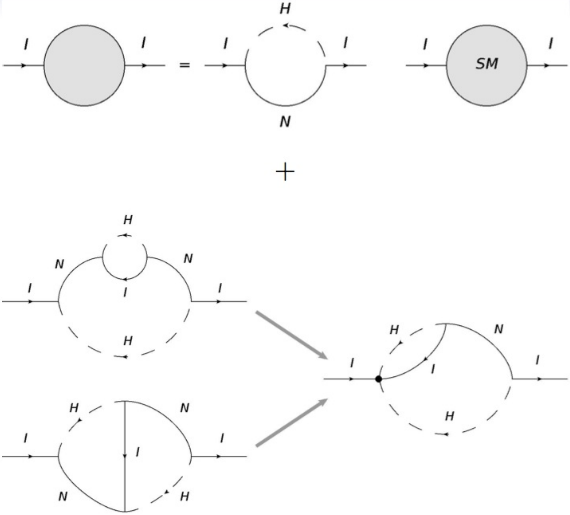
- define *lepton number matrix*

$$L_{\mathbf{k}ij}(t_1, t_2) = -\mathrm{tr}[\gamma^0 S^+_{\mathbf{k}ij}(t_1, t_2)].$$

- $L_{\mathbf{k}ii}(t, t)$ gives leptonic charge in flavour i at time t
- CP-violation comes from interference between LO and NLO terms

$\Rightarrow$ need to compute S^+ for leptons to NLO in Yukawa couplings!

Lepton Self-Energy



Non-equilibrium Majorana

The width

$$\Sigma_{\mathbf{p}}(\omega) = (a_{\mathbf{p}}(\omega)\not{p} + b_{\mathbf{p}}(\omega)\not{u})C^{-1}$$

$$\Gamma = -2\mathrm{Im}\left(b(\omega_{\mathbf{p}}) + \frac{a(\omega_{\mathbf{p}})M^2}{\omega_{\mathbf{p}}}\right)$$

Small width solution

$$G^{-}_{\mathbf{p}}(y) = \left(i\gamma_0 \cos[\omega_{\mathbf{p}} y] + \frac{M - \mathbf{p}\gamma}{\omega_{\mathbf{p}}} \sin[\omega_{\mathbf{p}} y]\right) e^{-\frac{\Gamma|y|}{2}} C^{-1}$$

$$G^{+}_{\mathbf{p}}(t, y) = -\left(i\gamma_0 \sin[\omega_{\mathbf{p}} y] - \frac{M - \mathbf{p}\gamma}{\omega_{\mathbf{p}}} \cos[\omega_{\mathbf{p}} y]\right)$$

$$\times \left[\frac{\tanh\left(\frac{\beta\omega}{2}\right)}{2} e^{-\frac{\Gamma|y|}{2}} + f_N^{eq}(\omega)e^{-\Gamma t}\right] C^{-1}$$

The Source of the Asymmetry

lepton self energy splits into a pure SM part and a part involving N :

$$\Pi^{\pm}_{\mathbf{k}ij}(t_1, t_2) = \Pi^{\pm,\mathrm{SM}}_{\mathbf{k}ij}(t_1 - t_2) + \delta\Pi^{\pm}_{\mathbf{k}ij}(t_1, t_2)$$

S^{+} can be split into a solution in the absence of N and a correction:

$$S^{\pm}_{\mathbf{k}ij}(t_1, t_2) = S^{\pm,\mathrm{SM}}_{\mathbf{k}ij}(t_1 - t_2) + \delta S^{\pm}_{\mathbf{k}ij}(t_1, t_2)$$

Only the correction can generate a non-zero leptonic charge!

Kadanoff-Baym Equation for δS^+

To leading order

$$
\begin{aligned}
(i\gamma_0\partial_{t_1} - \mathbf{k}\gamma)\,\delta S^+_{\mathbf{k}ij}(t_1,t_2) \quad &- \quad \int_0^{t_1} dt'\,\Pi^{-SM}_{\mathbf{k}ij}(t_1-t')\,\delta S^+_{\mathbf{k}ij}(t',t_2) \\
&= \quad \zeta^1_{\mathbf{k}ij}(t_1,t_2) + \zeta^2_{\mathbf{k}ij}(t_1,t_2) + \zeta^3_{\mathbf{k}ij}(t_1,t_2)
\end{aligned}
$$

Kadanoff-Baym Equation for δS^+

To leading order

$$\begin{aligned}(i\gamma_0\partial_{t_1} - \mathbf{k}\gamma)\,\delta S^+_{\mathbf{k}ij}(t_1,t_2) \quad &- \quad \int_0^{t_1} dt'\,\Pi^{-SM}_{\mathbf{k}ij}(t_1-t')\,\delta S^+_{\mathbf{k}ij}(t',t_2) \\ &= \quad \zeta^1_{\mathbf{k}ij}(t_1,t_2) + \zeta^2_{\mathbf{k}ij}(t_1,t_2) + \zeta^3_{\mathbf{k}ij}(t_1,t_2)\end{aligned}$$

The l.h.s. of the above equation is a homogeneous equation for $\delta S^+_{\mathbf{k}ij}$, and the sources are given by

$$\zeta^1_{\mathbf{k}ij}(t_1,t_2) = \int_0^{t_1} dt'\,\delta\Pi^-_{\mathbf{k}ij}(t_1,t')\,S^{+SM}_{\mathbf{k}ij}(t'-t_2),$$

$$\zeta^2_{\mathbf{k}ij}(t_1,t_2) = -\int_0^{t_2} dt'\,\delta\Pi^+_{\mathbf{k}ij}(t_1,t')\,S^{-SM}_{\mathbf{k}ij}(t'-t_2),$$

$$\zeta^3_{\mathbf{k}ij}(t_1,t_2) = -\int_0^{t_2} dt'\,\Pi^{+SM}_{\mathbf{k}ij}(t_1-t')\,\delta S^-_{\mathbf{k}ij}(t',t_2)$$

Solution for δS^+

To leading order in Π

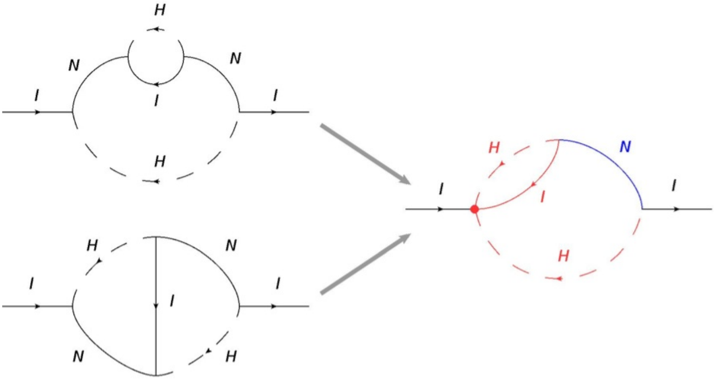
$$\delta S^+_{\mathbf{k}ij}(t_1, t_2) =$$

$$\int_0^{t_1} dt' \int_0^{t_2} dt'' S^{-,F}_{\mathbf{k}ij}(t_1 - t')\delta\Pi^+_{\mathbf{k}ij}(t', t'')S^{-,F}_{\mathbf{k}ij}(t'' - t_2)$$

$$- \int_0^{t_1} dt' \int_0^{t'} dt'' S^{-,F}_{\mathbf{k}ij}(t_1 - t')\delta\Pi^-_{\mathbf{k}ij}(t', t'')S^{+,F}_{\mathbf{k}ij}(t'' - t_2)$$

$$- \int_0^{t_2} dt'' \int_0^{t''} dt' S^{+,F}_{\mathbf{k}ij}(t_1 - t'')\delta\Pi^+_{\mathbf{k}ij}(t'', t')S^{-,F}_{\mathbf{k}ij}(t' - t_2)$$

CP-violating Part of the Lepton Self Energy



$$
\begin{aligned}
L_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\, 8\pi \int_{\mathbf{q},\mathbf{q}'} \frac{k\cdot k'}{|\mathbf{k}||\mathbf{k}'|\omega} f_{l\phi}(k,q) f_{l\phi}(k',q') f_N^{eq}(\omega) \\
\times \frac{\frac{1}{2}\Gamma}{((\omega-|\mathbf{k}|-|\mathbf{q}|)^2+\frac{\Gamma^2}{4})((\omega-|\mathbf{k}'|-|\mathbf{q}'|)^2+\frac{\Gamma^2}{4})} \\
\times \Big(\cos[(|\mathbf{k}|+|\mathbf{q}|-|\mathbf{k}'|-|\mathbf{q}'|)t] + e^{-\Gamma t} \\
-(\cos[(\omega-|\mathbf{k}|-|\mathbf{q}|)t] + \cos[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t])e^{-\frac{\Gamma t}{2}} \Big),
\end{aligned}
$$

$$L_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\, 8\pi \int_{\mathbf{q},\mathbf{q}'} \frac{k\cdot k'}{|\mathbf{k}||\mathbf{k}'|\omega} f_{l\phi}(k,q) f_{l\phi}(k',q') f_N^{eq}(\omega)$$

$$\times \frac{\frac{1}{2}\Gamma}{((\omega-|\mathbf{k}|-|\mathbf{q}|)^2+\frac{\Gamma^2}{4})((\omega-|\mathbf{k}'|-|\mathbf{q}'|)^2+\frac{\Gamma^2}{4})}$$

$$\times \Big(\cos[(|\mathbf{k}|+|\mathbf{q}|-|\mathbf{k}'|-|\mathbf{q}'|)t] + e^{-\Gamma t}$$

$$-(\cos[(\omega-|\mathbf{k}|-|\mathbf{q}|)t] + \cos[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t])e^{-\frac{\Gamma t}{2}}\Big)\,,$$

with $\mathbf{p} = \mathbf{q}+\mathbf{k} = \mathbf{q}'+\mathbf{k}'$

$$\int_{\mathbf{p}} \ldots = \int \frac{d^3p}{(2\pi)^3 2\omega_{\mathbf{p}}} \ldots$$

$$f_{l\phi}(k,q) = f_l(k)f_\phi(q) + (1-f_l(k))(1+f_\phi(q))$$
$$= 1 - f_l(k) + f_\phi(q)$$

Comparison to Boltzmann Result

$$L_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\,\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{q}'}\frac{k\cdot k'}{|\mathbf{k}'|\omega}f_{l\phi}(k,q)f_N^{eq}(\omega)f_{l\phi}(k',q')$$

$$\times\frac{\frac{1}{4}\Gamma}{((\omega-|\mathbf{k}|-|\mathbf{q}|)^2+\frac{\Gamma^2}{4})((\omega-|\mathbf{k}'|-|\mathbf{q}'|)^2+\frac{\Gamma^2}{4})}$$

$$\times\Big(\cos[(|\mathbf{k}|+|\mathbf{q}|-|\mathbf{k}'|-|\mathbf{q}'|)t]+e^{-\Gamma t}$$

$$-(\cos[(\omega-|\mathbf{k}|-|\mathbf{q}|)t]+\cos[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t])e^{-\frac{\Gamma t}{2}}\Big),$$

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'}k\cdot k' f_{l\phi}(k,q)f_N^{eq}(\omega)$$

$$\times\frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$

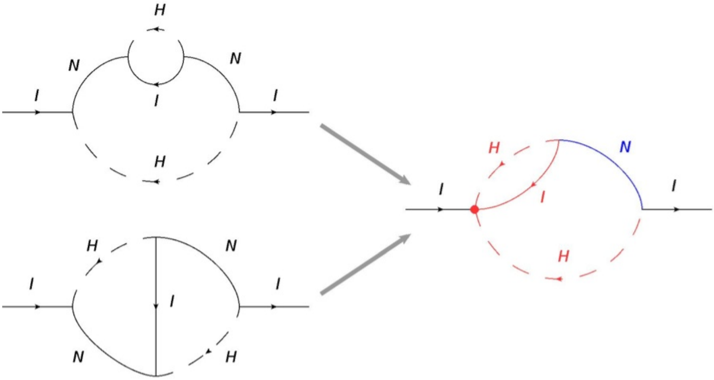
$$\times\left(1-e^{-\Gamma t}\right)$$

On-Shell Approximation (unjustified!)

$$L^{os}_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\frac{16\pi}{k}\int_{\mathbf{q},\mathbf{q}',\mathbf{p},\mathbf{k}'} k\cdot k' f_{l\phi}(k,q) f_N^{eq}(\omega) f_{l\phi}(k',q')$$
$$\times \frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$
$$\times \left(1-e^{-\frac{\Gamma t}{2}}\right)^2$$

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'} k\cdot k' f_{l\phi}(k,q) f_N^{eq}(\omega)$$
$$\times \frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$
$$\times \left(1-e^{-\Gamma t}\right)$$

Inclusion of SM widths



Inclusion of SM widths

$$\tilde{L}_{\mathbf{k}ij}(t,t) = -\epsilon_{ij}\,\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{q}'}\frac{k\cdot k'}{|\mathbf{k}'|\omega}f_{l\phi}(k,q)f_N^{eq}(\omega)f_{l\phi}(k',q')$$

$$\times\frac{1}{\Gamma}\frac{\frac{1}{4}\Gamma_{l\phi}\Gamma_{\phi}}{((\omega-k-q)^2+\frac{1}{4}\Gamma_{l\phi}^2)((\omega-k'-q')^2+\frac{1}{4}\Gamma_{\phi}^2)}$$

$$\left(1-e^{-\Gamma t}\right)$$

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'}k\cdot k' f_{l\phi}(k,q)f_N^{eq}(\omega)$$

$$\times\frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$

$$\times\left(1-e^{-\Gamma t}\right)$$

BUT: This is not yet a consistent treatment of gauge interactions!!!

Conclusions

- Quantum and non-Markovian effects can be crucial for leptogenesis.
- We computed the generated lepton asymmetry for hierarchical heavy neutrino masses and a constant (or very slowly changing) temperature without semi-classical approximations.
- We find significant deviations from Boltzmann equations due to off-shell effects, memory effects and temperature dependent corrections.
- We also find deviations from quantum corrected Boltzmann equations.
- The consistent inclusion of all SM corrections remains an issue.