Search for a Charged Higgs

Simon Grewe

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Max-Planck-Institut
für Physik

Charged Higgs

- In the SM, the Higgs sector has been chosen to be as simple as possible
- Alternative approaches predict existence of additional scalars:
  - Higgs singlet model
  - 2HDM
  - 3HDM
  - Higgs triplet model
- Both, the Higgs Doublet and Triplet models predict electrically charged scalars

2HDM(2 Higgs Doublet model)

- Introduces 2 complex SU(2) Doublets
- Type 2 2HDM is required for a MSSM
- 5Higgs Bosons
  - 2 Charged Higgs
  - 2 CP-Even Higss
  - 1 Pseudoscalar (A)
- 7 Free Parameters
  - 5 Higgs masses
  - $\tan(\beta) = \left(\frac{v_1}{v_2}\right)$
  - $\sin(\beta - \alpha)$(Mixing between Cp even Higgs)
    - $\sin(\beta - \alpha)$->1(light Higgs becomes SM-like)
    - $\sin(\beta - \alpha)$->0(heavy higgs becomes SM-like)

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}$$

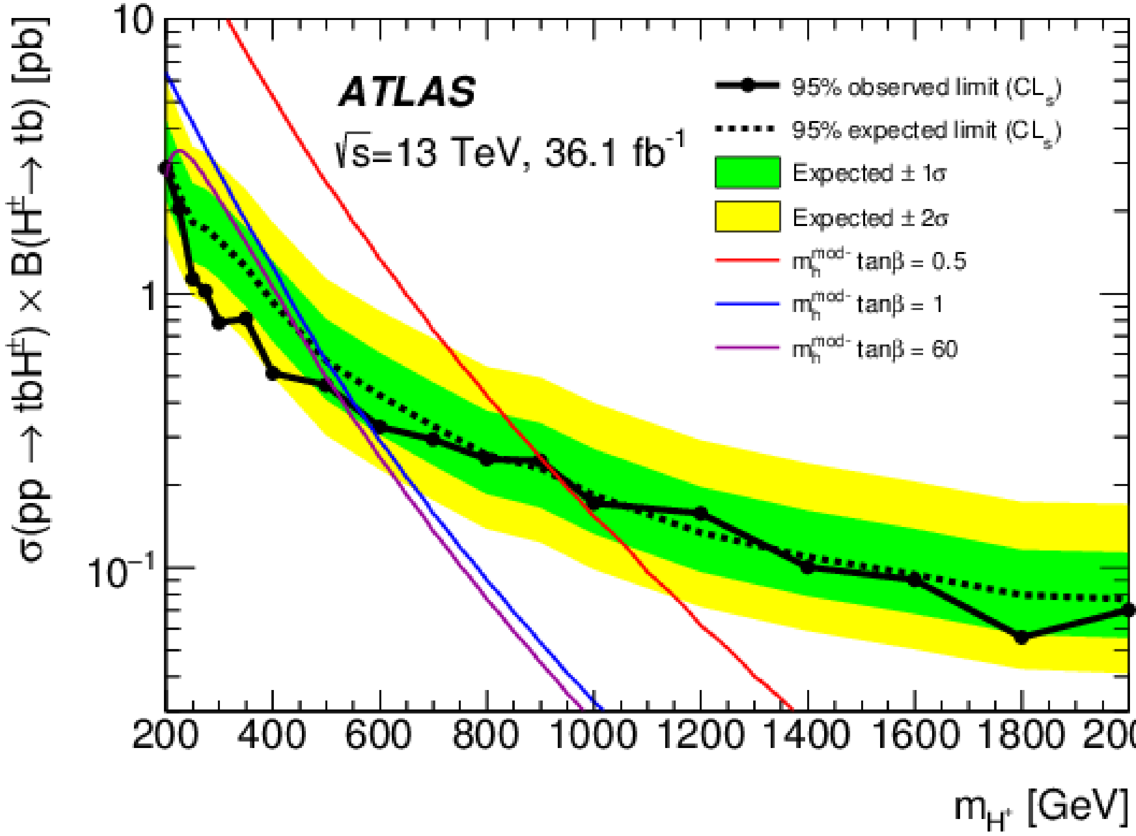
$$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix}$$

Previous Charged Higgs Searches

- Direct Searches
  - Searches at Lep
    - $e^+e^- \to \gamma\, Z \to H^+H^-$
    - Relative model independent
    - Limit $m(H^+) > 80GeV$
  - LHC
    - ATLAS/CMS Searches
    - $H^+ \to \tau\nu$ (Interesting for low Charged Higgs masses)
    - $H^+ \to tb$ (Interesting for high Charged Higgs masses)
- Indirect Searches
  - Flavour Physics
    - e.g. $B_u \to \tau\, \nu$ (sensitive to H^+ at Tree Level)
    - Highly model dependent

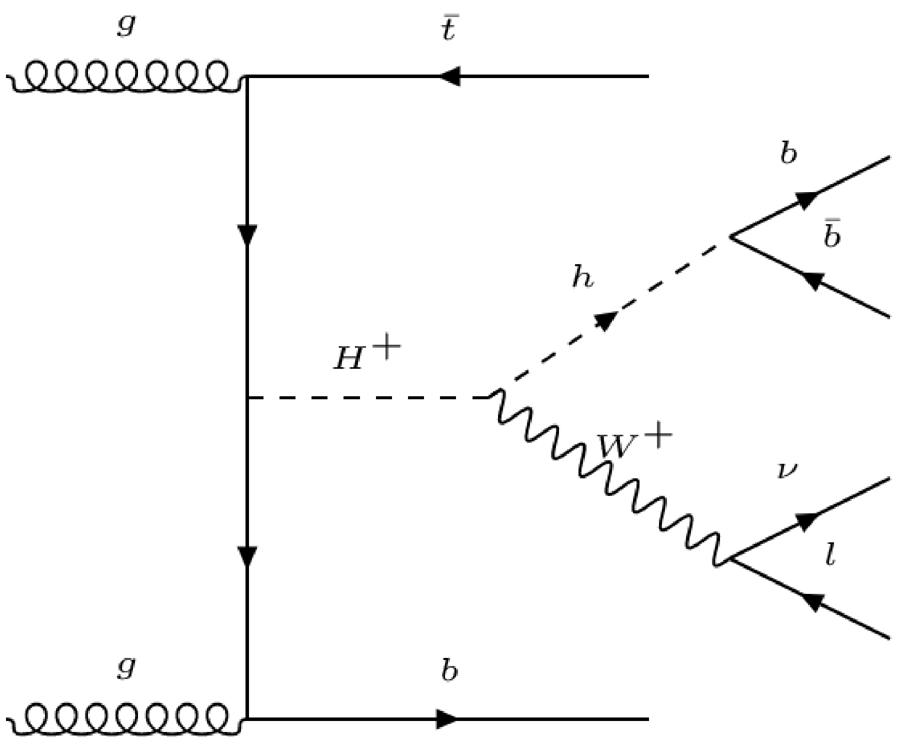
Search for charged Higgs bosons decaying into top and bottom quarks at √ s = 13 TeV with the ATLAS detector

- Investigates $pp \rightarrow tbH^{+} \rightarrow ttbb$
- Probes mass range of 200-2000GeV
- Uses 36.1 fb^{-1} of pp collision data at $\sqrt{s} = 13\text{TeV}$
- No excess above expected background observed
- Limits Br x Cross Section to 2.9-0.070 pb



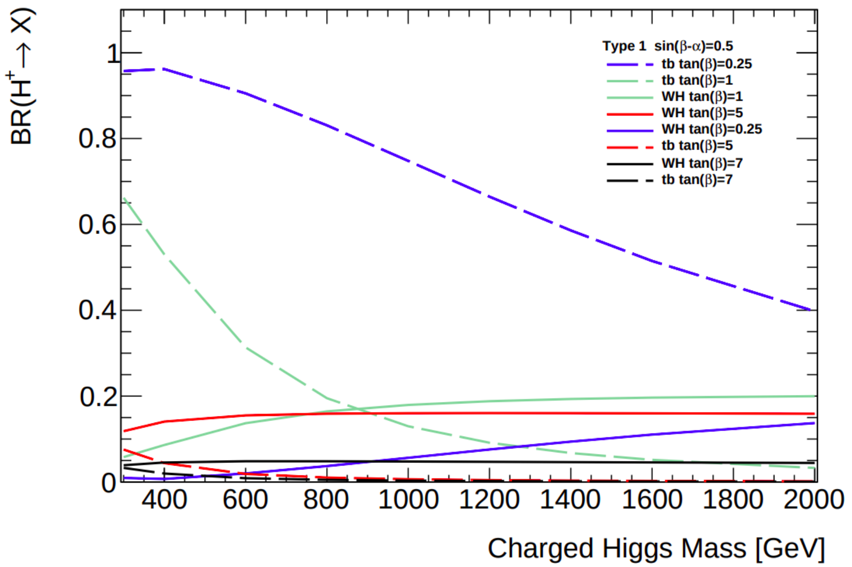
https://arxiv.org/abs/1808.03599

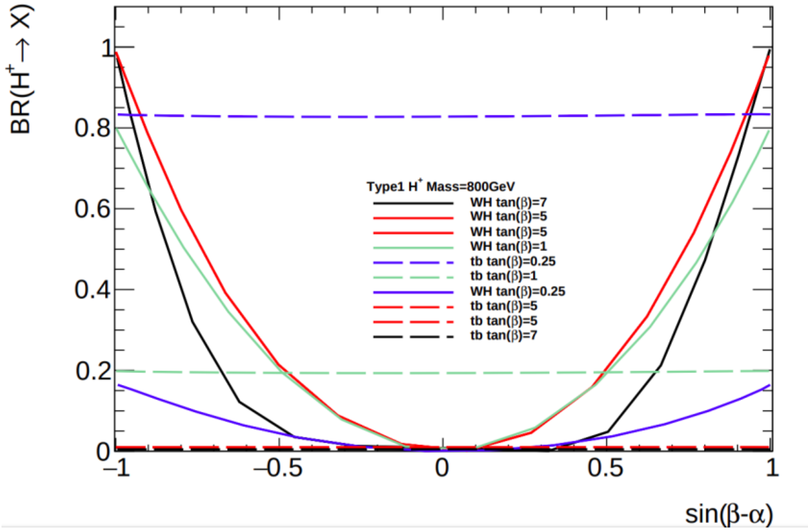
- Decay H^+-->Wh (h= 125 GeV SM Higgs)is so far uncovered
- Significant BR(H^+-->Wh) for:
  - N2HDM [https://arxiv.org/abs/1910.06858]
  - Georgi-Machacek model [https://arxiv.org/pdf/1908.00396.pdf]
  - 2HDM scenaria in which the 125GeV Higgs boson is the heaviest CP-even scalar
- Investigating the Case
  - h->bb
  - W->lv

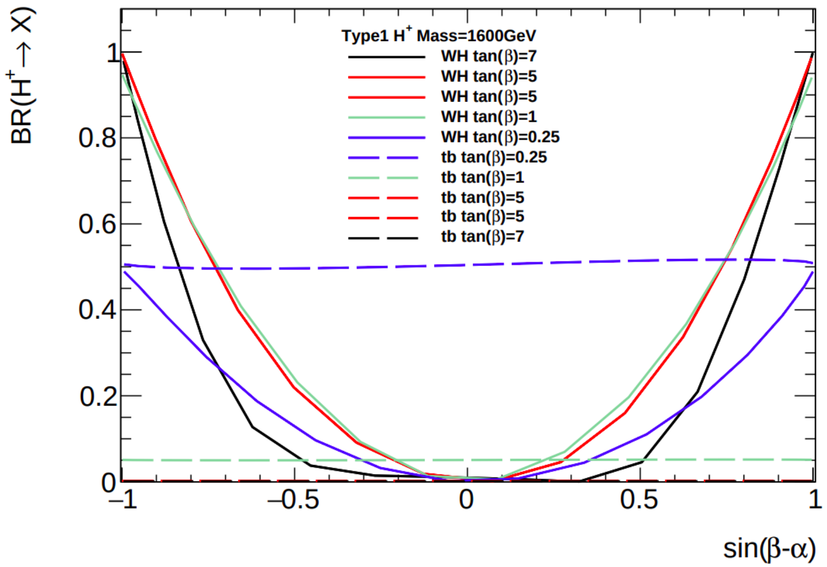


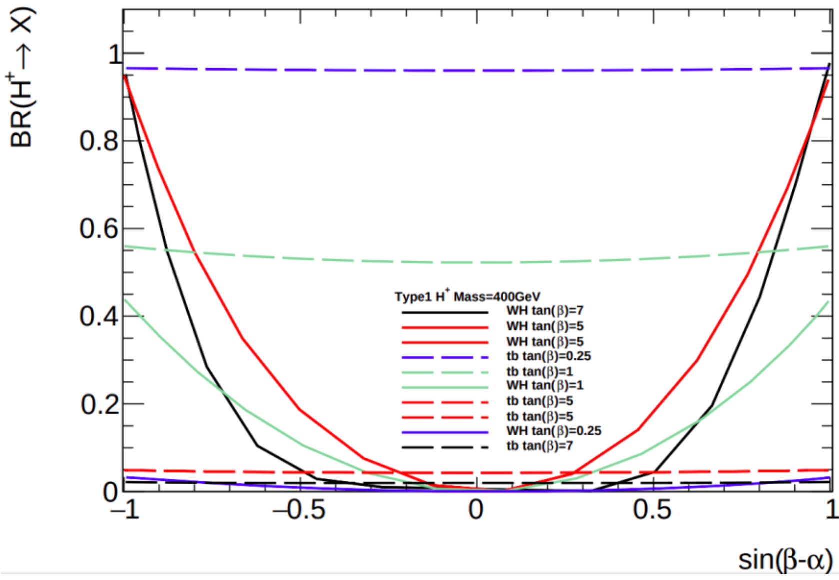
- BR for 2HDM model with a heavy 125 GeV Higgs
- Calculated with 2HDECAY[https://arxiv.org/abs/1810.00768]
- Large BR for hight sin(b-a)

$$g_{H^+W^+H} \propto \sin(\beta - \alpha)$$









Simulated Signal Events

- Study the Charged Higgs using Monte Carlo Events
- Simulation Data for Charged Higgs (H^+) was produced with MadGraph5
  - Parton Shower and Hadronisation Process were modeld by PYTHIA8.2
- 3 Mass Benchmark points
  - m= 400 GeV
  - m=800 GeV
  - m=1600 GeV
- Normalized to 1pb

Simulated Background Events

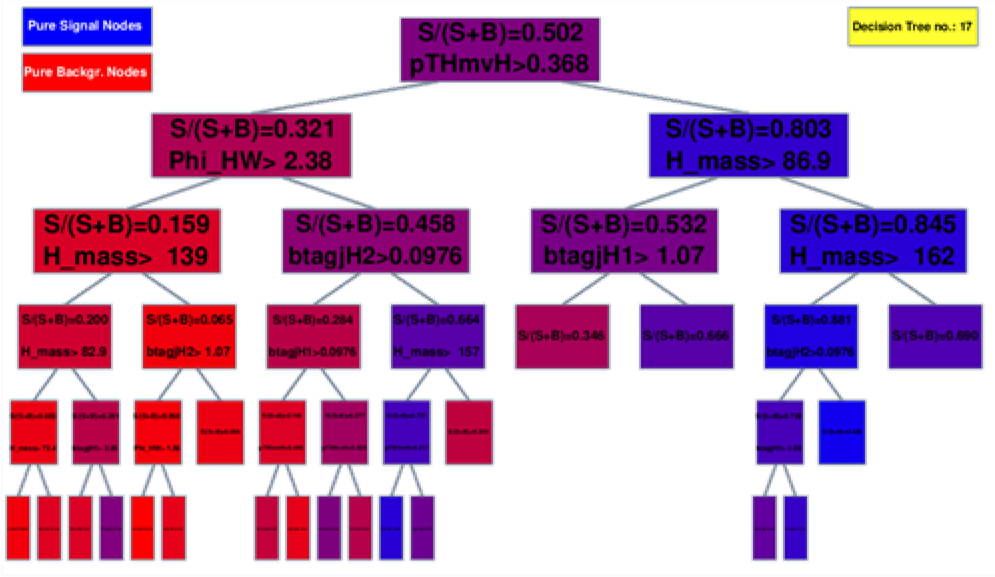
- tt pair (Powheg+Pythia8)
- Singel Top production (Powheg+Pythia8)
- W+ jets(Sherpa)
- Z+ jets(Sherpa)
- Di-boson(WW/ZZ) (Sherpa)

Event Reconstruction

- Events with 3 or more jets and only one positive charged lepton are considered
- Higgs Plus is reconstructed from a W and Higgs

- W is reconstructed from Lepton + Neutrino
  - Neutrino is reconstructed with Missing Transverse Energy technique using W boson mass constrain
- Higgs is reconstructed from any two Jets:
- **But**: Which jets come form the Higgs decay ?

Boosted Decision Trees



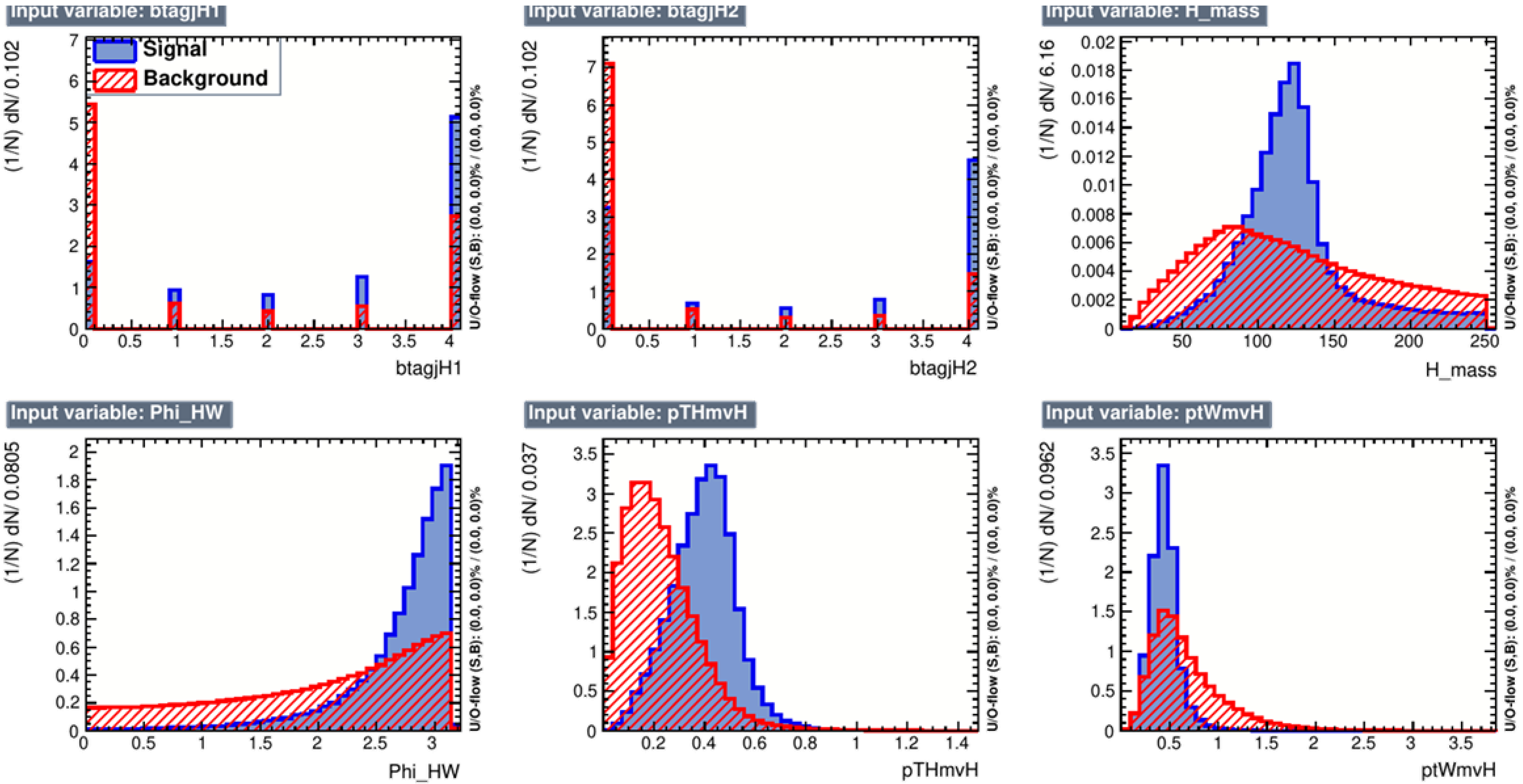
- Use Boosted Decision Trees(BDT)
- Decision Tree:
  - Asks a series of simple questions(e.g is m>86.9)
  - Next Question depends on Answer
- Boosted:
  - Have a lot of trees(to reduce impact of statistical fluctuations)
  - Give Events that are classified wrong higher weight
  - Next Tree will focus more on them, giving better performance
- BDT returns value between -1 and 1
- Choose the Jet pair Lepton Neutrino combination with the highest score as the Higgs and W

Boosted Decision Trees

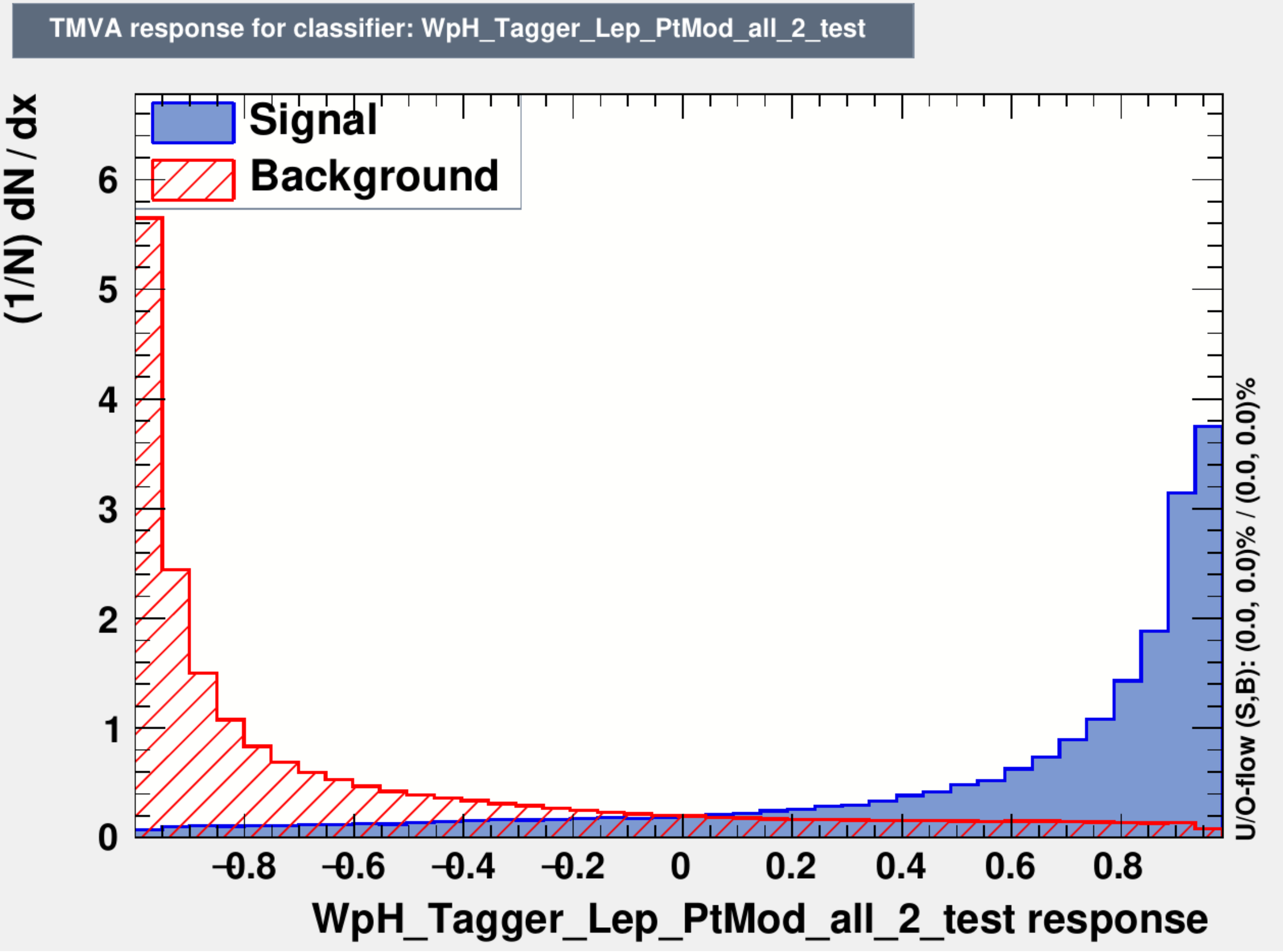
- Boosted Decision Trees provided by TMVA Root package were used
  - Max Depth=5
  - nTress=600
  - baggedBoost

- Signal: Jet pair and Lepton Neutrino combination correctly dR matched to MC truth h/W (the Jet Pair that is closest in dR to the truth Higgs is chosen, but dR<0.3 is requierd)
- Background: All the wrong combination of the signal sample
- Trained on:
  - Higgs Mass
  - B-tag score of Higgs jets
  - Delta Phi between Higgs and W candidate
  - pT_{Higgs}/m_{WH}
  - pT_{W}/m_{WH}

Distribution of Input variables for Signal/background



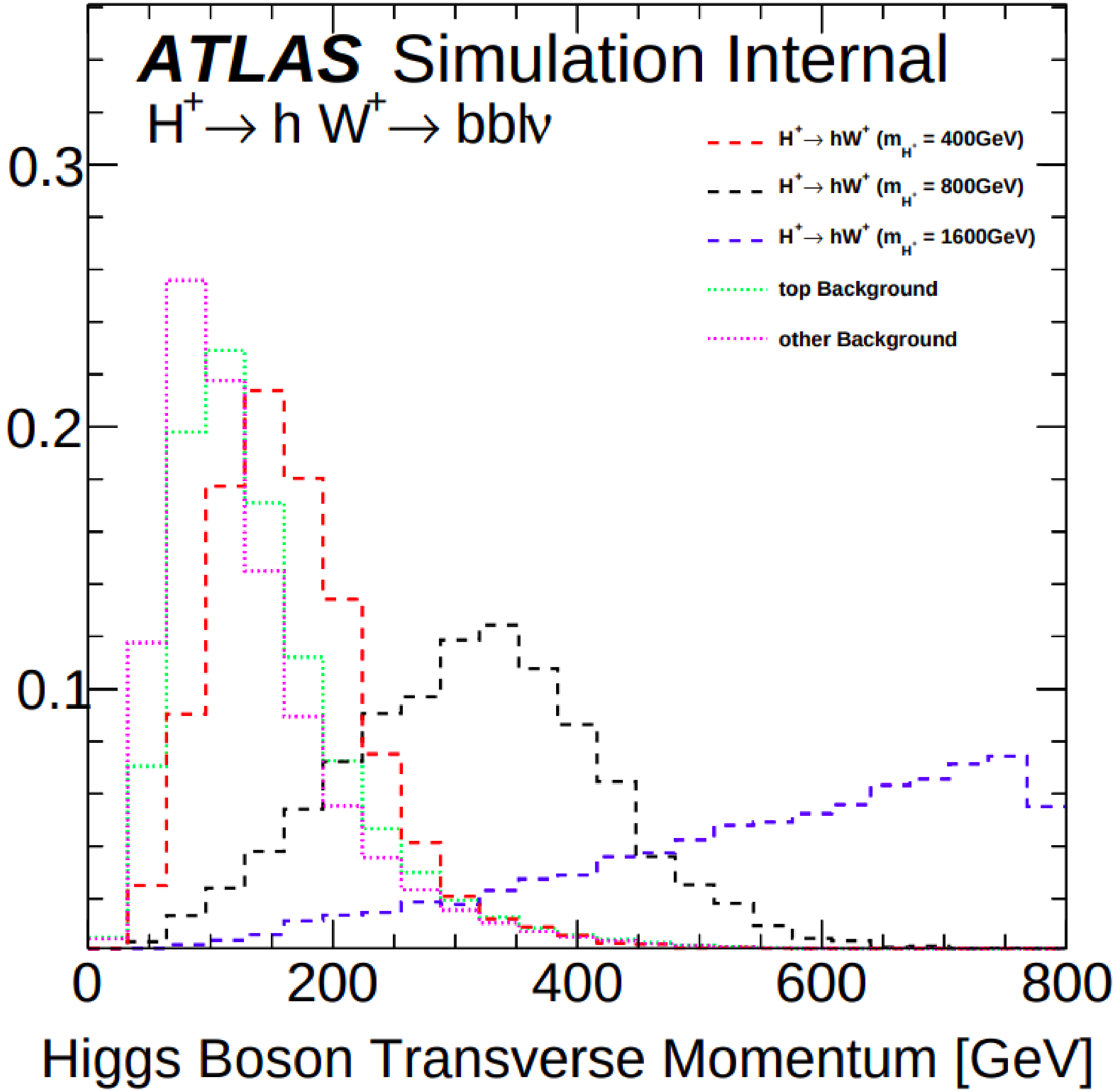
Training Results

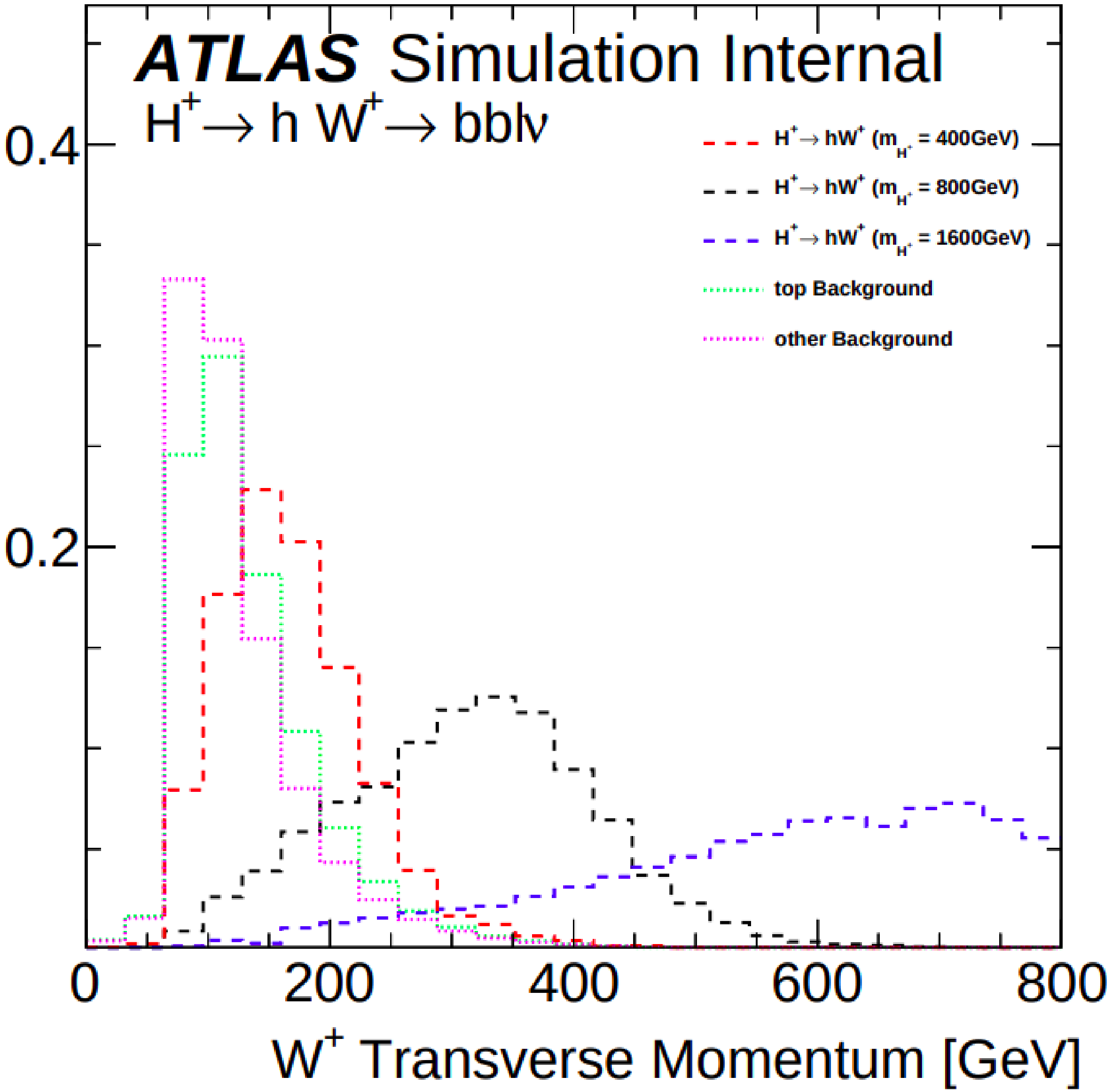


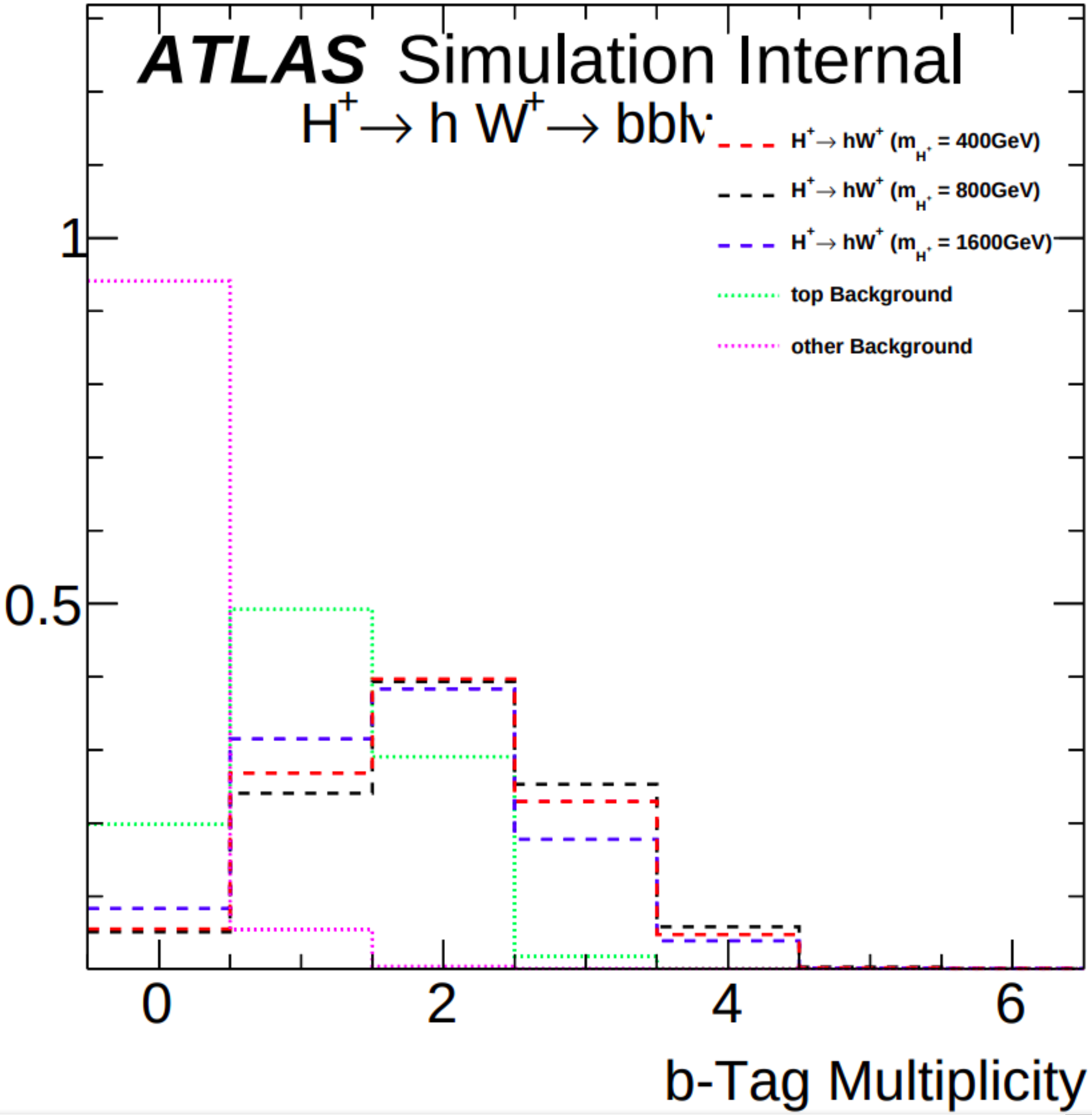
Variable name	Variable importance
$m_{j_{h1}j_{h2}}$	0.216
$p_{\mathrm{T}}^{h}/m_{Wh}$	0.200
$\Delta\Phi(h,W)$	0.183
$p_{\mathrm{T}}^{W}/m_{Wh}$	0.175
w_{MV1}^{h1}	0.123
w_{MV1}^{h2}	0.105

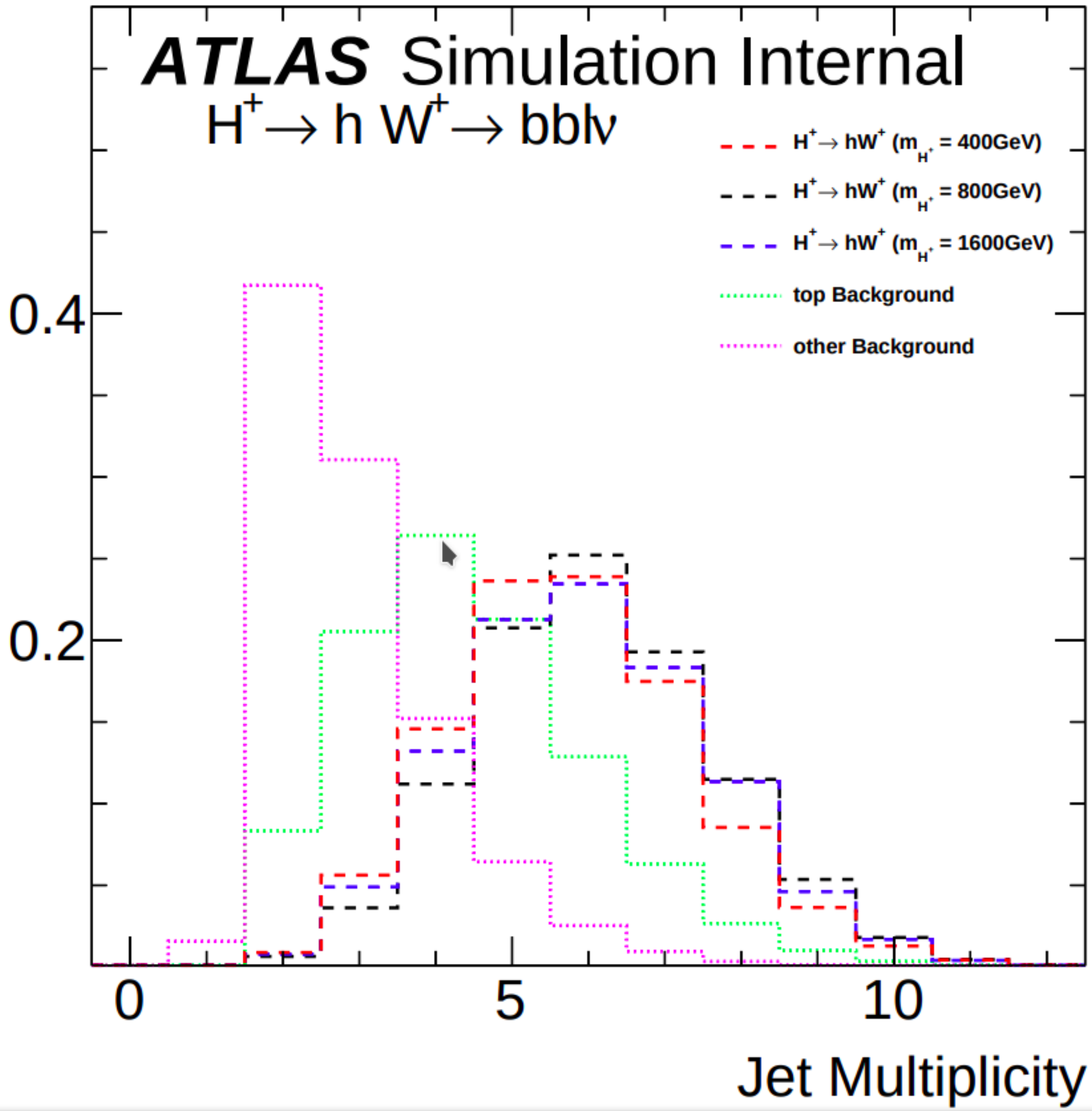
Event Selection

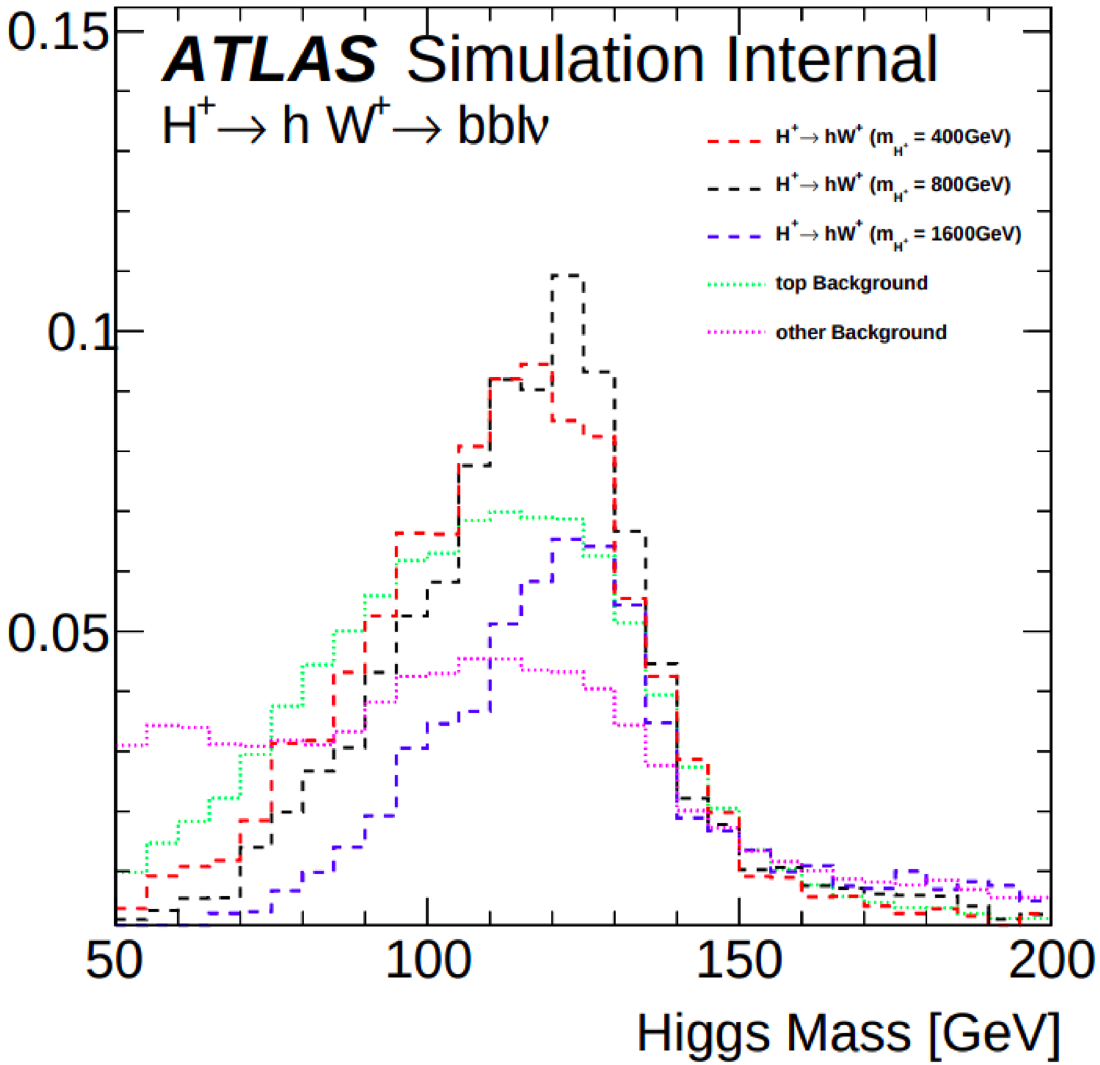
- Events have been classified by the number of b tagged Jets in the Categories
  - Two Tags
  - Three Tags
  - Four and more Tags
- To Enhance the Signal a way to distinguish between Signal and Background is needed
- Compare Shapes of different observables to find good candidates where the distribution for Signal and Background differ
- The following Observable were tested:
  - Missing Transverse Energy, Higgs/W Transvers Momentum,Higgs Mass, #Jets,
  BDT Response, Lepton Transverse Momentum, Missing Transvere Energy Significance, Delta Phi Higgs W, # b-Tags, b-Tag Category


ATLAS Simulation Internal
H+→ h W+→ bblν
H+→ hW+ (mH+ = 400GeV)
H+→ hW+ (mH+ = 800GeV)
H+→ hW+ (mH+ = 1600GeV)
top Background
other Background
0.3
0.2
0.1
0
200
400
600
800
Higgs Boson Transverse Momentum [GeV]

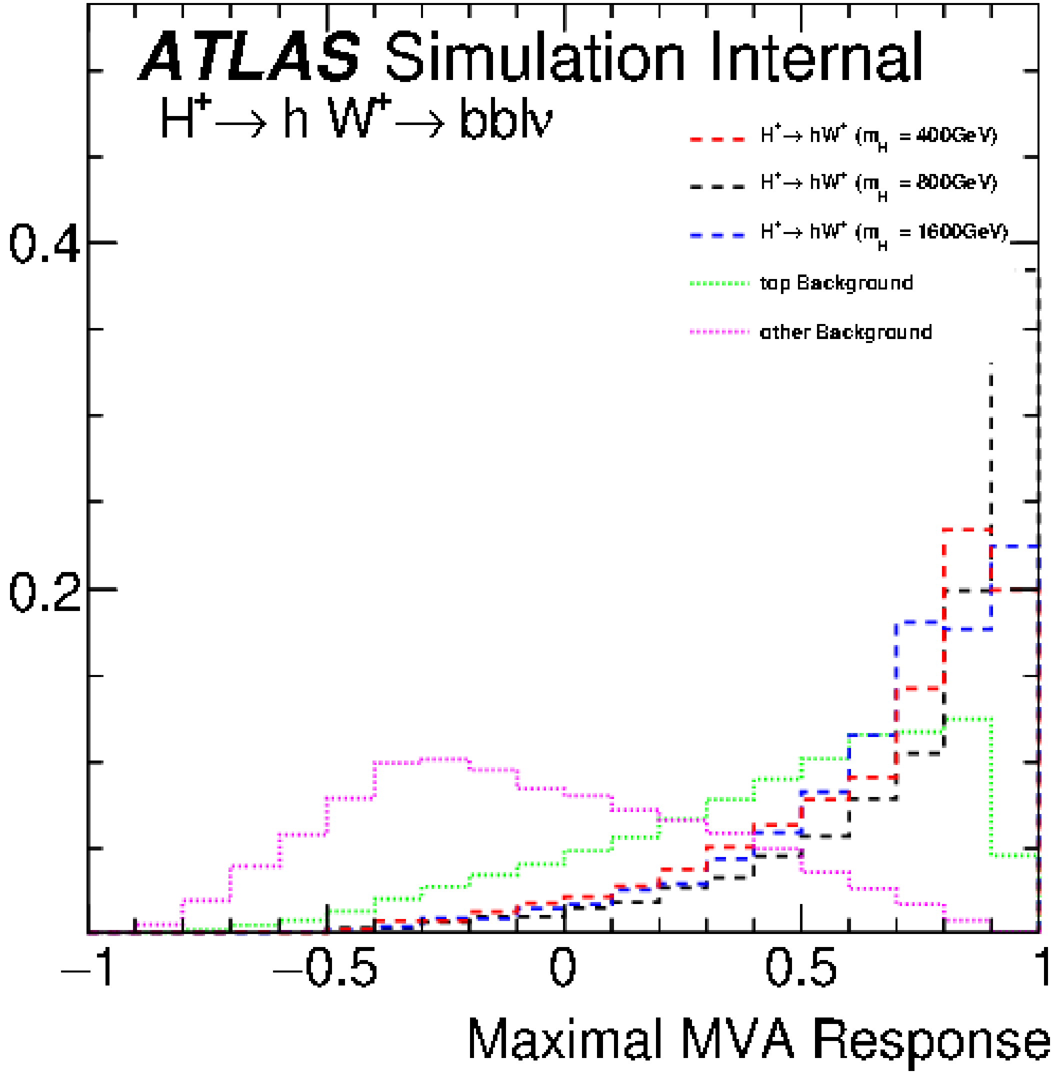

ATLAS Simulation Internal
H+→ h W+→ bblν
H+→ hW+ (mH+ = 400GeV)
H+→ hW+ (mH+ = 800GeV)
H+→ hW+ (mH+ = 1600GeV)
top Background
other Background
0.4
0.2
0
200
400
600
800
W+ Transverse Momentum [GeV]


ATLAS Simulation Internal
$H^+ \to h W^+ \to bbl\nu$
$H^+ \to hW^+$ (m_{H^+} = 400GeV)
$H^+ \to hW^+$ (m_{H^+} = 800GeV)
$H^+ \to hW^+$ (m_{H^+} = 1600GeV)
top Background
other Background
1
0.5
0
2
4
6
b-Tag Multiplicity

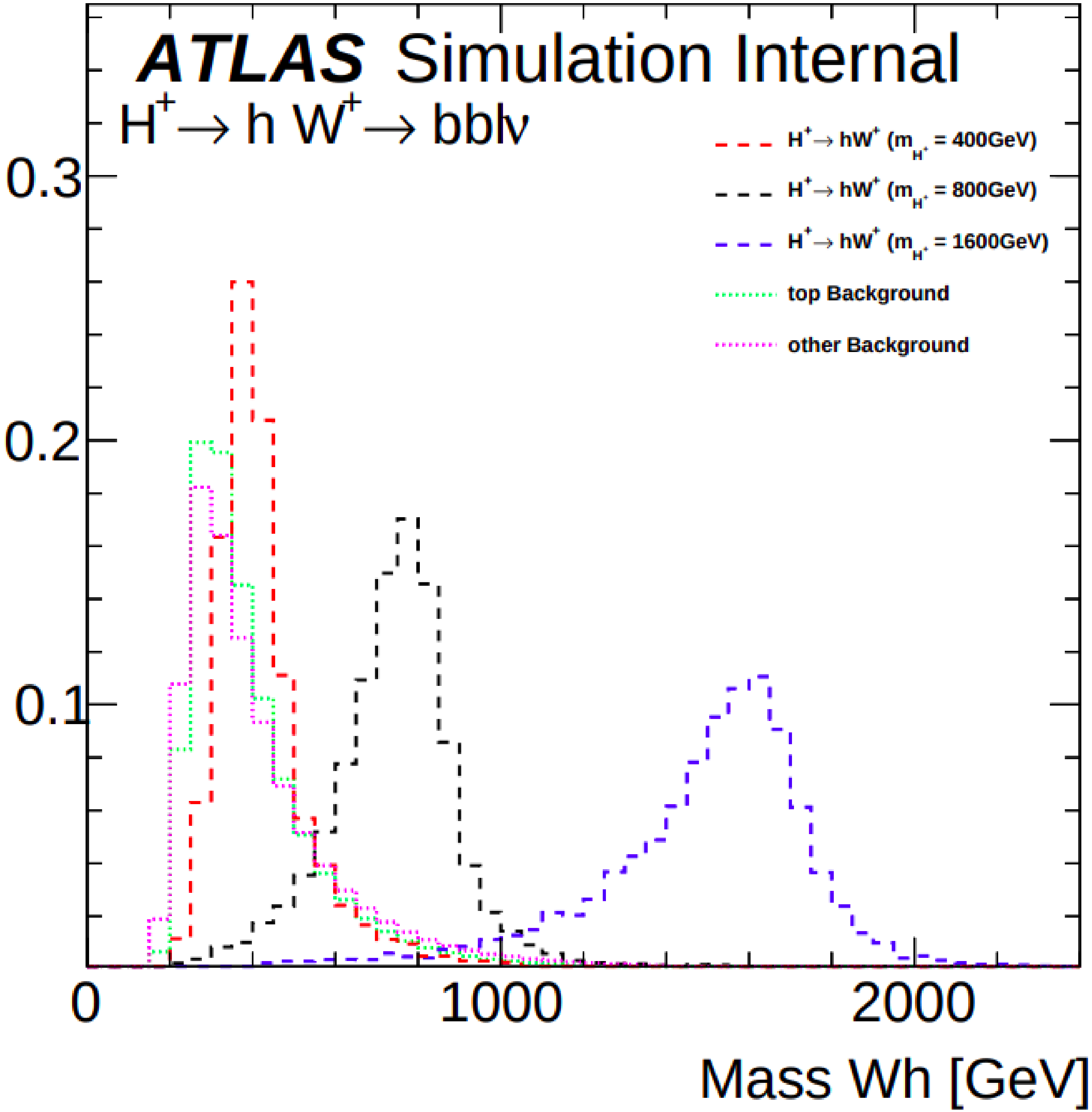

ATLAS Simulation Internal
$H^+ \to h W^+ \to bbl\nu$
$H^+ \to hW^+$ (m_{H^+} = 400GeV)
$H^+ \to hW^+$ (m_{H^+} = 800GeV)
$H^+ \to hW^+$ (m_{H^+} = 1600GeV)
top Background
other Background
0.4
0.2
0
5
10
Jet Multiplicity



ATLAS Simulation Internal
H+→ h W+→ bblν
H+→ hW+ (mH+ = 400GeV)
H+→ hW+ (mH+ = 800GeV)
H+→ hW+ (mH+ = 1600GeV)
top Background
other Background
0.15
0.1
0.05
50
100
150
200
Higgs Mass [GeV]



ATLAS Simulation Internal
H+→ h W+→ bblν
H+→ hW+ (mH = 400GeV)
H+→ hW+ (mH = 800GeV)
H+→ hW+ (mH = 1600GeV)
top Background
other Background
0.4
0.2
−1
−0.5
0
0.5
1
Maximal MVA Response


ATLAS Simulation Internal
H+→ h W+→ bblν
H+→ hW+ (mH+ = 400GeV)
H+→ hW+ (mH+ = 800GeV)
H+→ hW+ (mH+ = 1600GeV)
top Background
other Background
0.3
0.2
0.1
0
1000
2000
Mass Wh [GeV]

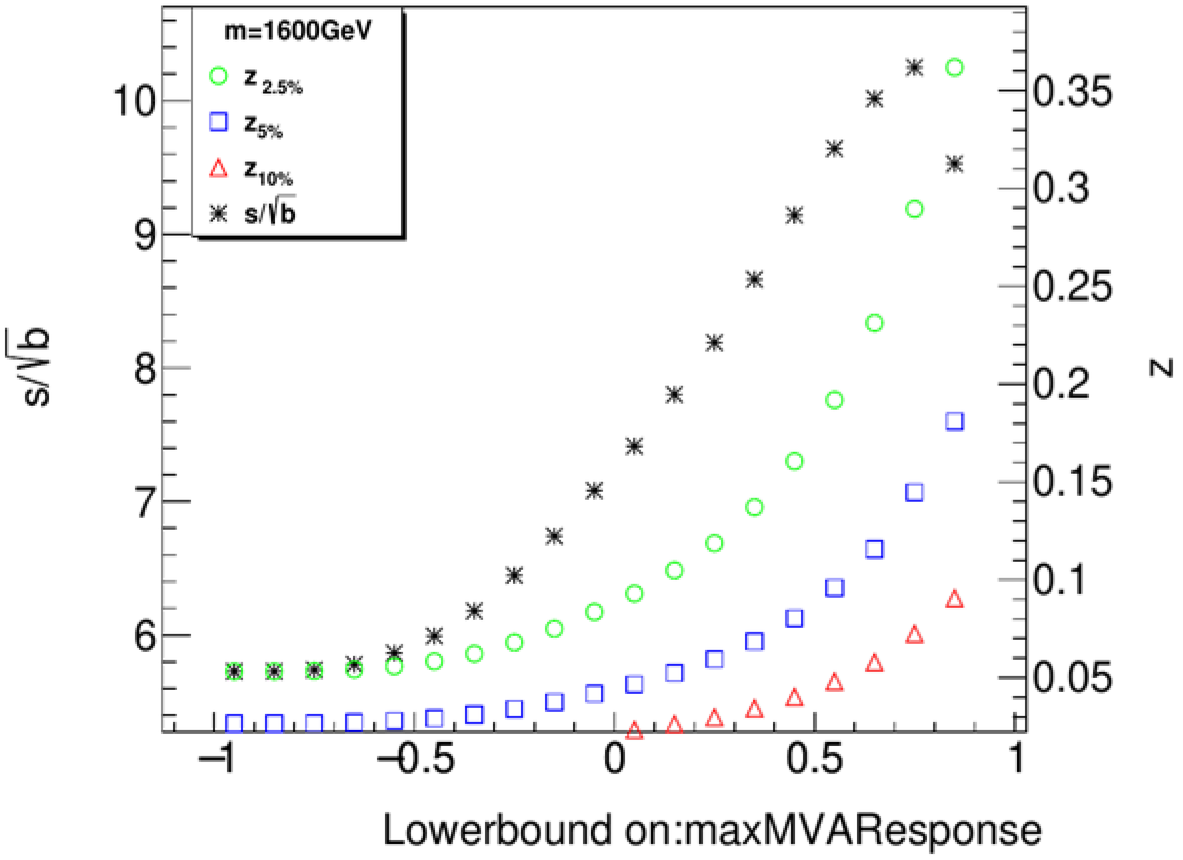
Event Selection Optimization

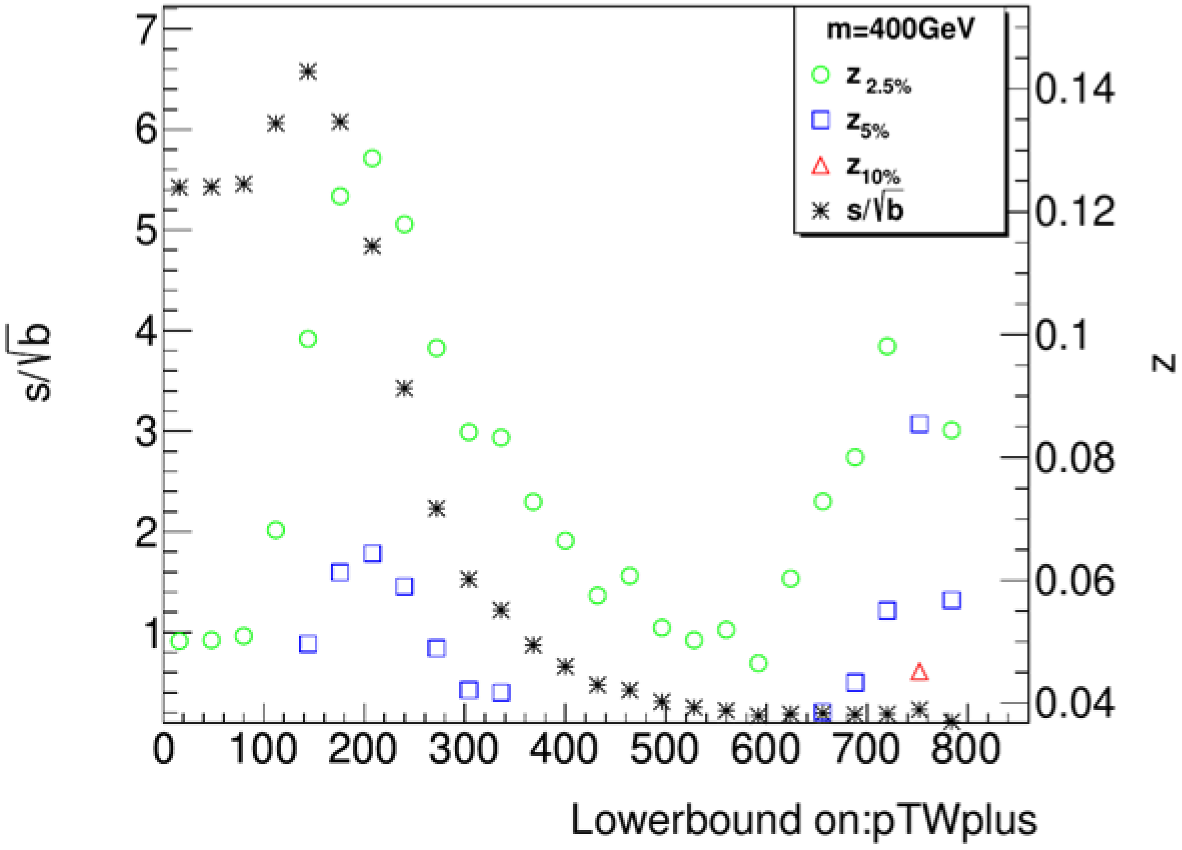
- To find the exact Regions where the Signal is enhanced relative to the Background calculate:

$$\frac{\#s}{\sqrt{\#b}}$$

$$z = \sqrt{2 \cdot (n \log \frac{n(b+\sigma^2)}{b^2 + n\sigma^2}) - \frac{b^2}{\sigma^2} \log(1 + \frac{\sigma^2 + (n-b)}{b \cdot (b+\sigma^2)})}$$

  - For all intervals between max(Observable) and lower Bound(Observable)
  - And min(Observable) to upperBound(Observable)



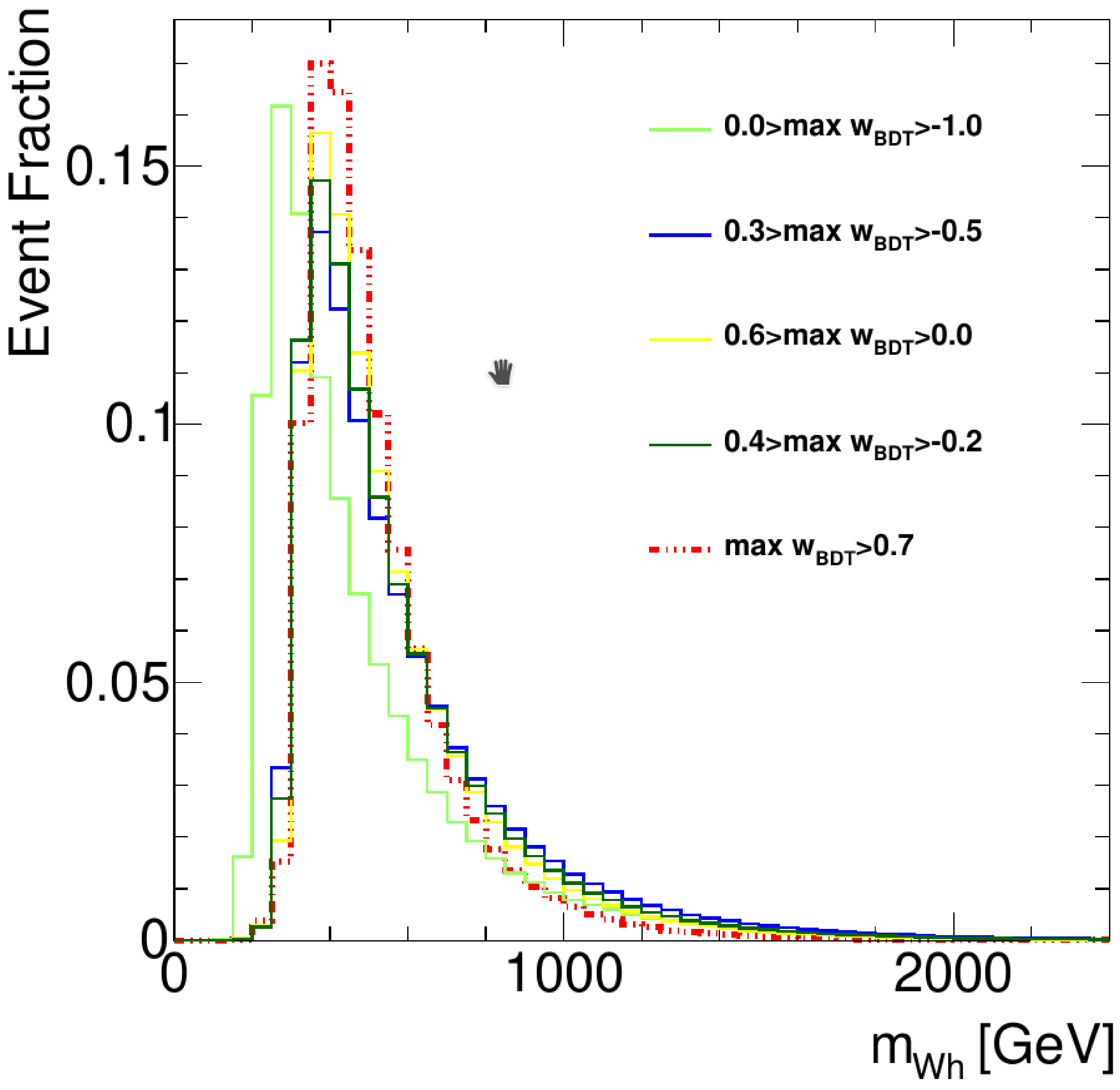


MaxMVA> 0.7

pT W> 120

The followig cuts were chosen to define the Signal Region

- pT_{Higgs}>100
- pT_{W}>120
- # Jets>=5
- BDT Response>0.7

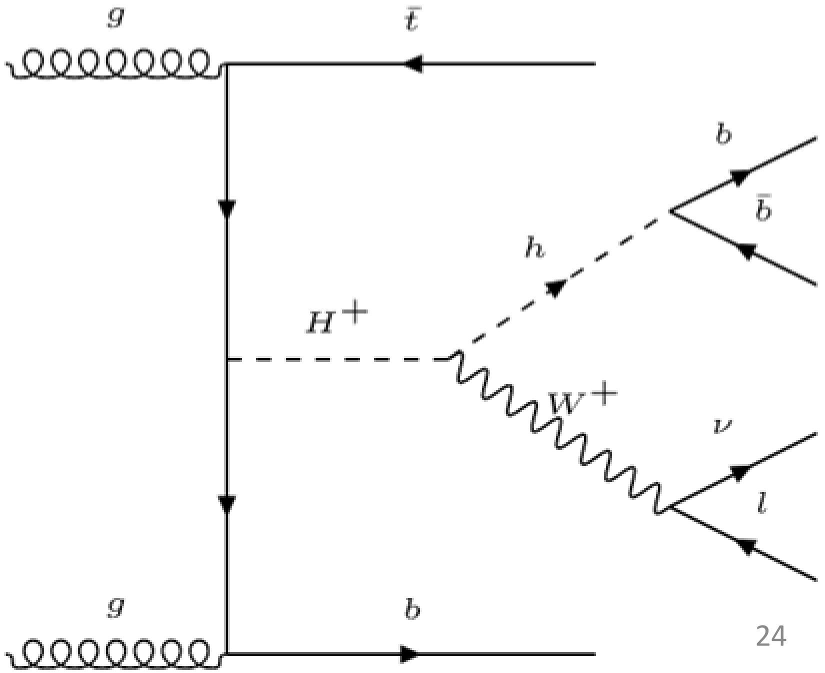


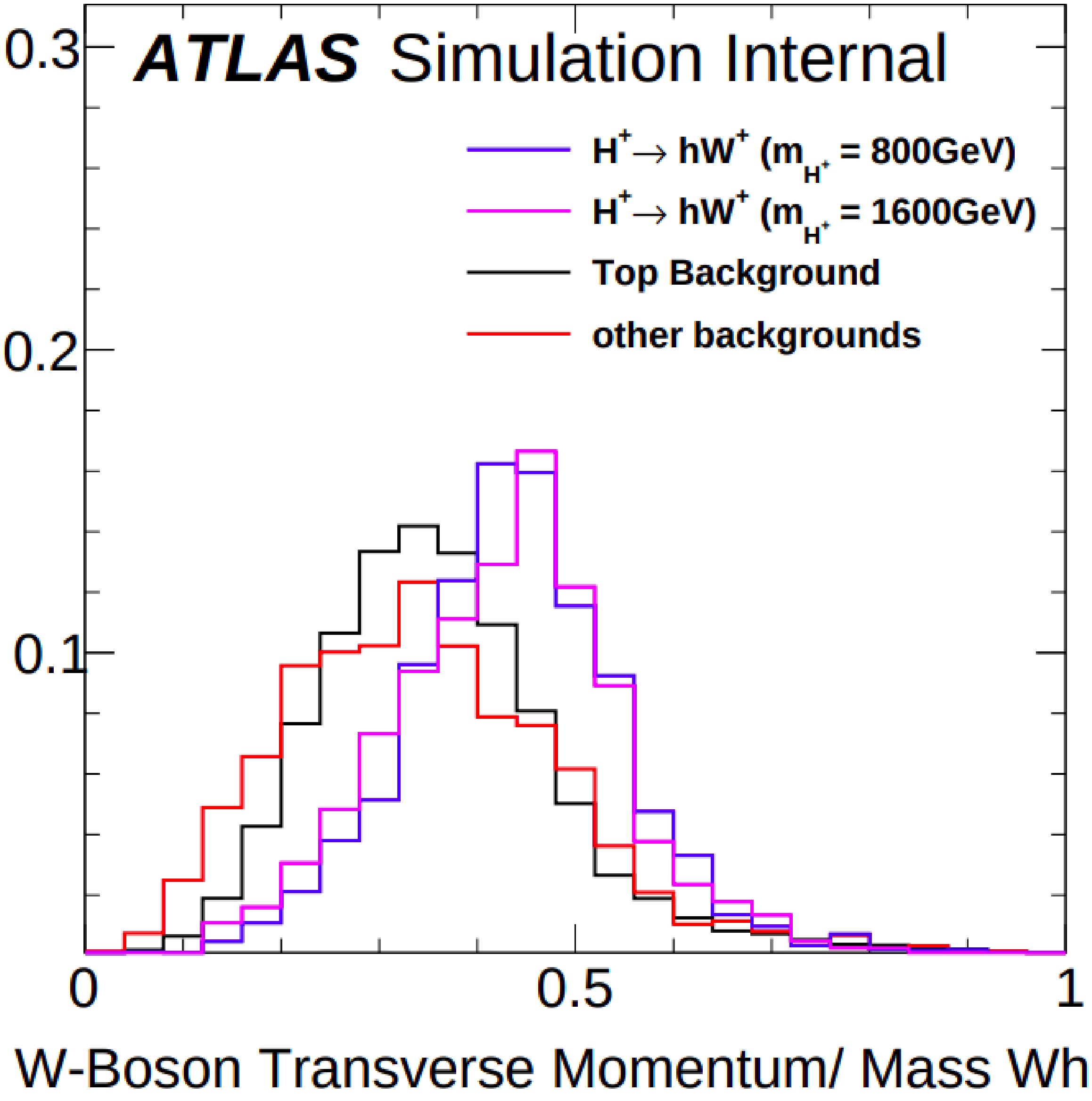
- The Region with BDT Response > 0.7 is defined as the signal Region
- Find a Region where m_{hW} distribution of the background is similar wrt. m_{hW} distribution in the SR
- For most Regions in BDT Response this is the case
- Choose the one where the the background is most enhanced over the signal

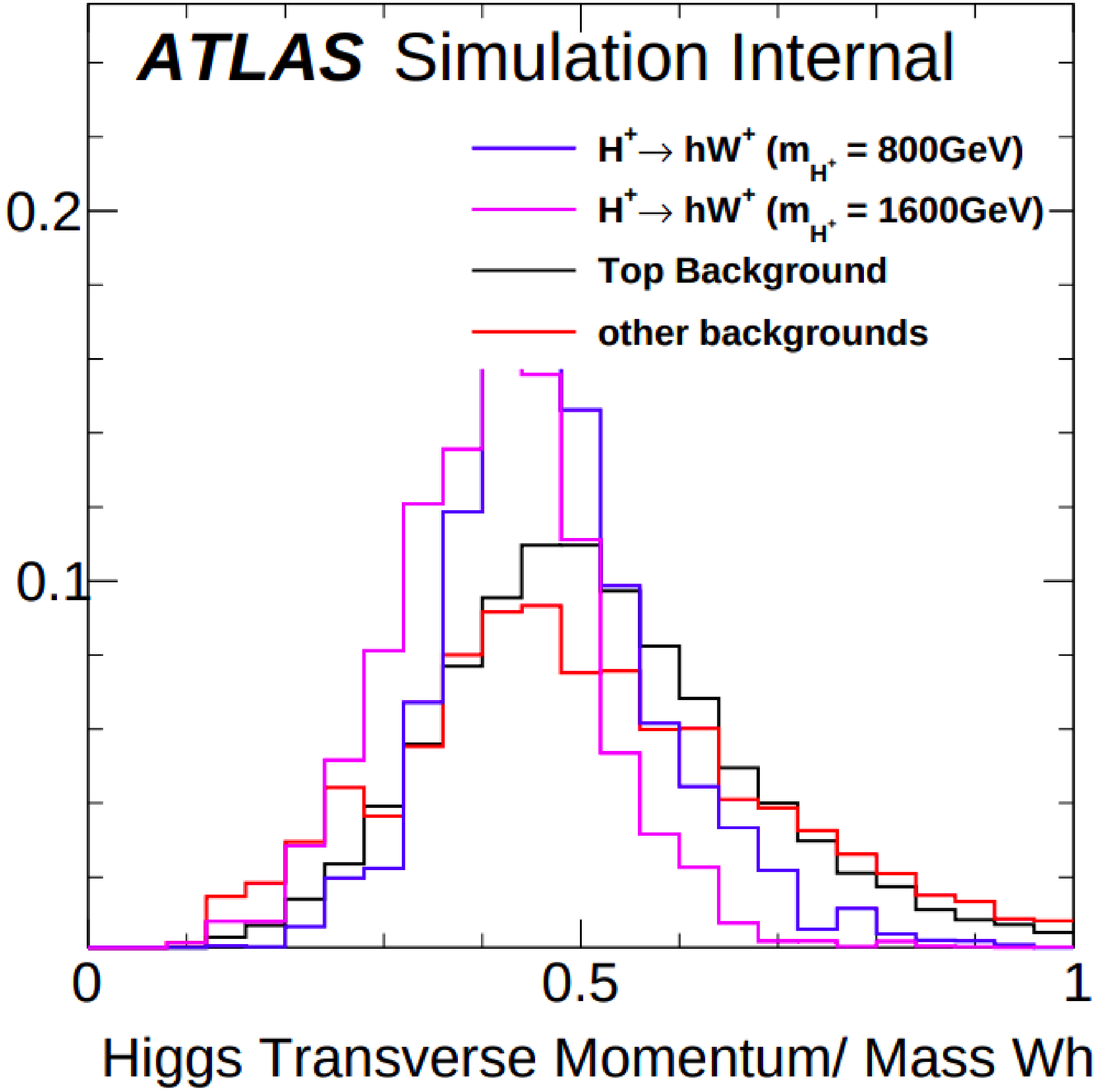
→ -0.5< max BDT<0.3

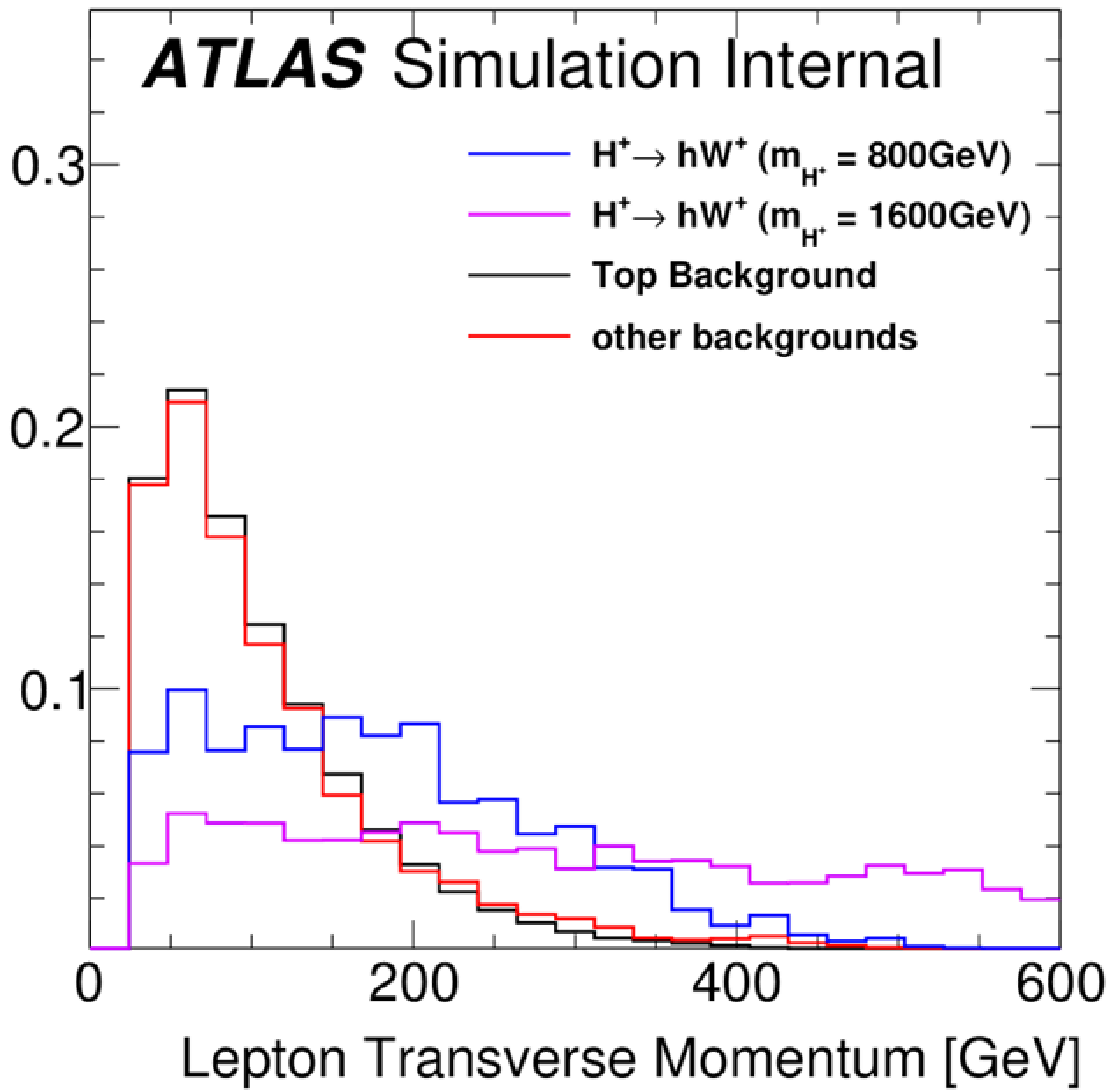
Merged Jet event selection

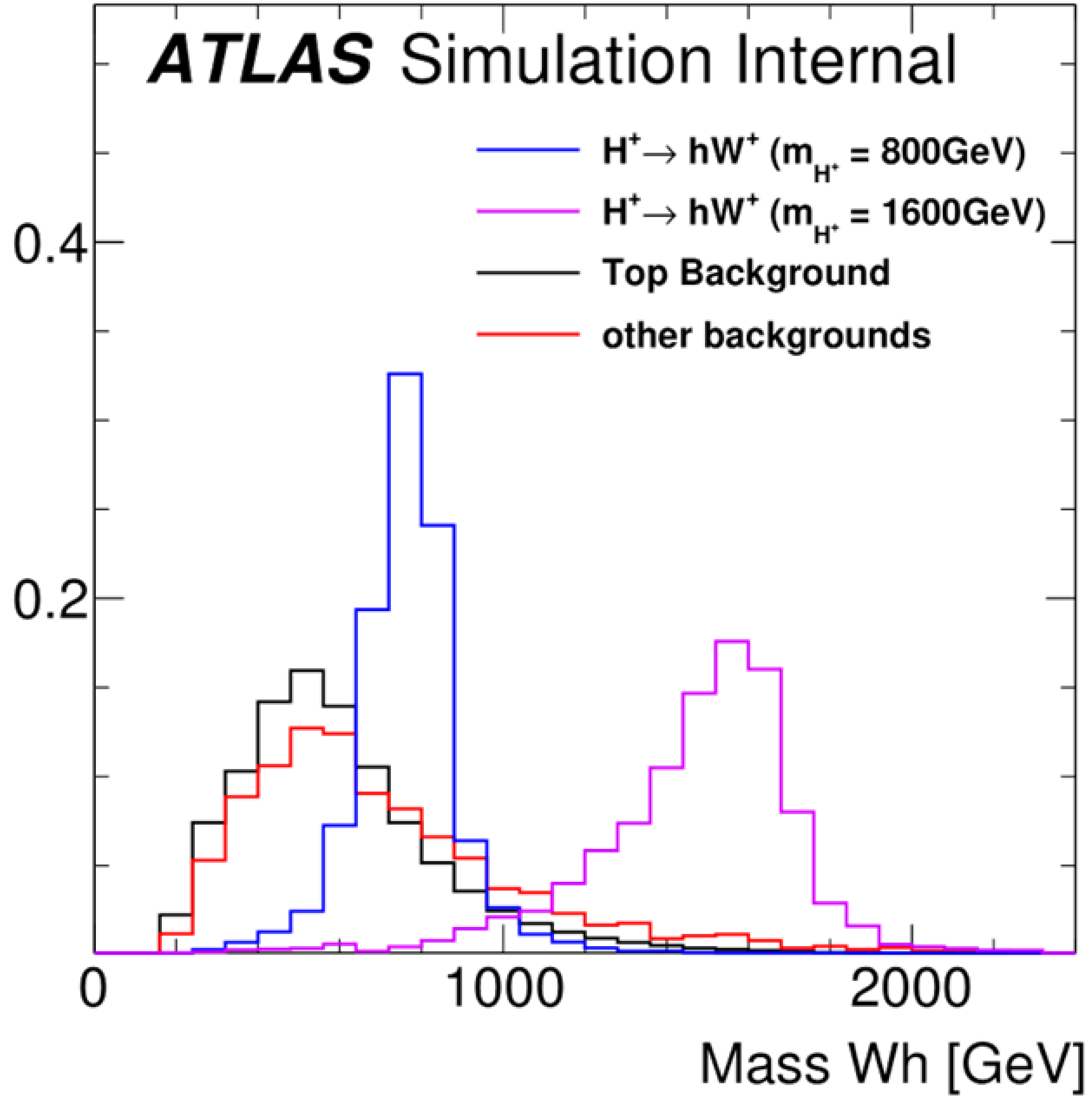
- Especially for high Charged Higgs masses the jets can be merged into one
- If a fat jet is present choose the one with highest pT as (SM) Higgs
  - If Jet mass is compatible with Higgs mass (90-140) GeV -> SR
  - Else CR
- Reconstruct W from Lepton + MET
- Based on Studies of Patrick Bongratz




ATLAS Simulation Internal
H⁺→ hW⁺ (m_H⁺ = 800GeV)
H⁺→ hW⁺ (m_H⁺ = 1600GeV)
Top Background
other backgrounds
0.3
0.2
0.1
0
0.5
1
W-Boson Transverse Momentum/ Mass Wh


ATLAS Simulation Internal
H⁺→ hW⁺ (m_H⁺ = 800GeV)
H⁺→ hW⁺ (m_H⁺ = 1600GeV)
Top Background
other backgrounds
0.2
0.1
0
0.5
1
Higgs Transverse Momentum/ Mass Wh


ATLAS Simulation Internal
$H^+ \rightarrow hW^+$ (m_{H^+} = 800GeV)
$H^+ \rightarrow hW^+$ (m_{H^+} = 1600GeV)
Top Background
other backgrounds
0.1
0.2
0.3
0
200
400
600
Lepton Transverse Momentum [GeV]

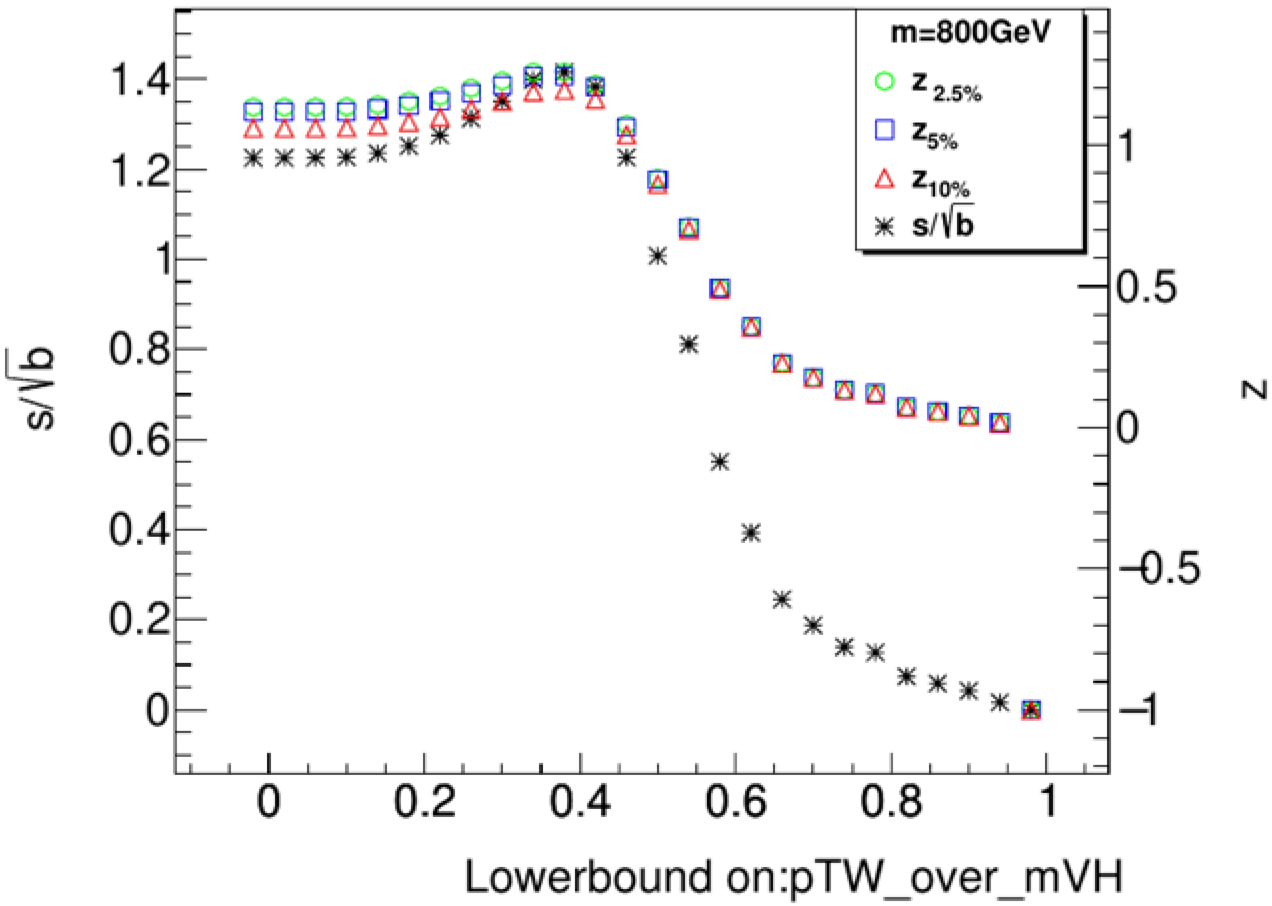

ATLAS Simulation Internal
$H^+ \rightarrow hW^+$ (m_{H^+} = 800GeV)
$H^+ \rightarrow hW^+$ (m_{H^+} = 1600GeV)
Top Background
other backgrounds
0.2
0.4
0
1000
2000
Mass Wh [GeV]

- To find the exact Regions where the Signal is enhanced relative to the Background calculate:

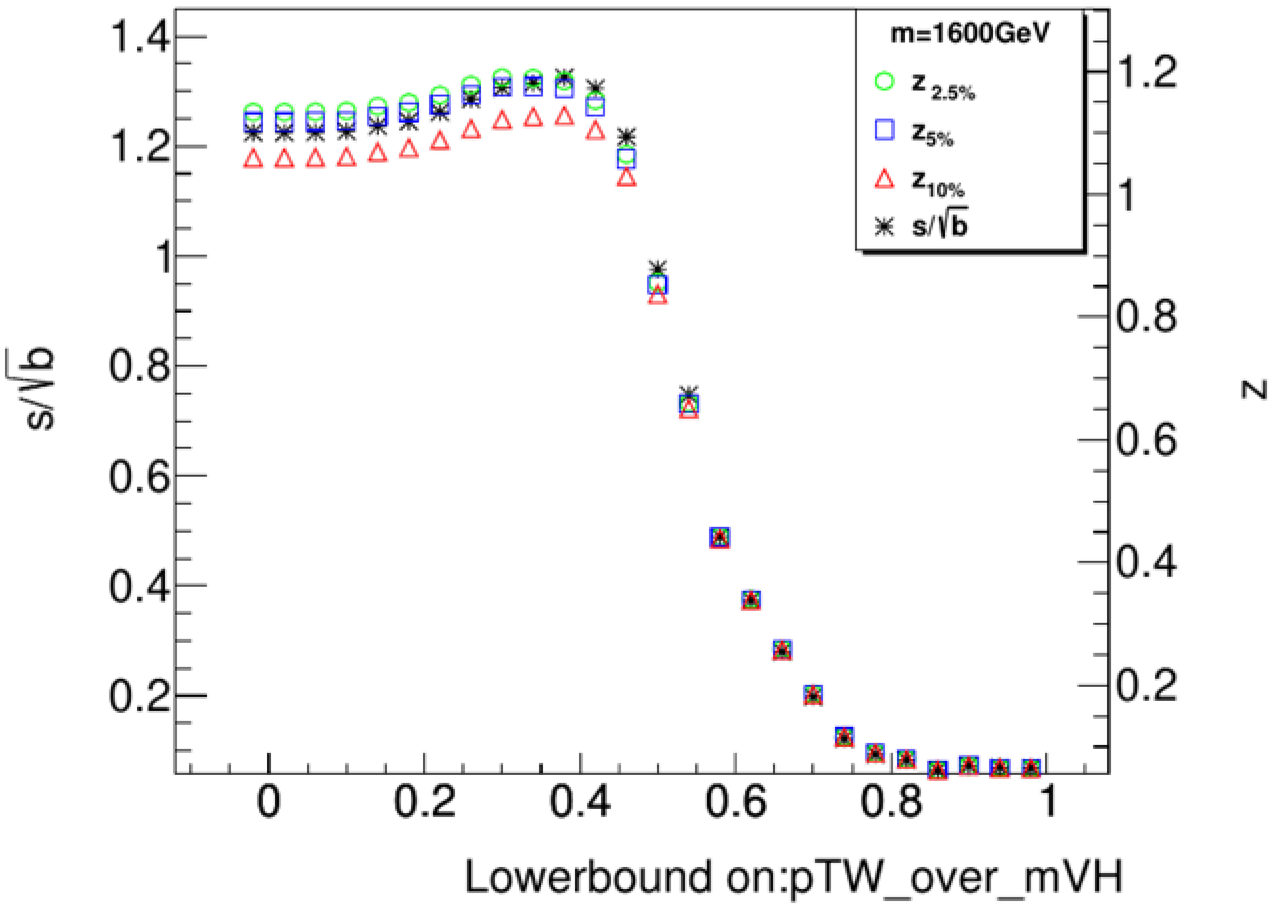
$$\frac{\#s}{\sqrt{\#b}}$$

$$z = \sqrt{2 \cdot (n \log \frac{n(b+\sigma^2)}{b^2 + n\sigma^2}) - \frac{b^2}{\sigma^2} \log(1 + \frac{\sigma^2 + (n-b)}{b \cdot (b+\sigma^2)})}$$

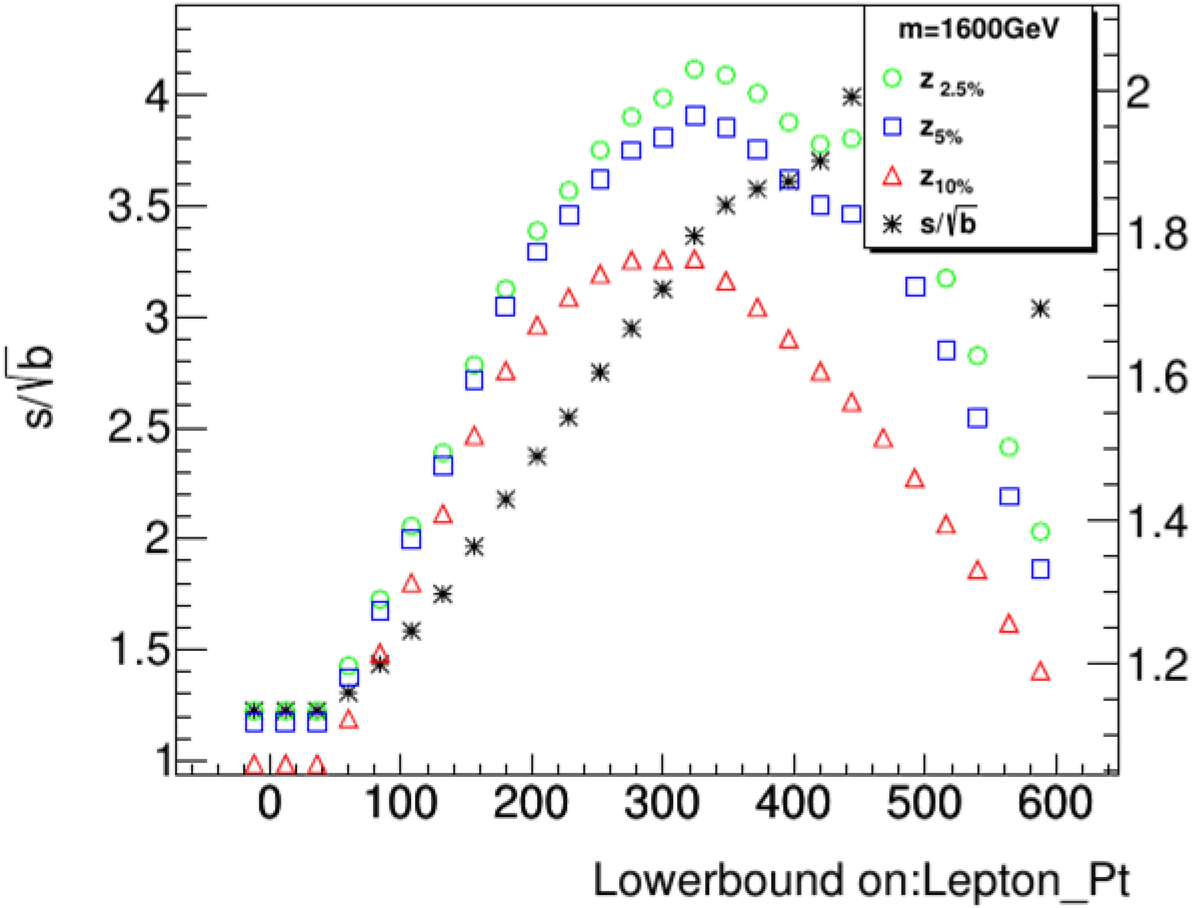
  - For all intervals between max(Observable) and lower Bound(Observable)
  - And min(Observable) to upperBound(Observable)

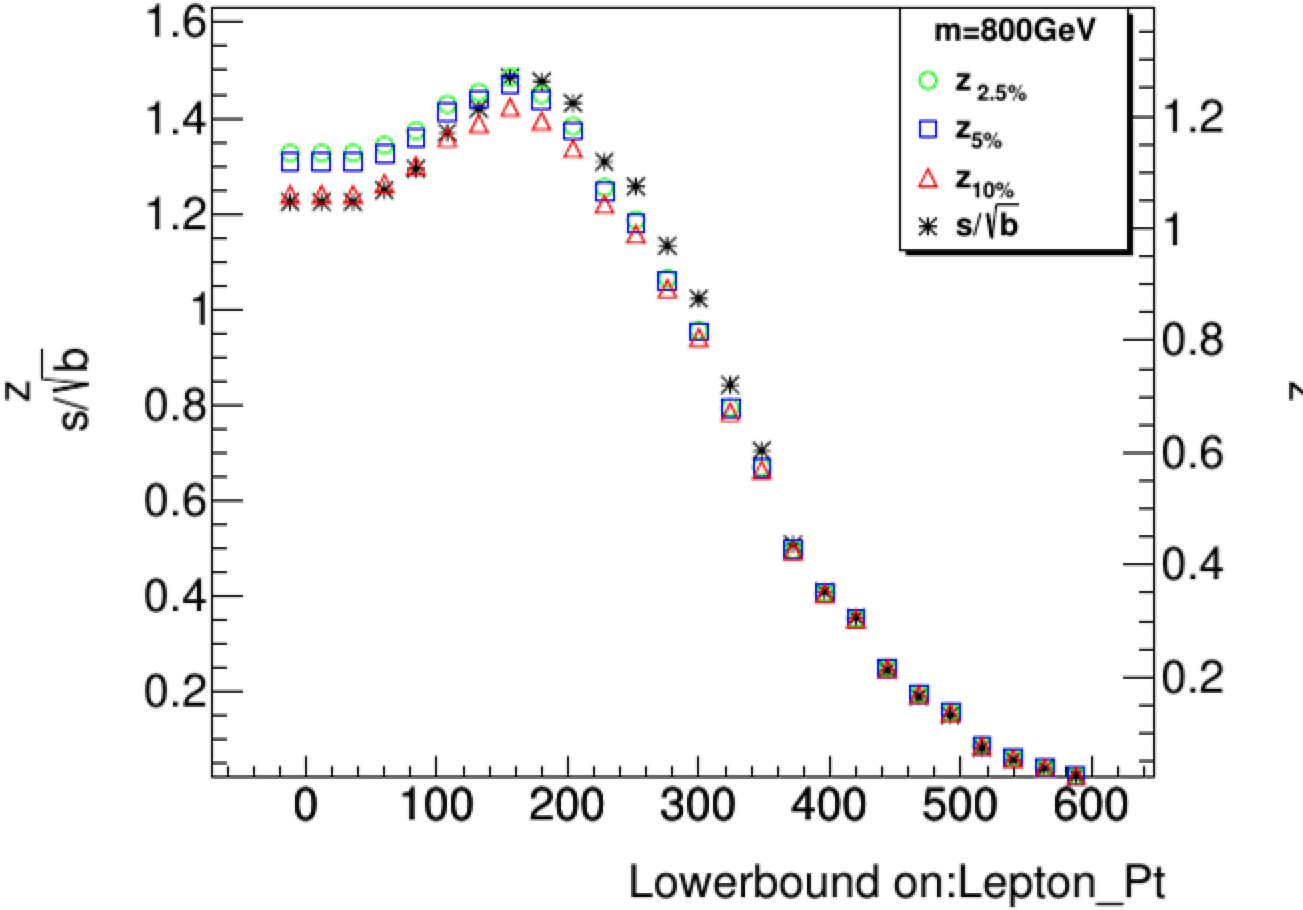


m=800GeV
z 2.5%
z5%
z10%
s/√b
s/√b
z
Lowerbound on:pTW_over_mVH



m=1600GeV
z 2.5%
z5%
z10%
s/√b
s/√b
z
Lowerbound on:pTW_over_mVH





Lepton Pt $\geq$ 150 GeV ?

of Background Events and z:

	#Events(600-1000)GeV no pt cut	#Events(600-1000)GeV pT Lepton> 150 GeV	z_{05}no pt cut	z_{05}pt cut
Two Tags	4682	2618	1.87	3.11
Three Tags	3507	1854	7.73	8.66
Four+ Tags	858	533	8.60	7.40

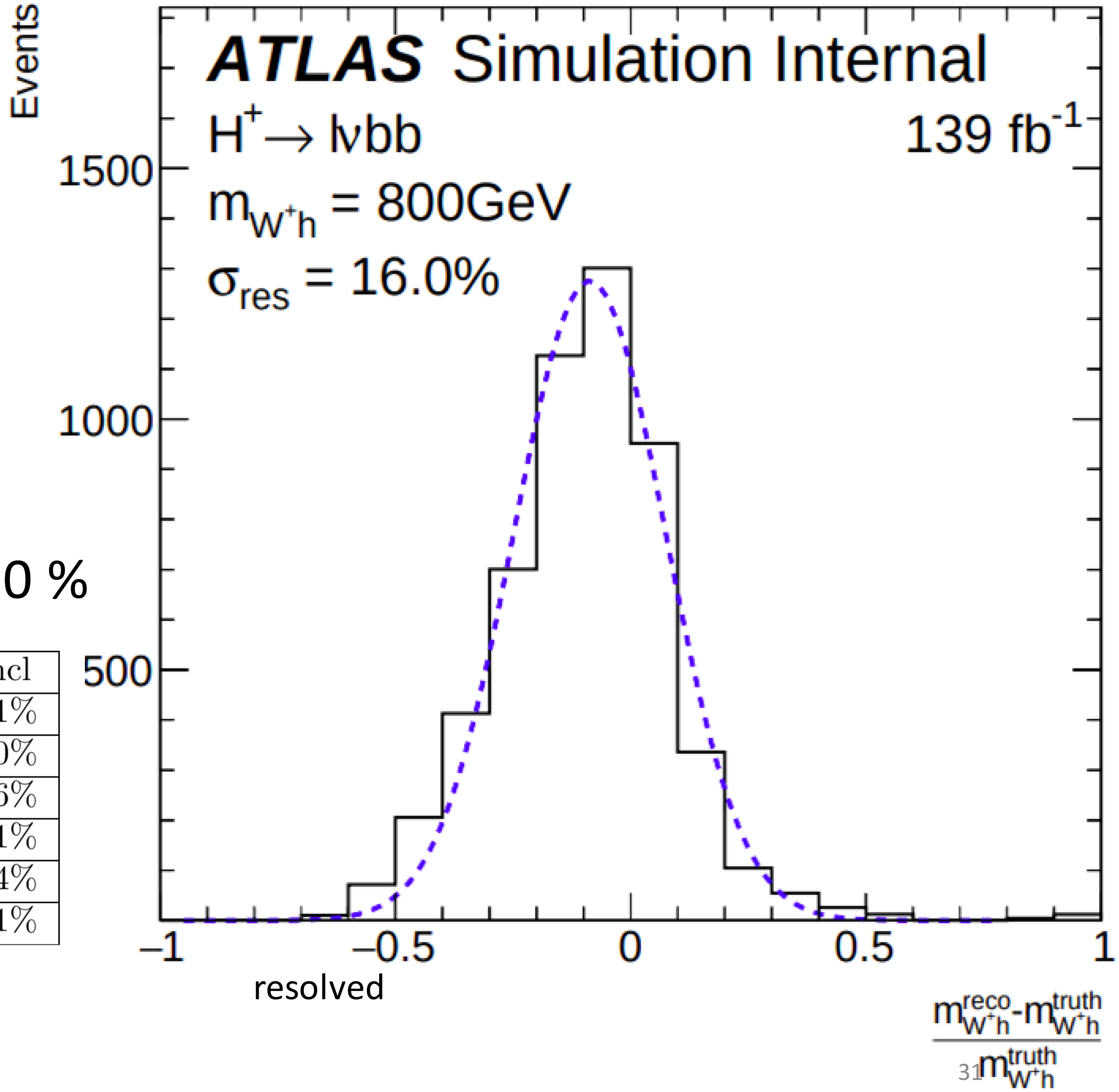
	#Events(1000-2000)GeV no pt cut	#Events(1000-2000)GeV pT Lepton> 150 GeV	z_{05}no pt cut	z_{05} pt cut
Two Tags	5263	4135	7.72	8.68
Three Tags	2958	2241	7.95	6.94
Four+ Tags	708	545	5.53	4.9

Significance is reduced in 4 tag Region(„right Region“) → Do not choose cut

Mass Resolution

- Calculate $\frac{m_{W^+h}^{reco} - m_{W^+h}^{truth}}{m_{W^+h}^{truth}}$
- Fit Gaussian to data
- Take Variance
- Mass resolution approx. 10-20 %

	TwoTags	ThreeTags	Four+Tags	Incl
1600 Resolved	10%	11%	15%	11%
1600 Merged	9%	10%	11%	10%
800 Resolved	16%	16%	18%	16%
800 Merged	11%	11%	11%	11%
400 Resolved	14%	14%	16%	14%
400 Merged	35%	31%	20%	31%



resolved

Event Yield Tables

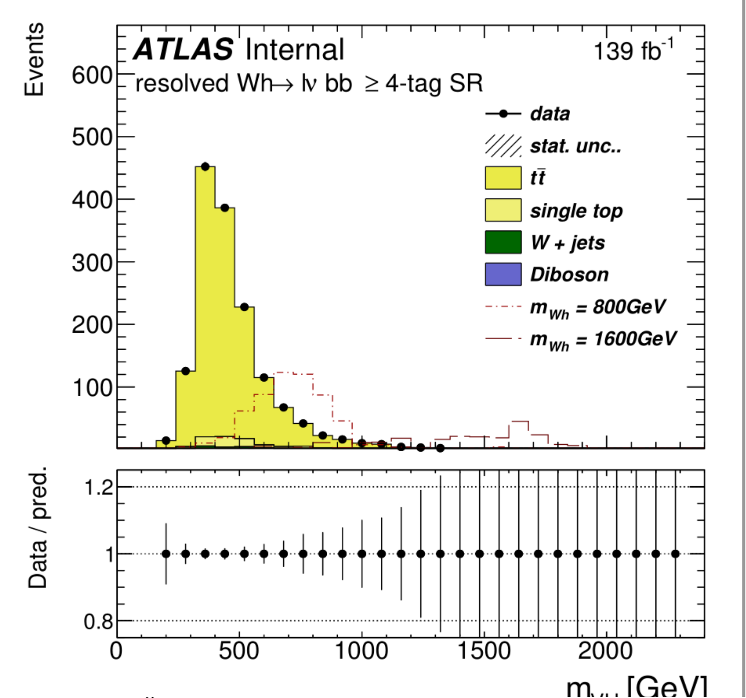
Resolved:

	2-tag SR	3-tag SR	4+ tag SR	2-tag CR	3-tag CR	4+ tag CR
H→hW→lνbb(m=400GeV)	3450±110	2600±100	620±50	520±40	71± 18	6± 4
H→hW→lνbb(m= 800GeV)	2700±100	2076± 91	580± 40	540± 40	118± 19	8± 5
H→hW→lvbb(m=1600GeV)	2660±110	1170 ±70	225±29	120± 30	51 ±14	7± 4
ttbar+singletop	243440±160	26390±50	1478± 13	60040±90	1530± 14	45.0± 2.5
W+jets	6530± 40	399± 7	20.8±1.5	4226±25	76.4±3.1	1.15±0.29
DiBoson	469± 5	30± 1	2.83±0.29	125.0±2.9	4.1± 0.6	0.025± 0.014

Merged:

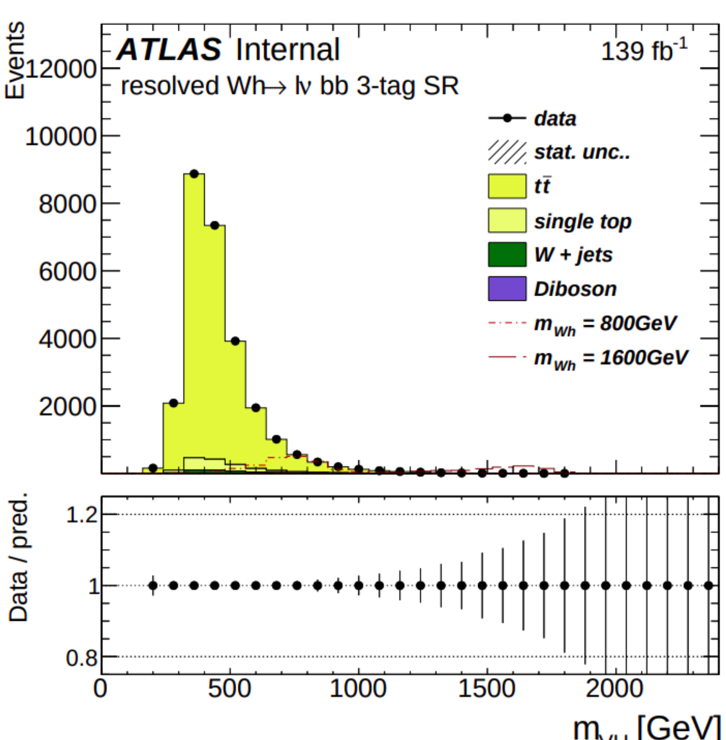
	2-tag SR	3-tag SR	4+ tag SR	2-tag CR	3-tag CR	4+ tag CR
H→hW→lνbb(m=400GeV)	600±50	490±40	165±23	249 ±31	83±18	17.0±6.3
H→hW→lνbb(m=800GeV)	5000±140	3670±120	910±60	1110±70	320±40	24±13
H→hW→lνbb(m=1600GeV)	5570±150	3090±110	740±50	3320±120	1130±70	131±23
ttbar+singletop	21000±100	8950±31	690±9	56840±80	3500±20	171±4
W+jets	3303±22	187±4	13±1	4033±22	170.9±3.8	5.3±0.5
DiBoson	262±4	17±1	1.65±0.24	266±4	13.6±0.9	0.63±0.15

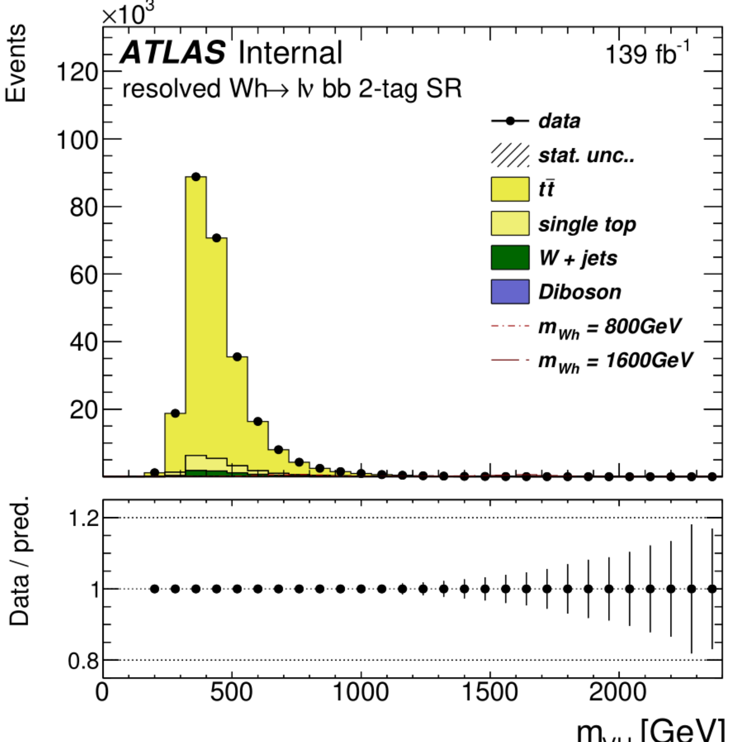
- Background mostly from Tops
- The 3, and 4 Tag Region are the most sensitive
- Merged Topology become important for higher masses

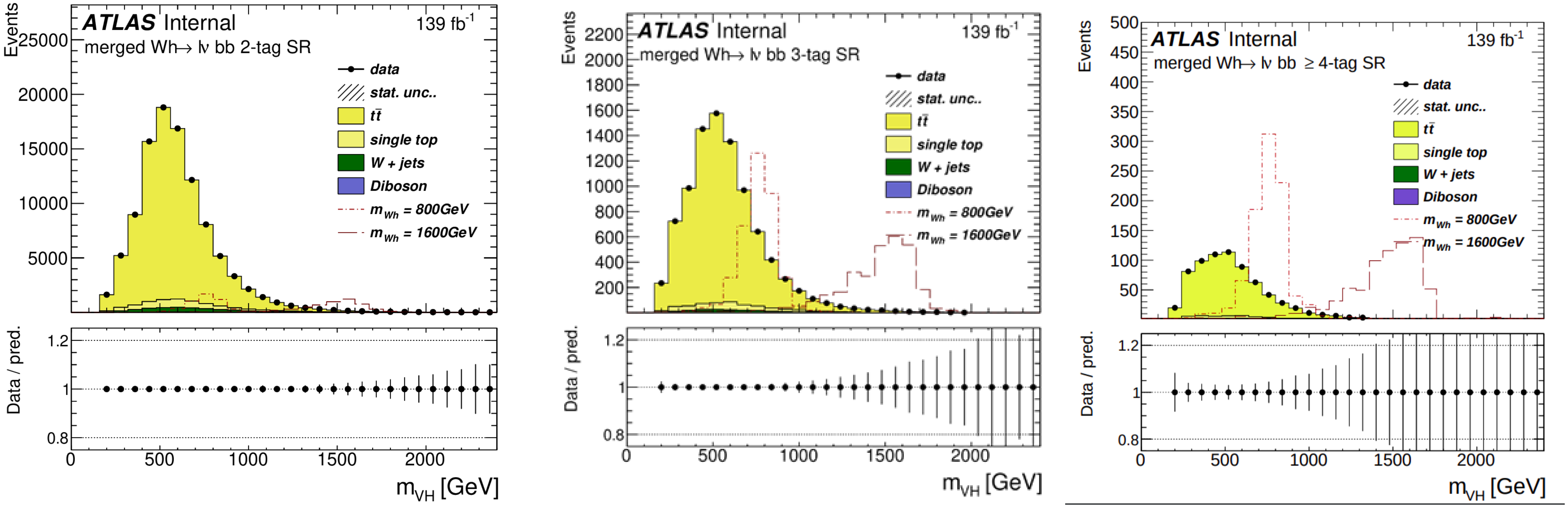


$S/\sqrt{B}$ under the Peak($\pm 2\sigma$)

Resolved	m=400GeV	m=800GeV	m=1600GeV
2Tags	6.75	9.587	69.19
3Tags	16.08	21.60	79.73
4+Tags	15.85	24.07	46.57
merged			
2Tags	1.63	18.81	95.68
3Tags	4.25	48.77	183.20
4+Tags	5.94	45.91	151.19

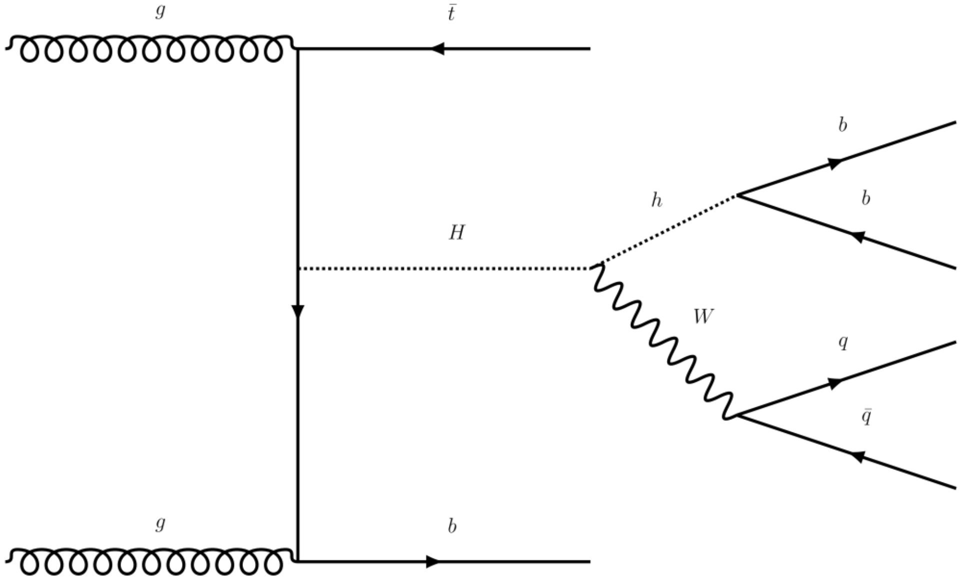






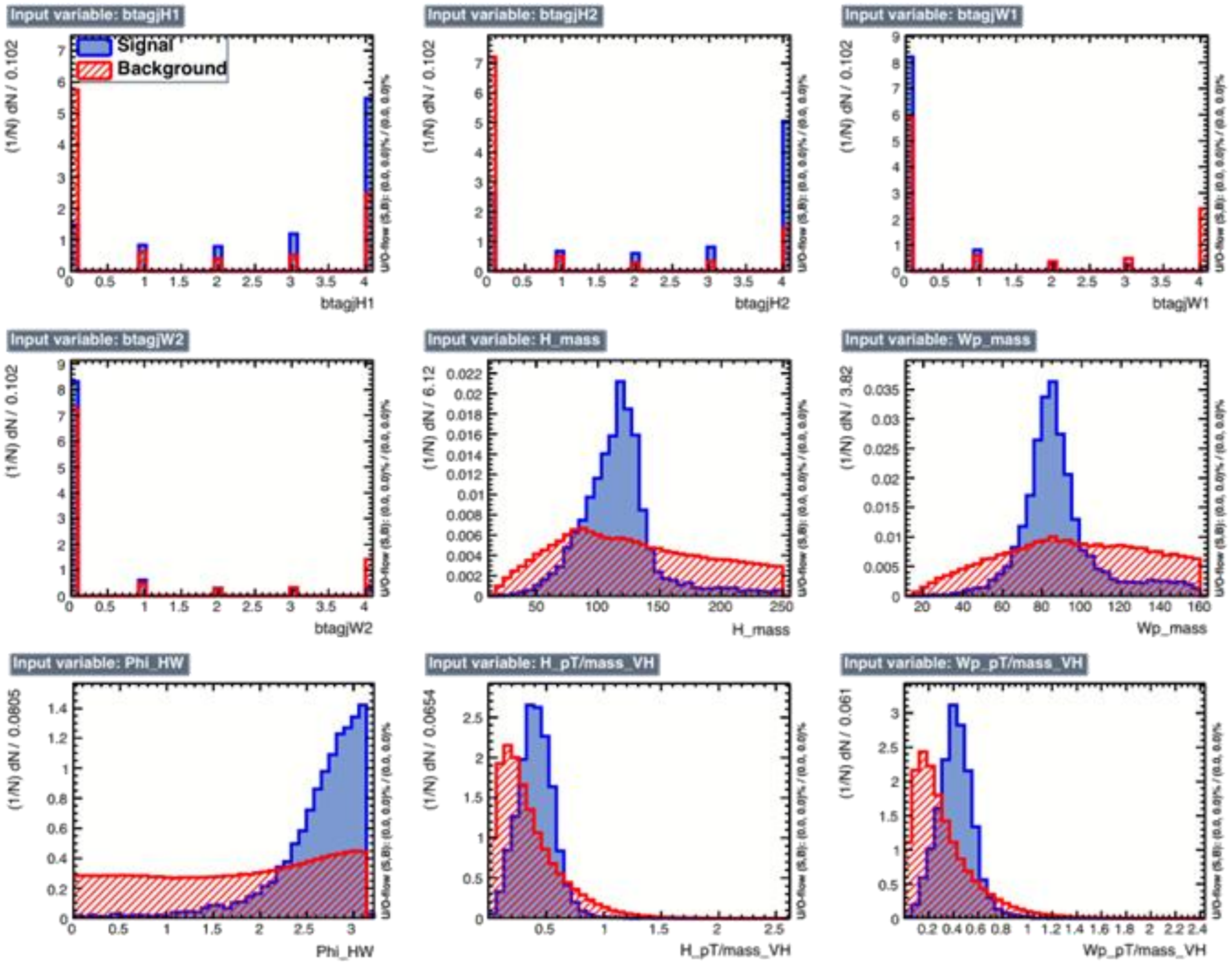
ATLAS Internal
139 fb-1
merged Wh→ lν bb 2-tag SR
data
stat. unc..
tt̄
single top
W + jets
Diboson
m_Wh = 800GeV
m_Wh = 1600GeV
Events
Data / pred.
m_VH [GeV]
ATLAS Internal
139 fb-1
merged Wh→ lν bb 3-tag SR
ATLAS Internal
139 fb-1
merged Wh→ lν bb ≥ 4-tag SR

Hadronic Channel



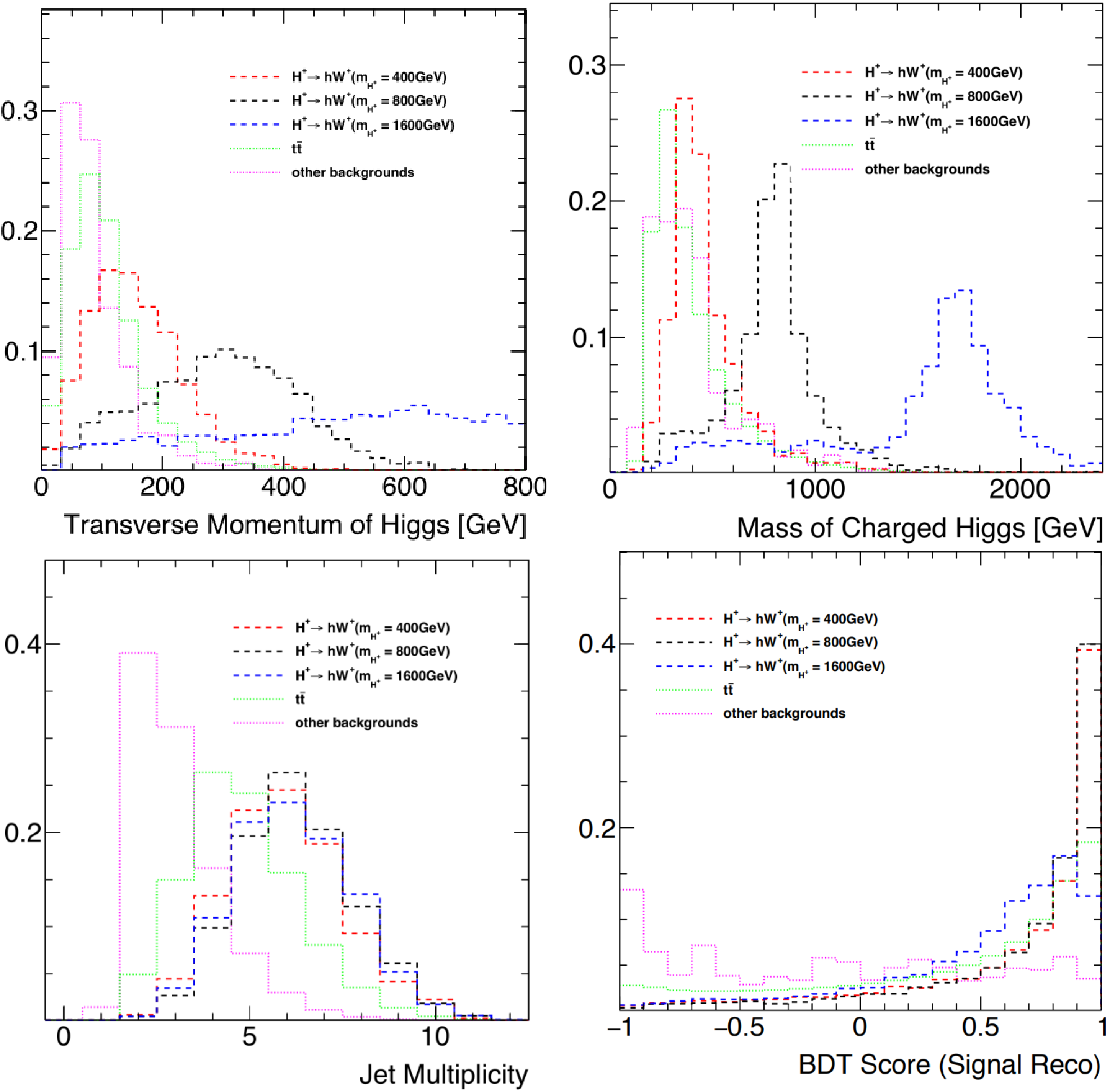
- W decays fully hadronic
- Top quark decays semi-leptonic
- Similar Approach as in Leptonic Channel
  - BDT to reconstruct H+
    - BDT is more complicated Find 4 instead of 2 jets
  - Shape Comparison to Optimize Event Selection
- Currently studied by Shubham Bansal[Uni Bonn]

BDT Hadronic Channel



- **Signal**: : Jet pair and Lepton Neutrino combination correctly dR matched to MC truth h/W
- **Background**: All wrong combinations

1 : H_mass : 1.516e-01
2 : Phi_HW : 1.482e-01
3 : Wp_mass : 1.439e-01
4 : Wp_pT/mass_VH : 1.326e-01
5 : H_pT/mass_VH : 1.297e-01
6 : btagjH1 : 1.033e-01
7 : btagjH2 : 7.514e-02
8 : btagjW1 : 6.635e-02
9 : btagjW2 : 4.927e-02

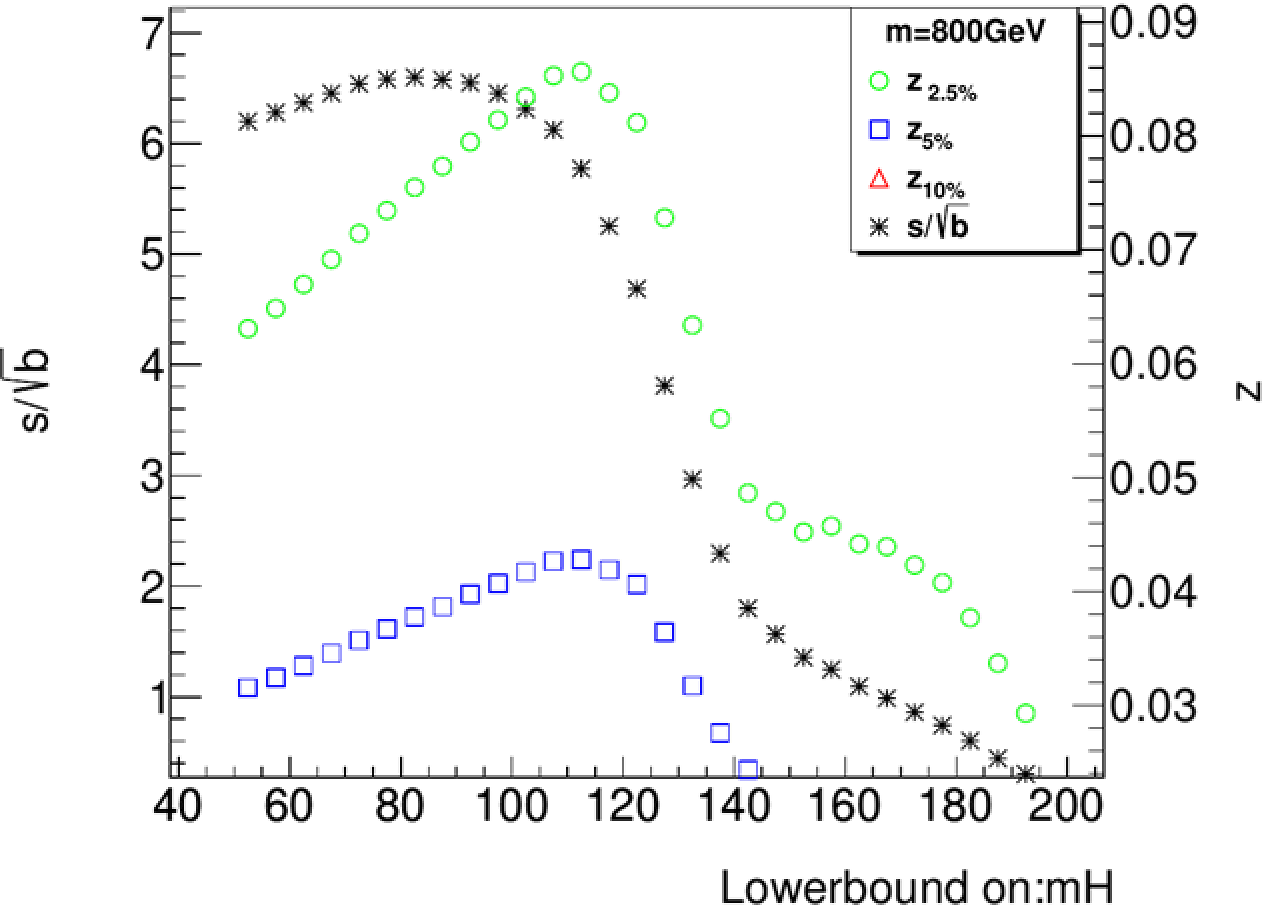


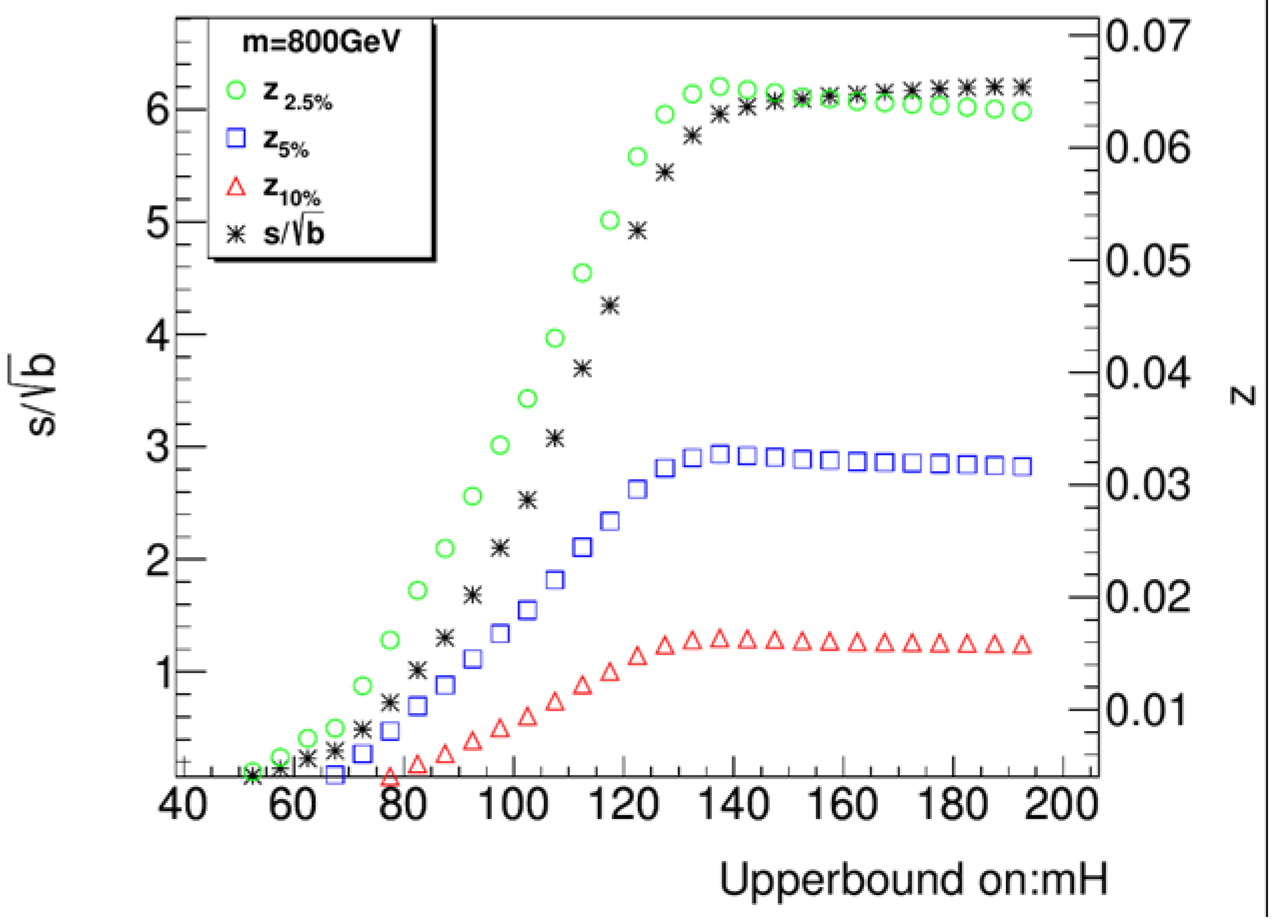
- Signal Region
  - pT_Higgs>140GeV
  - pT_W>100GeV
  - N_jets>6
  - BDT_Score>0.9
- Currently no CR defined
- Still Work in Progress

Summary/Outlook

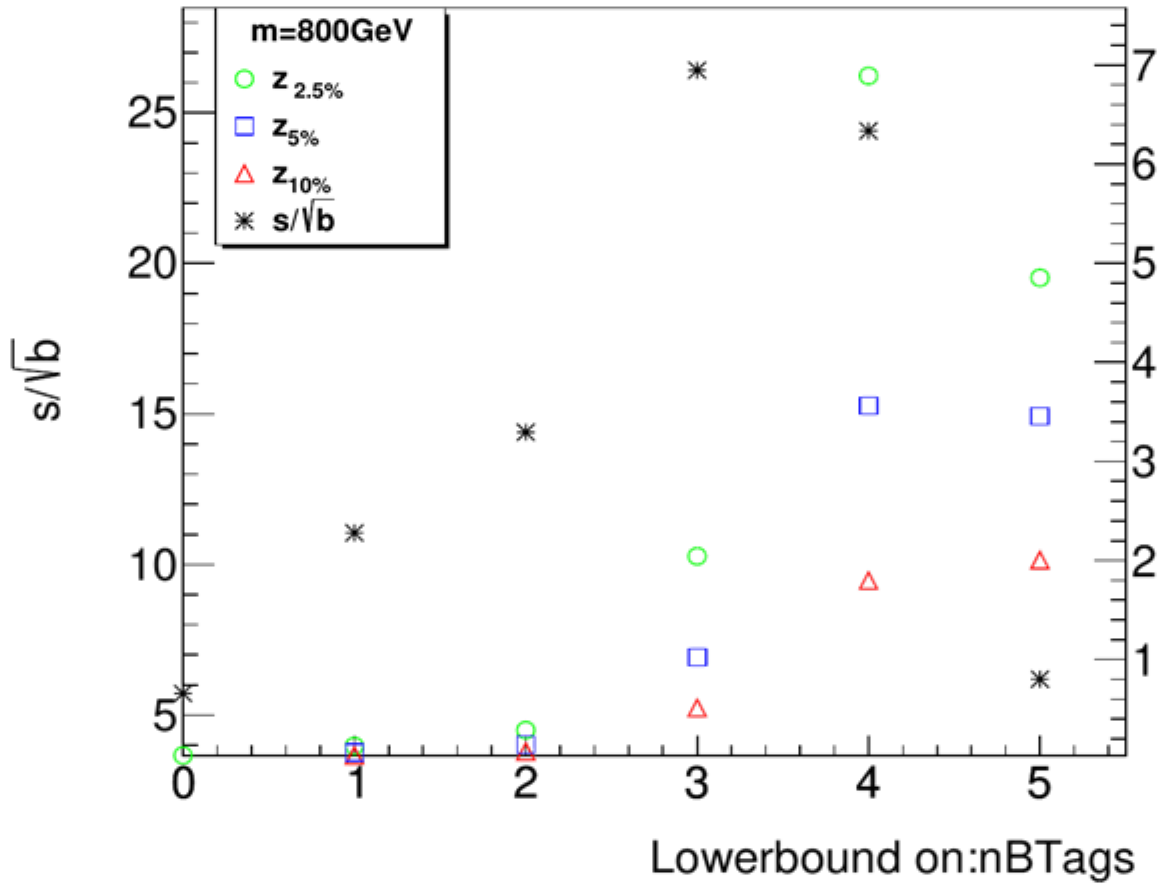
- A first Event Selection has been built
- A Signal and Control Region have been defined
- Next:
  - MC/Data comparison studies in the CR
  - Calculate Asimov limits

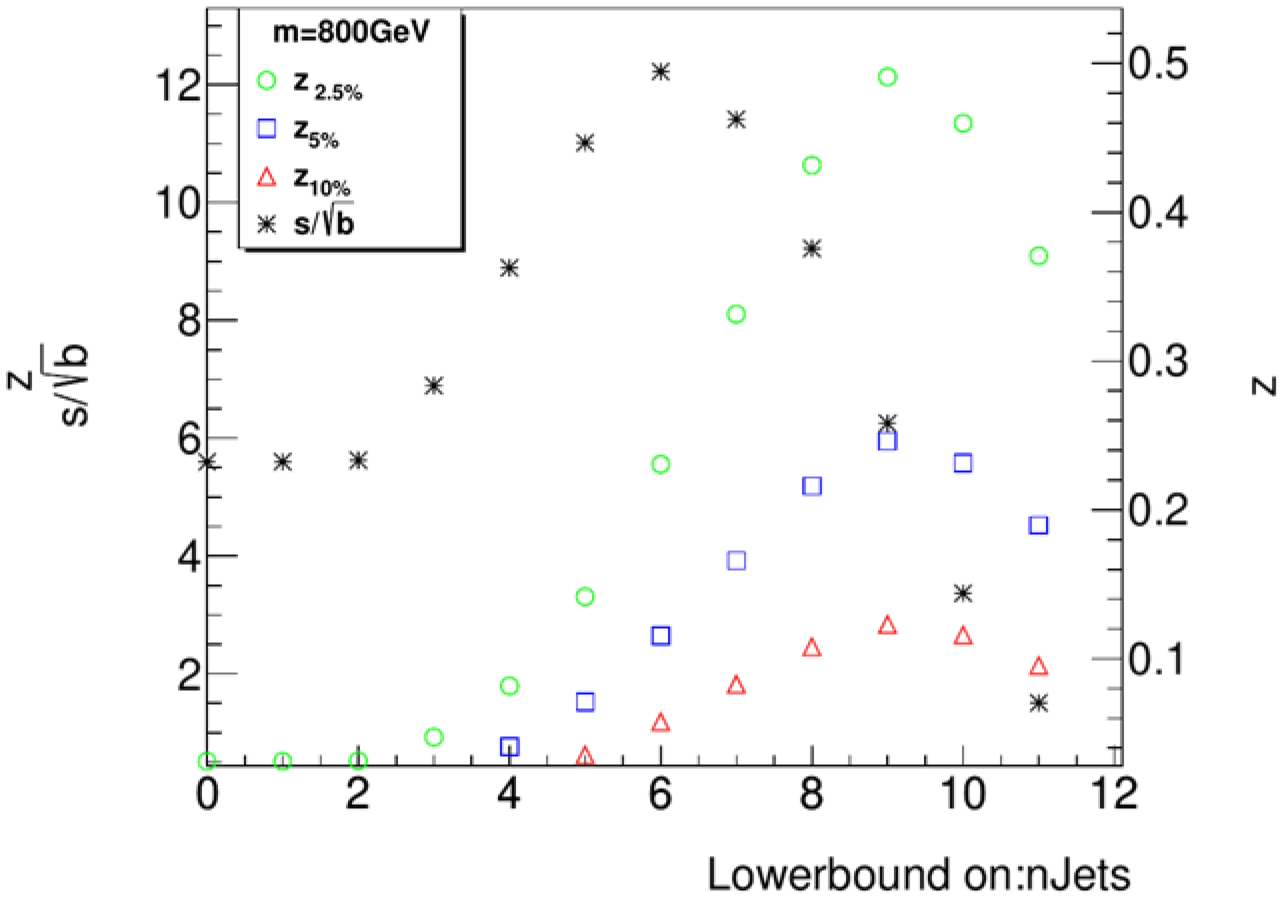
Backup

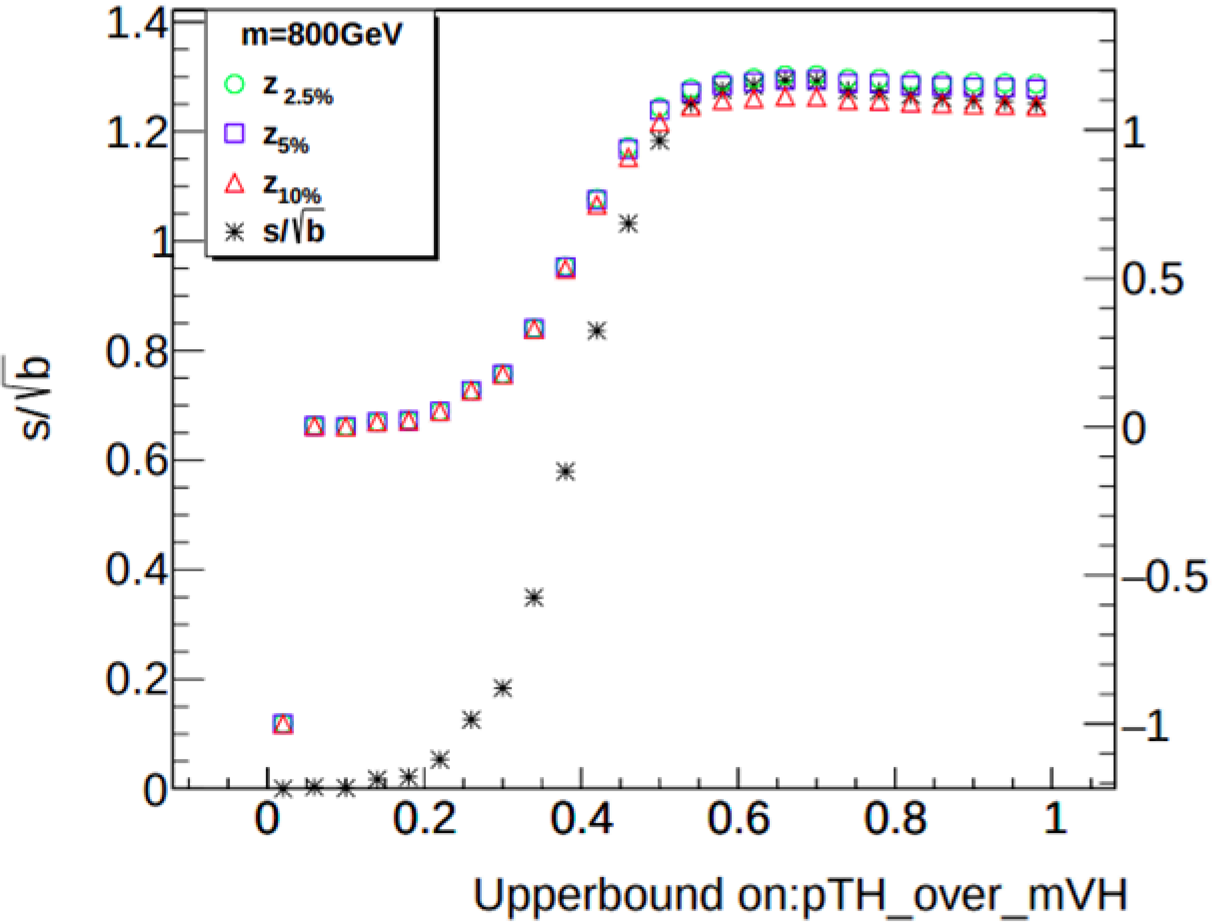


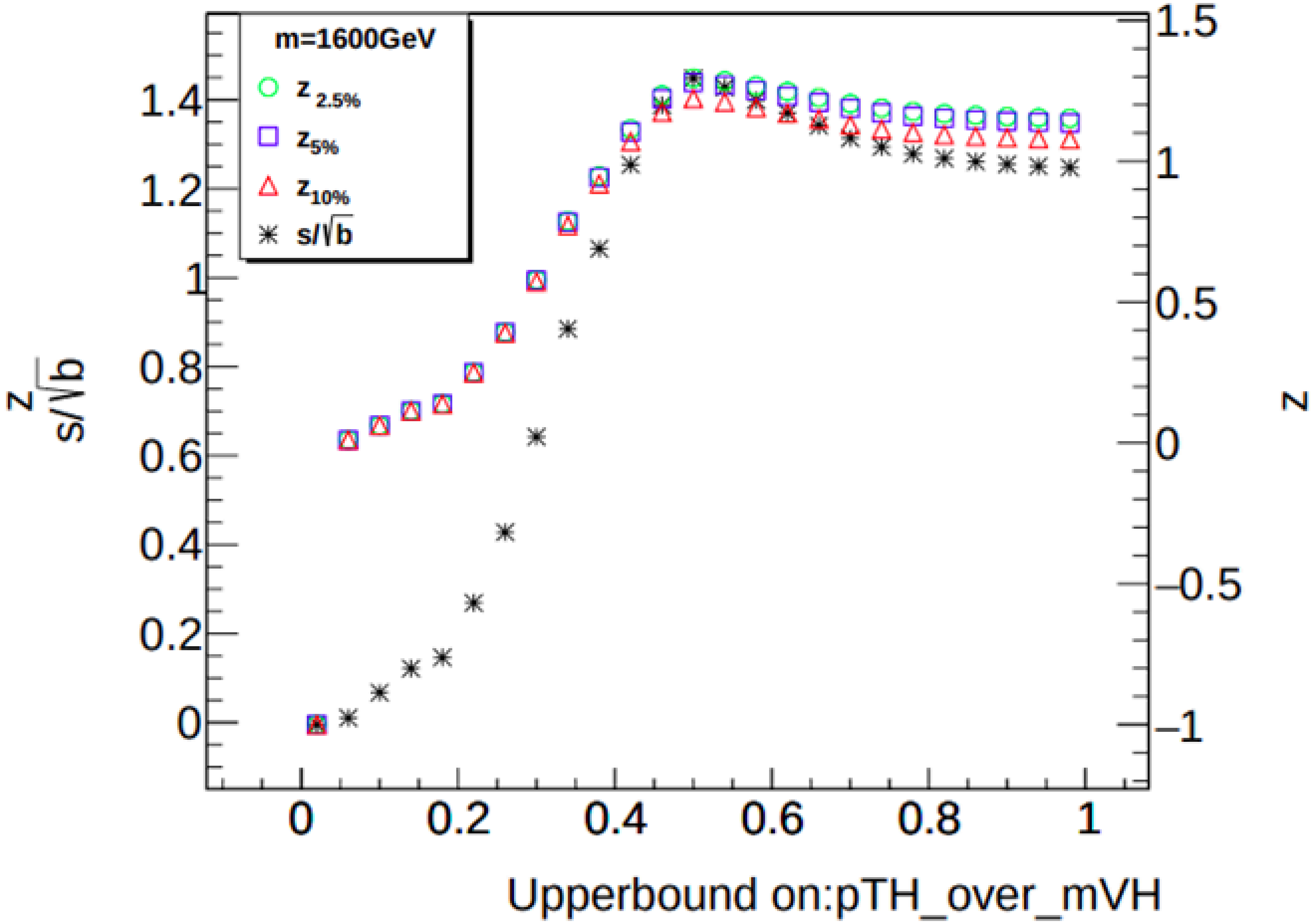


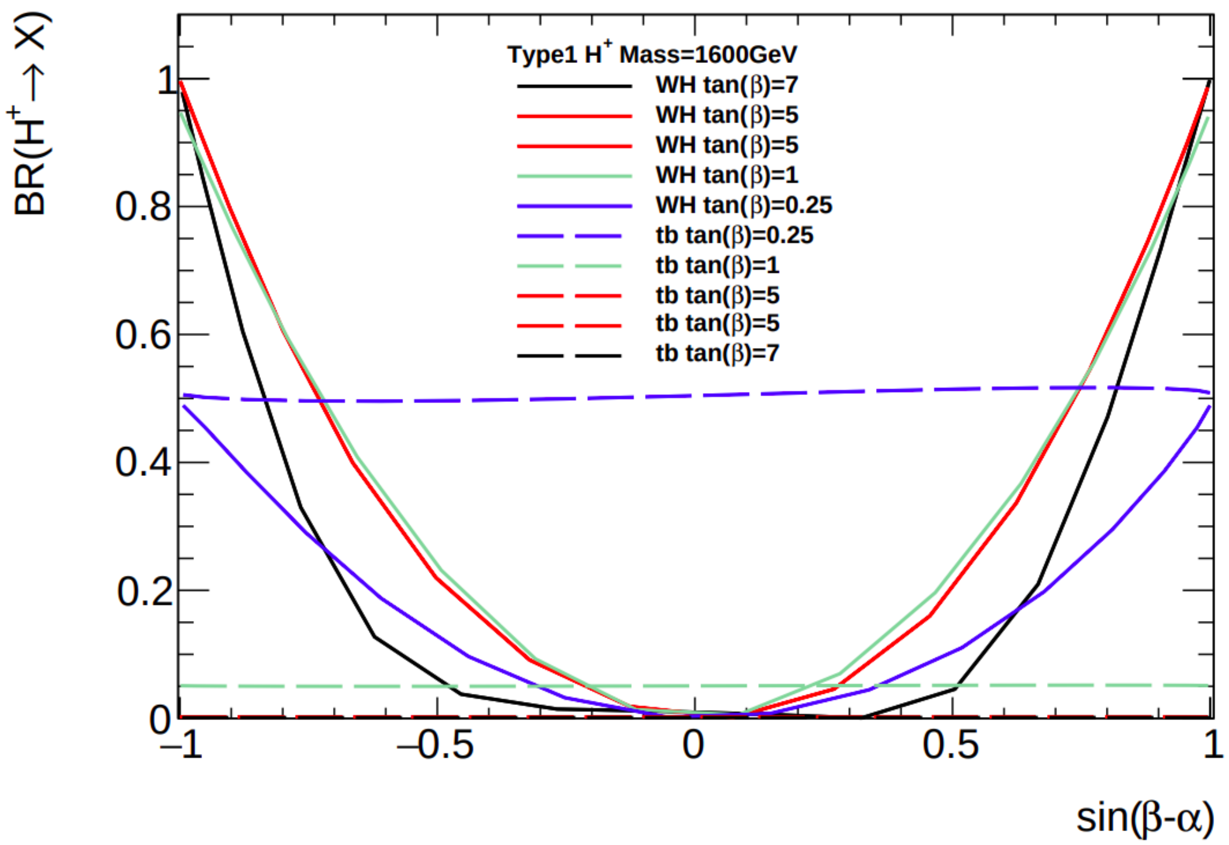
These Calculations have been done both scanning
right to left(Lowerbound)
and left to right(Upperbound)

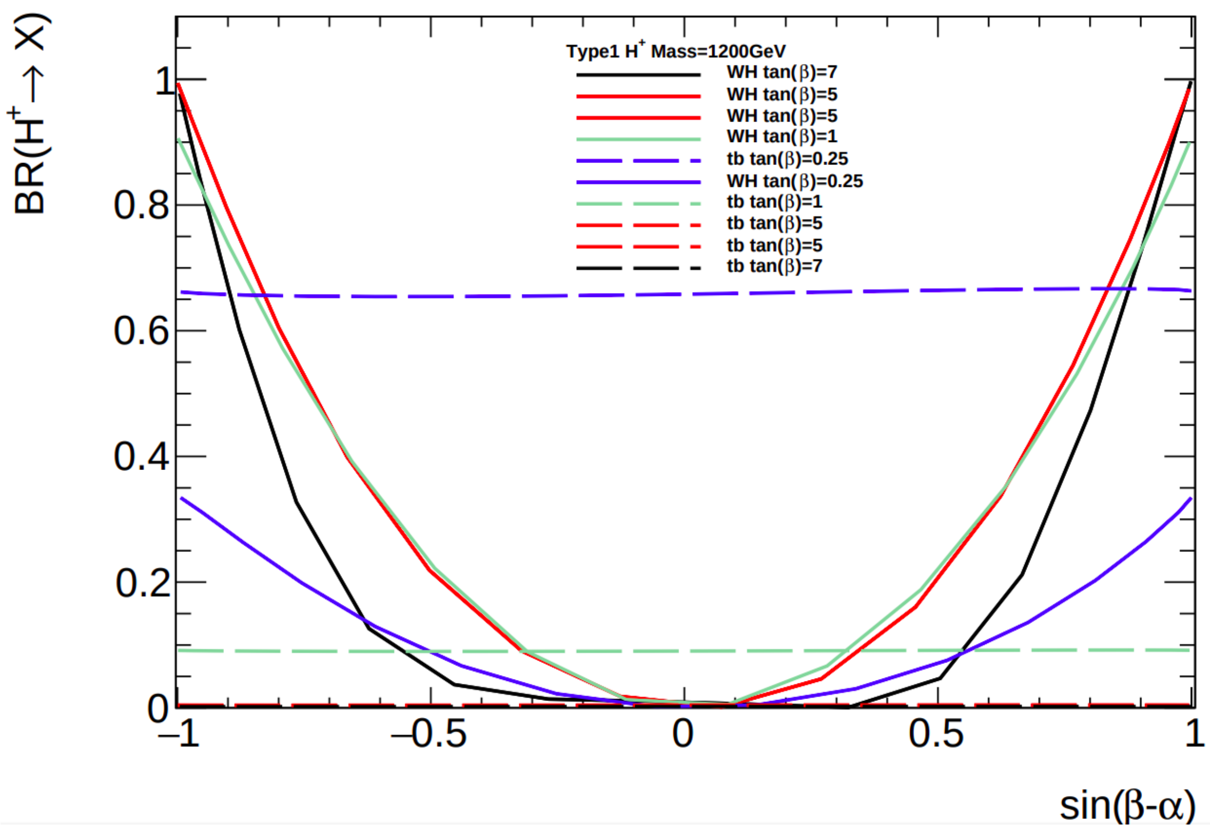

m=800GeV
$z_{2.5\%}$
$z_{5\%}$
$z_{10\%}$
$s/\sqrt{b}$
Lowerbound on:nBTags


m=800GeV
$z_{2.5\%}$
$z_{5\%}$
$z_{10\%}$
$s/\sqrt{b}$
Lowerbound on:nJets


m=800GeV
z 2.5%
z5%
z10%
s/√b
s/√b
Upperbound on:pTH_over_mVH

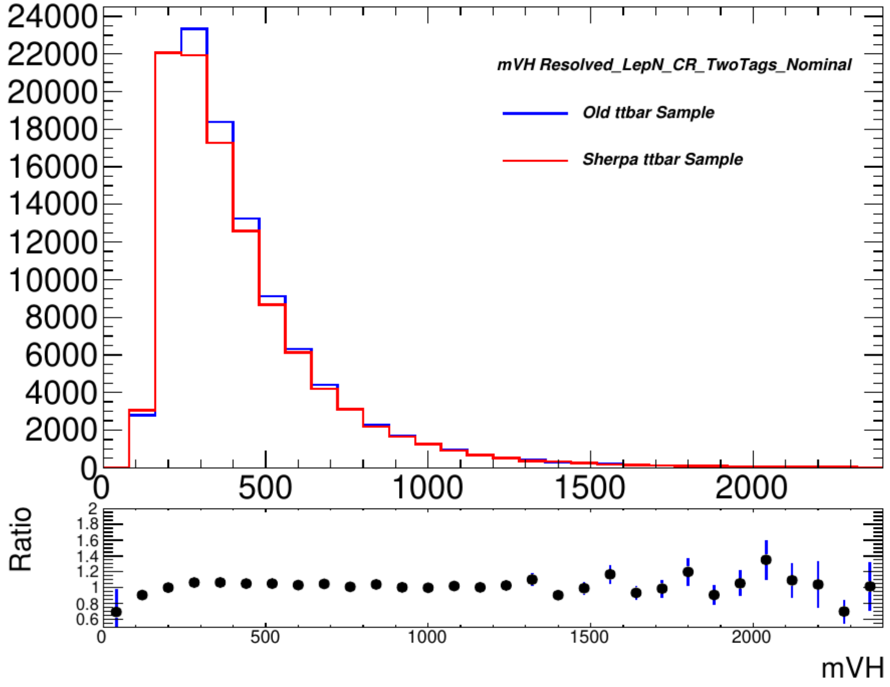

m=1600GeV
z 2.5%
z5%
z10%
s/√b
s/√b
z
Upperbound on:pTH_over_mVH


Type1 H+ Mass=1600GeV
WH tan(β)=7
WH tan(β)=5
WH tan(β)=5
WH tan(β)=1
WH tan(β)=0.25
tb tan(β)=0.25
tb tan(β)=1
tb tan(β)=5
tb tan(β)=5
tb tan(β)=7
BR(H+→ X)
sin(β-α)

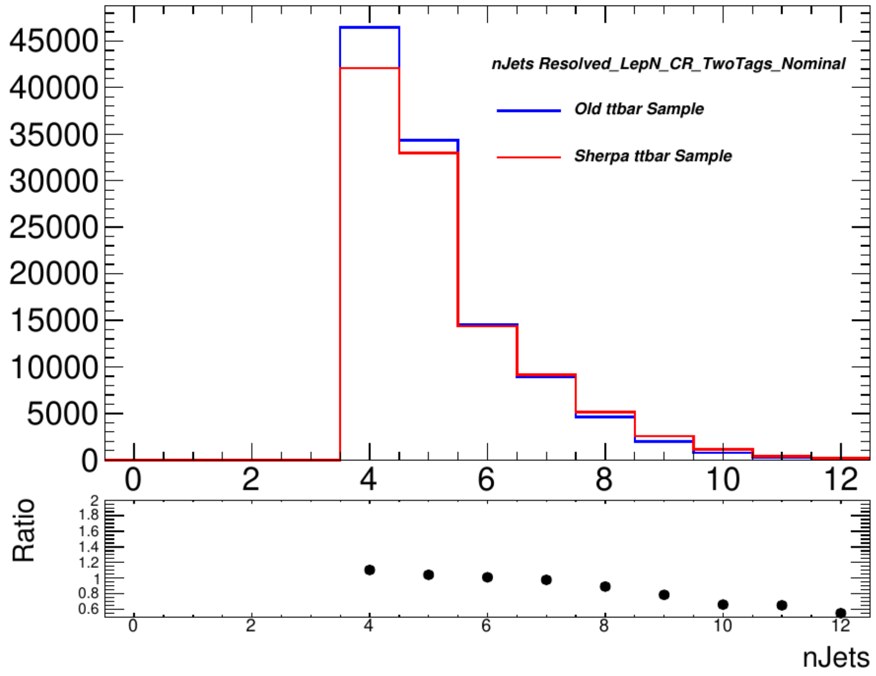

Type1 H+ Mass=1200GeV
WH tan(β)=7
WH tan(β)=5
WH tan(β)=5
WH tan(β)=1
tb tan(β)=0.25
WH tan(β)=0.25
tb tan(β)=1
tb tan(β)=5
tb tan(β)=5
tb tan(β)=7
BR(H+→ X)
sin(β-α)

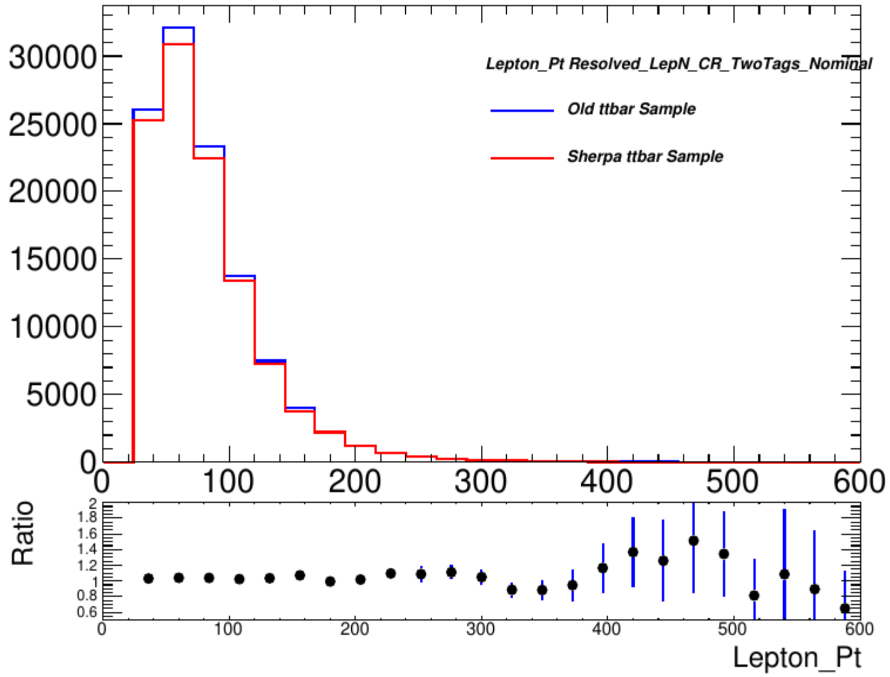
- How well does the highest pT jet overlap with the Higgs?
- Calculate # of jets where dR with respect to the Higgs is smaller than 1.0, 0.8, 0.6 over # of all jets
- In the 4-Tag CR region the statistics are too low , to calculate a sensible value

	2-Tag SR	3-Tag SR	4-Tag SR	2-Tag CR	3-Tag CR	4-Tag CR
m=1600 GeV dR=1	0.978	0.968	0.9789	0.980	0.968	
m=1600 GeV dR=0.8	0.977	0.967	0.975	0.979	0.966	
m=1600 GeV dR=0.6	0.972	0.963	0.975	0.978	0.962	
m=800 GeV dR=1	0.962	0.960	0.967	0.922	0.960	
m=800 GeV dR=0.8	0.956	0.949	0.964	0.919	0.953	
m=800 GeV dR=0.6	0.948	0.942	0.970	0.894	0.939	
m=400 GeV dR=1	0.717	0.758	0.804	0.736	0.541	
m=400 GeV dR=0.8	0.675	0.734	0.780	0.706	0.556	
m=400GeV dR=0.6	0.595	0.730	0.766	0.631	0.532	



- New Samples for ttbar have been made with Sherpa
- Compare Shapes and Calculate Ration
- For Lepton Pt, MET, nJets,mVH





- Compare Normalization of two samples in the different regions
- Calculate:

$$\frac{Integtal(Sherpa) - Integral(Powheg + Pythia8)}{Integral(Powheg + Pythia8)}$$

	SR_2Tag	SR_3Tag	SR_4+Tag	CR_2Tag	CR_3Tag	CR_4+Tag
Merged Lep_P	-1.1%	-8.1%	-7.1%	-1.7%	-4.5%	+2.6%
Resolved Lep_P	-7.9%	-16.7%	-4.3%	-1.5%	+0.9%	-34.3%
Merged Lep_N	+23.2%	+26.3%	+36.9%	+30.7%	+19.8%	-100.0%
Resolved Lep_N	-2.96%	-2.15%	+0.6%	-3.4%	-0.8%	+0.6%

	m=400 GeV	m=800 GeV	m=1600 GeV
dR<0.2			
MVA >=0.9	23.8%	57.3 %	76.2%
MVA >=0.8	21.4%	54.5%	75.4%
MVA >= 0.7	20.0%	52.2%	74.3%
MVA >=0.6	19.0%	50.3%	73.0%
MVA >=0.5	18.0%	49.2%	71.4%
all	15.2%	44.5%	65.8%
dR<0.3			
MVA >=0.9	36.5%	70.2%	85.8%
MVA >=0.8	32.9%	67.4%	84.8%
MVA >=0.7	31.6%	65.3%	83.9%
MVA >=0.6	30.4%	63.5%	83.2%
MVA >=0.5	29.1%	62.6%	82.1%
all	25.3%	57.8%	77.4%
dR<0.4			
MVA >=0.9	46.9%	79.1%	90.0 %
MVA >=0.8	42.6%	76.4%	88.9 %
MVA >=0.7	40.7%	74.3%	88.5%
MVA >=0.6	39.4%	72.5%	88.1%
MVA >=0.5	38.1%	71.7%	87.2 %
all	33.9%	66.7%	83.3%
dR<0.5			
MVA >=0.9	54.6%	84.9%	93.4%
MVA >=0.8	49.8%	82.4%	92.1%
MVA >=0.7	47.5%	80.1%	91.5%
MVA >=0.6	46.22%	78.4%	91.0%
MVA >=0.5	44.8%	77.6%	90.2%
all	40.4%	73.0%	86.8%
dR<0.6			
MVA >=0.9	61.1%	88.7%	95.3%
MVA >=0.8	56.6%	86.5%	94.0%
MVA >=0.7	54.0%	84.2%	93.4%
MVA >=0.6	52.5%	82.7%	93.2%
MVA >=0.5	51.1%	81.9%	92.3%
all	46.3%	77.4%	89.1%

Fraction of chosen di-jet/ Lepton/MET combination within a certain dR to the truth Higgs/W

$$\frac{\#\text{Events in CR}}{\#\text{ of all Event * Intergral Lenght}}$$

	m=400GeV	m=800GeV	m=1600GeV	Background
$0.0 < Maxw_{BDT} < 0.5$	0.36	0.24	0.32	0.77
$-0.1 < Maxw_{BDT} < 0.5$	0.32	0.21	0.28	0.73
$-0.2 < Maxw_{BDT} < 0.5$	0.28	0.19	0.26	0.699
$0.0 < Maxw_{BDT} < 0.4$	0.30	0.20	0.26	0.75
$-0.1 < Maxw_{BDT} < 0.4$	0.27	0.18	0.23	0.7
$-0.2 < Maxw_{BDT} < 0.4$	0.24	0.16	0.21	0.67
$-0.3 < Maxw_{BDT} < 0.4$	0.21	0.14	0.19	0.63
$0.0 < Maxw_{BDT} < 0.6$	0.42	0.29	0.39	0.8
$-0.1 < Maxw_{BDT} < 0.6$	0.38	0.26	0.35	0.76
$-0.2 < Maxw_{BDT} < 0.6$	0.34	0.23	0.32	0.72
$-1.0 < Maxw_{BDT} < 0.0$	0.05	0.02	0.03	0.43
$-0.5 < Maxw_{BDT} < 0.0$	0.06	0.04	0.06	0.39
$-0.5 < Maxw_{BDT} < 0.3$	0.13	0.09	0.12	0.51
$-0.2 < Maxw_{BDT} < 0.3$	0.19	0.13	0.17	0.62
$-0.3 < Maxw_{BDT} < 0.3$	0.17	0.11	0.15	0.58