

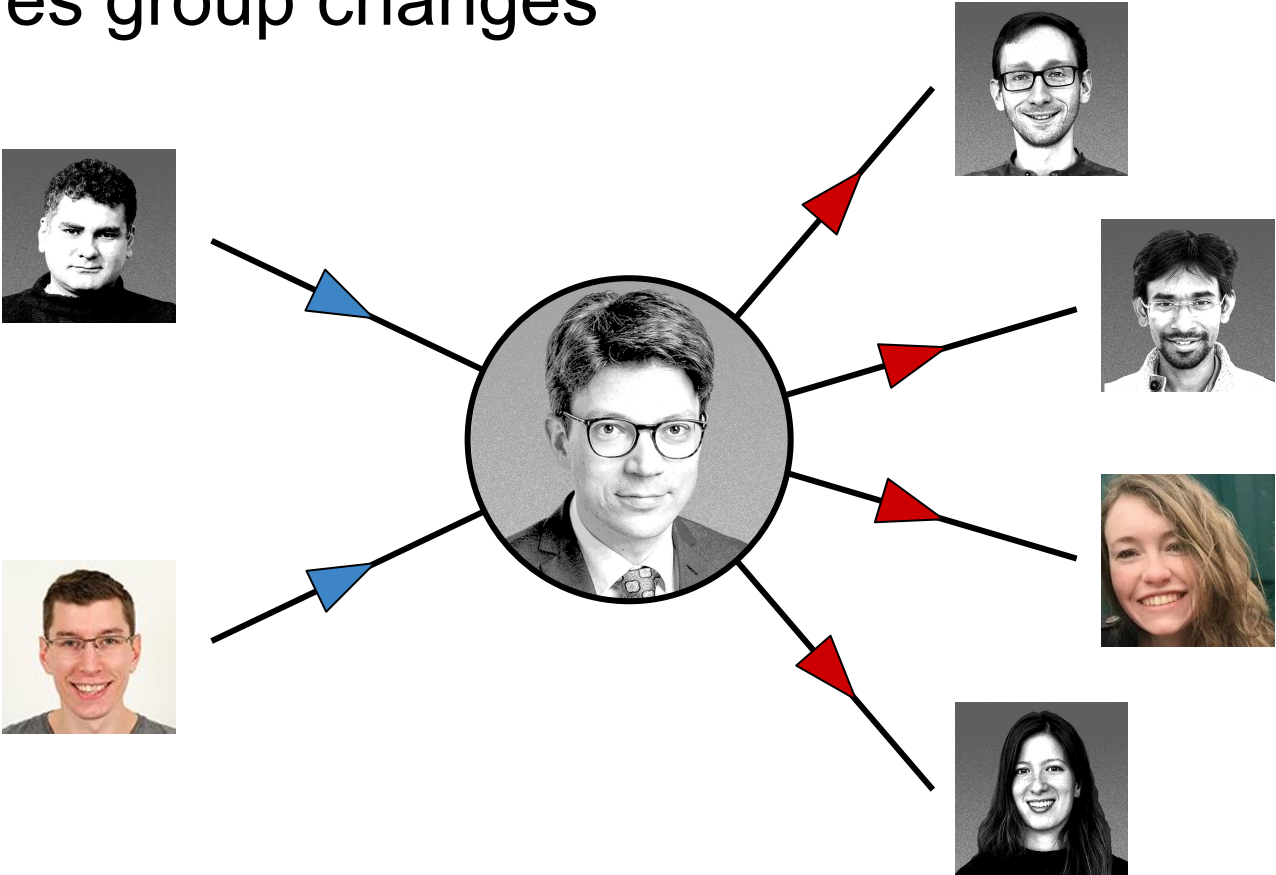
# Amplitudes Group Projects



Alexander Tumanov

Annual project review  
MPP - Quantum Field Theory  
December 15, 2020

# Amplitudes group changes

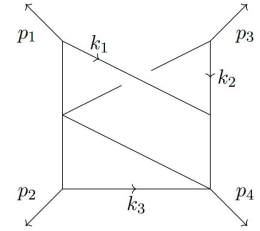


# Amplitudes group

Johannes Henn



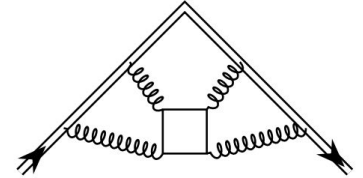
Group leader  
Canonical differential equations approach  
Conformal and dual conformal symmetries of amplitudes and Wilson loops  
and many more...



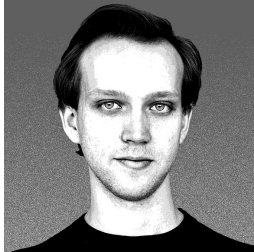
Christoph Dlapa



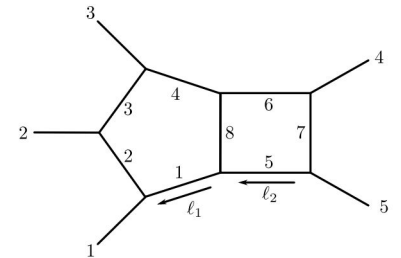
Calculating Feynman integrals via canonical differential equation approach  
4-loop cusp anomalous dimension



Vasily Sotnikov

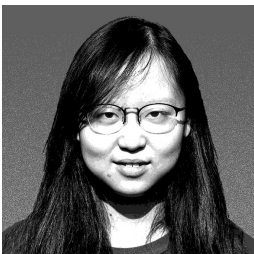


Bridging scattering amplitudes and HEP phenomenology  
Multi-scale Feynman integrals and transcendental functions  
Numerical approaches for analytic calculations

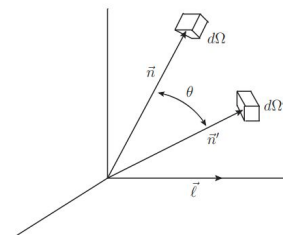


# Johannes Henn's group

Kai Yan



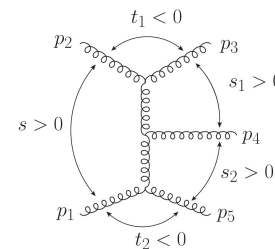
Event shapes in QCD  
Calculating Feynman integrals via canonical differential equation approach



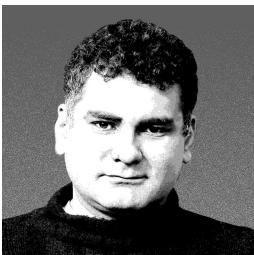
Dmitry Chicherin



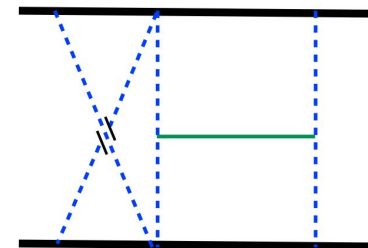
Scattering amplitudes, NNLO calculations,  
perturbative QCD, conformal symmetry,  
supersymmetric gauge theories



William Torres  
Bobadilla



Post-Newtonian corrections to Newton potential  
Manifestly casual representation of Scattering  
Amplitudes  
Numerical evaluation of Scattering Amplitudes  
Local IR methods at NNLO

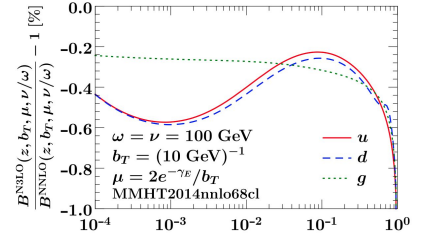


# Johannes Henn's group

Markus Ebert



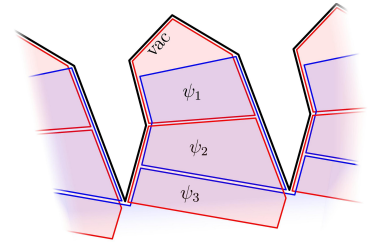
LHC pheno, resummation, transverse momentum distributions, parton distributions from lattice



Alex Tumanov



Supersymmetric gauge theories  
Non-perturbative calculations of scattering amplitudes and form factors  
Integrability



Sorana Scholtes



Outreach



scattering-amplitudes.  
mpp.mpg.de

# Constructing d-log integrands and computing master integrals for three-loop four-particle scattering



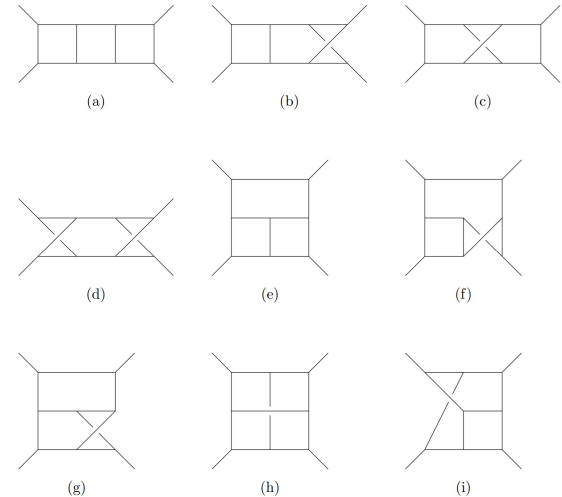
Johannes Henn<sup>a</sup> Bernhard Mistlberger<sup>b</sup> Vladimir A. Smirnov<sup>c</sup> Pascal Wasser<sup>d</sup>

Computed all master integrals for massless three-loop four-particle scattering amplitudes required for processes like di-jet or di-photon production at the LHC.

Presented an algorithm that allows to construct a basis of master integrals integrals with integrands that only have logarithmic poles - called dlog forms.

dlog forms integrate to functions of uniform transcendental weight.

Finally, applied the algorithm to determine a basis of master integrals required to express any amplitude for the scattering of four massless particles at three loops with only massless virtual particles and evaluated these integrals using the method of canonical differential equations.



Diagrams that describe all master integrals for 3-loop massless 4pt scattering



# Pentagon Functions for Scattering of Five Massless Particles



D. Chicherin<sup>a</sup> V. Sotnikov<sup>a</sup>

<sup>a</sup>Max Planck Institute for Physics (Werner Heisenberg Institute), D-80805 Munich, Germany

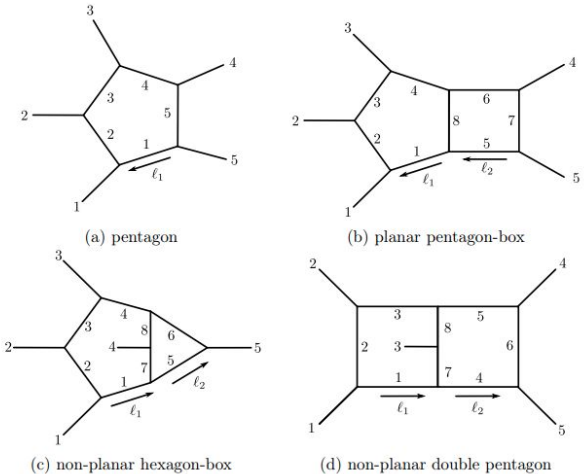
Constructed a minimal set of transcendental functions, *the pentagon functions*, that is sufficient to express all (planar and non-planar) master integrals for 5-point massless scattering at two loops.

Knowing the pentagon functions is crucial for techniques like perturbative bootstrap and for studying the asymptotic limits of the corresponding amplitudes.

The pentagon functions are obtained by applying the canonical differential equations method to all 5-point 2-loop integral topologies.

Mathematica: **PentagonMI**

C++: **PentagonFunctions++**



5-point 2-loop integral topologies



# Multi-Regge Limit of the Two-Loop Five-Point Amplitudes in $\mathcal{N} = 4$ Super Yang-Mills and $\mathcal{N} = 8$ Supergravity



Simon Caron-Huot<sup>a</sup> [Dmitry Chicherin](#)<sup>b</sup> [Johannes Henn](#)<sup>b</sup> Yang Zhang<sup>c,d</sup> [Simone Zoia](#)<sup>b</sup>

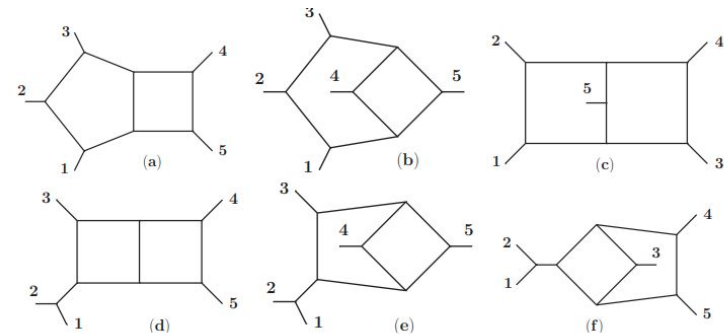
Computed the full functional forms, as well as their multi-Regge limits, of the 5-point non-planar amplitudes in N=4 SYM and N=8 SUGRA.

Calculation of two-loop non-planar correction to 5-parton amplitudes in QCD (NNLO approximation) is at the frontier of modern perturbative calculations.

In the multi-Regge limit

$$|p_3^+| \gg |p_4^+| \gg |p_5^+|, \quad |p_3^-| \ll |p_4^-| \ll |p_5^-|.$$

the space of pentagon functions drastically simplifies. There exists an alternative description of the Regge limit, the BFKL effective theory, which the results have been found to be in perfect agreement with.



Integrands of 2-loop 5pt amplitudes





# The full angle-dependence of the four-loop cusp anomalous dimension in QED

R. Brüser\*

*Theoretische Physik 1, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, 57068 Siegen, Germany*

C. Dlapa,<sup>†</sup> J. M. Henn,<sup>‡</sup> and K. Yan<sup>§</sup>

*Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, 80805 München, Germany*

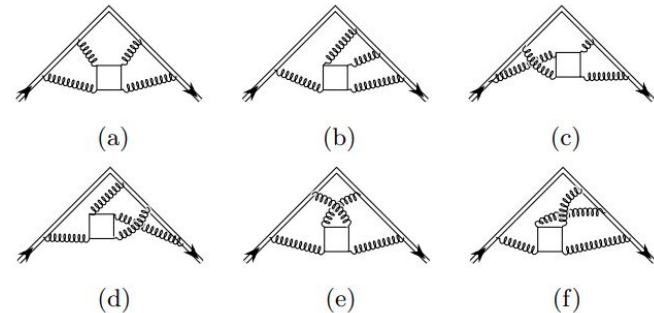


Computed the quartic Casimir correction to angle-dependent cusp anomalous dimension at four loops and used them to obtain the full 4-loop expression for the QED cusp anomalous dimension.

Quartic Casimir term in QCD = Complete QED result

This is the first truly non-planar calculation of a cusp anomalous dimension correction (all lower loop non-planar contributions can be reduced to the planar ones).

This result has disproven the previously existing conjecture that the higher loop cusp anomalous dimension corrections can be reconstructed from the lower loop ones.



Diagrams contributing to the quartic Casimir term at four loops

# Efficient resummation of high post-Newtonian contributions to the binding energy



Stefano Foffa<sup>a</sup>, Riccardo Sturani<sup>b</sup>, William J. Torres Bobadilla<sup>c,d</sup>

Computed post-Newtonian corrections to Newton potential to 5PN order.

$$\mathcal{S} = \mathcal{S}_{\text{pp}} + \mathcal{S}_{\text{bulk}}$$

$$\mathcal{S}_{\text{pp}} = - \sum_{i=1,2} m_i \int \sqrt{g_{\mu\nu}(x_i)} dx_i^\mu dx_i^\nu$$

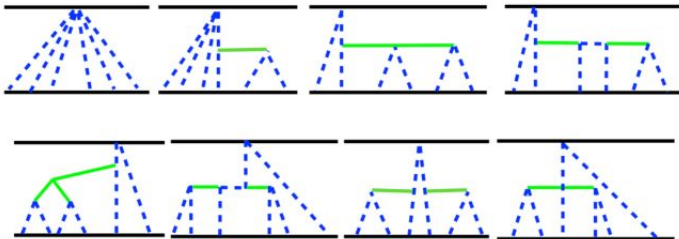
$$\mathcal{S}_{\text{bulk}} = 2\Lambda^2 \int d^{d+1}x \sqrt{-g} \left( R - \frac{1}{2} \Gamma_\mu \Gamma^\mu \right)$$

Newtonian level:



$$V(r) = -\frac{Gm_1m_2}{r}$$

5PN (Post Newtonian) contribution:



$$V_{(\text{static})}^{(5PN)}(r) = \frac{5}{16} \frac{G^6 m_1^6 m_2}{r^6} + \frac{91}{6} \frac{G^6 m_1^5 m_2^2}{r^6} + \frac{653}{6} \frac{G^6 m_1^4 m_2^3}{r^6} + (m_1 \leftrightarrow m_2)$$

Kaluza-Klein parametrization:

$$g_{\mu\nu} = e^{\frac{2\phi}{\Lambda}} \begin{pmatrix} -1 & & & \frac{A_j}{\Lambda} \\ & & & \\ \frac{A_i}{\Lambda} & & e^{-\frac{c_d \phi}{\Lambda}} (\delta_{ij} + \frac{\sigma_{ij}}{\Lambda}) & -\frac{A_i A_j}{\Lambda^2} \\ & & & \end{pmatrix}$$

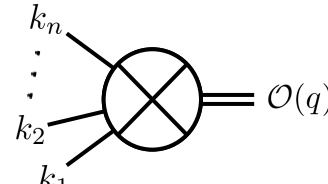
$$c_d = 2 \frac{d-1}{d-2} \quad \text{Static limit: } A_i \rightarrow 0$$



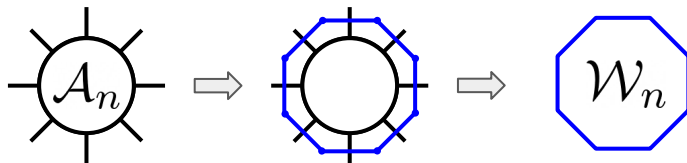
# An Operator Product Expansion for Form Factors

Amit Sever<sup>⌚</sup>, Alexander G. Tumanov<sup>◇</sup>, Matthias Wilhelm<sup>□</sup>

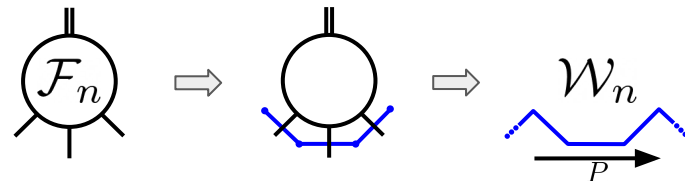
FF = amplitude of a local operator to create an n-particle asymptotic state

$$F_{\mathcal{O}}(k_1, \dots, k_n) = \langle k_1, \dots, k_n | \mathcal{O}(q) | 0 \rangle =$$


Amplitude = closed Wilson loop



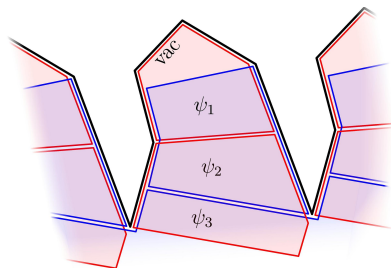
FF = periodic Wilson loop



Wilson loop OPE

Pentagon transitions  
FF transition

Bootstrap



$$= \sum_{\psi_i} \left[ \prod_i e^{-E_i \tau_i + i p_i \sigma_i + i m_i \phi_i} \right] P(0|\psi_1) P(\psi_1|\psi_2) P(\psi_2|\psi_3) \times F(\psi_3)$$

All-loop results for N=4 SYM FF

All-loop results for the maximally transcendental part of the QCD FF

# Summary

Two-loop five-point amplitudes is the new frontier of modern perturbative QCD, and our group has made some great advancements in this area this year.

New members that joined our group (William & Markus) have expanded our group interests into areas like post-Newtonian physics and Parton distributions.

Projects in many other areas have been completed (4-loop cusp anomalous dimension calculations, event shapes in QCD, all-loop form factor calculations, etc...).

Looking forward to next year!

**Thank you!**