Axions in the Early Universe

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The Strong CP Problem

- The θ-vacuum term violates CP symmetry

$$\mathcal{L}_\theta = \bar{\theta}\frac{\alpha_s}{8\pi}G^{a\ \mu\nu}\widetilde{G}^a_{\mu\nu}$$

$$\text{with} \quad \widetilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{a\ \rho\sigma} \quad \text{and} \quad \bar{\theta} = \theta + \operatorname{Arg}\det M$$

- Leads to an electric dipole moment of the neutron
- Experimental constraint:

[PDG,'08]

$$|\bar{\theta}| < 10^{-9}$$

The Peccei-Quinn Solution

[Peccei,Quinn,'77]

- New global chiral $U(1)_{PQ}$ symmetry
- $U(1)_{PQ}$ broken spontaneously at f_a
- Axion a is the associated Goldstone boson

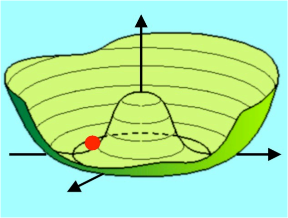
[Weinberg,'78;Wilczek,'78]

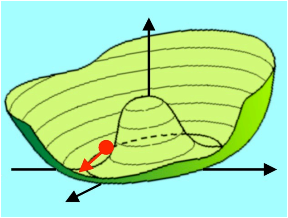
- The additional term serves as a potential for the axion enforcing CP conservation

$$\mathcal{L}_a = a \frac{\alpha_s}{8\pi} \frac{1}{f_a} G^{a\ \mu\nu} \widetilde{G}^a_{\mu\nu}$$

The Potential

- Mexican hat
- Hat gets tilted by instanton effects at $T \approx \Lambda_{\mathrm{QCD}}$: axion acquires a mass
- Lowest point is the CP conserving value: $\langle a \rangle = -\bar{\theta} f_a$





Axion Properties

- Coupling:

$$\mathcal{L}_a = a\frac{\alpha_s}{8\pi}\frac{1}{f_a}G^{a\ \mu\nu}\widetilde{G}^a_{\mu\nu}$$

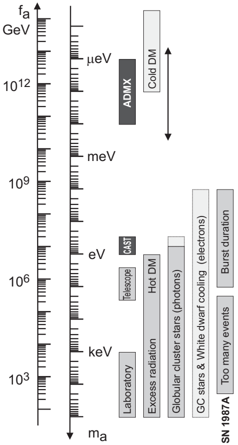
- Axion mass:

$$m_a \simeq 0.60 \text{ meV} \left(\frac{10^{10} \text{ GeV}}{f_a}\right)$$

- All limits imply

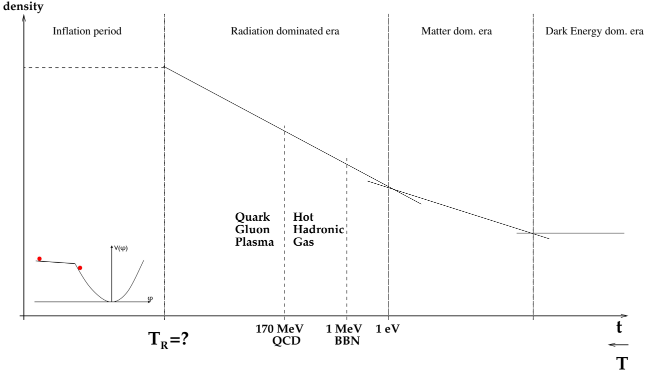
$$f_a \gtrsim 6 \times 10^8 \text{ GeV}$$

$$m_a \lesssim 10 \text{ meV}$$



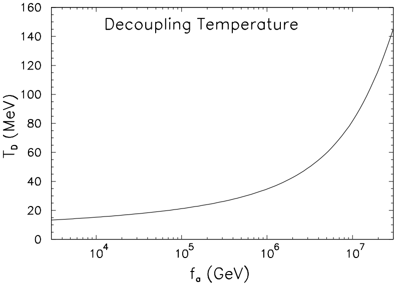
[Raffelt,'06]

Standard Thermal History



Here we assume $f_a > T_{\mathrm{R}}$ to avoid domain walls

Axions in the Primordial Hot Hadronic Gas



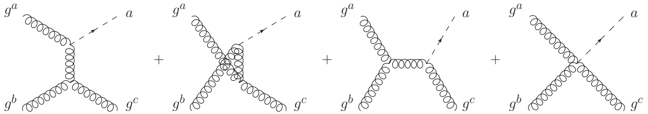
[Hannestad,Mirizzi,Raffelt,'05]

- $T_\mathrm{R} > T_\mathrm{D}$: Axions are in thermal equilibrium
- Freeze out if interaction rate becomes too slow compared to expansion rate of the Universe $\Gamma_a \approx H$
- $T_\mathrm{R} < T_\mathrm{D}$: Axions are thermally produced via

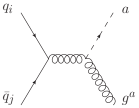
$$\pi + \pi \rightarrow \pi + a \quad \text{and} \quad N + \pi \rightarrow N + a$$

Thermal Axion Production in the Quark Gluon Plasma

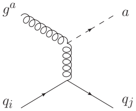
- $g^a + g^b \longrightarrow g^c + a$



- $q_i + \bar{q}_j \longrightarrow g^a + a$



- $q_i + g^a \longrightarrow q_j + a$



Results for Squared Matrix Elements

Label i	Process i	$\lvert M_i\rvert^2 / \left(\frac{\alpha_s^3}{f_a^2}\frac{1}{\pi}\right)$
A	$g^a + g^b \to g^c + a$	$-2\frac{(s^2+st+t^2)^2}{st(s+t)}\lvert f^{abc}\rvert^2$
B	$q_i + \bar{q}_j \to g^a + a$	$\frac{1}{2}\left(\frac{2t^2}{s} + 2t + s\right)\lvert T_{ij}^a\rvert^2$
C	$q_i + g^a \to q_j + a$	$\frac{1}{2}\left(-\frac{2s^2}{t} - 2s - t\right)\lvert T_{ij}^a\rvert^2$

depending on $s = (P_1 + P_2)^2$ and $t = (P_1 - P_3)^2$

IR divergences

Processes A and C: logarithmic IR divergence

Regularization using QCD Debye mass done by
[Masso,Rota,Zsembinszki,'02] $\longleftarrow$ gauge-dependent

HTL resummation and Braaten-Yuan ['91] Prescription

introduce $gT \ll k_{\text{cut}} \ll T$ to separate $\frac{d\Gamma_a}{d^3p} = \frac{d\Gamma_a}{d^3p}\Big|_{\text{hard}} + \frac{d\Gamma_a}{d^3p}\Big|_{\text{soft}}$

$$\frac{d\Gamma_a}{d^3p}\Big|_{\text{hard}} = \frac{1}{2(2\pi)^3 E}\int \frac{d\Omega_p}{4\pi}\int\left[\prod_{j=1}^{3}\frac{d^3p_i}{(2\pi)^3 2E_i}\right] \times (2\pi)^4\delta^4(P_1+P_2-P_3-P) \times\{f_1(E_1)f_2(E_2)[1\pm f_3(E_3)] \times[1+f_a(E)]|M_{1+2\to 3+a}|^2\}$$

$$\frac{d\Gamma_a}{d^3p}\Big|_{\text{hard}} = A_{\text{hard}} + B\log\left(\frac{T}{k_{\text{cut}}}\right)$$

$$\frac{d\Gamma_a}{d^3p}\Big|_{\text{soft}} = -\frac{1}{(2\pi)^3}\frac{f_B(E)}{E}\,\text{Im}\,\Sigma_a(E+i\epsilon,\vec{p})_{k<k_{\text{cut}}}$$

$$\frac{d\Gamma_a}{d^3p}\Big|_{\text{soft}} = A_{\text{soft}} + B\log\left(\frac{k_{\text{cut}}}{m_g}\right)$$

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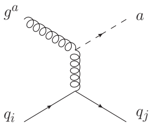

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$$\frac{d\Gamma_a}{d^3p}\Big|_{\rm soft} = A_{\rm soft} + B\log\left(\frac{k_{\rm cut}}{m_g}\right)$$

$$\frac{d\Gamma_a}{d^3p}\Big|_{\rm hard} = A_{\rm hard} + B\log\left(\frac{T}{k_{\rm cut}}\right)$$

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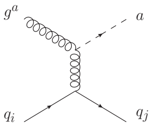

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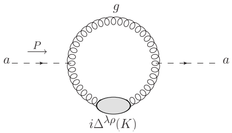


$$\frac{d\Gamma_a}{d^3p}\Big|_{\text{hard}} = A_{\text{hard}} + B\log\left(\frac{T}{k_{\text{cut}}}\right)$$

$$\frac{d\Gamma_a}{d^3p}\Big|_{\text{soft}} = A_{\text{soft}} + B\log\left(\frac{k_{\text{cut}}}{m_g}\right)$$

Thermal Axion Number Density

$$\frac{dn_a}{dt} + 3Hn_a = C_a$$

$$C_a = \int d^3p \left[\frac{d\Gamma_a}{d^3p} \right] = \frac{\alpha_s^3}{f_a^2} T^6 \zeta(3) \frac{1}{\pi^4} \left[\ln\left(\frac{T^2}{m_g^2} \right) + 0.3298 \right]$$

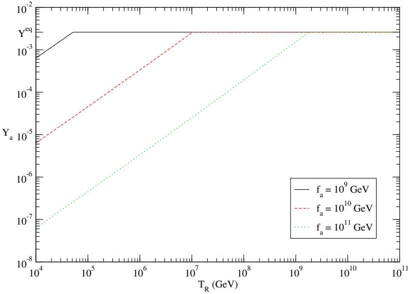
with the thermal gluon mass

$$m_g^2 = \frac{g^2 T^2}{9} \left(N + \frac{n_f}{2} \right)$$

Axion Yield and Energy Density

$$Y_a(T_0) = \frac{n_a(T_0)}{s} \simeq 18.63 \times g^6 \ln\left(\frac{1.444}{g}\right)\left(\frac{10^{10}\ \mathrm{GeV}}{f_a}\right)^2\left(\frac{T_{\mathrm{R}}}{10^{10}\mathrm{GeV}}\right)$$

$$Y_a^{\mathrm{eq}} \approx 2.6 \times 10^{-3}$$



$$\Omega_a h^2 = \frac{m_a Y_a s(T_0) h^2}{\rho_c}$$

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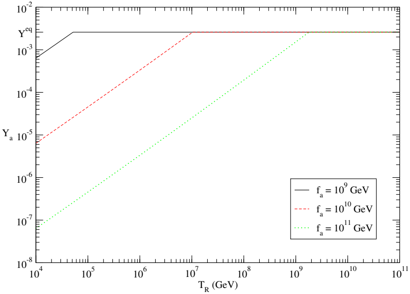
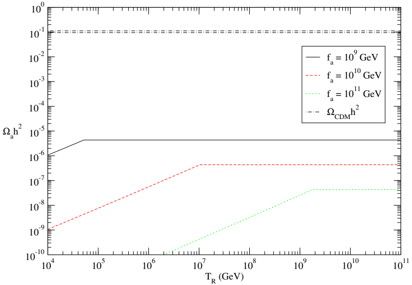
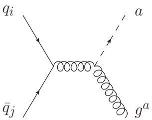




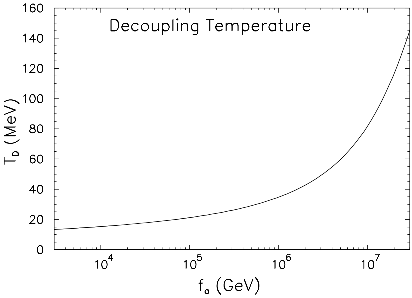
Decoupling Temperature

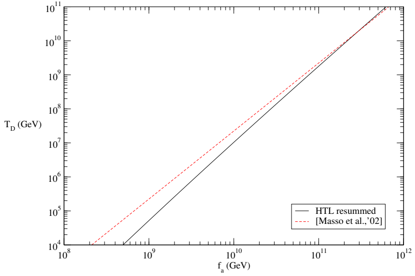
$$\pi + \pi \rightarrow \pi + a$$



[Hannestad,Mirizzi,Raffelt,'05]

[P.G.,Steffen,*in prep.*]





Conclusion

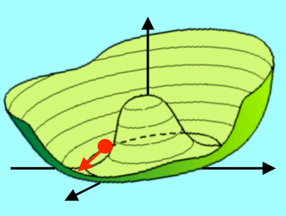
- HTL resummation and Braaten-Yuan prescription allow for a gauge-invariant treatment of thermal axion production
- $\Omega_a^{\mathrm{TP/thermal}} \ll \Omega_{\mathrm{CDM}}$
- First gauge-invariant calculation of the axion decoupling temperature T_{D} for $f_a \gg 10^7$ GeV in the primordial quark gluon plasma

T_{D}

$$T_{\mathrm{D}} = 9 \times 10^6 \left(\frac{f_a}{10^{10}\ \mathrm{GeV}} \right)^2\ \mathrm{GeV}$$

Misalignment Production

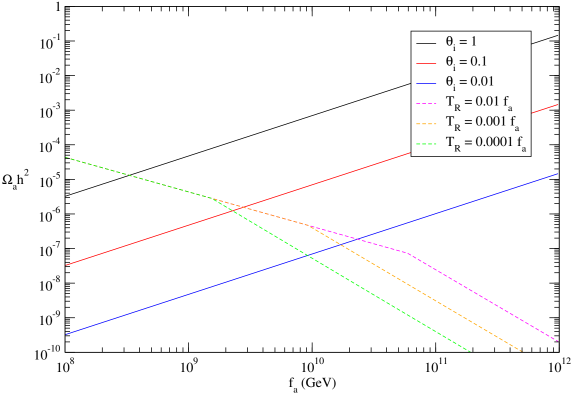
- At $T \approx \Lambda_{\mathrm{QCD}}$: Axion mass switches on
- Axion field performs coherent oscillations around CP conserving minimum



Resulting energy density:

$$\Omega_a h^2 \approx 0.15 \chi f(\bar{\theta}_i^2) \bar{\theta}_i^2 \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{7/6}$$

Misalignment Production



The Hadronic Axion

[Kim,'79;Shiftman,Vainshtein,Zakharov,'80]

- Idea: Construct a model such that interaction imply the counterterm
- A complex scalar field ϕ and an exotic heavy quark with mass $M_Q = h\langle\phi\rangle = hf_a/\sqrt{2}$
- Coupling to gluons give effective term in the Lagrangian