# Towards a Swampland Global Symmetry Conjecture

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based work with **T. Daus / J. March-Russell / S. Leonhardt**

(for wormhole part see also review with **T. Mikhail / P. Soler** )

## Outline

- Introduction / Motivation
- Different types of approximate global symmetries
- Constraining the gauge-derived case
- Relation to euclidean wormhole arguments
  and general wormhole issues

# Introduction / Motivation

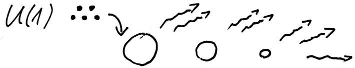
- Are there low-energy phenomena relevant to QG?
- One approach: **Swampland Program**

  (Search for necessary features of consistent low-energy EFTs).
- One example: **No Global Symmetries**

  see e.g. Banks/Dixon '88, Kamionkowksi/March-Russell, Holman et al. '92, Kallosh/Linde$^2$/Susskind '95, Banks/Seiberg '10
- But is this really a 'Swampland Conjecture'?

  – Consider an EFT with a global symmetry.

  – Standard BH evaporation physics will induce the expected violation.




  – the EFT is not constrained by this!

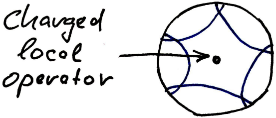
## Introduction/Motivation (continued)

- Clearly, ‘standard’ BH evaporation physics is an overstatement. In, fact, at least in AdS (and with some assumptions), an independent argument for

  **No (exact) global symmetries**

  can be given.

Harlow/Ooguri ’18




- However, our interest will be approximate global symmetries.
- Those are not forbidden and it is crucial to constrain their quality. In a sense, the low-energy effective field theorist has no other way to approach the issue.

# Introduction/Motivation (continued)

- Again: Our interest is in quantitative conjectures against approximate global symmetries.

- One such approach is, of course, the Weak Gravity Conjecture:

$$\begin{array}{rcl} \text{Gauge symmetry} & \rightarrow & \text{global symmetry} \\ g & \rightarrow & 0 \end{array}$$

  (Ideal) claim of WGC: $g \gtrsim m/M_P$, where $m$ is the mass of the lightest charged particle.

- Such a strong statement has not yet been proven.
- Rigorous progress has only been made in the context of the BH mass spectrum (i.e. masses of highly charged particles)

Cheung/Remmen .... Hamada/Noumi/Shiu .... '16..'20

## Introduction/Motivation (continued)

- We want to consider a second route for approaching exact global symmetries:

$$\begin{array}{ccccc} \text{gauge symm.} & \rightarrow & \text{global symm.} & \leftarrow & \text{approx. global symm.} \\ g & \rightarrow & 0 \qquad\qquad 0 & \leftarrow & c \text{ (operator coeffs.)} \end{array}$$

- This second way of approaching a global symmetry is fundamentally different: no light vector is part of the EFT.

- Arguably, it is in fact the practically most useful way to think about a global symmetry

  (B-L, flavor symmetries, DM stability, flat axion potentials, ...)

## What is the definition of an approximate global symmetry?

- Consider EFT with some (global) group action.
- Approximate Symmetry: All non-singlet operators are either irrelevant or have small coefficients ($c \ll 1$).
- Our goal: Quantify the smallness.

see also Coleman/Lee, Rey $\sim$'90 .... Alonso/Urbano '17
AH/Mikhail/Soler '18 ..... Alvey/Escudero '20
(relies on wormholes – more details later...)

see also Fichet/Saraswat '19

( New conjecture inspired by BH evaporation:
In a thermal plasma, the BH-induced violation effect should not exceed the effect of symmetry-violating local operators. )

- We want a derivation instead of a new conjecture (at least for a subclass of global symmetries).

# Types of approximate global symmetries

- (1) Gauge derived

  Start with gauged U(1); 'Higgs' it using an axion

  $\Rightarrow$ vector and axion become heavy

  $\Rightarrow$ any light charged particle now sees an approximate global symmetry.

- (2) Accidental

  Spacetime and gauge symmetries forbid all relevant and marginal non-singlet operators.

- (3) Fine-tuned

  Coefficients of relevant and marginal non-singlet operators are small by landscape-type tuning.

**Our focus will be on the gauge-derived case.**

# Minimal setting / basic idea

$$\mathcal{L} \supset \frac{1}{g^2} F_{\mu\nu}^2 + |D\Phi|^2 + m^2|\Phi|^2 + f^2|\partial_\mu\varphi + A_\mu|^2$$

- If $m \ll gf$, the light field $\Phi$ sees a surviving global $U(1)$.
- $\varphi$ started out as an axion, i.e. a scalar with gauged discrete shift symmetry $(\varphi \to \varphi + 2\pi n)$.
- Instanton (wormhole?) effects break the associated continuous shift symmetry very weakly (non-perturbatively).
- Natural question: Can this be used to apply the Weak Gravity/Swampland logic to quantitatively constrain global symmetry violation? (of our lin.-realized global $U(1)$)

For this purpose, recall the

## Generalized Weak Gravity Conjecture:

- Consider a $p$-form gauge theory ($p \neq 1$):

$$S \;\sim\; \int (F_{p+1})^2 \,+\, T \int_{p-dim.} dV \,+\, g \int_{p-dim.} A_p \,.$$

It is claimed that $T/M_P \lesssim g$ .

- In particular, the axionic ($p = 0$) case reads:

$$S \;\sim\; \int (d\varphi)^2 \;+\, S_{inst.} \,+\, g\,\varphi(x_{inst.})\,.$$

or, for gauge instantons,

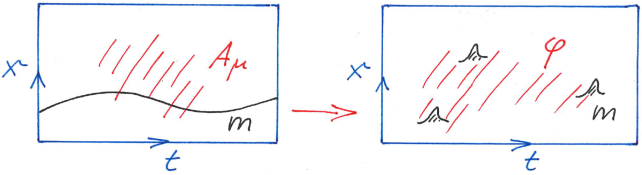
$$\text{cf.} \quad S \sim \int (d\varphi)^2 + \int \text{tr}(F^2) + \int \frac{1}{f}\,\varphi\,\text{tr}(F\tilde{F})\,.$$

## Generalization to axions / instantons

- Thus, with the substitutions $g \to 1/f$ and $T \to S_{inst.}$, the WGC now says:

$$T/M_P < g \quad \Rightarrow \quad \boxed{S_{inst.} < M_P/f} .$$

- This implies a lower bound on the strength of instanton effects: $\exp(-S_{inst.}) > exp(-M_P/f)$.



- We want to gauge the axion by a $U(1)$.
- Recall more generally how a $p$-form is gauged by a $(p+1)$-form:

$$\frac{1}{g_p^2}|dA_p|^2 + \frac{1}{g_{p+1}^2}|dA_{p+1}|^2 \quad \to \quad \frac{1}{g_p^2}|dA_p + A_{p+1}|^2 + \frac{1}{g_{p+1}^2}|dA_{p+1}|^2$$

- Crucially, in the gauged/Higgsed version, the charged $(p-1)$-branes of the $p$-form theory cease to exist as independent objects:

  They would break the gauge invariance
  $\delta A_{p+1} = d\chi_p\,,\;\; \delta A_p = -\chi_p$

- Instead, those brains can appear
  only as boundaries of the $p$-branes $B_p$ charged under $A_{p+1}$:

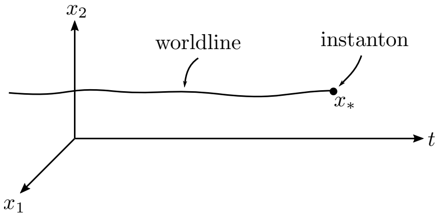
$$S \supset \int_{B_p} A_{p+1} + \int_{\partial B_p} A_p\,.$$

- In our case of **Gauge-derived global symmetries**, we gauge an axion (0-form) with a $U(1)$ vector (1-form).

- Thus, instantons become boundaries of worldlines.

- In other words: Instantons automatically destroy globally-charged particles

  (cf. many stringy examples: Ibanez/Marchesano/Rabadan '01 .... Antoniadis/Kiritsis/Rizos .... Uranga ... Blumenhagen/Cvetic/Kachru/Weigand .... Martucci '15)




  By the WGC for axions, this particle-number violation is suppressed by $\exp(-S_{inst.}) \sim \exp(-M_P/f)$

- Moreover, according to the magnetic WGC for axions (for the dual $B_2$-theory with strings) the string tension is bounded by $T \lesssim M_P f$.

AH/Soler '17

- This implies a UV-cutoff for the EFT:

$$\Lambda \;\sim\; \sqrt{M_P f}$$

Hence, in total, the global-symmetry violation is bounded below by

$$\exp(-S_{inst.}) \;\sim\; \exp(-M_P^2/\Lambda^2)$$

- Very intriguingly, this is the same as the plasma-motivated bound of Fichet/Saraswat and as the bound expected from wormholes:

$$S_{WH} \;\sim\; M_P^2 \int \mathcal{R} \;\sim\; M_P^2/\Lambda^2\,.$$

## An example with 'UV-complete' instantons:

$$\mathcal{L}_1 = -\frac{1}{e^2}F^2 + \overline{\psi} i \not{D} \psi \quad , \qquad \mathcal{L}_2 = -f^2(\partial\varphi)^2 - \frac{1}{g^2}\mathrm{tr}\,G^2 + \frac{\varphi\,\mathrm{tr}\,G\tilde{G}}{8\pi^2}$$

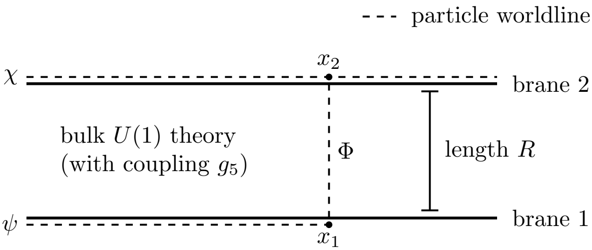
- Gauge: $\partial\varphi_\mu \;\rightarrow\; \partial_\mu\varphi + A_\mu$ and take $\psi$ in the $N$ of $SU(N)$.
- Now standard $SU(N)$ instantons induce a 't Hooft vertex

$$\mathcal{O} = e^{-S_I}\,\overline{\psi}_L\psi_R\,e^{i\varphi} + \mathrm{h.c.}$$

- After gauge-fixing to $\varphi = 0$, as appropriate in the IR, this is precisely the effect we claimed on general grounds.

For more general situations and stringy origins of such models see e.g. Anastasopoulos/Bianchi/Dudas/Kiritsis '06

# A simple 5d example on $S^1/\mathbb{Z}_2$:




- The 5d $U(1)$ is Higgsed on brane 2.
- The field $\psi$ on brane 1 becomes globally charged.
- This global $U(1)$ is broken exponentially weakly (by the massive charged 5d particle $\Phi$, required by the WGC)
- The resulting toy-model 'exotic instanton' has an action consistent with our general result.

# A potential loophole

- If the light charged particle has $U(1)$-charge $n \gg 1$, the low-energy observers may be mislead:

  They see an $n$-instanton effect $(-\exp(-n\, M_P^2/\Lambda^2))$ and take it for a single-instanton effect $(-\exp(-M_P^2/\tilde{\Lambda}^2))$.

  So they expect a cutoff $\tilde{\Lambda} = \Lambda/\sqrt{n}$ that is too low and suspect a violation of our bound.

- In examples we studied, light high charges can only be constructed at the price of lowering the EFT-cutoff.
  $\Rightarrow$ probably no loophole (but more work needed).

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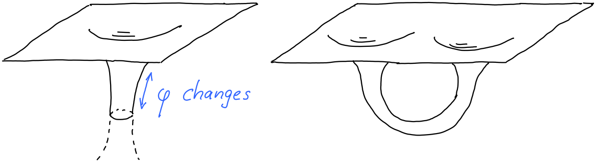
- We already mentioned the parametrically similar wormhole-based arguments against global symmetries – let us develop this line of thought ....

# Euclidean wormholes / gravitational instantons

- In Euclidean Einstein gravity, supplemented with an axionic scalar $\varphi$ , instantonic solutions exist:

Giddings/Strominger '88
. . .




- The 'throat' is supported by the kinetic energy of $\varphi = \varphi(r)$, with $r$ the radial coordinate of the throat/instanton.

- A wormhole-end looks like an instanton to the low-energy observer

  (recently revived in the Swampland/WGC context by

  Montero/Uranga/Valenzuela, Heidenreich/Reece/Rudelius '15
  AH/Mangat/Theisen/Witkowski '17, ....)

## Euclidean wormholes (continued)

- The underlying lagrangian is simply

$$\mathcal{L} \sim M_P^2\, \mathcal{R} + f^2 |d\varphi|^2 \,, \quad \text{now with} \quad \varphi \equiv \varphi + 2\pi \,.$$

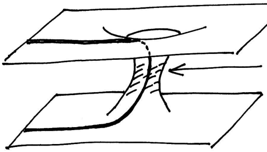
- This can be dualized $(dB_2 \equiv f^2 * d\varphi)$ to give

$$\mathcal{L} \sim M_P^2\, \mathcal{R} + \frac{1}{f^2} |dB_2|^2 \,.$$

- The 'throat' exists due the compensation of these two terms:
  Placing one unit of flux (of $H_3 = dB_2$) on the transverse $S^3$ of radius $R$, we have

$$M_P^2\, R^{-2} \sim \frac{1}{f^2} R^{-6} \quad \Rightarrow \quad M_P\, R^2 \sim \frac{1}{f} \,.$$

- Thus, the instanton action is $S \sim M_P/f$
- This coincides parametrically with
  the lowest-action instanton of the WGC.
- The maximal WH-curvature scale is $\sqrt{f\, M_P}$, which should not exceed the UV cutoff:
  $$f\, M_P < \Lambda^2 \quad \Rightarrow \quad S \sim M_P^2/\Lambda^2$$
- This agrees with our WGC-bound on global-symm.-violation
- Also technically (cf. our Appendix), one finds a new class of wormholes carrying our gauge-derived global charge:



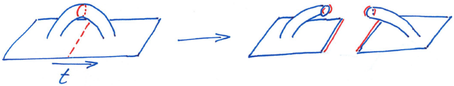
# Euclidean wormholes - conceptual issues

- However, euclidean wormholes come at the price of deep conceptual issues.

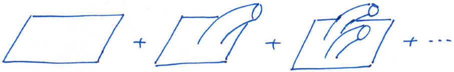
  Hawking '78..'88, Coleman '88, Preskill '89
  Giddings/Strominger/Lee/Klebanov/Susskind/Rubakov/Kaplunovsky/..
  Fischler/Susskind/...

  Recent review: AH, P. Soler, T. Mikhail '18

- First, with wormholes come baby universes:




- Second, with baby universes comes a 'baby universe state' ($\alpha$ vacuum) encoding information on top of our 4d geometry.

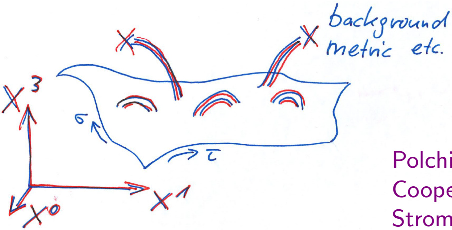


## Conceptual issues (continued)

- In our concrete (single-axion) case, an $\alpha$ parameter now governs the naively calculable $e^{-S_I}$-effects.
- Most naively, 4d measurements collapse $\alpha$ parameters to random constants.
- However, one should really include the full quantum dynamics of $\alpha$ parameters ...

# Conceptual issues (continued)

- In 1+1 dimensions this corresponds to the target-space-dynamics of string theory.




Polchinski, Banks/Lykken/O'Loughlin, Cooper/Susskind/Thorlacius, Strominger '89...'92

- What is the analogue in 3+1 dimensions?

- Another key problem is a possible clash with locality on the CFT-side of AdS/CFT (factorization problem)

  Maldacena/Maoz '04, Arkani-Hamed/Orgera/Polchinski '07, ...., 'SYK'

# Conceptual issues (continued)

- With all these problems in mind, maybe one should dismiss wormholes altogether?

- One option is to forbid topology change, but certainly (?) not in $d = 2$.

- Is there a reason to forbid topology change just in $d > 2$ ?

- A different argument is that these wormhole solutions have negative modes and should hence be dismissed.

Rubakov/Shvedov '96, Maldacena/Maoz '04, see however Alonso/Urbano '17, ...

In particular: Van Riet et al. '04 ... '17/'18 (and talk)

- But, while this is even technically still an open issue, it does not appear to be a strong enough objection ....

## Conceptual issues (continued)

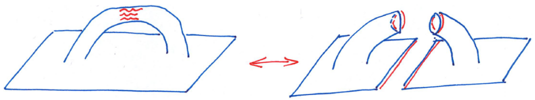
- Indeed, once a non-zero amplitude

  universe $\rightarrow$ universe $+$ baby-universe

  is accepted, the reverse process is hard to forbid.

- As a result, one gets all the wormhole effects.

- The negative mode issue may be saying:
  'Giddings-Strominger' does not approximate the amplitude well.



- ..hard to see how it would dispose of the problem altogether..

# Recent developments related to Wormholes

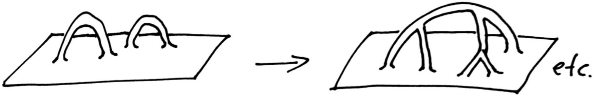
- Recently, a concrete proposal for calculating the entropy of an evaporating BH has emerged (method of 'Islands')

  Penington, Almheiri/Engelhardt/Marolf/Maxfield, Almheiri/Mahajan/Maldacena/Zhao, .... '19/20

- The concrete mechanism by which entropy leaves the BH in this approach is related to euclidean WHs

- Motivated by this, a new 2d toy model developing Coleman's baby universe calculation has been suggested

  Marolf/Maxfield '20




(For a different model see Ambjorn/Sato/Watabiki '21)

## Recent developments related to wormholes (continued)

- In particular, Marolf/Maxfield proposed to mod out the naive BU Hilbert space by a certain equivalence (related to 1 BU $\rightarrow$ 2 BU transitions, etc.)

- It has then been proposed that, in $d \geq 4$, this equivalence should be so strong that the BU Hilbert space is 1-dimensional
McNamara/Vafa '20

- This would not remove the effect of BUs completely, but it would get rid of the arbitrariness of $\alpha$ parameters

- But can we do a proper calculation in $d \geq 4$ ?

## Summary/Conclusions

- The WGC for axions demands certain minimal-action instantons.

- This leads to a universal bound on the quality of gauge-derived global symmetries: $\gtrsim \exp(-M_P^2/\Lambda^2)$.

  (In agreement with other effects, such as wormholes.)

- But the latter come at the price of $\alpha$ vacua (and other disasters).

- Keep struggling with these fundamental unresolved issues!