Do Coleman's Euclidean wormholes contribute?

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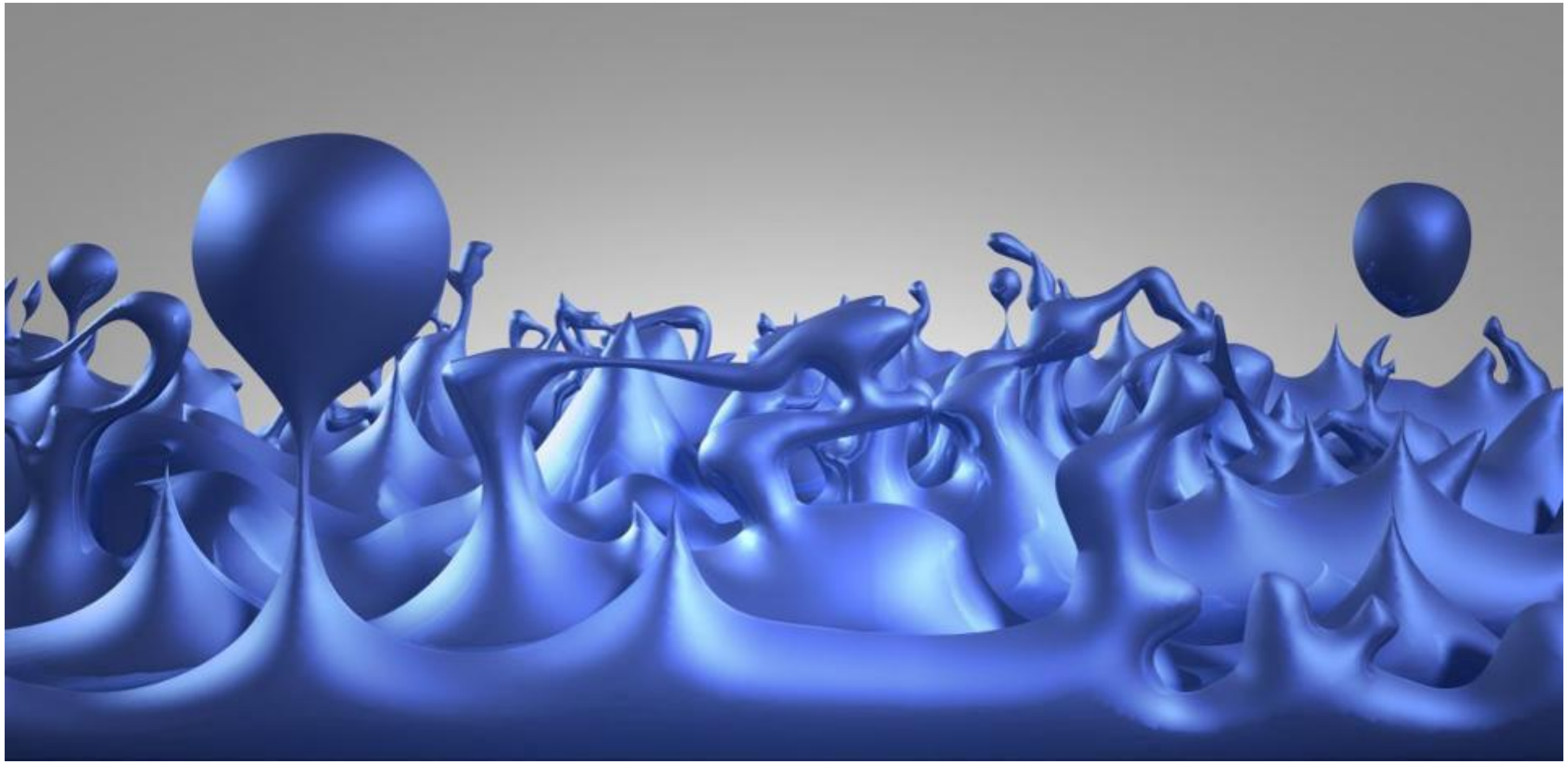

Munich 2021

Based on

- VR, arXiv:2004.08956
- Katmadas, Trigiante, Ruggeri, VR: 1812.05986 & Astesiano, Trigiante, Ruggeri, VR, *in progress*.
- Hertog, Truijen, VR, arXiv: 1811.12690 & Hertog, Maenaut, Tielemans, VR, *in progress*.
- Andriolo, Shiu, Soler, VR, in progress.

Can we define the ***Euclidean*** path integral with gravity at least semi-classically? Which saddles contribute? Do we include non-trivial topologies?

This talk; for concrete model we try to work towards some answers

Disclaimer : I do not regard models in 2D, 3D [Shenker, Stanford, Marold, Maxfield, Maldacena,… …., 2020] or "constrained instanton" models. [Cotler&Jenssen 2020]

Euclidean Wormholes à la Coleman

General relativity action (M_p=1): $S = \int \sqrt{-g}\,(\mathcal{R} + \mathcal{L}_{\mathrm{matter}})$

Minimalism: $\mathcal{L}_{\mathrm{matter}} = \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi)$ $\Rightarrow$ $\mathcal{L}_{\mathrm{matter}} = \frac{1}{12}(F_{\mu\nu\rho}F^{\mu\nu\rho})$

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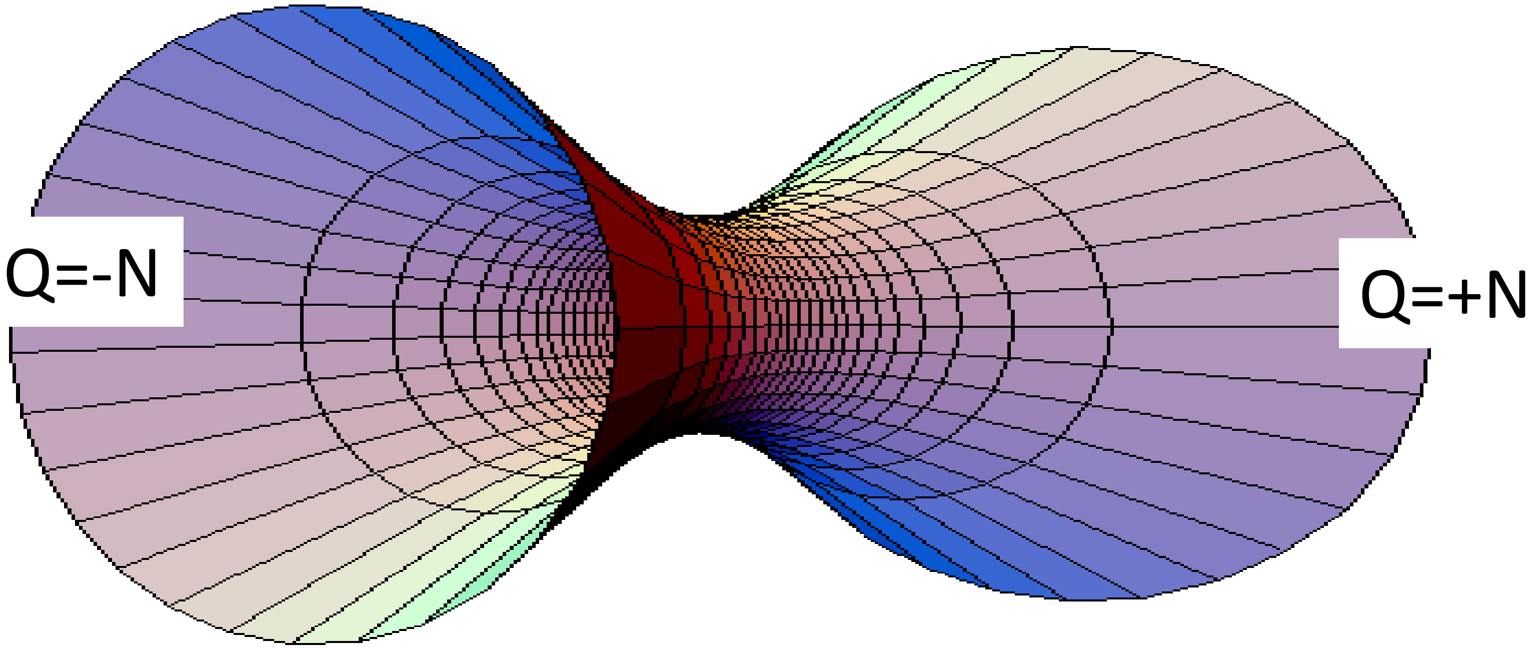
Wormhole? In gauge f=1, a(t) should grow, reach a minimum and then grow again.
Other gauge is easier:

$$ds^2 = \left(1 + \frac{\tau^2}{\ell^2} - \frac{Q^2}{\tau^8}\right)^{-1} d\tau^2 + \tau^2 d\Omega_3^2 .$$

Where I allowed AdS space asymptotics in case of negative cc $\Lambda = -\frac{1}{\ell^2}$

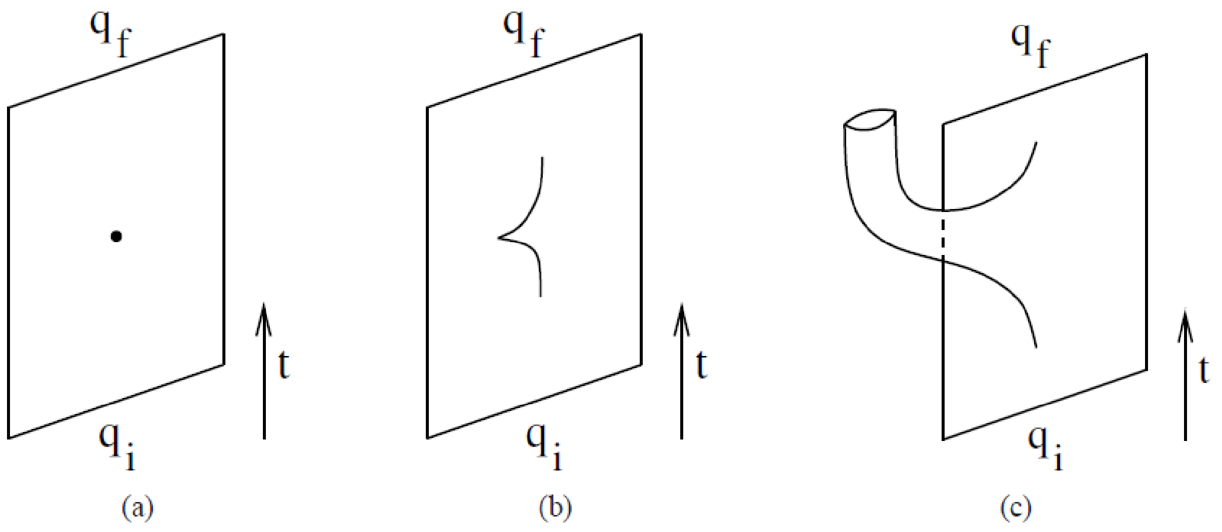


Wormhole is a dipole. There is no monopole axion charge, only locally at one side.

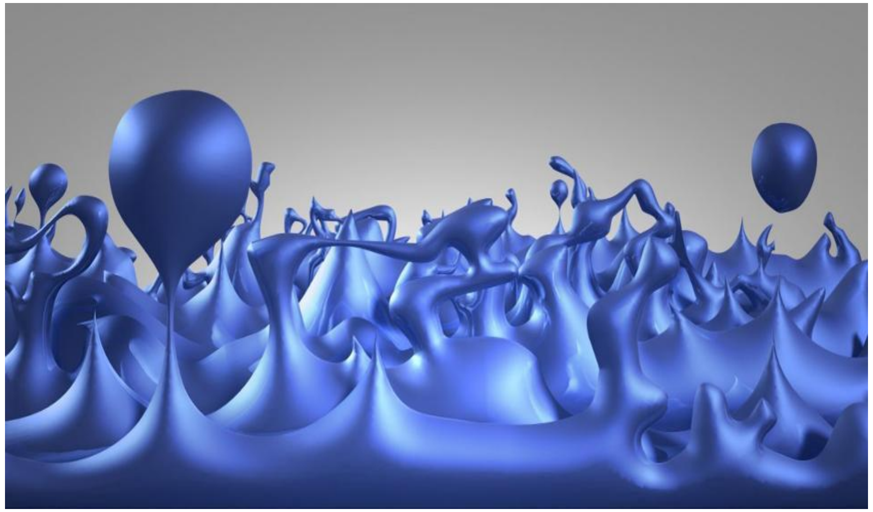
Finite action: $\boxed{S \sim |Q|}$

Very rich and long history in quantum gravity, prior to string theory. Recent revival in string theory due to Swampland discussions & holography. See [Hebecker, Mikhail, Soler 2018] for comprehensive review

Interpretation as tunneling instantons describing nucleation of baby universes → only if cut in half:



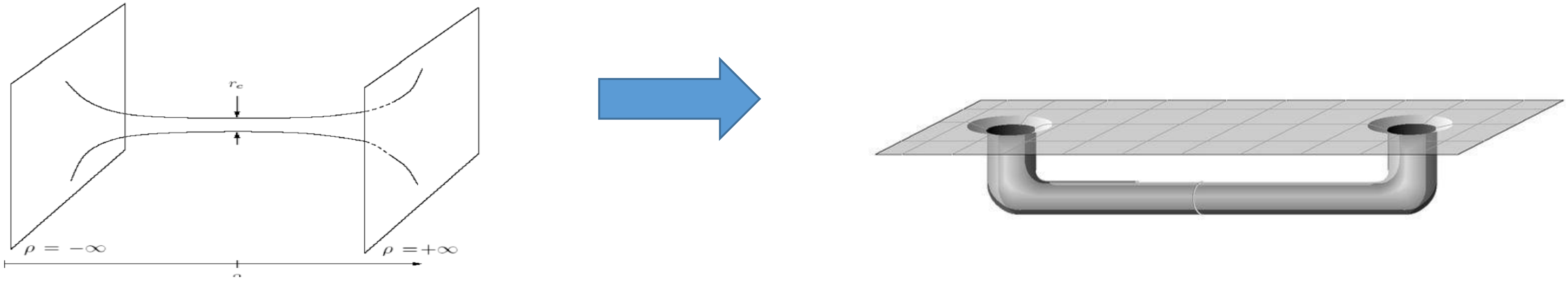
[Giddings/Strominger 1987, Lavrelashvili/Tinyakov/Rubakov 1998, Hawking 1987, …]



→Full wormhole describes emission *and* subsequent absorption of baby universe. Tunneling probability Planckian suppressed. (Planckian sized universes only)

An observer detects a violation of axion charge conservation. (Not surprising since it is global symmetry.) Related phenomenom of ***non-unitarity***.

If one glues the two boundaries into one space-time:



then wormholes represent a breakdown of (macroscopic) locality : the effective action would include operators of the form

$$S_{WH} = -\frac{1}{2}\sum_{IJ}\int d^D x\, d^D y\, \mathcal{O}_I(x) C_{IJ} \mathcal{O}_J(y)\ ;$$

[Coleman 1998]: Not really since

$$e^{-S_{WH}} = \int d\alpha_I\, e^{-\frac{1}{2}\alpha_I (C^{-1})_{IJ}\alpha_J}\, e^{-\int d^D x \sum_I \alpha_I \mathcal{O}_I(x)}\ .$$

ENSEMBLES

Euclidean Stability

Interpretation of instantons depends on **stability**

Perform Gaussian approximation around saddle point:

$$Z = e^{-S[\Phi_0]} \int \mathrm{D}\phi \, e^{-\delta^2 S[\Phi_0,\phi] + \mathcal{O}(\phi^3)} \qquad \delta^2 S = \frac{1}{2} \int \phi \hat{\mathrm{M}} \phi$$

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Solve eigenvalue problem: $$\frac{1}{X} \hat{\mathrm{M}} \phi_n = \lambda_n \phi_n \,, \qquad \int X \, \phi_n \phi_m = \delta_{nm}$$

To find: $$Z \sim e^{-S[\Phi_0]} \int \Pi_n \mathrm{d}z_n \, e^{-\frac{1}{2} \sum_n \lambda_n z_n^2} \sim \frac{e^{-S[\Phi_0]}}{\sqrt{\Pi_n \lambda_n}} \,.$$

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*Coleman: in QM & QFT we have standard instantons (all eigenvalues positive) or "bounces" with **one** negative eigenvalue. The latter describe tunneling amplitudes. **Multiple** negative eigenvalues means instanton is spurious.*

- Literature: there is possibly one negative eigenmode, which is expected from tunneling interpretation [Rubakov 1989, Kim&Lee&Myung 1997, Kim&Kim&Hetrick2003, Alonso&Urbano 2017].

- [Hertog, Truijen, VR 2018] Computations did not use the right gauge-invariant variables + Interpretation as path integral for axion-charge transitions is crucial.

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Boundary conditions

We want matrix elements from charge eigenstates = momentum eigenstates

$$|\Pi\rangle = |Q\rangle$$

So we wish to evaluate

$$K \equiv \langle \Pi_F | \exp(-HT) | \Pi_I \rangle \qquad |\Pi\rangle = \int \mathrm{d}[\chi]\, e^{i\int_\Sigma \chi \Pi} |\chi\rangle$$

$$\Rightarrow \quad K = \int d[\chi] \mathrm{e}^{-\frac{1}{\hbar}\int L[\chi,\partial\chi]} \mathrm{e}^{-i\int_{\Sigma_F} \Pi_F \chi + i\int_{\Sigma_I} \Pi_I \chi} .$$

NO BC

Saddles obey: $\Box\chi = 0$ $(\star \mathrm{d}\chi - i\Pi)|_{\Sigma_{I,F}} = 0$

$\Rightarrow$ "Euclidean free field action with wrong sign kinetic term"

Equivalent to

$$Z = \int_{bc} d[F]d[\chi]\mathrm{e}^{-\frac{1}{\hbar}\int \star F\wedge F + i\chi \mathrm{d}F} \qquad F_{I,F} = \star\Pi_{I,F}$$

→ Boundary conditions! Dirichlet for momentum.

Taking this into account gives well-behaved quadratic action. *No conformal factor problem (no Hawking-Perry rotation)*! Reason: homogenous modes non-dynamical.

Use formalism of cosmological perturbation theory [Gratton-Turok 1999]

$$\begin{aligned} \mathrm{d}s^2 =& b^2\Big(-(1+A)^2\mathrm{d}\eta^2 + \partial_i B \mathrm{d}x^i \mathrm{d}\rho + \\ & [(1-2\psi)\gamma_{ij} + \partial_i\partial_j E]\mathrm{d}x^i\mathrm{d}x^j\Big) \ , \end{aligned}$$

We focus on scalar perturbations and use the following gauge invariant observable:

$$\boxed{\mathcal{X} = \psi + \frac{b'}{b\chi'}\delta\chi}$$

After a mode decomposition and lengthy computation (software) :

$$S_2 = \frac{\mathrm{Vol}(S^3)}{\kappa^2}\int \mathrm{d}\rho\left(A_n\dot{\mathcal{X}}_n^2 - B_n\mathcal{X}_n^2\right)$$

With A_n, B_n certain functions of Euclidean time.

Crucially we need the quadratic action for the momenta instead! Formal manipulation;

$$\boxed{S_2 = \frac{\mathrm{Vol}(S^3)}{\kappa^2} \int \mathrm{d}\rho \left(-B_n^{-1} (\dot{\Pi}_{\mathcal{X}}^n)^2 + A_n^{-1} (\Pi_{\mathcal{X}}^n)^2\right) .}$$

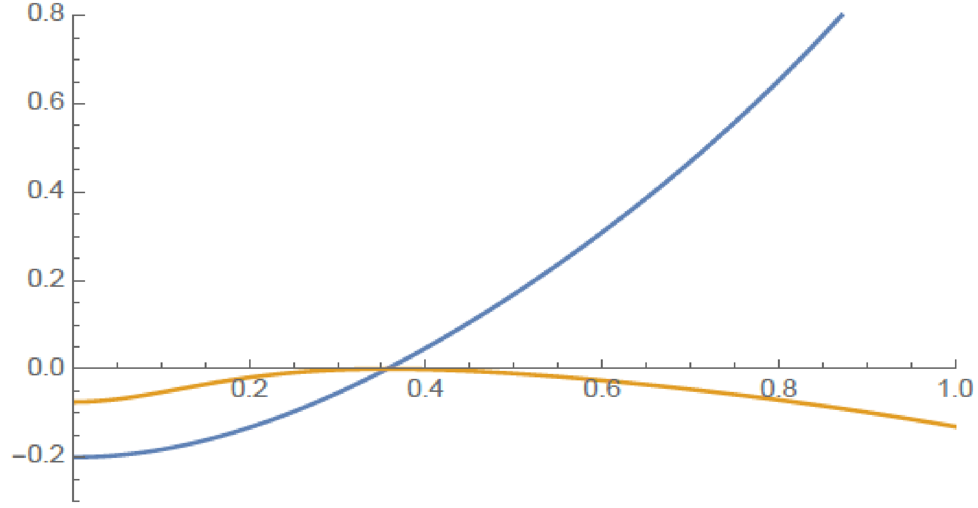
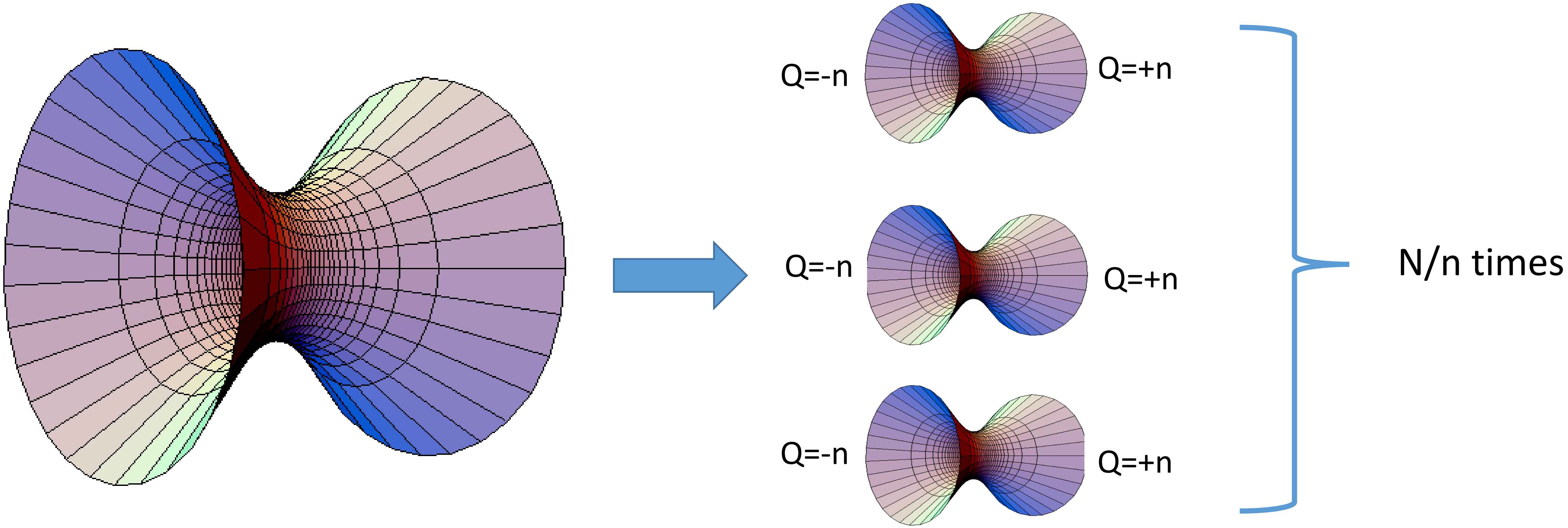


FIG. 1: *The coefficients* A_n^{-1} *(blue) and* B_n^{-1} *(orange) entering in the action for perturbations about axion wormholes, shown here for* $n = 3$ *(and with* $c = 1$*).*

Kinetic term positive. Potential bounded from below and negative only near neck. But enough to find square integrable test functions that lower the total action. Only for n>2.

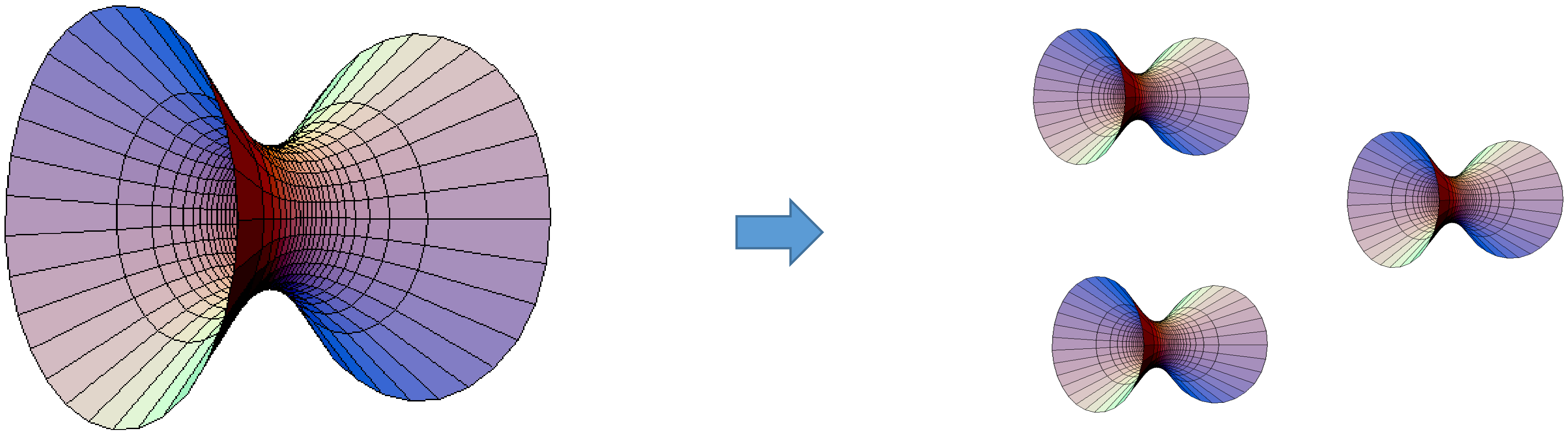
Infinitely many modes lower the action. All centered close to the neck and probe the non-trivial topology. For very small wormholes those modes become sub-planckian.

→ Macroscopic wormholes do not contribute. There is a lower action saddle with same boundary conditions? Which one? → wormhole fragments into smaller wormholes.



Wormhole defragmentation

- Consistent with picture of [McNamara&Vafa, 2020]: dimension baby universe Hilbert space equals 1.
- In line with recent paper of [Marolf&Santos 2021] on general instabilities of wormholes from string theory.
- **Microscopic** instantons cannot be argued to be spurious. But they are not wormholes



Criticism?

- Kinetic term has a zero. Well-behaved self-adjoint operator only required when computing determinants in case of stability. No reason for “bad instantons” to have nice operator.

- Rotation 1: We Wickrotated cosmological (Lorentzian) perturbation theory. Is there a catch?

- Rotation 2: The axion is known to Wickrotate, but we Wickrotated the gauge invariant combination

$$\chi \to i\chi \qquad \xrightarrow{??} \qquad \mathcal{X} \to i\mathcal{X}$$

Fluctuation theory directly in Euclidean space with form field having normal kinetic term can be shown to yield identical results. [Hertog, Maenaut, Tielemans, VR, in progress]

Stability questions for the (near) future

- *Multiple field analysis*. String theory provides extended axion-saxion models. Changes stability conclusion? Preliminary computations suggest no:

→ Using Riemann normal coordinates $$G_{ij}(\phi^i + q^i) = \eta_{ij} - \frac{1}{3} R_{ikjl} q^k q^l$$

Computation effectively boils down to decoupled scalar system. Instability remains.

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- Coleman wormholes lead to *ensemble* theories. Here ensemble would be integration over marginal couplings in the CFT. Could wormhole stability be saved in an ensemble?

→Suggestion M. Montero [Montero, VR unpublished]: Ensemble theories are dual to AdS bulk solutions with *mixed* boundary conditions. Then stability of wormholes *naively* seem possible due to results of [Urbano et al (2017)]? Preliminary computations *do not* support it.

Interpretation & extremality

Recall notion of **extremality** in GR. Charged black holes.

The metric:

$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega_2^2$$

$$V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r}$$

- *Sub-extremal* : Q < M (eg Schwarzschild)
- *Extremal* : Q = M (Inner and outer horizon coincide)
- *Super-extremal*: Q > M (Naked singularity)

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Extremal solution allows a generalization to multiple centers:

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Physics? Gravitational attraction equals coulomb repulsion. Over-extremal solutions have stronger repulsion than attraction. All charged particles in the Standard Model! But they cannot be seen as black holes. *Microscopic versus macroscopic*. **Same applies to charged branes […]. What about instantons?**

You need some extra bells and whistles. Inspiration from reducing black hole in D+1 dimensions “over time” to instanton in D. The reduction of vector potential gives axion, size of extra dimension gives “saxion”:

$$G_{ij}\partial\phi^i\partial\phi^j = (\partial\phi)^2 - e^{b\phi}(\partial\chi)^2$$

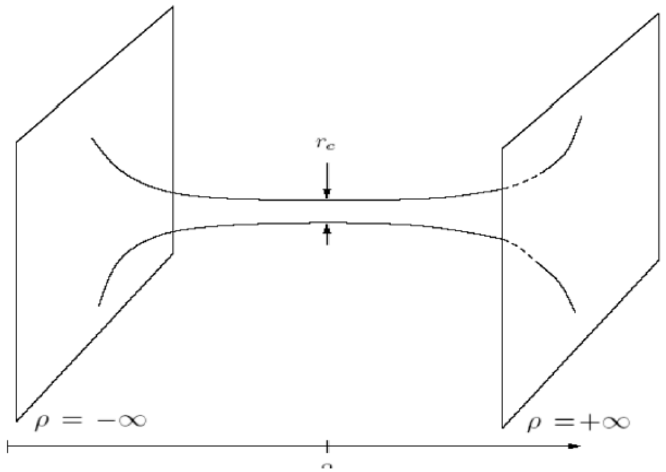
Solutions?:

$$\boxed{\mathrm{d}s^2 = \left(1 + \frac{\tau^2}{\ell^2} + \frac{c}{2(D-1)(D-2)}\tau^{-2(D-2)}\right)^{-1}\mathrm{d}\tau^2 + \tau^2\mathrm{d}\Omega^2}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}h^2}\phi^i + \Gamma^i_{jk}\frac{\mathrm{d}}{\mathrm{d}h}\phi^j\frac{\mathrm{d}}{\mathrm{d}h}\phi^k = 0 \quad\Rightarrow\quad \boxed{G_{ij}\frac{\mathrm{d}}{\mathrm{d}h}\phi^i\frac{\mathrm{d}}{\mathrm{d}h}\phi^j = c}$$

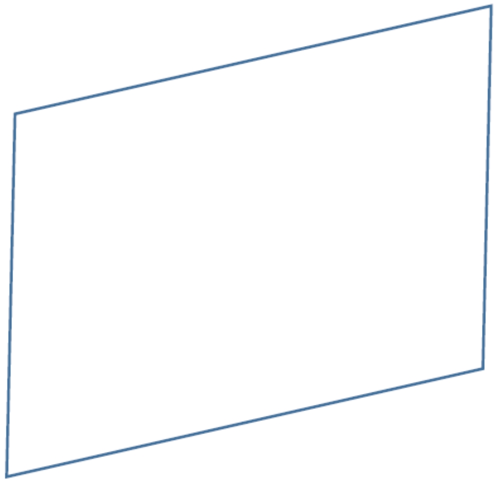
Works for ANY sigma model.

“Over-extremal” c < 0



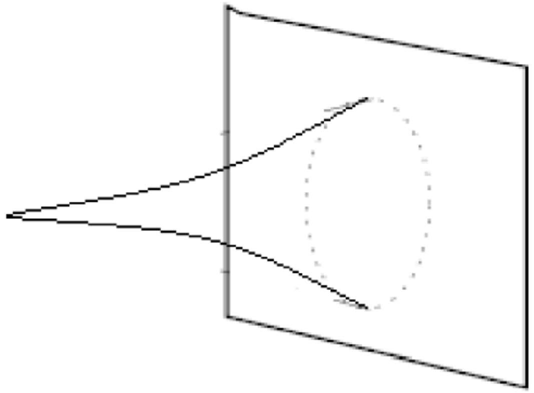
``Time-like” geodesics

“Extremal” c = 0 :



``Light-like’’ geodesics

“Under-extremal” c > 0 :



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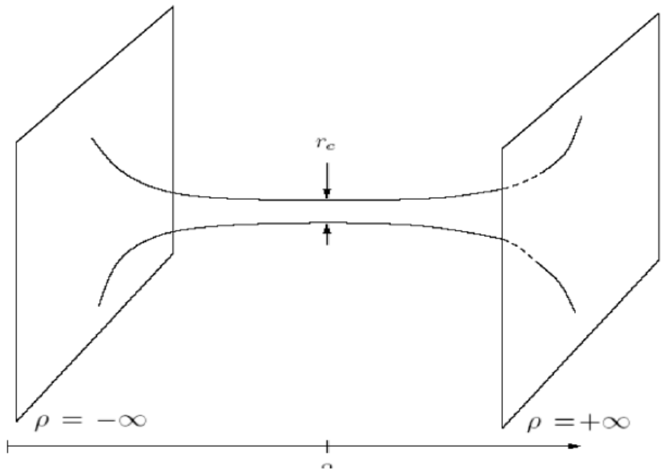

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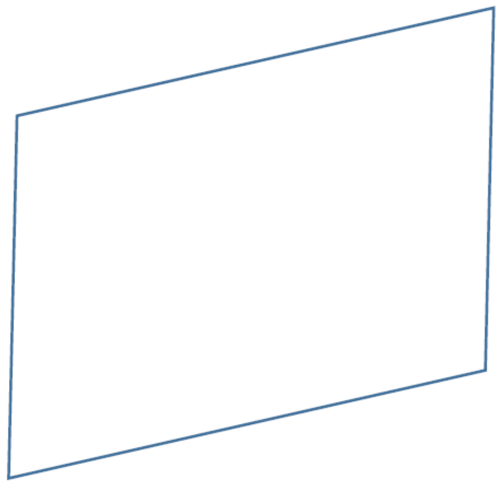

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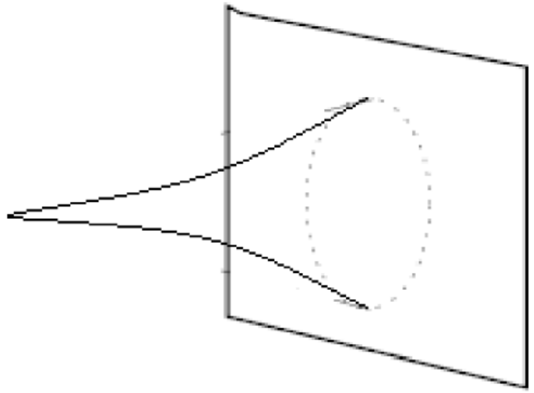

``Space-like" geodesics

→ **With the "saxion" we can introduce notion of extremality.**

This way embedding in string theory/holography can be made very explicit.

Extremality for instantons, how exactly? [VR, 2019, 2020]

- On-shell action (or direct dimensional reduction of black holes)

$$\boxed{S \sim |Q| e^{-b\phi(\infty)/2} \sqrt{1 + \frac{c}{Q^2} e^{b\phi(\infty)}},}$$

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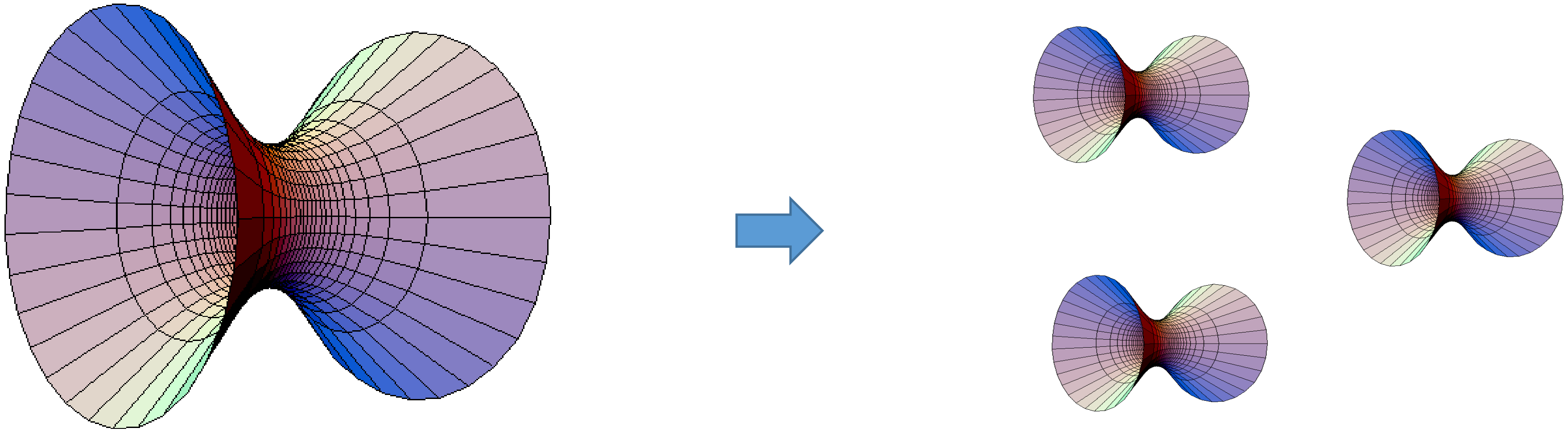
Over-extremal black holes unphysical. Not over-extremal particles. What about over-extremal instantons (Colemans wormholes)? There is no naked singularity to warn us.

It is the instability in the path integral that makes them unphysical. Instability is in non-homogenous sector: signals fragment into smaller pieces to lower action. Just like super-extremal "black holes" shatter into super-extremal particles that cannot decay anymore. Microscopic over-extremal instantons are physical?

Wormhole fragmentation



Holography

Coleman wormholes have no support from AdS/CFT [Arkani-Hamed/ Orgera/ Polchinski 2007, Maldacena/ Maoz 2004] Dual field theory has no sign of Coleman's α parameters.

Can we say something new? Regard the wormholes as one of the 3 instanton classes by adding saxions (moduli).

- Moduli inside AdS are coupling constants for exactly marginal operators in the dual field theory: they label the family of CFT's = *conformal manifold.*

- Metric Gij on moduli space corresponds to the `Zamolodchikov' metric gij defined by the two-point functions:

$$g_{ij}(\varphi) = x^{2\Delta} \langle O_i(x) O_j(0) \rangle_{S[\varphi]}$$

- Holography suggest that some *geodesic curves on the conformal manifold correspond to instantons of the CFT*.

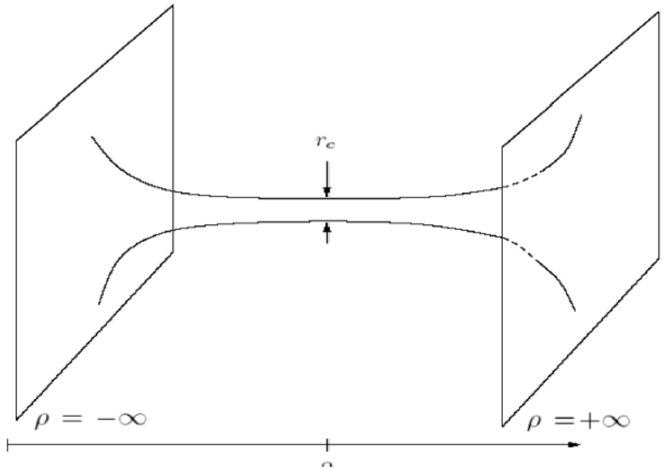
We [Katmadas, Ruggeri, Trigiante, VR, 2018] studied $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_\mathrm{k}$

Dual theory is N=2 "necklace quiver CFT" [Kachru, Silverstein '98] and has k gauge nodes → hence k complex couplings (k theta-angles), which form the conformal manifold.

$$\mathcal{L} \supset \sum_{\alpha=0}^{k-1} \left(-\frac{1}{4g_\alpha^2} \mathrm{Tr}[F_\alpha^2] - i\frac{\theta_\alpha}{32\pi^2} \mathrm{Tr}[F_\alpha \wedge F_\alpha] \right)$$

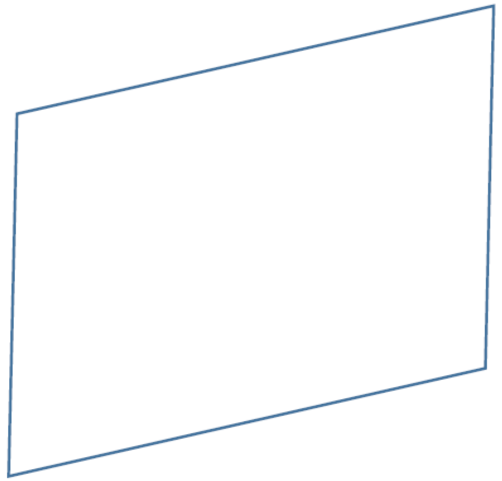
moduli space $\dfrac{\mathrm{SU}(1,k)}{\mathrm{S}[\mathrm{U}(1) \times \mathrm{U(k)}]} \Longrightarrow \dfrac{\mathrm{SL}(k+1,\mathbb{R})}{\mathrm{GL}(k,\mathbb{R})}$. 2k real scalars.

“Over-extremal” c < 0



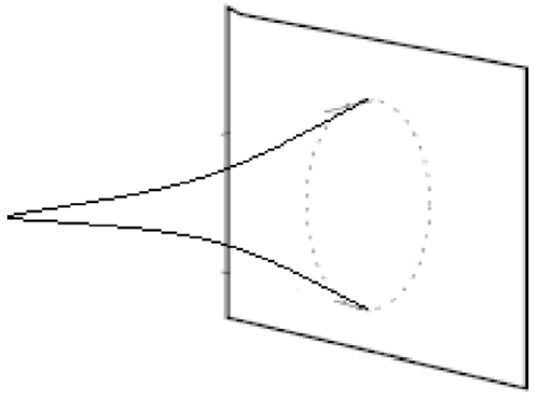
``Time-like” geodesics

“Extremal” c = 0 :



``Light-like” geodesics

“Under-extremal” c > 0 :



``Space-like” geodesics

Main results are

- <u>**SUSY solutions**</u> match SUSY gauge theory instantons. (One point functions & on-shell actions)

- <u>**non-SUSY solutions but extremal**</u>: Some of them can be interpreted and match so called "quasi-instantons" [Imaanpur 2008]. *These are solutions which are self-dual in each separate gauge node, but orientations differ from node to node. Very simple way of SUSY-breaking!*

$$\mathrm{Tr}[F_\alpha^2] = \mathrm{sign}(N_\alpha)\mathrm{Tr}[F_\alpha \wedge F_\alpha]$$

Potryagin index = axion charge quantum

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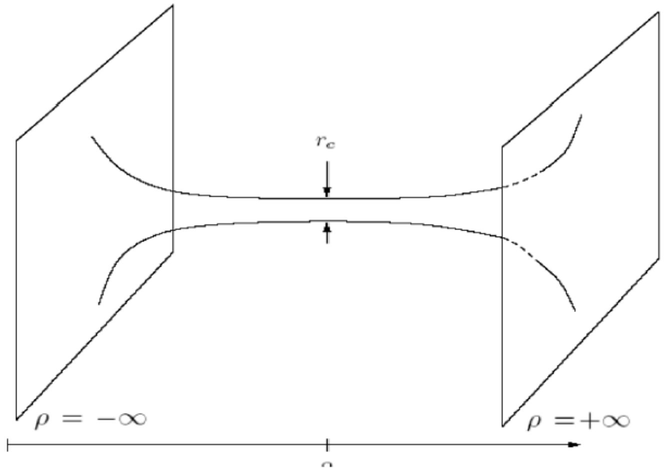

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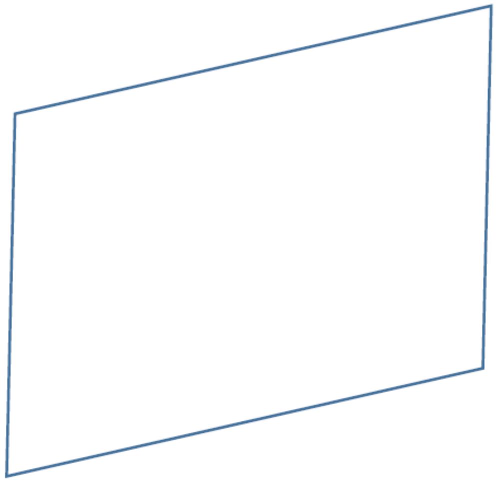

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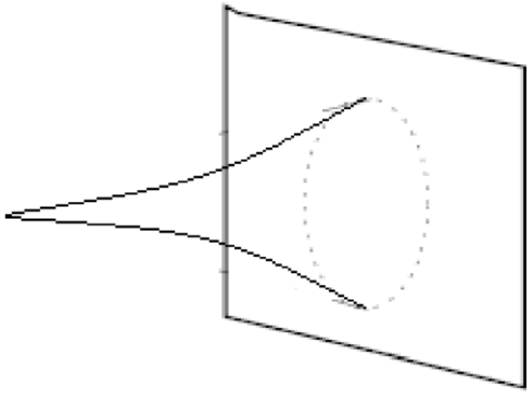


``Space-like” geodesics

- Solution is singular, but singularity seems ok?

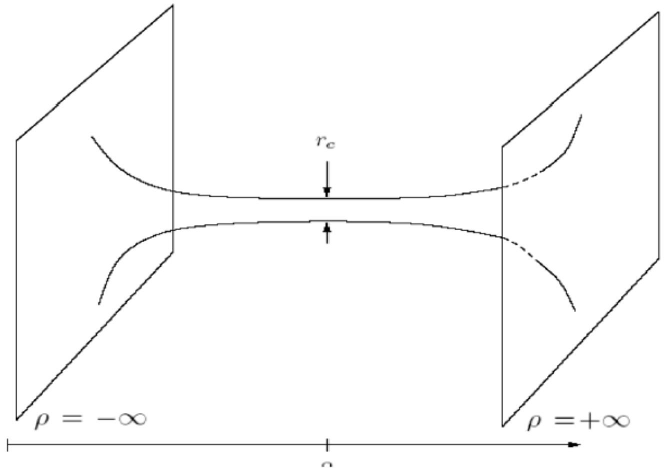
- Suggestion for holographic dual from computing one point functions & action.

non-self dual YM instantons…

[Bergshoeff, Collinucci, Ploegh, Vandoren, VR 2005]

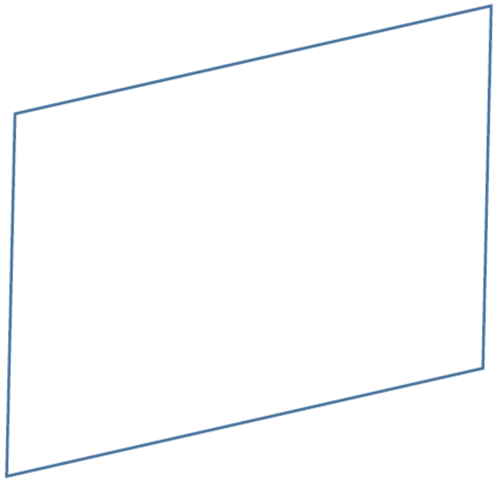
$$A_\mu^{\mathrm{SU(N)}} = \begin{pmatrix} A_\mu^{\mathrm{SU(2)}} & 0 & \cdots & 0 \\ 0 & A_\mu^{\mathrm{SU(2)}} & & 0 \\ \vdots & & \ddots & \\ 0 & & & \overline{A}_\mu^{\mathrm{SU(2)}} \end{pmatrix} .$$

"Over-extremal" c < 0



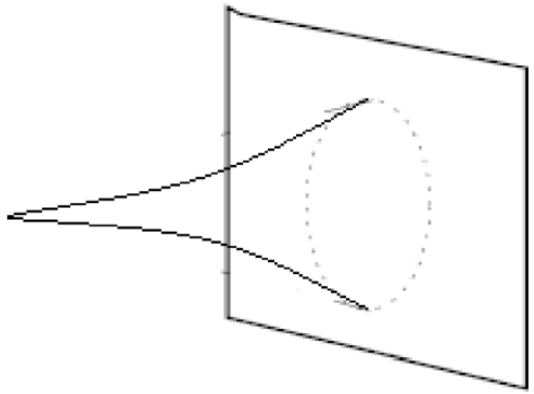
``Time-like" geodesics

"Extremal" c = 0 :



``Light-like" geodesics

"Under-extremal" c > 0 :



``Space-like" geodesics

- First examples of smooth Euclidean axion wormholes in AdS! Despite claim in [Arkani-Hamed &Orgera & Polchinski 2007], **no** smooth examples in D1-D5 system (AdS3xS3xT4 or AdS3xS3xK3) [Astesiano, Trigiante, Ruggeri, VR, in progress.]

- First examples of smooth Euclidean axion wormholes in AdS! Despite claim in [Arkani-Hamed &Orgera & Polchinski 2007], **no** smooth examples in D1-D5 system (AdS3xS3xT4 or AdS3xS3xK3) [Astesiano, Trigiante, Ruggeri, VR, in progress.]

- Our explicit embedding provides another paradox: holographic one-point function give violation of positivity [Katmadas, Ruggeri, Trigiante, VR, 2018]:

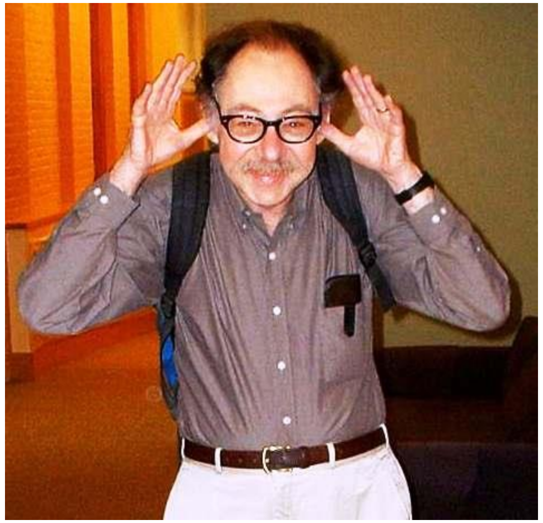
$$|\mathrm{Tr}[F_\alpha^2]| < |\mathrm{Tr}[F_\alpha \wedge F_\alpha]|\,.$$

- Field theories without gravity do not allow a notion of super-extremality. BPS bounds cannot be violated. It requires gravity. But AdS gravity = CFT.

→ **Again evidence for spurious nature of wormholes.**

Summary

Are Coleman's Euclidean wormholes real?



(S. Coleman 1937-2007)

Probably not as they are self-repulsive in a Euclidean sense. So macroscopically sized wormholes (bigger than Planck scale) won’t contribute in the path integral. We gave direct evidence through computation. And then a GR-like interpretation plus evidence from holography.

Remaining questions

- Multifield analysis
- Averaging over marginal couplings makes stable?
- End-point wormhole fragmentation? → study corrections to small wormholes
  [Andriolo, Ooguri, Shiu, Soler 2020], [Andriolo, Shiu, Soler, VR, in progress]

Thank you!

EXTRA

$$S[A] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p$$

$$S[F, B] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p + dF_p \wedge B_{D-p-1} \qquad dB = G_{D-p}$$

With partial integration, and dropping a boundary term, we can get:

$$S[F, B] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p + (-1)^{p+1} F_p \wedge G_{D-p}$$

the EOM for F gives:

$$\star F_p = (-1)^{p(D-p)} G_{D-p} \quad \Longrightarrow \quad S = \int \star R + \tfrac{1}{2}(-1)^t \star G_{D-p} \wedge G_{D-p}$$

$$
\begin{aligned}
Z &= \int \mathcal{D}q \; exp\left[-\int dt \left(-\frac{A}{2}\dot{q}^2 + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt \left(\frac{A^{-1}}{2}(p - A\dot{q}^2) - \frac{A}{2}\dot{q}^2 + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt \left(\frac{A^{-1}}{2}p^2 + \dot{p}q + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt \left(\frac{A^{-1}}{2}p^2 + \frac{B}{2}(q + B^{-1}\dot{p})^2 - \frac{B^{-1}}{2}\dot{p}^2\right)\right] \\
&= \int \mathcal{D}p \; exp\left[-\int dt \left(-\frac{B^{-1}}{2}\dot{p}^2 + \frac{A^{-1}}{2}p^2\right)\right]
\end{aligned}
$$

Boundary

$$\mathcal{Z}_{\mathrm{QG}}(X) = \mathcal{Z}_{\mathrm{CFT}}(X).$$

$$\mathcal{H}_{\mathrm{QG}}(M_1 \sqcup M_2) = \mathcal{H}_{\mathrm{QG}}(M_1) \otimes \mathcal{H}_{\mathrm{QG}}(M_2)$$

$$\mathcal{H}_{\mathrm{BU}} = \mathcal{H}_{\mathrm{QG}}(\varnothing)$$

$$\varnothing \sqcup M = M,$$

$$\mathcal{H}_{\mathrm{QG}}(M) = \mathcal{H}_{\mathrm{BU}} \otimes \mathcal{H}_{\mathrm{QG}}(M)$$