# The OPE Randomness Hypothesis & Euclidean Wormholes

Based on:

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Workshop on Quantum Gravity, Holography and Quantum Information

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# The AdS/CFT correspondence:

$$\boxed{Z_{CFT} = Z_{AdS}}$$

For ex: IIB String Theory on $AdS_5 \times S^5$

At large N, strong coupling:

$$Z_{AdS} \approx \int \mathcal{D}\phi \, e^{-S_{EFT}}$$

$$S_{EFT} = S_{GR} = -\frac{1}{16\pi G_N} \int d^{d+1}x \sqrt{g} \, (R - 2\Lambda) + S_{matter}$$

The focus of my talk today:

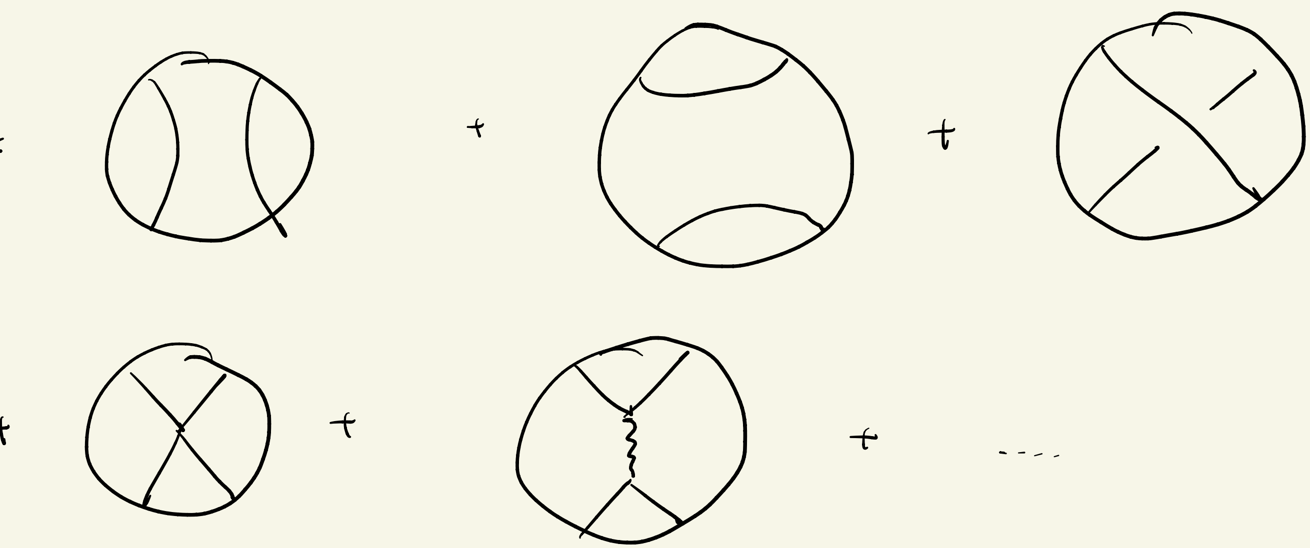
- What part of the CFT dynamics does $S_{EFT}$ have access to?

- What is the "CFT dual" of $S_{EFT}$

## $\boxed{1}$ Standard/Perturbative EFT

$$S_{EFT} : \quad g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$$

$\Rightarrow$ Compute low-point correlation functions of the dual operators

$\langle O \, O \, O \, O \rangle =$ 

**Exact** map between parameters of $S_{EFT}(m, \lambda)$ and the CFT data $(\Delta_O, C_{O_1 O_2 O_3})$ in the $1/N$ expansion.

$\boxed{2}$ Beyond perturbative EFT, black holes

$S_{EFT} \longrightarrow$ Classical non-linear solutions to the E.O.M.

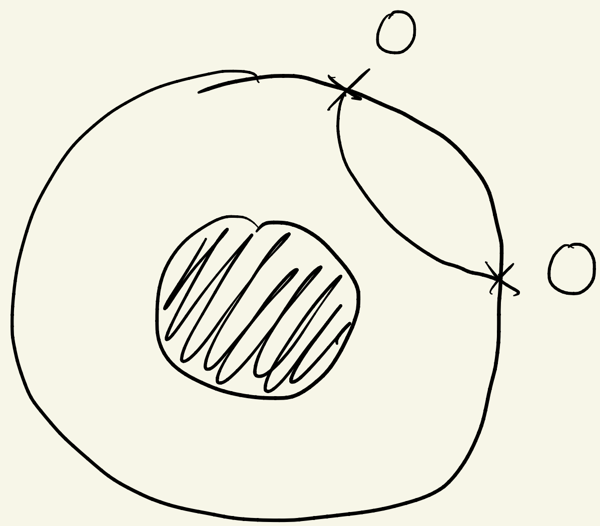
$$Z_{CFT} \approx e^{-S_{on\text{-}shell}} \Rightarrow S_{BH} = \frac{A_H}{4 G_N}$$

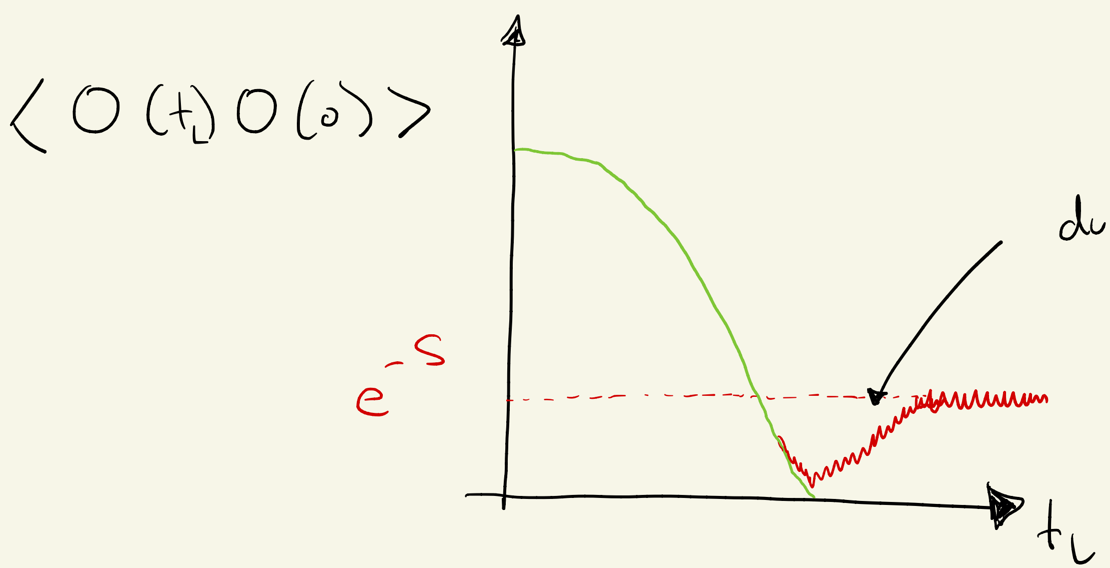
$$\rho_{bulk}(E) = e^{S_{BH}(E)}$$ smooth function

$$\rho_{bulk}(E) \overset{?}{\Longleftrightarrow} \rho_{CFT}(E)$$

$$\rho_{CFT}(E) = \sum_{E_i} \delta(E - E_i)$$

Coarse-graining: $\quad \rho_{bulk}(E) = \int_{E-\delta E}^{E+\delta E} \rho_{CFT}(E)$



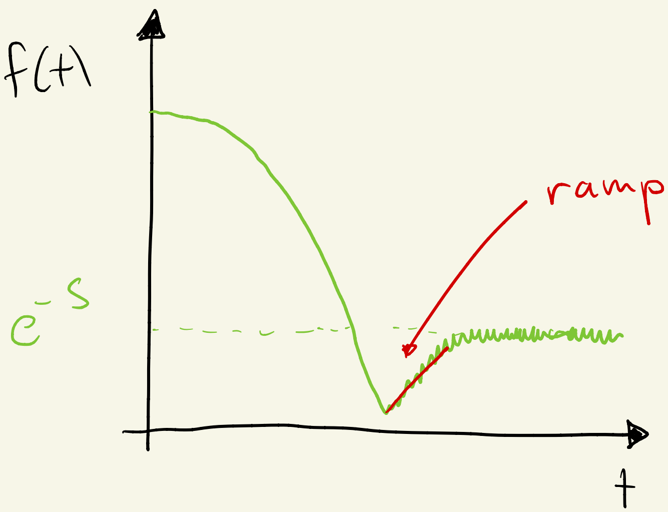

due to discreteness, cannot capture

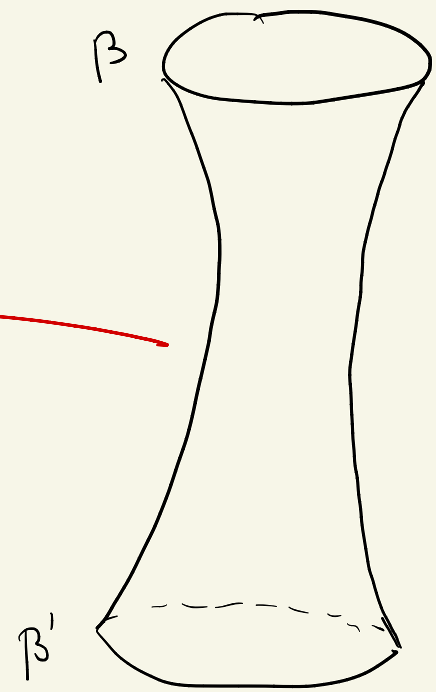
[Maldacena]

## $\boxed{3}$ Way beyond EFT, Euclidean Wormholes, discreteness

$$f(t) = Z(\beta+it)\,Z(\beta-it)$$

[SSS]




JT gravity

Can capture a universal feature of quantum chaos involving <u>discreteness</u> from semi-classical path integral.

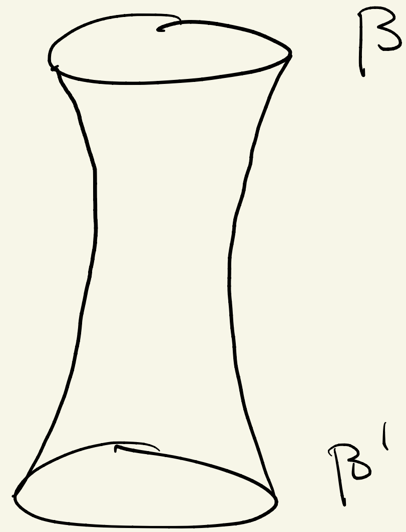
$\Rightarrow$ Not unrelated to progress on the Information Paradox

# Price to Pay

[Maoz, Maldacena]

Factorization problem

$Z(\beta)\, Z(\beta') \xrightarrow{\text{gravity}}$




$= f(\beta,\beta') \neq \tilde{f}(\beta)\,\tilde{f}(\beta')$

Missmatch

$Z_{bulk} \overset{?}{=}$ ensemble average

## Goal for today:

- Give framework that can capture $\boxed{1}, \boxed{2}, \boxed{3}$
- Provide perspective on the lack of factorization.

## Outline:

1. Introduction
2. Review of ETH, Introduce ORH
3. Application 1: genus-2 wormhole
4. Application 2: no global symmetries in Q.G.

# Review of ETH

$$\langle E_i | O^a | E_j \rangle = f^a(\bar{E})\,\delta_{ij} + g^a(\bar{E},\omega)\, R_{ij}$$

$$g^a(\bar{E},\omega) \rightarrow \tilde{g}^a(\bar{E},\omega)\, e^{-S(\bar{E})/2}$$

$$\bar{E} = \frac{E_i + E_j}{2}$$

$$\omega = E_i - E_j$$

$f^a$, $g^a$ } smooth functions

$R_{ij}$, erratic numbers, Gaussian random variables with 0 mean, unit variance

# 2 comments:

- In any fixed theory, $R_{ij}$ are <u>not</u> random.

For many practical purposes, treat them as random variables, $\overline{R_{ij}} = 0$ $\overline{R_{ij} R_{kl}} = \delta_{il}\delta_{jk}$

- $R_{ij}$ are not Gaussian. Non-gaussianities are needed to reproduce k-point correlation functions.

# ETH in CFT

$$\langle E_i | O^a | E_j \rangle = \langle O_i \, O^a \, O_j \rangle = C_{ij}{}^a$$

$O^a$ light, $\Delta_a$ fixed

$O_{i,j}$ heavy, $\Delta_{i,j} \to \infty$

$C_{ij}{}^a$ HHL

$C_i{}^{ab}$ HLL

$C_{ijk}$ HHH

What about these?

# OPE Randomness Hypothesis (ORH)

In a chaotic CFT:

$$C_i^{ab} = \sqrt{f^{ab}}\, R_i$$

$$C_{ijk} = \sqrt{f}\, R_{ijk}$$

$R_i$, $R_{ijk}$ are Gaussian random variables with unit variance and 0 mean.

In 2d CFTs:

$f^{ab} \longrightarrow$ 4pt fcts on the plane, crossing

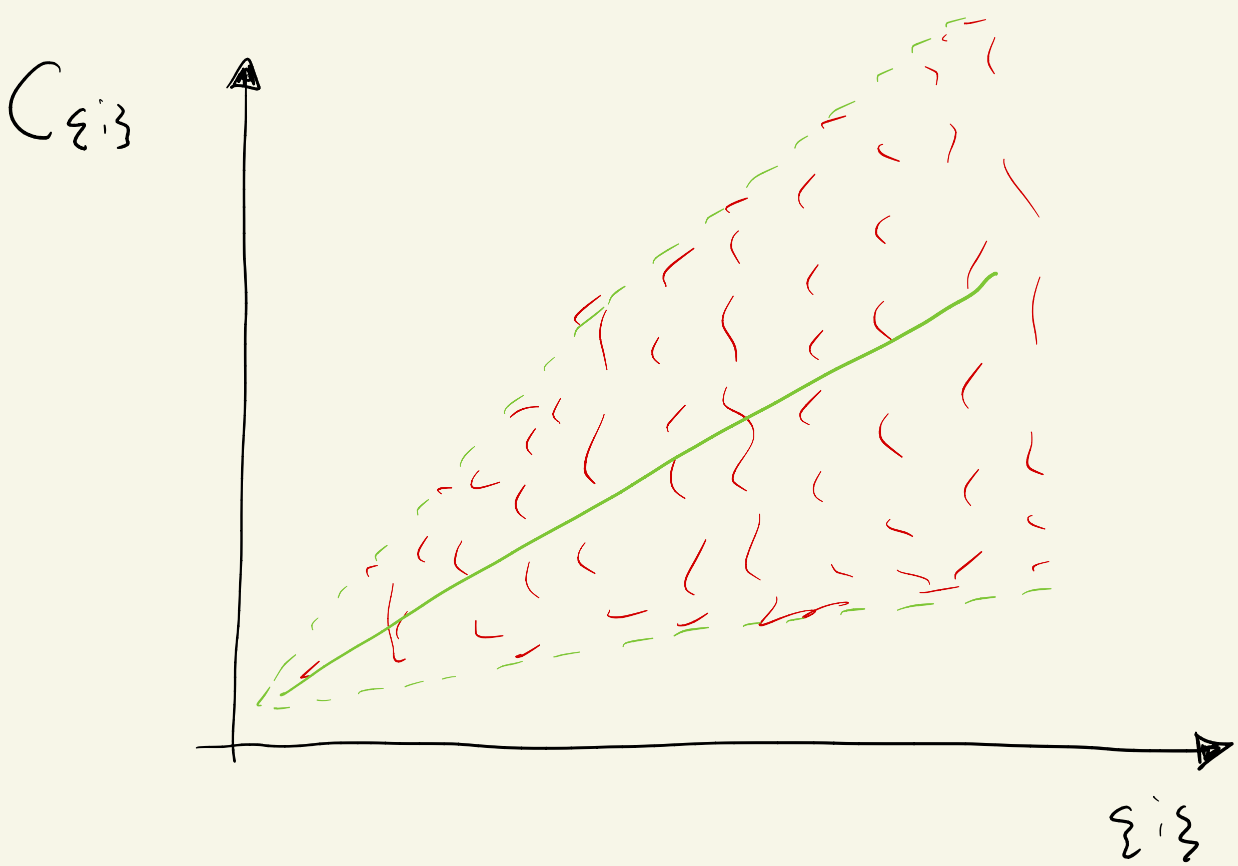
$f \longrightarrow$ genus-2 partition function, mod. inv. $\quad f(\Delta) = \sum |C_{ijk}|^2 = \left(\frac{27}{16}\right)^{3\Delta} e^{-3\pi\sqrt{\frac{c}{3}\Delta}}$

[Collier, Maloney, Maxfield, Tsiares; many more]

# ORH in Holography

Euclidean Path Int. of GR $\Longleftrightarrow$ ORH

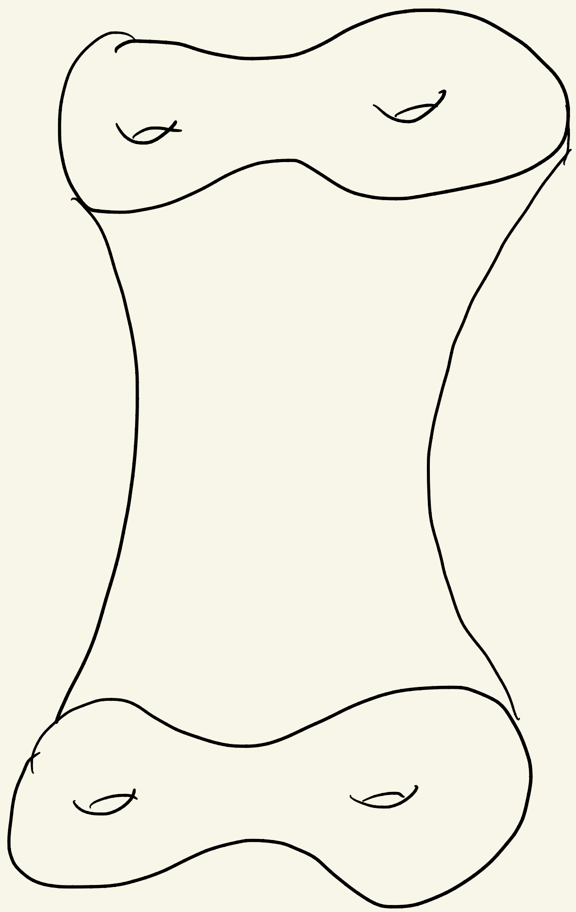
GR cannot resolve black hole microstates.




$\Rightarrow$ can capture only moments

$\Rightarrow$ Makes a mistake! Wormholes

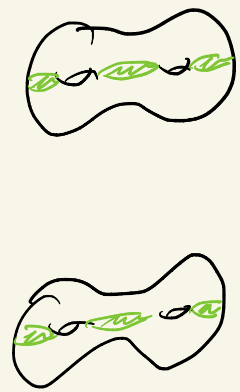
# Application 1: the genus-2 wormhole



$$ds^2 = d\tau^2 + \cosh^2\tau \, d\Sigma_{g=2}^2$$

$$Z_{g=2} \approx Z_{g=2}\big|_{\text{gravity}}$$

$=$



$+$

$\beta$

$\beta'$

$$= e^{\frac{c}{2}\frac{\pi^2}{\beta}} + 1 \times f(\beta, \beta')$$

$$Z_{g=2} \times Z_{g=2} \Big|_{\text{microsc.}} \neq Z_{g=2} \times Z_{g=2} \Big|_{\text{gravity}}$$

$$Z_{g=2} = \sum_{i,j,k} |C_{ijk}|^2 e^{-\beta(\Delta_i + \Delta_j + \Delta_k)}$$

$$Z_{g=2} \times Z_{g=2} \Big|_{\text{ORH}}$$

$$= \sum_{\substack{i,j,k \\ l,m,n \\ o,p,q \\ r,s,t}} \delta_{il}\, \delta_{jm}\, \delta_{kn}\, \delta_{or}\, \delta_{ps}\, \delta_{qt}\, C_{ijk}\, C_{lmn}\, C_{opq}\, C_{rst}\, e^{-\beta(\Delta_i + \Delta_j + \Delta_k) + \beta'(\Delta_r + \Delta_s + \Delta_t)}$$

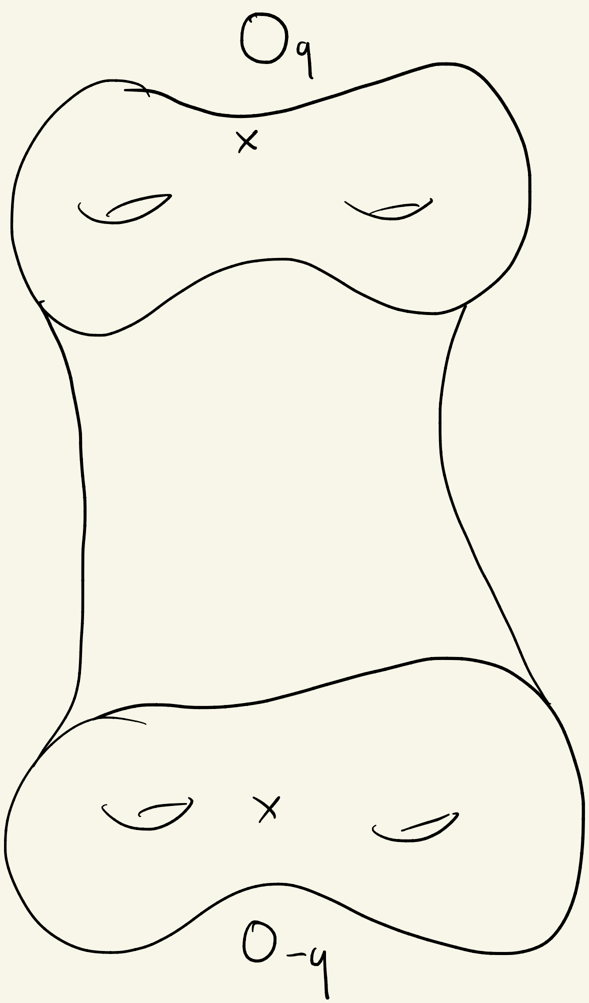
$$= \underline{\underline{e^{\frac{c}{2}\frac{\pi^2}{\beta}}}} + \underline{\underline{1}} + f(\beta, \beta')$$

$\Leftarrow$ matches the answer from gravity

## ④ Application 2: No global symmetries in Q.G.

See also [Chen, Lin] [Hsin, Iliesiu, Yang]

$$S_{EFT} = S_{EH} + \frac{1}{2}\int (\partial\varphi)^2 + m^2(\varphi)^2$$




$\neq 0$

$\Rightarrow$ This computes variance of $\langle O_q \rangle_{g=2}$

Problem / Puzzle

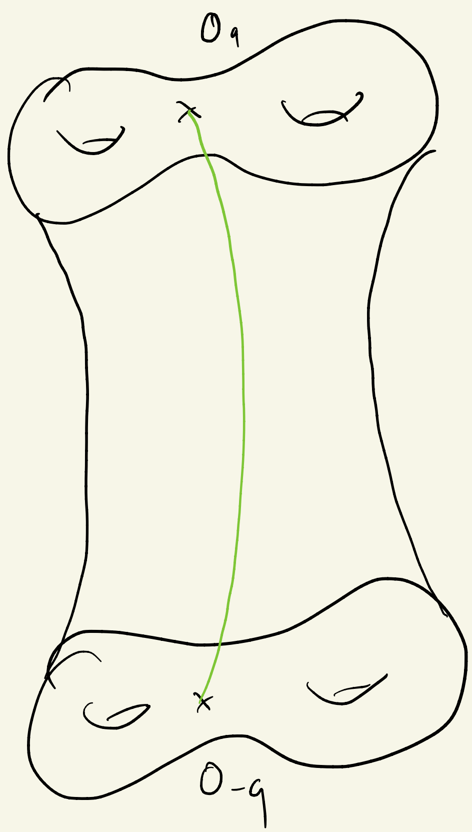
$$\langle O_q \rangle_{\Sigma_{g=2}} = 0$$

Exactly! (By charge conj.)

No erratic signal

# 2 possibilities

The symmetry is gauged in the bulk




$= 0$

The symmetry is broken (at least) by non-perturbative effects.

$$\text{Breaking} \sim e^{-S}$$

# ETH and charge

V1 : Exact charge conservation

$$\langle E_i, Q_i | O^a_q | E_j, Q_j \rangle = \delta_{E_i,E_j} \delta_{Q_i,Q_j} \delta_{q,0} f^a(E, Q_i)$$
$$+ \delta_{Q_i, q+Q_j} g^a(\bar{E}, \omega, Q_i, Q_j) R_{ij}$$

V2 : Charge violating ETH

$$\langle E_i, Q_i | O^a_q | E_j, Q_j \rangle = \delta_{E_i,E_j} \delta_{Q_i,Q_j} \delta_{q,0} f^a(\bar{E}, Q_i)$$
$$+ \tilde{g}^a(\bar{E}, \omega, q, \bar{Q}, \delta Q) R_{ij}$$

$$\langle O_q O_{-q} \rangle_{\beta,\mu} \Big|_{V1} \approx \langle O_q O_{-q} \rangle_{\beta,\mu} \Big|_{V2}$$

$$\left| \overline{C_{iiq}} \right|^2 \Big|_{V1} = 0$$

$$\left| \overline{C_{iiq}} \right|^2 \Big|_{V2} = e^{-S} \neq 0$$

allows non-zero variance, but breaks symmetry in BH microstates.

This is how the wormhole computation should be interpreted.

# Conclusion

- The ORH is an extension of ETH for chaotic CFTs, treats heavy operators as random variables
- In holography, the moments of such distributions is what GR has access to. It is an approximation, explains lack of factorization.
- When the EFT has a global symmetry, wormholes show this symmetry cannot be exact.

# Open Questions

- How is factorization restored? Strings, branes? What if wormholes are unstable? [CF Thomas' talk]
- No average over theories in this talk. But, by averaging over theories, can we make ORH exact?
- What controls the non-gaussianities of $C_{ijk}$?
- How to use argument against global symmetries to bound operators in SEFT? [CF Arthur's talk]

Thank You!