Comments on Euclidean Wormholes and Holography

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Work in collaboration with E. Kiritsis, O. Papadoulaki
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and in progress (+ C. Bachas + I. Lavdas)

Workshop on
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Wormholes: Plethora of Setups

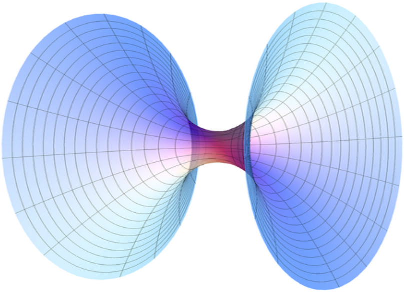
Wormholes are interesting exotic solutions of GR + QFT

They challenge our physical intuition

- Their role in the path integral and low energy physics?
- Holographic interpretation?
- One should make clear distinctions:
  - Lorentzian vs Euclidean
  - Geometries with single or multiple large asymptotic regions
  - Macroscopic vs. Microscopic "gas of wormholes"
  - Different characteristic scales
    $L_P \ll L_W \sim L_{A(dS)}$ vs. $L_P \leq L_W \ll L_{A(dS)}$
- In string theory a further separation: Wormholes on the worldsheet vs. Wormholes on the target space/time: focus on the later
  (2d: see also the talk by [Papadoulaki])

Lorentzian Wormholes

- Einstein - Rosen Bridge: Connects the two sides of the eternal black hole
- Holography ⇒ interpret such a geometry as describing a pair of decoupled but entangled QFT's
- We cannot communicate a message between the two sides
- The two boundaries are always separated by a horizon, once the null energy condition is assumed for the gravitational theory
- Upon analytically continuing to Euclidean signature the space smoothly caps off disconnecting the different asymptotic boundaries
- Traversable Wormholes: Lorentzian signature solutions for which the null energy condition is violated ⇒ Signals can pass through the wormhole!
- Recent efforts to construct long-lived macroscopic traversable Lorentzian wormholes with reasonable matter content... [Maldacena-Milekhin ...]



- Euclidean Wormholes:
  There is no time, only space
- Analogous to instantons (?) of YM theories
- To have such solutions, one needs locally negative Euclidean Energy to support the throat from collapsing
- Such energy can be provided by axions or "magnetic fluxes" etc...

General Holographic comments

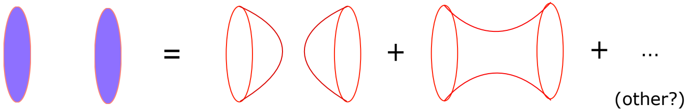
- Decoupled QFTs on $\partial\mathcal{M} = \cup_i \partial\mathcal{M}_i$? Run into problems...
- No time $\Rightarrow$ No entaglement
- Global symmetries for the boundary theories? $\rightarrow$ F.G.: $\partial_\mu J_1^\mu = \partial_\mu J_2^\mu = 0$
- Common Bulk gauge fields and constraints!

The factorisation problem

[Maldacena - Maoz ...]

The problem that $Z(J_1, J_2) \neq Z_1(J_1)Z_2(J_2)$ is :

The factorisation problem



Ideas on the market:

- After summing over bulk topologies $(+$ other?$)$ the correlators factorise
  $\Rightarrow$ Need non-perturbative info ($c = 1$ Liouville: talk by [Papadoulaki])
- The bulk QGR path integral corresponds to an average over QFT's (JT)
  $\Rightarrow$ Unitarity? and higher d? (ETH?: talk by [Belin])
- The partition function could be of the form (S some "sector" ?)

$$Z(J_1, J_2) = \sum_S Z_S^{(QFT1)}(J_1) Z_S^{(QFT2)}(J_2)$$

- Interactions between QFT's $\Rightarrow$ subtle properties

Part I : Bulk perspective

Euclidean Wormholes - Solutions with two boundaries

Existence of several on-shell (non-vacuum) Euclidean solutions with two asymptotic AdS boundaries

- Axionic wormholes: Talks by [Hebecker, Van-Riet]
- Solutions of Einstein - Maxwell - Dilaton in two dimensions
- Solutions of Einstein - Dilaton theory with hyperbolic slicings in three dimensions
- Solutions of Einstein - Yang - Mills theory with spherical slices in four dimensions

...

A subset of such solutions is expected to be perturbatively stable [Marolf, Santos]

$EAdS_2$, Einstein - Maxwell - Dilaton Solutions

[Cvetič- Papadimitriou'16]

$$S = \frac{1}{2\kappa_2^2}\int_{\mathcal{M}} d^2x\sqrt{g}e^{-\phi_D}\left(R + \frac{2}{L^2} - \frac{1}{4}e^{-2\phi_D}F_{\mu\nu}F^{\mu\nu}\right) + \int_{\partial\mathcal{M}}\sqrt{\gamma}e^{-\phi_D}2K$$

- The 2D EMD action arises from reduction of higher dimensional theories
- Gravity in 2D is non dynamical $\Rightarrow$ Extra fields are needed to support AdS_2
- There exist non-trivial solutions of the EMD theory either with running dilaton, or with a running gauge field

$$ds^2 = \frac{du^2 + d\tau^2}{\cos^2 u}, \quad u \in (-\pi/2, \pi/2)$$

$$A_\tau = \mu - Q\tan u$$

with Q the conserved charge, μ the chemical potential

- $F_{u\tau}$ provides the appropriate flux to support the throat

3D Einstein - Dilaton solutions

$$S = \int d^{d+1}x\sqrt{g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

- We find wormhole solutions for constant potential

$$V = -\frac{d(d-1)}{\alpha^2}$$

- They exist only in the case of hyperbolic slicings, the metric is

$$ds^2 = dr^2 + e^{2A(r)}dH_2^2, \qquad r \in [-\infty, \infty]$$

$$e^{2A} = Z + \frac{\tilde{C}}{Z} + 2\alpha^2 \quad , \quad Z \equiv \alpha^2 e^{2\frac{r}{\alpha}} \quad , \quad \tilde{C} = \alpha^4 - 4C^2$$

- These solutions are also accompanied by a running dilaton ϕ

$$\phi = \phi_0 - 2\text{arctanh}\frac{Z + \alpha^2}{2C}$$

- We should quotient the hyperboloid H_2/Γ with Γ a finite discrete subgroup of $PSL(2, \mathbb{R})$, to render the global slice compact

4D Einstein - Yang - Mills Solutions

[Hosoya-Ogura'89]

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{16\pi G} R + \Lambda + \frac{1}{4g_{YM}^2} \left(F^a_{\mu\nu} \right)^2 \right)$$

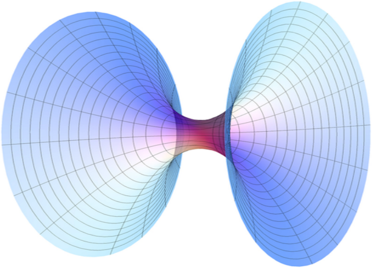
- The metric is $ds^2 = dr^2 + \left(B \cosh(2r) - \frac{1}{2} \right) d\Omega_3^2, \; r \in [-\infty, \infty]$
- with $B = \sqrt{\frac{1}{4} - r_0^2 H^2}$, $r_0^2 = 4\pi G / g_{YM}^2$, $H^2 = 8\pi G \Lambda / 3$
- The throat is supported by a background gauge field A^α: the Meron configuration ("half-instanton")
- It is convenient to use Euler angles (t_3-fiber)

$$d\Omega_3^2 = \frac{1}{4} \left(dt_1^2 + dt_2^2 + dt_3^2 + 2 \cos\, t_1 \, dt_2 dt_3 \right) = \frac{1}{4} \omega^a \omega^a$$
$$0 \le t_1 < \pi, \quad 0 \le t_2 < 2\pi, \quad -2\pi \le t_3 < 2\pi$$

$$A^a = \frac{1}{2} \omega^a \,, \qquad \text{with} \quad F^a = \frac{1}{8} \epsilon^{abc} \omega^b \wedge \omega^c$$

ω^a is the Maurer-Cartan form of $SU(2)$

Correlators: Two boundaries

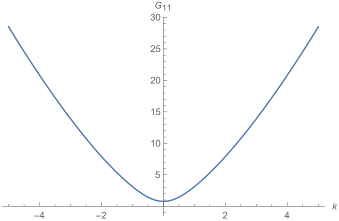


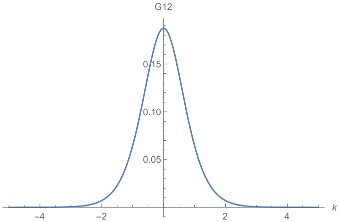
- To study correlators for boundary operators ⇒ Study the (2nd order) bulk fluctuation equation
- For a single UV boundary :
  - regularity in the IR fixes a linear combination of the asymptotic solutions
  - We are left with only one constant to fix

- We have two boundaries, where the solution can potentially diverge or become a constant
- The extra freedom provides for two types of correlation functions, one on a single boundary which we label by $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ or $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$, and one cross-correlator across the two boundaries $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$

- $EAdS_2$ we can compute the correlators analytically, other cases: numerical results

Scalar Correlators: Universal properties





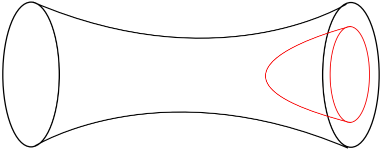
- The $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$ have a similar behaviour in the UV as when there is only one boundary (power law divergence)
- In the IR they saturate to a constant positive value
- The cross correlator $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$ goes to zero in the UV and has a finite maximum in the IR
- In position space $(EAdS_2)$ they behave as $\sim 1/\sinh^{2\Delta_+}(\tau)$ and $\sim 1/\cosh^{2\Delta_+}(\tau)$ respectively $\Rightarrow$ No short distance singularity for the cross-correlator
- The qualitative behavior of the correlators is the same for all the types of solutions $\Rightarrow$ Universality

Non-local observables: Wilson Loops

- Interesting to study non-local observables such as correlators of Wilson loops

$$W(C) = \mathrm{tr}\left(\mathcal{P} \exp\ i \oint_C A_\mu dx^\mu\right)$$

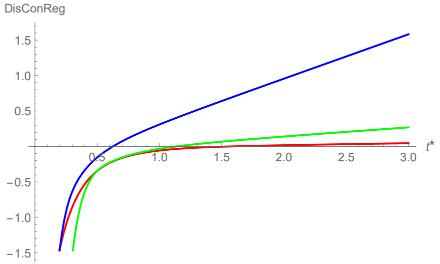
- In holography: Find the string worldsheet ending on the corresponding loop on a boundary and minimize its area



- The disk slices are three spheres and the loop is a circle sitting on the S^3

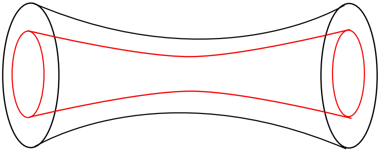
- Simplest observable: expectation value of a Wilson loop $\langle W(C) \rangle$ on one of the two boundaries
- In the limit of a large loop we can probe the IR properties of the boundary dual
- Large loops on the boundary penetrate further in the bulk
- The dual bulk string minimal surface, does not pass through the throat

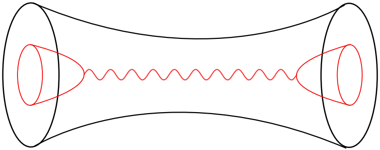
- Plot of the regularised action S^{reg} as a function of t^*
- $0 < t^* < \pi$ governs the boundary loop size
- $B \sim L_{throat}/L_{AdS} = 0.6,\, 0.7,\, 0.9$ from red to blue



- For large loop ($t^* \to \pi$) the action scales approximately linearly with t^*
- This scaling is an Area law
- Normally: indicative of IR confining behaviour only in the infinite volume limit of the S^3 (else kinematical effect) [Witten]
- Here two S^3 boundaries

Wilson Loop correlators





- In both cases the correlator scales as $O(1)$
- In the disconnected case the loops can interact only via exchange of perturbative bulk modes

- Study loop - loop correlators $\langle W(C_1)W(C_2)\rangle$, with the two loops residing on different boundaries
- In the regime of large Wilson loops, the leading contribution originates from a single surface connecting the two loops having a cylinder topology $S^1 \times R$
- As we shrink the boundary loops, we find that the leading configuration of lowest action is the one for two disconnected loops
- Large loops $\Rightarrow$ Strong IR cross-coupling!

Part II: QFT models

Universal properties of a putative dual

- Bottom up proposal: Take two Euclidean theories S_1 and S_2 and local operators $O_1(x) \in S_1$ and $O_2(x) \in S_2$. Introduce effective (non-local) cross-interactions

$$S = S_1 + S_2 + \lambda \int d^d x \; d^d y \, O_1(x) O_2(y) \, f(x-y)$$

- Explore theories whose cross interactions are softer at shorter distances, they increase and become strong in the IR

- The two theories interact "mildly" and the mixed correlators do not have short distance singularities

- The cross-correlator is (G_{ii} are undeformed correlators)

$$\langle O_1(x) O_2(y) \rangle_\lambda = \lambda \int \frac{d^d p}{(2\pi)^d} \frac{G_{11}(p) G_{22}(-p) \tilde{f}(p) e^{ip(x-y)}}{1 + \lambda^2 \tilde{f}(p)^2 \; G_{11}(p) G_{22}(-p) \tilde{f}(p)} + \cdots$$

where in the UV $\tilde{f}(p) \sim p^{-a}\,, a > 2\Delta_1 + 2\Delta_2 - d$ for absence of short distance singularities

A toy QFT model

- We can realise such features in a simple model

$$S = \int \frac{d^4q}{(2\pi)^4} \left[\phi_1(q^2+m^2)\phi_1 + \phi_2(q^2+m^2)\phi_2 + \phi_1 \frac{2}{q^2+\Lambda^2}\phi_2 \right]$$

$$G_{11} = \frac{1}{q^2+m^2-\frac{1}{(q^2+m^2)(q^2+\Lambda^2)^2}} \quad , \quad G_{12} = \frac{q^2+\Lambda^2}{(q^2+m^2)^2(q^2+\Lambda^2)^2-1}$$

- These correlators exhibit the desired universal properties of wormhole correlators i.e. $G_{12} \sim 1/q^6$ (UV)
- Diagonal field basis ($\phi_\pm = \phi_1 \pm \phi_2$), the propagators take the form

$$G_\pm = \frac{q^2+\Lambda^2}{q^4+q^2(m^2+\Lambda^2)+m^2\Lambda^2 \pm 1}$$

- While in the UV $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ in the IR the $\pm$ is a good basis of states ($G_\pm$ admit a KL spectral rep)

A toy QFT model: further properties

- Fourier transforming to position space we find that we need $\Lambda^2 \geq m^2 + 2$, $m^2\Lambda^2 > 1$ in order to satisfy reflection positivity for the $G_\pm$ correlators

- Reflection positivity can be used to put a bound on the amount of non-locality allowed

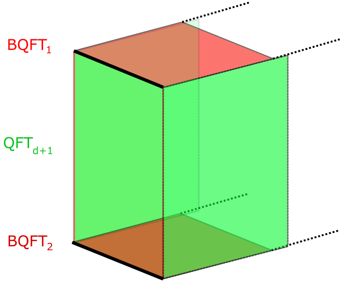
- We cannot resolve the non-local interaction by integrating in 4-dim local fields (the one is a ghost)

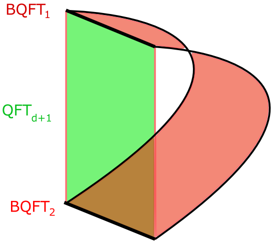
$$G_\pm^{-1} = q^2 + m^2 \pm \frac{1}{q^2 + \Lambda^2}$$

- The analytic continuation $q_0 \to iq_0$ produces ghosts in Lorentzian signature (Osterwalder-Schrader theorem?)

"Sandwich" construction

[van Raamsdonk]





- Another interesting class of models: Two d-dim (holographic) BQFT's coupled through a $d+1$-dim intermediate theory
- The hope: The dual bulk gravity can localise on $d+1$-dim EOW branes that bend and connect in the IR
- The relevant regime is $c_{d+1} \ll c_d$. It was argued that the system should flow to a gapped/confining theory in the IR
- SUSY should be completely (almost?) broken
- Similar top-down BCFT (SUSY preserving) setup by [Bachas - Lavdas]: Almost factorised Quiver with a very weak node $n \ll N_{rest}$ coupling the “two sectors"

Toy model (II)

- A simple model of this type ($S_{1,2}$ are separated by $|y_1 - y_2| = L$)

$$S_{1,2} = -\frac{1}{2}\int d^dx\ \phi_{1,2}(x)\left(\Box_d - \tilde{m}^2\right)\phi_{1,2}(x)\ +\ \ldots$$

$$S_3 = -\frac{1}{2}\int d^dxdy\ \Phi(x,y)\left(\Box_{d+1} - \tilde{M}^2\right)\Phi(x,y)$$

$$S_{int} = g\int d^dx\ \phi_1(x)\Phi(x,y_1) + g\int d^dx\ \phi_2(x)\Phi(x,y_2)$$

- Integrate out $\Phi(x,y)$ to obtain

$$S = \int \frac{d^dp}{(2\pi)^d}\left[\phi_+(p)D_+(p)\phi_+(-p) + \phi_-(p)D_-(p)\phi_-(-p)\right] + \ldots$$

$$\phi_\pm = \frac{\phi_1 \pm \phi_2}{\sqrt{2}} \quad , \quad D_\pm(p) = p^2 + m^2 + \frac{g^2\ \left(1 \pm e^{-L\sqrt{p^2+M^2}}\right)}{2\sqrt{p^2+M^2}}$$

- $D_\pm^{-1}(p)$ are well defined $\forall g, L$ and admit KL spectral rep with positive weight
- All the composite operators $O^m_{1,2} = \phi^m_{1,2}$ have appropriately UV soft cross correlators $\langle O^m_1 O^m_2\rangle$

Microscopic “sandwich" model $(2d - 1d)$

- Take a $2d$ gYM or BF theory (τ, z) and couple it to two $1d$ $U(N)$ matrix quantum mechanics theories at the endpoints of an interval I $(z = \pm L)$.
- The action is

$$S = \int d\tau \, \mathrm{tr} \left(\frac{1}{2} (D_\tau M_{1,2})^2 \, - V(M_{1,2}) \right) \, , \; D_\tau M_{1,2} = \partial_\tau M_{1,2} + i [A_\tau^{1,2}, M_{1,2}]$$

$$S_{gYM} = \frac{1}{g_s^2} \int_\Sigma \mathrm{tr} \, BF \, + \, \frac{\theta}{g_s^2} \int_\Sigma \mathrm{tr} \, B \, d\mu - \frac{p}{2g_s^2} \int_\Sigma \mathrm{tr} \, \Phi(B) \, d\mu$$

where $F = dA + A \wedge A$ and $A_\tau(\tau, z = \pm L) = A_\tau^{1,2}(\tau)$ is the restriction of the 2d gauge field on the two boundaries ($A_z = 0$ gauge)

- The two MQM's are coupled via rep theory “selection rules" giving rise to subtle correlations

Microscopic "sandwich" model

- Take $\Phi(B) = B^2$ (2d YM) and place the system on $I \times S^1$
- The partition function on the cylinder is

$$Z(\beta) = \sum_R e^{-Ag_s^2 p C_R^2 + i\theta C_R^1} Z_R^{MQM_1}(\beta) Z_R^{MQM_2}(\beta)\,,$$

$$Z_R^{MQM}(\beta) = \mathrm{tr}_{\mathcal{H}_R}\, e^{-\beta \hat{H}_R^{MQM}}$$

  with β the S^1 size and R a $U(N)$ representation and $C_R^{1,2}$ its Casimirs
- The two systems are coupled, by carrying common representations (What we called the sectors!)
- One MQM (taking a double scaling limit) is dual to $2d$ linear dilaton background ($c = 1$-Liouville) $\Rightarrow$ One asymptotic region of space
- Non trivial reps are related to long strings and (possibly) black holes $\Rightarrow$ Understand the dual bulk geometry!

Part III: α-parameters

Wormholes and α-parameters (80's-90's)

see the review by [Hebecker et al.]

- Other setup: "Gas" of microscopic wormholes that affect the low energy IR physics
- Argued that they lead to bi-local interactions in the low energy effective Lagrangian $E \ll M_w \leq M_P$

$$S_w = \frac{1}{2} \int d^d x dz \sqrt{g} \int d^d y dz' \sqrt{g} \sum_{ij} C_{ij} \mathcal{O}_i(x,z) \mathcal{O}_i(y,z')$$

- Violation of bulk-locality? Possible to introduce α-parameters

$$e^{S_w} = \int \prod_i d\alpha_i \, e^{-\frac{1}{2}\sum_{ij} \alpha_i C^{-1}_{ij} \alpha_j} e^{\sum_i \alpha_i \int d^d x dz \sqrt{g} \mathcal{O}_i(x,z)}$$

- This leads to shifting coupling constants $S[g;\lambda_i] \rightarrow S^{\alpha}_{eff}[g;\lambda_i - \alpha_i]$
- Any (bulk) correlator is an α-state average

$$\langle \mathcal{O} \rangle = \int \prod_i d\alpha_i \, P(\alpha_i) \langle \mathcal{O} \rangle_{\lambda_i - \alpha_i}$$

- The α-parameter source functional is (ϕ_i wormhole affected bulk fields)

$$Z_{w.d}(J_i(x)) = \int d\alpha_\Lambda \prod_i d\alpha_i \, e^{-\frac{1}{2}\alpha C^{-1}_{ij}\alpha_j - \frac{1}{2}\alpha_\Lambda C^{-1}_\Lambda \alpha_\Lambda}$$

$$\int_{\phi_i(z,x)\to z^{\Delta_-(\alpha)} J_i(x)} \mathcal{D}g \, \mathcal{D}\phi_i \, e^{-S[g,\phi_i] + \alpha_\Lambda \int d^d x dz \sqrt{g} + \alpha_i \int d^d x dz \sqrt{g}\phi_i(x,z)}$$

- One should first find the α-state gravitational saddle solving

$$G_{MN} = T^{w.d.}_{MN}\,, \quad -\Box\phi_i + \frac{\partial V^{w.d.}}{\partial \phi_i} = 0\,, \quad V^{w.d}(\phi_i) = V(\phi_i) - \alpha_\Lambda - \alpha_i\phi_i$$

  $V^{w.d}$ is the α-state potential

- The saddle point solution depends on the α-parameters ex: AdS radius
- At quadratic level $\Delta_i(\alpha)$, interactions lead to $C_{ijk}(\alpha)$

The wormhole gas in Holography - properties

- At face value we get a collection of CFT's labelled by α
- All observables are then computed as α parameter averages (quenched vs. annealed)
- Is this a proxy for the path integral of a Chaotic/“random" QFT? (talk by [Belin])
- Minimise also the α dependent $S_{eff}(\alpha) \Rightarrow$ find a collection of saddle point values $\alpha^*(C)$.
- In each “superselection sector" it is like having a unique dual

Wormholes and α-parameters - pitfalls

- The bilocal action and dilute gas approximate at best: Gaussian dist. in $\alpha \to P(\alpha)$, $C_{ij}(x, y)$, mouths can interact...
- Problem I: The [Fischler-Susskind-Kaplunovsky] catastrophe Proliferation of large wormholes and violation of dilute gas approximation...
- Solution: "Small wormholes crowd out large ones" (phase space) [Preskill or "bleed off" their charges (destabilisation) [Coleman]
- Problem I': Violation of the bulk Wilsonian RG (Decoupling)? Not so, overdensity of wormholes persists at each scale [Polchinski]
- Problem II: Ambiguities in Euclidean QG path integral
- Hard to make sense of the α-parameter computations ("3rd quantisation"?)

Summary and Future

Summary

- Euclidean wormholes can appear in various setups/regimes
- It seems that there are various possible resolutions of the factorisation problem depending on the context
  $\Rightarrow$ Quantum gravity seems to be extremely rich
- One of them is in terms of a system of interacting QFT's
- Compatibility with geometric dual constrains the properties of correlation functions
- This is a specific example of the general discussion on emergent gravity [PB - Kiritsis - Niarchos]: $QFT_1 + QFT_2 +$ "messengers"
- For wormholes $QFT_{1,2}$ have a similar number of dofs $>>$ "messenger" dofs
- In a different limit one obtains the effective braneworld models
- It would be nice to have top-down constructions or No-Go theorems

Stability and Top-down constructions?

- A subset of ad-hoc solutions is expected to be perturbatively stable [Marolf, Santos]
- On the other hand it is not clear if top down SUSY preserving (stable) solutions exist
  Yes/NO-go?
- Important to thoroughly examine Euclidean SUGRA setups with reduced SUSY!
- Promising Euclidean setup: $\mathcal{N} = 1^*$ on S^4 (mass deformation) and the dual $\mathcal{N} = 8$ gauged SUGRA truncation by [Bobev - et al.]

"IR confining" properties?

- Bottom-up holography: Cross coupling strong in the IR, IR “Area law" for the loop operator
- [van Raamsdonk] construction: the EOW branes need to reconnect (mass gap formation/confinement/susy breaking)
- Toy model (I): diagonal field basis ($\phi_\pm = \phi_1 \pm \phi_2$), the propagators take the form

$$G_\pm = \frac{q^2 + \Lambda^2}{q^4 + q^2(m^2 + \Lambda^2) + m^2\Lambda^2 \pm 1}$$

- Curious fact: Similar to the "Refined Gribov-Zwanziger" [Dudal et. al.] and "SD-FRG" gluon propagators [Pawlowski et. al.]
- An idea: A (Euclidean) system with two decoupled sectors in the UV and $SU(N) \times SU(N)$ gauge symmetry, which flows to a strongly cross-coupled IR theory with only the $SU_{diag}(N)$ left...
- Might resemble cascading gauge theories (that do exhibit confining IR behaviour)...

Other Directions

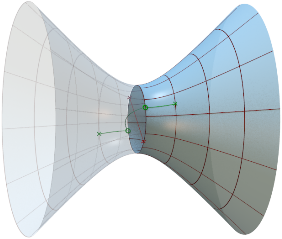
- Analytic continuation of Euclidean wormhole solutions into Lorentzian signature?
- If we Wick rotate one of the transverse directions $\Rightarrow$ Energy conditions?
- If we Wick rotate the radial direction we have a bang/crunch universe
- What about Osterwalder-Schrader axioms? (for the boundary dual)
- Hard to make sense of the α-parameter computations - many caveats
- What can Holography contribute to this? (RG in the bulk vs. RG on the boundary)
- Applications in condensed matter systems? (bi-layer BCFT's)...

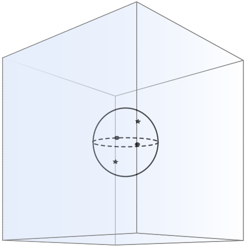
- Many promising directions ahead!

Thank you!

Vacuoles

- It is also possible to perform a $\mathbb{Z}_2$ (antipodal) identification on wormhole solutions
- This turns them into "vacuoles" [PB - Gaddam - Papadoulaki, 't Hooft]





- A prototype geometry is ($r \in (-\infty, \infty)$, $x \in (0, \infty)$)

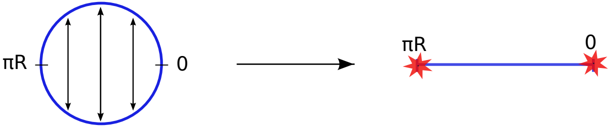
$$ds^2 = dr^2 + (r^2 + 4r_1^2)d\Omega_3^2\,, \quad \Rightarrow \quad ds^2 = \left(1 + \frac{r_1^2}{x^2}\right)^2 \delta_{\mu\nu} dx^\mu dx^\nu$$

- and the identification $(r, \Omega) \sim (-r, \Omega^P)$ or (inversion in x^μ)
- Can this project out unstable fluctuations? General consistency?

MQM on S^1/Z_2

[PB - Gursoy - Papadoulaki]

- One can study MQM on the orbifold $S^1/Z_2 \Rightarrow$ Euclidean time interval



$$M(-\tau) \sim \Omega M(\tau)\Omega^{-1}\,, \quad \Omega = \begin{pmatrix} I_{n\times n} & 0 \\ 0 & I_{(N-n)\times(N-n)} \end{pmatrix}$$

- $U(N) \to U(n) \times U(N-n)$ at the endpoints $\Rightarrow$ non-singlets get liberated (like the UV of the wormhole geometry...)
- Equivalent description: $U(n)$ MQM $+\ U(N-n)$ MQM coupled via bi-fundamental instantons at the endpoints
- Upon analytic continuation this might describe a Bang/Crunch type of universe
- Important to understand the bulk geometry

Analytic continuation to Lorentzian?

Example of the Meron wormhole

- If we Wick rotate one of the spherical directions $\Rightarrow S_3 \to dS_3$
- The gauge field now has imaginary components. Have not found a gauge transformation that can remove them!

- If we Wick rotate in the radial direction we have a bang/crunch universe
  [Maldacena-Maoz'04, PB-Gaddam-Papadoulaki'17]

$$ds^2_{BC} = \left(B - \frac{1}{2}\right) \mathrm{cn}^2(\tilde{u}, k') \left(-d\tilde{u}^2 + \frac{d\Omega_3^2}{2B}\right)$$

- The lifetime of this universe can be described in terms of spatial S^3's that start at zero size, expand and then contract to a Crunch
- The universe has zero size at $\tilde{u} = (2m+1)K$ and maximal finite size at $\tilde{u} = 2mK$, m being an integer
- It is not clear how one can interpret such a continuation from a dual field theory point of view since it involves the RG direction

Prescription for Correlators

- For a probe scalar

$$S[\phi] = \frac{1}{2}\int_{\mathcal{M}} d^{d+1}x\sqrt{g}\;\phi\left(-\Box + m^2\right)\phi - \frac{1}{2}\int_{\partial\mathcal{M}} d^d x\sqrt{\gamma}\;\phi\;\vec{n}\cdot\partial\phi$$

- Two asymptotic falloffs $\sim u^{\Delta^i_\pm}$ near each of the boundaries, the boundary to Bulk (btB) propagators satisfy

$$(-\Box + m^2)K^i(u,\Omega;\Omega') = 0$$

$$K^i(u,\Omega;\Omega')|_{\partial\mathcal{M}_i} = \epsilon_i^{\Delta^i_-}\delta_{\epsilon_i}(\Omega-\Omega'), \quad K^i(u,\Omega;\Omega')|_{\partial\mathcal{M}_j} = 0\,, \quad \text{for } i\neq j$$

$$\phi(u,\Omega) = \sum_i \int_{\partial\mathcal{M}_i} d^d\Omega' K^i(u,\Omega;\Omega')\phi_i^s(\Omega')$$

- The last formula gives the reconstruction of a bulk field from given sources ϕ_i^s at the boundaries
- The correlators are expressed as:

$$\langle\mathcal{O}_1\mathcal{O}_1\rangle = \left[u^{-d}u\partial_u f_k^1(u)\right]_{\partial\mathcal{M}_1}^{u=\epsilon}\,, \quad \langle\mathcal{O}_1\mathcal{O}_2\rangle = \left[u^{-d}u\partial_u f_k^2(u)\right]_{\partial\mathcal{M}_1}^{u=\epsilon}$$

with $f_k^i(u) = \epsilon_i^{\Delta^i_-}\frac{\sqrt{\gamma_i(\epsilon_i)}}{\sqrt{\gamma_i(u)}}\frac{K^i(k,u)}{K^i(k,\epsilon_i)}$

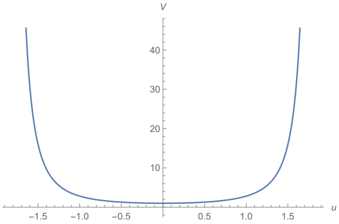
Correlators for the Meron Wormhole

- Euclidean Einstein Static Universe coordinates $\Rightarrow$ Bring the fluctuation equation for the scalar into a Schrödinger form
- The metric (elliptic modulus $k^2 = (B + 1/2)/2B$ and K the elliptic period)

$$ds^2 = \frac{(B - \frac{1}{2})}{\mathrm{cn}^2(u,k)} \left(\frac{du^2}{2B} + d\Omega_3^2 \right), \quad u \in [-K, K]$$

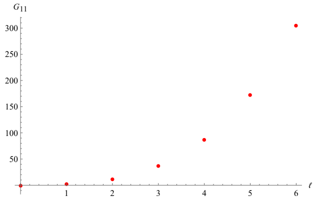
- Fluctuation equation is a Lamé type equation

$$-\frac{d^2}{du^2}\Psi(u) + \frac{k'^2\left(m^2+2\right)}{\mathrm{cn}^2\,u}\Psi(u) = -\frac{(\ell+1)^2}{2B}\Psi(u)$$

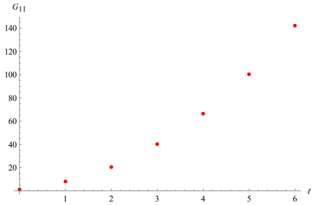


- The potential $1/\,\mathrm{cn}^2\,u$ for one period of elliptic functions
- The energies are below the minimum of the potential $\Rightarrow$ no bound states - uniqueness of the boundary value problem

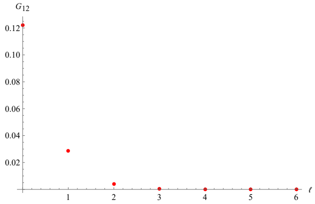
Meron Wormhole correlators: Numerical results



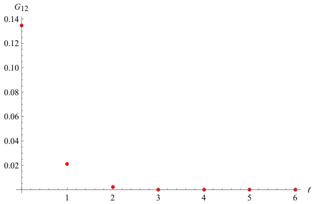
- $m^2 = 1, B = 1$ as a function of l (irrelevant)



- $m^2 = -1, B = 1$ as a function of l (relevant)



- $m^2 = 1, B = 1$ as a function of l



- $m^2 = -1$, $B = 1$ as a function of l

Correlators in 3D Einstein Dilaton Wormholes

- Euclidean Einstein Static Universe $\Rightarrow$ bring the fluctuation equation for the scalar into a Schrödinger form
- parametrising the metric with elliptic functions with $k^2 = (B - 1/2)/2B < 1$

$$\frac{ds^2}{\alpha^2} = \frac{B + \frac{1}{2}}{\mathrm{cn}^2\, u} \left(\frac{du^2}{2B} + dH_2^2 \right) , \qquad u \in [-K(k), K(k)]$$

- The scalar field can be decomposed as $\phi = \xi(u) F_s(H_2/\Gamma)$ with $-\Box_{\mathcal{M}_T} F_s = s(1-s) F_s$
- The fluctuation equation for the radial part in Schrödinger form is

$$-\Psi''(u) + \left(\frac{k'(m^2+1)}{\mathrm{cn}^2\, u} \right) \Psi(u) = -\frac{1}{2B} (s(1-s)) \Psi(u)$$

- The potential and the correlators are qualitatively similar to the ones of the other examples
- The behavior of these correlators is Universal

Loop Parametrisation

- Minimise the Nambu-Goto action (fixing as. loop size)

$$S_{NG} = \frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{\gamma}\,, \qquad \gamma_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N G_{MN}$$

- The target space has topology $R \times S^3$
- Study circular Wilson-loops located on the S^3 $\Rightarrow$ the dual bulk world-sheet must have circular symmetry
- Gauge-fix the world-sheet coordinates as follows (Euler angles):

$$X^0 \equiv r = \tau\,, \qquad t_2 = \sigma\,, \qquad t_{1,3} = t_{1,3}(\tau)$$

- Use left over symmetry to set $t_3 = const$
- The NG action is

$$S_{NG} = \frac{1}{2\alpha'} \int dr \sqrt{B\cosh(2r) - \frac{1}{2}} \sqrt{1 + \frac{\dot{t}_1^2}{4}\left(B\cosh(2r) - \frac{1}{2}\right)}$$

Wilson Loop Solutions

- The EOM can be integrated once to give

$$\dot{t}_1 = \pm \frac{8C}{\sqrt{B\cosh(2r) - \frac{1}{2}}\sqrt{\left(B\cosh(2r) - \frac{1}{2}\right)^2 - 16C^2}}$$

- $C = 0 \Rightarrow$ A constant size loop that connects the two boundaries
- There is a competing solution that ends in the bulk smoothly at $r = r_m$ where the loop acquires zero size
- At this point $\dot{t}_1(r_m) = \infty$
- This fixes

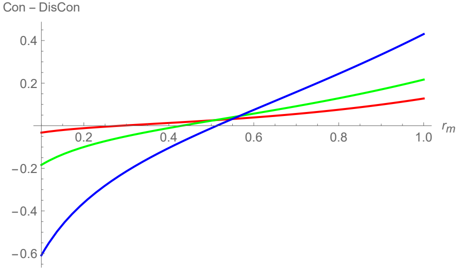
$$C = \frac{B\cosh 2r_m - \frac{1}{2}}{4}$$

- There is still a one parameter freedom. Performing the following integral

$$t_1(\infty) = \pm \int_{r_m}^{\infty} dr \dot{t}_1 = t^*$$

- We define $t_1(\infty) = t^*$ as the asymptotic value of the angle at the boundary

- Plot of the difference of the connected to the disconnected action $S_{conn} - S_{disc}$ as a function of r_m
- For $B = 0.6,\ 0.7,\ 0.9$ from red to blue



- For large r_m/small loop size the disconnected dominates
- For small r_m/large boundary loop size the connected dominates

- For a more detailed analysis we need to include the exchange of bulk perturbative modes between the disconnected loops

- Nevertheless from the analysis of Wilson loops it is clear that the boundary theory/ies become strongly cross-coupled in the IR

Quantum Field Theory Dual

- The cross correlator is to first order in λ

$$\langle O_1(x)O_2(y)\rangle = \lambda \int \frac{d^d p}{(2\pi)^d} G_{11}(p)G_{22}(-p)\tilde{f}(p)\ e^{ip\cdot(x-y)} + \cdots$$

- In the UV

$$G_{11}(p) \sim p^{2\Delta_1 - d} \quad , \quad G_{22}(p) \sim p^{2\Delta_2 - d}$$

- For short distance singularities to be absent,

$$\tilde{f}(p) \sim p^{-a} \quad , \quad \langle O_1(x)O_2(y)\rangle \sim \frac{1}{|x-y|^{2\Delta_1+2\Delta_2-d-a}}$$

with $a > 2\Delta_1 + 2\Delta_2 - d \geq d - 4$

- The all order result in λ is

$$\langle O_1(x)O_2(y)\rangle = \lambda \int \frac{d^d p}{(2\pi)^d} \frac{G_{11}(p)G_{22}(-p)\tilde{f}(p)e^{ip(x-y)}}{1 + \lambda^2 \tilde{f}(p)^2\ G_{11}(p)G_{22}(-p)\tilde{f}(p)} + \cdots$$

- The higher orders in λ induce softer and softer interactions
- The ellipsis are corrections due to higher point functions that are subleading at large-N

The wormhole gas in Holography

- Then one has to average the boundary correlators computed in each of the α-state saddles

$$\langle \mathcal{O}_i(x) \rangle = \int d\alpha_\Lambda \prod_i d\alpha_i \, P(\alpha_\Lambda, \alpha_i) \langle \mathcal{O}_i(x) \rangle_{\alpha_\Lambda, \alpha_i}$$

- One has to check whether this average over QFT's makes sense (annealed or quenched?)

- In lower dimensional examples (ex: JT gravity or Liouville theory), it has been recently argued that the gravitational path integral is dual to an ensemble of boundary theories [Shenker-Stanford-Maldacena]

- This construction seems to give a concrete way of generalising this idea to higher dimensions...

- Bulk cluster decomposition is violated, but now not a single boundary QFT (only on a a^* saddle)