Phases of Holographic Interfaces

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Based on 2101.12529 *w.* Vassilis Papadopoulos

See also 2006.11333 *w.* Shira Chapman, Dongsheng Ge, Giuseppe Policastro

1. INTRODUCTION

Domain walls between AdS vacua are ubiquitous:

- String landscape; gravity localization; de Sitter embedding . . .

- Holographic duals to conformal interfaces

- Models of unitary black-hole evaporation

Transparent (permeable) conditions for Hawking radiation

Most (back-reacting) domain walls are **thick.** But since the original work of Coleman & De Lucia a frequently-used approximation is that of thin walls.

The minimal action

$$I_{\rm gr} = -\frac{1}{2}\int_{\mathbb{S}_1} d^3x\sqrt{g_1}\,(R_1+\frac{2}{\ell_1^2}) - \frac{1}{2}\int_{\mathbb{S}_2} d^3x\sqrt{g_2}\,(R_2+\frac{2}{\ell_2^2})$$

$$+\lambda\int_{\mathbb{W}} d^2s\sqrt{\hat{g}_w} + \text{GHY terms} + \text{ct}\,.$$

depends on 3 dimensionless parameters ℓ_1, ℓ_2, λ (with $8\pi G = 1$)

1. INTRODUCTION

Work in 2+1 dimensions where calculations are analytically tractable

cf Simidzija & Van Raamsdonk '20

Expect that qualitative conclusions extend to higher dims

Restrict below to static walls (equilibrium states): surprisingly rich

Time dependence, and/or walls hidden behind horizons: very interesting but not for today

e.g. Balasubramanian et al '20

Take $\ell_1 \leq \ell_2$. A simple calculation shows that static domain walls exist for

true false

CB ’02

$$\underbrace{\frac{1}{\ell_1} - \frac{1}{\ell_2}}_{\lambda_{\rm min}} < \lambda < \underbrace{\frac{1}{\ell_1} + \frac{1}{\ell_2}}_{\lambda_{\rm max}}$$

← *False vacuum unstable to bubble nucleation*

↓ *BPS values for flat walls*

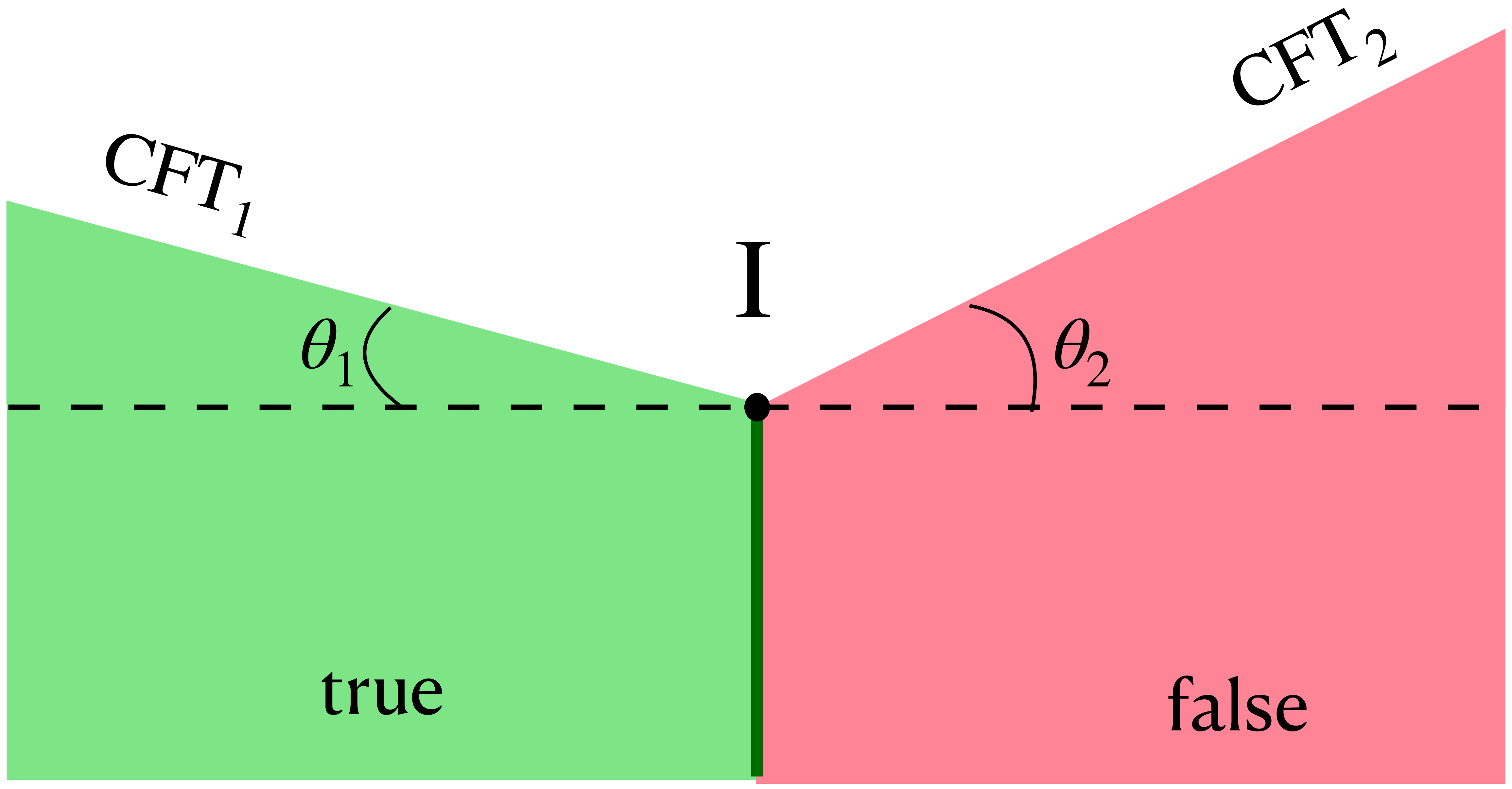
Cvetic, Griffies, Rey ’92
Cardoso, Dall’Agata, Lust ’02
Ceresole *et al* ‘06

→ *Domain wall inflates*

Vilenkin ’81
Ipser, Sikivie ’83
Karch, Randall ‘01

In static solutions the wall hits the AdS boundary at the location of a conformal interface.

Karch, Randall '01
CB, de Boer, Dijkgraaf, Ooguri '02



$$\cos\theta_j = \frac{\ell_j}{\ell_w}$$

$$\tan\theta_1 + \tan\theta_2 = \lambda \ell_w$$

Holographic dictionary:

$$c_j = 12\pi\,\ell_j$$

Brown, Henneaux ’86

Entropy

$$\log g_{\rm I} \;=\; 2\pi\ell_1\ell_2\left[\lambda_{\max}\tanh^{-1}\!\left(\frac{\lambda}{\lambda_{\max}}\right) - \lambda_{\min}\tanh^{-1}\!\left(\frac{\lambda_{\min}}{\lambda}\right)\right]$$

Simidzija, Van Raamsdonk ’20

Azeyanagi, Karch, Takayanagi, Thompson ‘07

Energy transport

$$\mathcal{T}_{1\to2} = \frac{\lambda_{\max}+\lambda_{\min}}{\lambda_{\max}+\lambda}\,, \qquad \mathcal{T}_{2\to1} = \frac{\lambda_{\max}-\lambda_{\min}}{\lambda_{\max}+\lambda}$$

CB, Chapman, Ge, Policastro ’20

For $\lambda \in [\lambda_{\min}, \lambda_{\max}]$

$$\log g_{\mathrm{I}} \in [-\infty, \infty]$$

$$\mathcal{T}_{1\to 2} \in \left[1, \frac{\lambda_{\max} + \lambda_{\min}}{2\lambda_{\max}}\right]$$

$$\mathcal{T}_{2\to 1} \in \left[\frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}}, \frac{\lambda_{\max} - \lambda_{\min}}{2\lambda_{\max}}\right]$$

BCFT limit:

$$c_1 \sim \ell_1 \to 0 \implies \begin{cases} \mathcal{T}_{1\to 2} \to 1 \\ \mathcal{T}_{2\to 1} \to 0 \end{cases}$$

cf Takayanagi '11

2. INTERFACE CFT

- For general interfaces, the *entropy* and *energy transport* coefficient are independent parameters

  Ex. Topological interfaces have $\mathscr{T}_{1\to 2} = \mathscr{T}_{2\to 1} = 1$ but arbitrary $\log g_{\rm I}$

  In the **minimal thin-brane model** they both depend on the unique parameter λ

- Transport coefficients satisfy bound coming from (A)ANEC

  Meineri, Penedones, Rousset '19

  Stricter than the (Euclidean) Reflection-positivity bound

  Billo, Gonçalves, Lauria, Meineri '16

The minimal model captures the universal operators of **ICFT**:

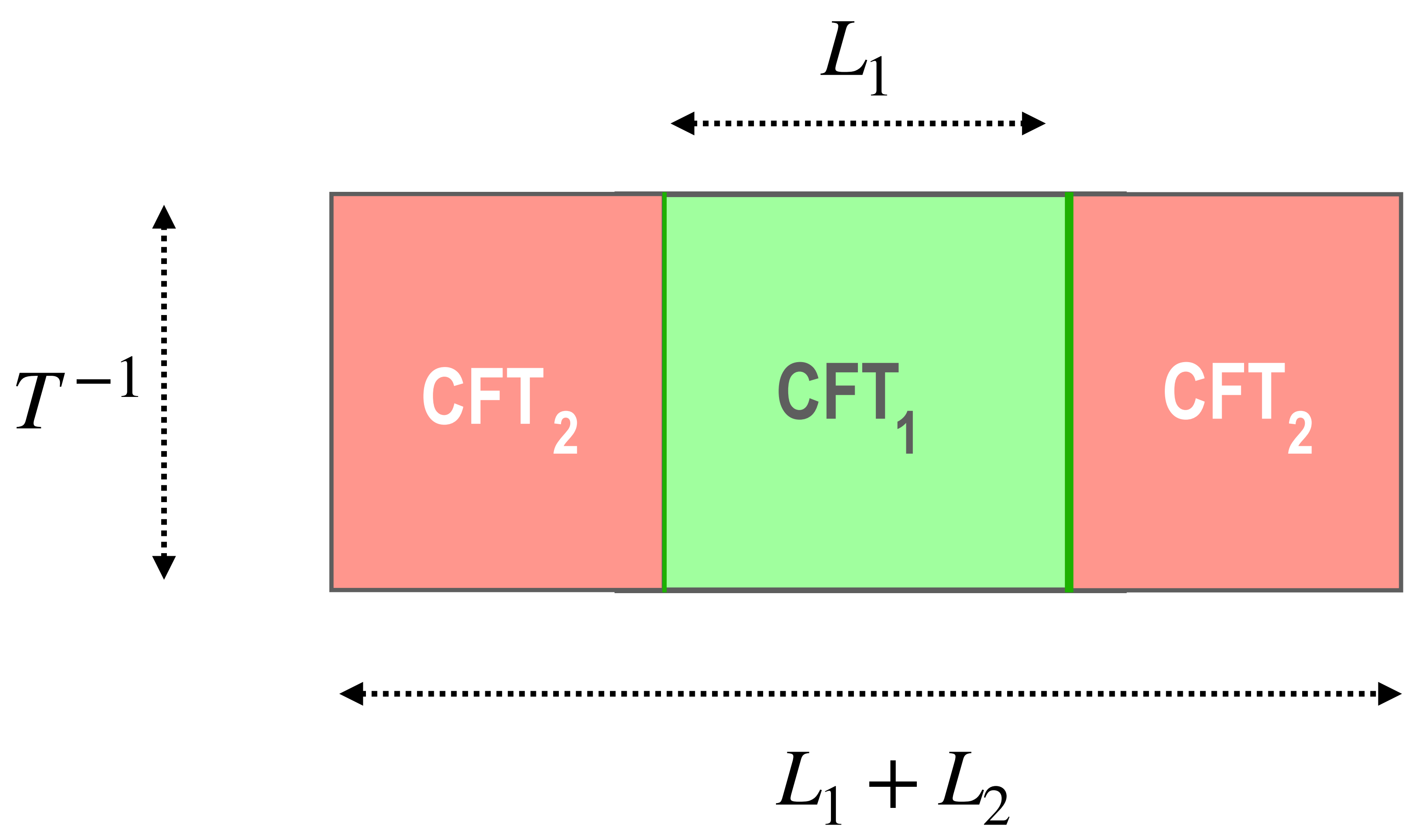
$$T^{(1)}_{ab}\ ,\ \ T^{(2)}_{ab}\ \text{\& displacement}$$

After folding, it has a second spin-2 primary $\Longrightarrow$ $\neq$ **BCFT**

Quella, Runkel, Watts ’06

Similar to pure Einstein gravity for homogeneous CFT

We now place the ICFT on a torus:



This introduces two new dimensionless parameters:

$$\tau_1 := TL_1 \,, \quad \tau_2 := TL_2 \qquad \text{or} \qquad \gamma := \frac{L_1}{L_2} = \frac{\tau_1}{\tau_2}$$

Bnry, or Thermodynamic

The gravity bulk consists of two spacetime slices, glued along the thin domain wall.

The metric of each slice has the universal static BTZ form

$$ds^2 = \frac{\ell^2 dr^2}{r^2 - M\ell^2} - (r^2 - M\ell^2)\, dt^2 + r^2 dx^2$$

If $x \sim x + L$ then:

$$\begin{cases} M = -(2\pi/L)^2 & \text{thermal AdS} \\ M = (2\pi T)^2 & \text{non-rotating BH} \end{cases}$$

Energy density: $\frac{E}{L} = \frac{1}{2} M\ell$

Entropy density: $\frac{S}{L} = \begin{cases} 0 \\ \sqrt{M}\,\ell/4G \end{cases}$

The homogeneous CFT undergoes a Hawking-Page transition at $\tau := LT = 1$

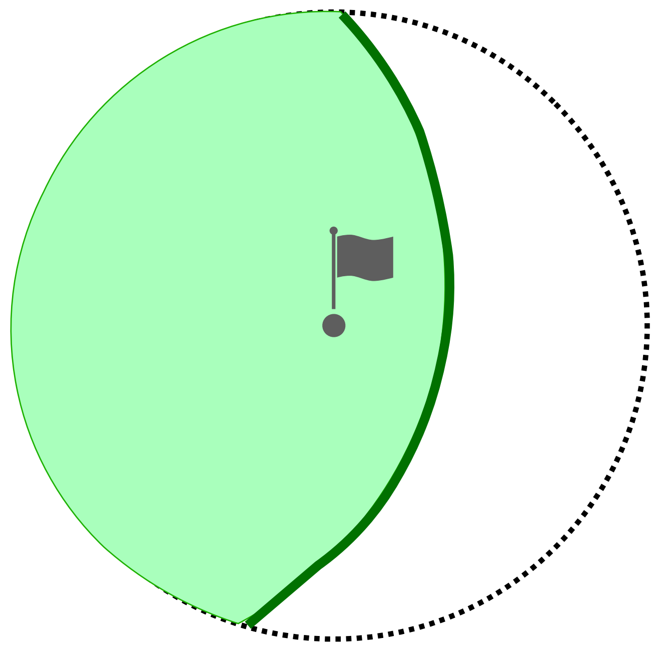
$$F_{\text{BTZ}} - F_{\text{TAdS}} = -2\pi^2 \ell \left(LT^2 - \frac{1}{L} \right)$$

We will study the phase diagram in the (τ_1, τ_2) plane

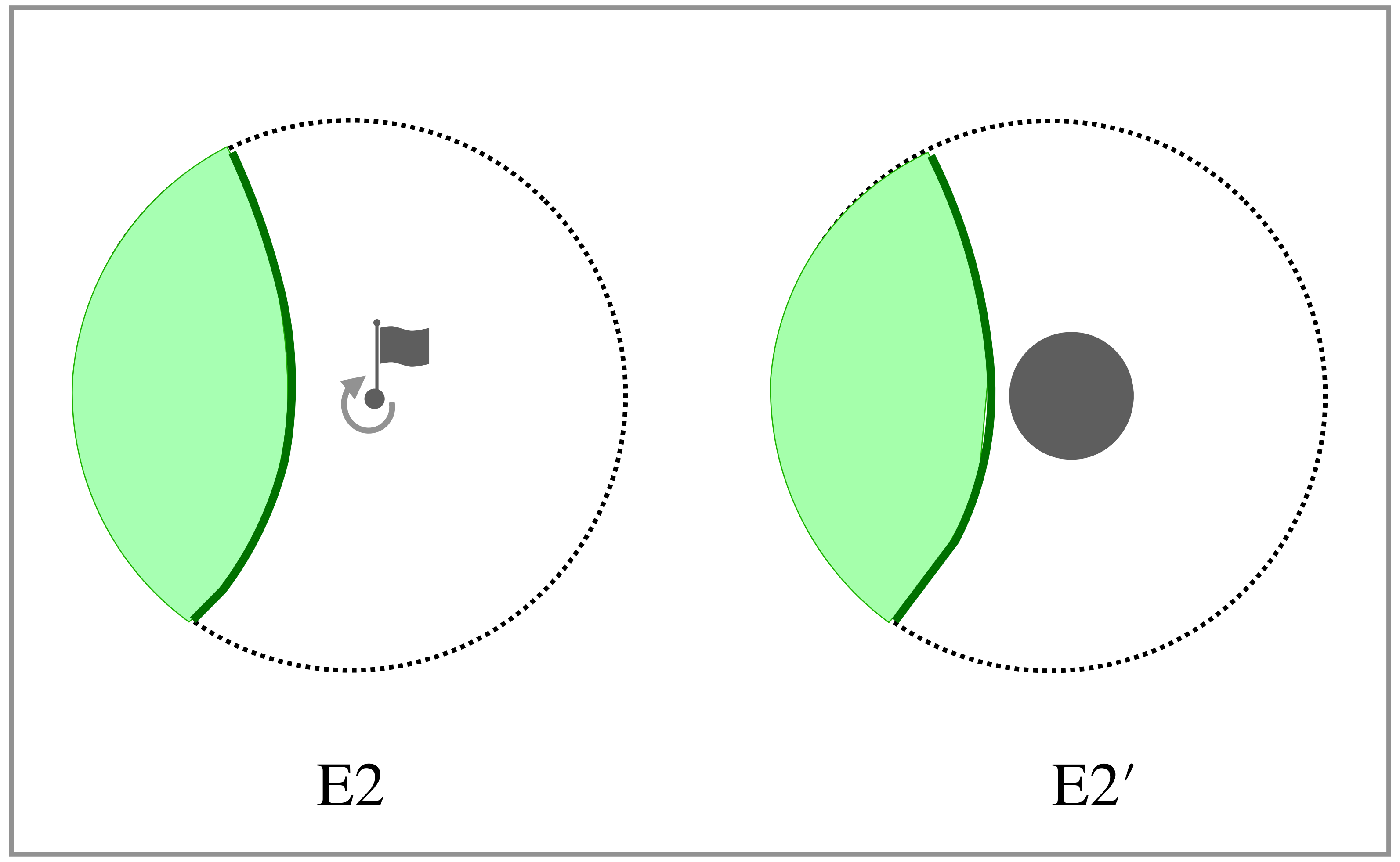
We glue two slices together. Each slice has one of the following topologies

3. FINITE TEMPERATURE & VOLUME

Empty



E1

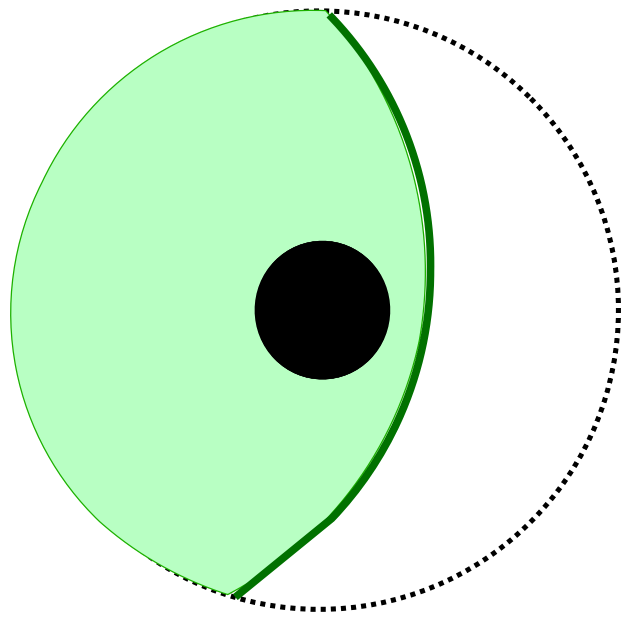


E2 E2′

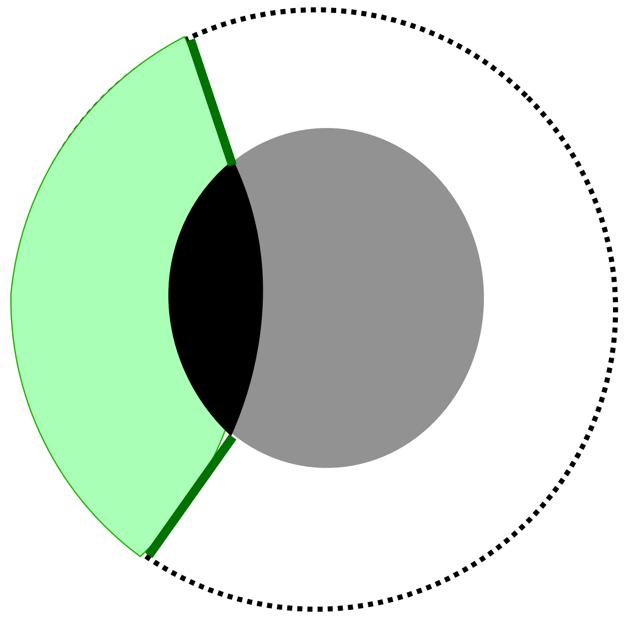
$$M \in (-\infty, \infty)$$

$$x \sim x + \frac{2\pi}{\sqrt{-M}} \qquad \text{for E1}$$

Horizon



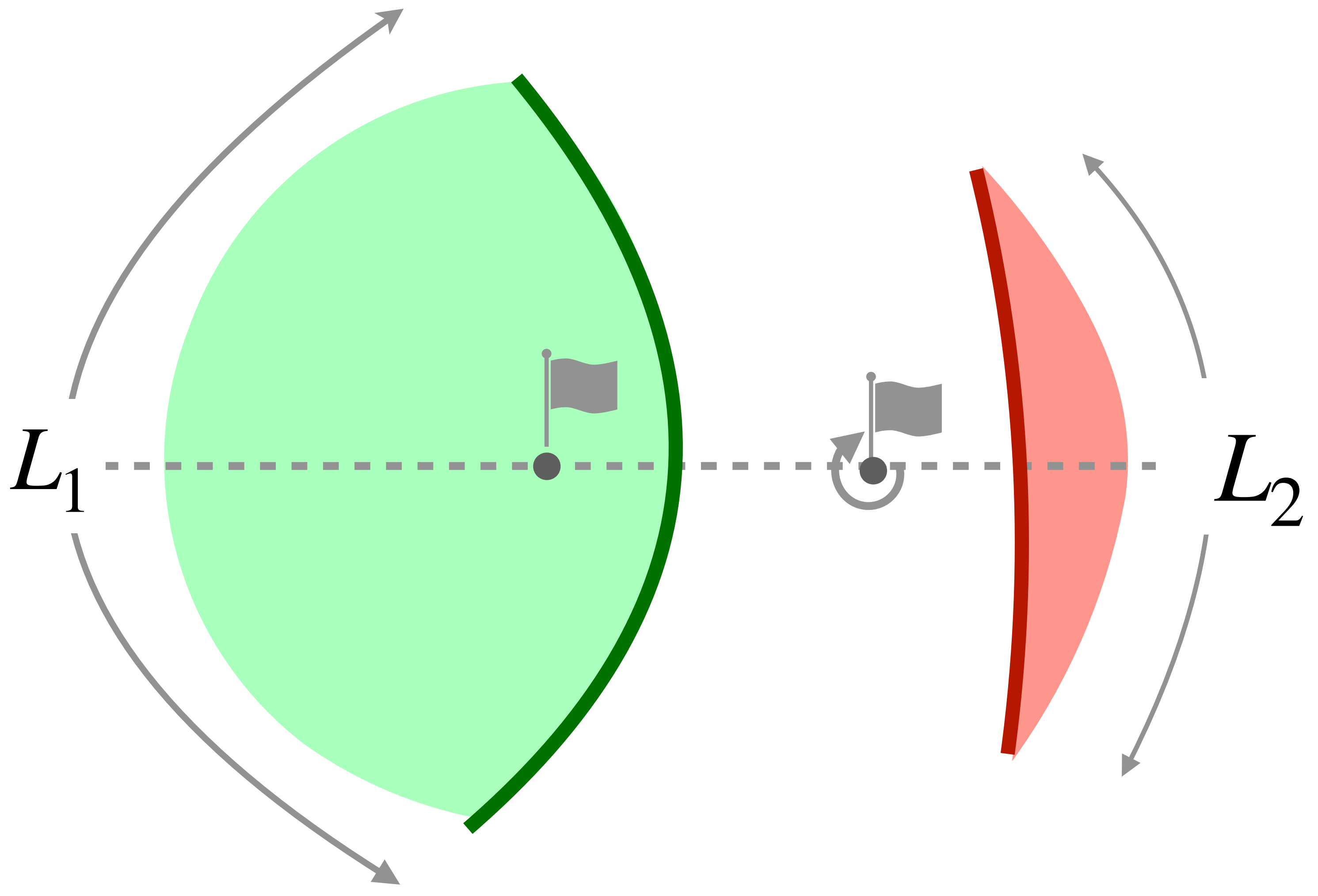
H1



H2

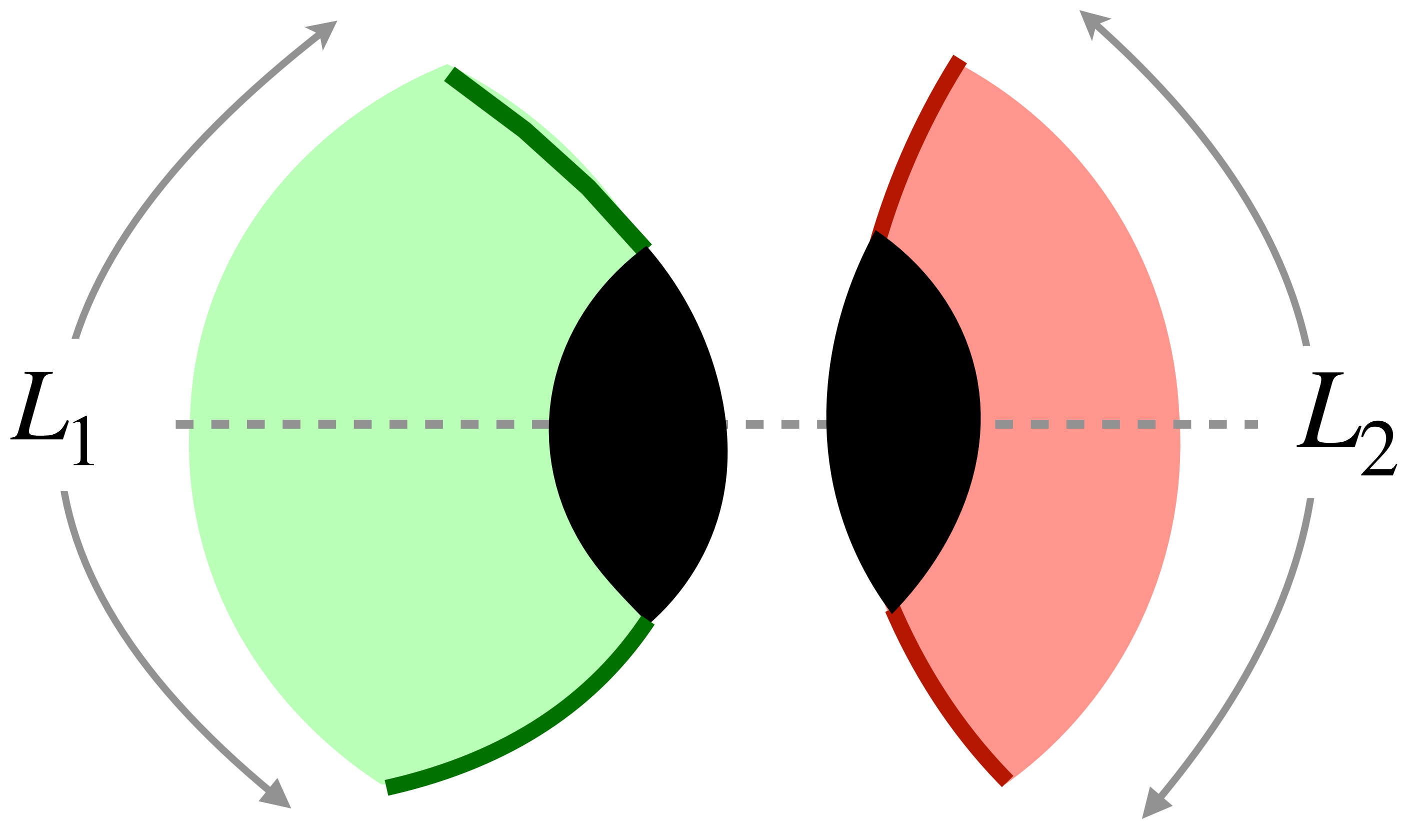
$$M = (2\pi T)^2$$

Examples of complete space time:



[E1,E2]

cold



[H2,H2]

hot

Embedding of static wall in each coordinate slice: $r_j(\sigma), x_j(\sigma)$

3 equations:

Continuity of metric	$[\hat{g}_{ab}] = 0$
Israel matching	$[\mathrm{tr}K] = -2\lambda$

Parametrize with blue-shift factor: $|\sigma| = r_1^2 - M_1\ell_1^2 = r_2^2 - M_2\ell_2^2$

Induced metric on domain wall: $d\hat{s}^2 = -|\sigma|\,dt^2 + g(\sigma)\,d\sigma^2$

4. SOLVING THE EQUATIONS

solution is algebraic:

$(\sigma \geq \sigma_+ \geq 0 > \sigma_-)$

$$\frac{x_1'}{\ell_1} = -\frac{\sigma(\lambda^2 + \lambda_0^2) + M_1 - M_2}{2(\sigma + M_1\ell_1^2)\sqrt{A\sigma(\sigma - \sigma_+)(\sigma - \sigma_-)}}$$

$$\frac{x_2'}{\ell_2} = -\frac{\sigma(\lambda^2 - \lambda_0^2) + M_2 - M_1}{2(\sigma + M_2\ell_2^2)\sqrt{A\sigma(\sigma - \sigma_+)(\sigma - \sigma_-)}}$$

In terms of Lagrangian parameters &

$\mu = M_2/M_1$

with

$$\sigma_\pm = \frac{-B \pm (B^2 - AC)^{1/2}}{A} \;;$$

$$A = (\lambda_{max}^2 - \lambda^2)(\lambda^2 - \lambda_{min}^2)$$

$$B = \lambda^2(M_1 + M_2) - \lambda_0^2(M_1 - M_2)$$

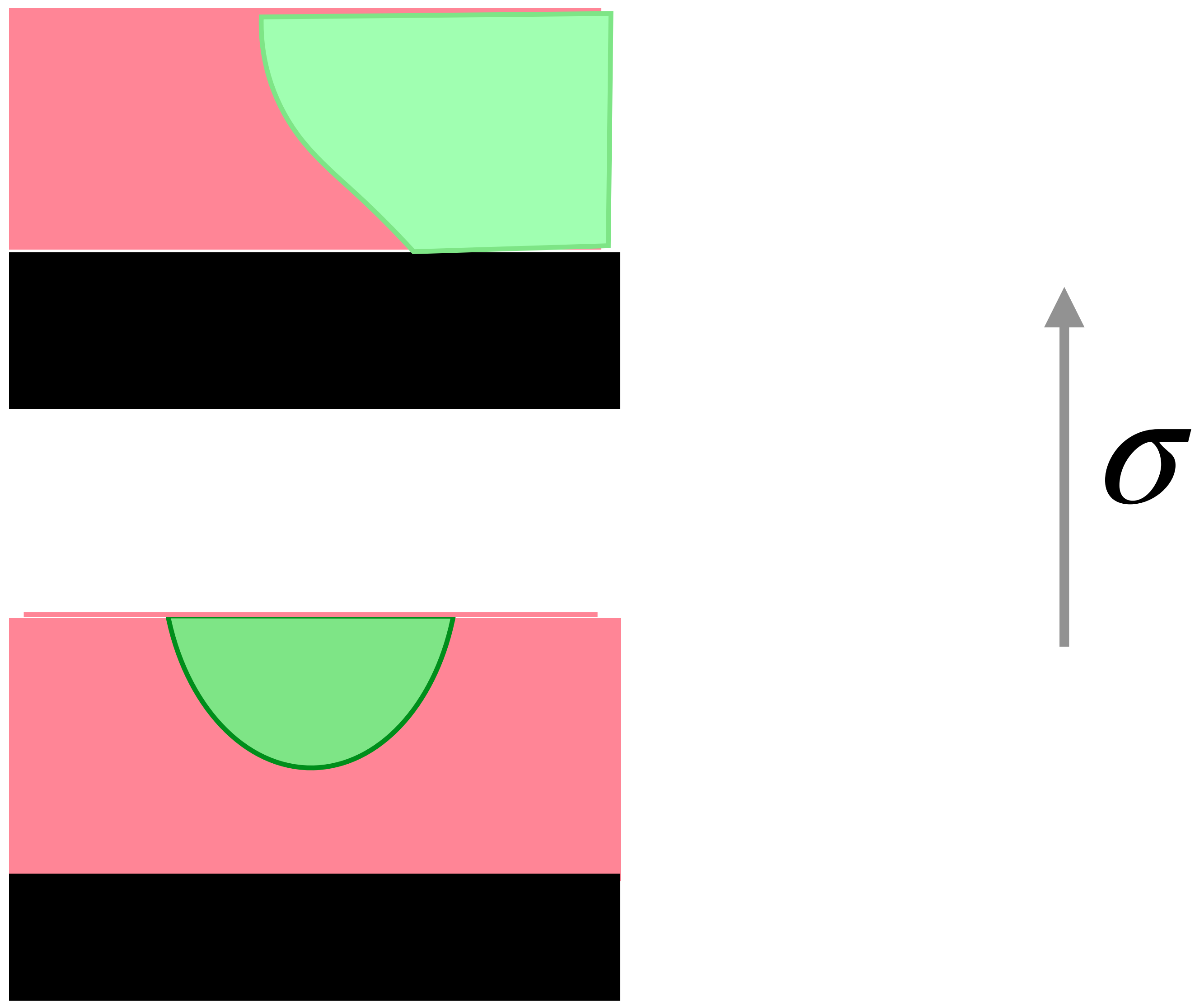
$$C = -(M_1 - M_2)^2$$

4. SOLVING THE EQUATIONS

Two possibilities:

$\sigma_+ = 0$ wall enters the horizon

$\sigma_+ > 0$ wall turning point



$$\mathrm{sign}\left(x_j'\Big|_{\sigma\approx\sigma_+}\right) = \begin{cases} + \ \text{for E2, E2}' & \textit{centerless} \\ - \ \text{for E1, H1} & \textit{center} \end{cases}$$

4. SOLVING THE EQUATIONS

Boundary conditions:

$$L_j = 2\int_{\sigma_+}^{\infty} d\sigma\, x_j' \qquad \text{for} \quad E2,\ E2'$$

$$L_j = \mathrm{Per}\,(x_j) + 2\int_{\sigma_+}^{\infty} d\sigma\, x_j' \qquad \text{for} \quad E1,\ H1$$

$$L_j = \Delta x_j\Big|_{\mathrm{Hor}} + 2\int_{\sigma_+}^{\infty} d\sigma\, x_j' \qquad \text{for} \quad H2$$

elliptic of 3rd kind, evaluated numerically

Assume no wall intersections; wall junctions are an interesting extension

Thermodynamic dictionary:

$T:$ Temperature

$L_j:$ Volumes

$\frac{1}{2}\ell_j M_j:$ Energy densities

$\frac{r_j^{\mathrm{H}}}{4G}\Delta x_j\Big|_{\mathrm{Hor}}:$ Entropies

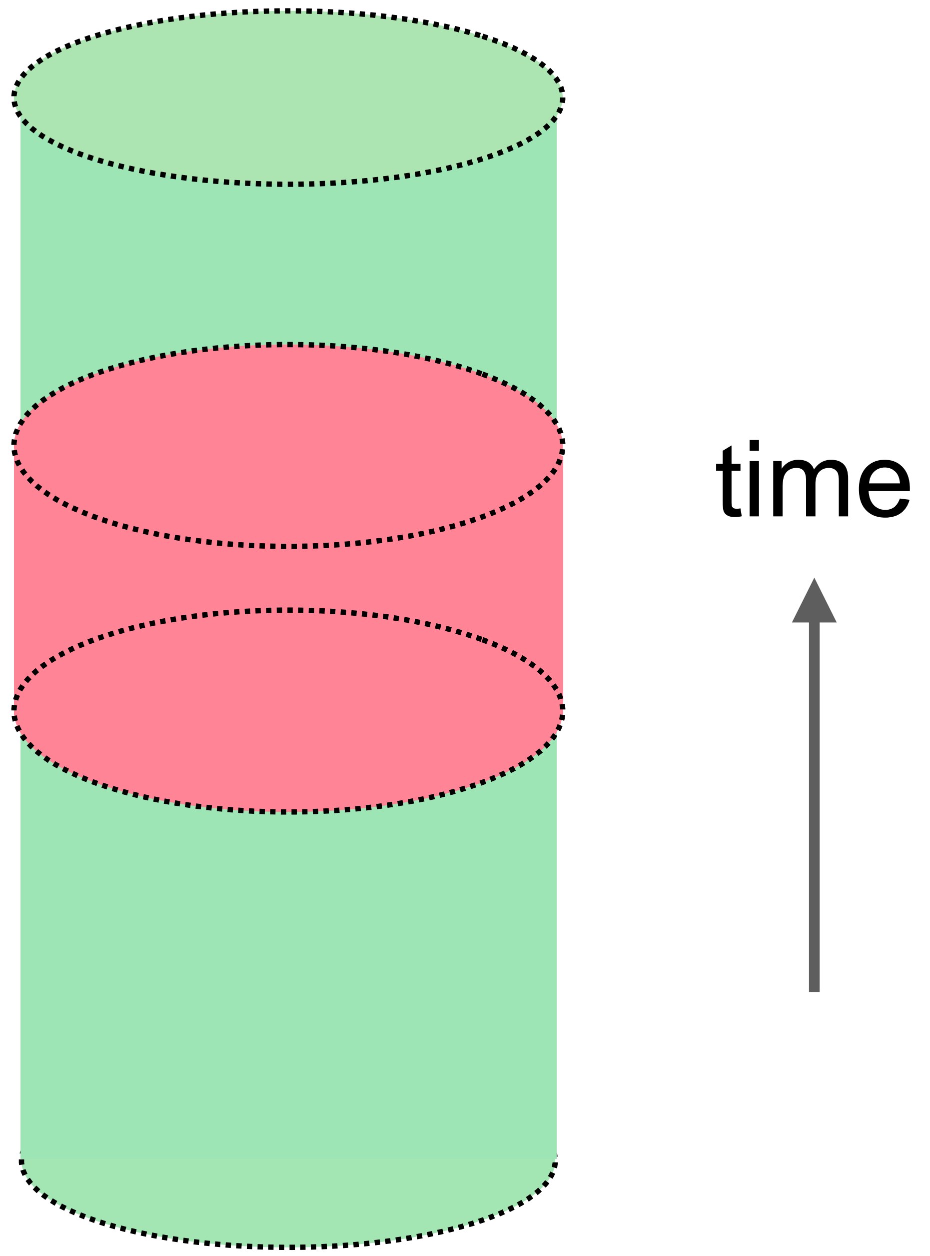
Boundary conditions = pair of coupled equations of state

that should be inverted in terms of canonical variables

Closely-related to

Simidzija & Van Raamsdonk

'Holo-ween' 2006.13943



`prepare CFT_2 states that mimic CFT_1 geometry'

NB: double Wick rotation (special to 2+1 dims); in Euclidean $L_1 \sim \infty$

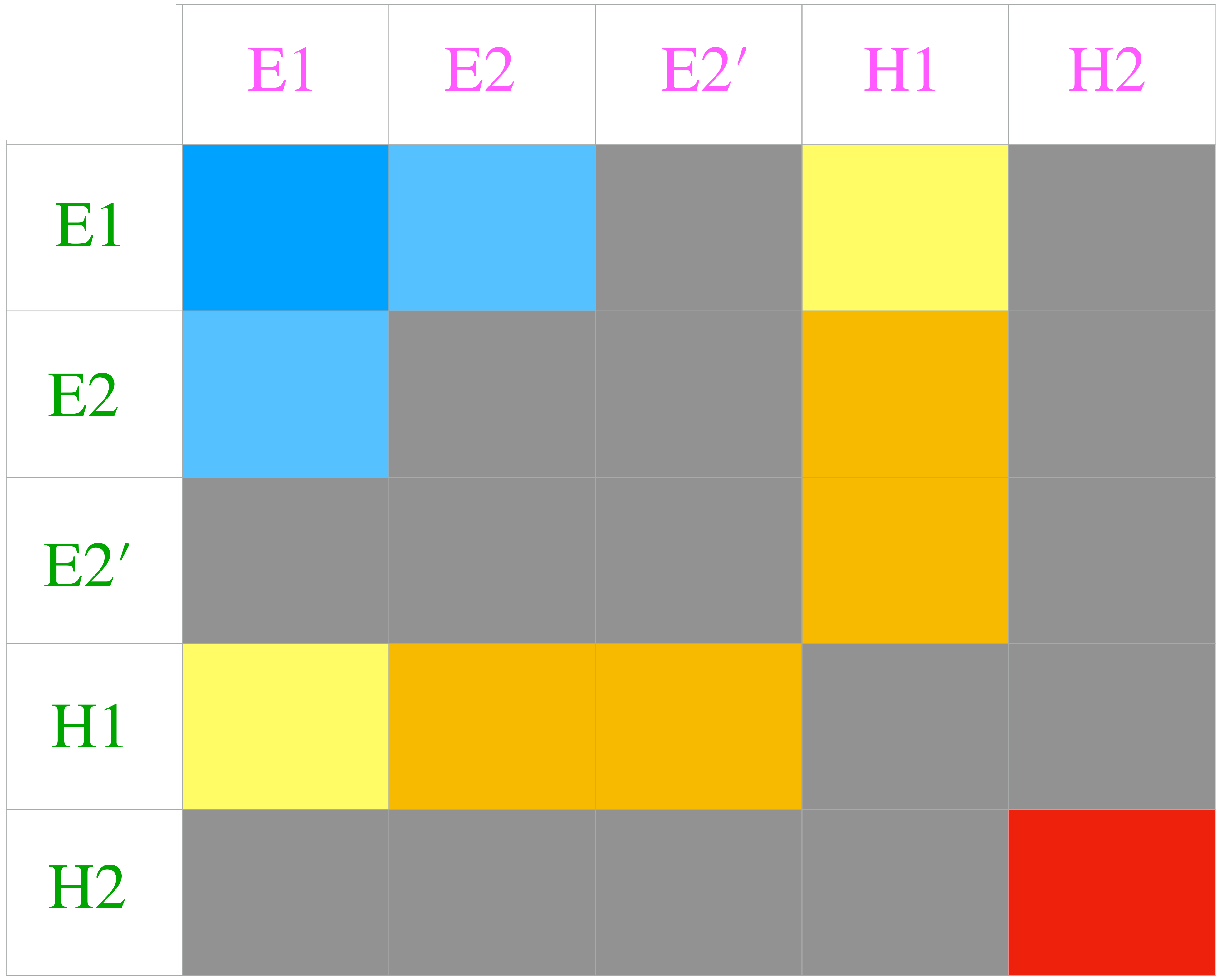
By studying $\operatorname{sign}\left(x_j'\big|_{\sigma\approx\sigma_+}\right)$ we can rule out the following slice pairs:

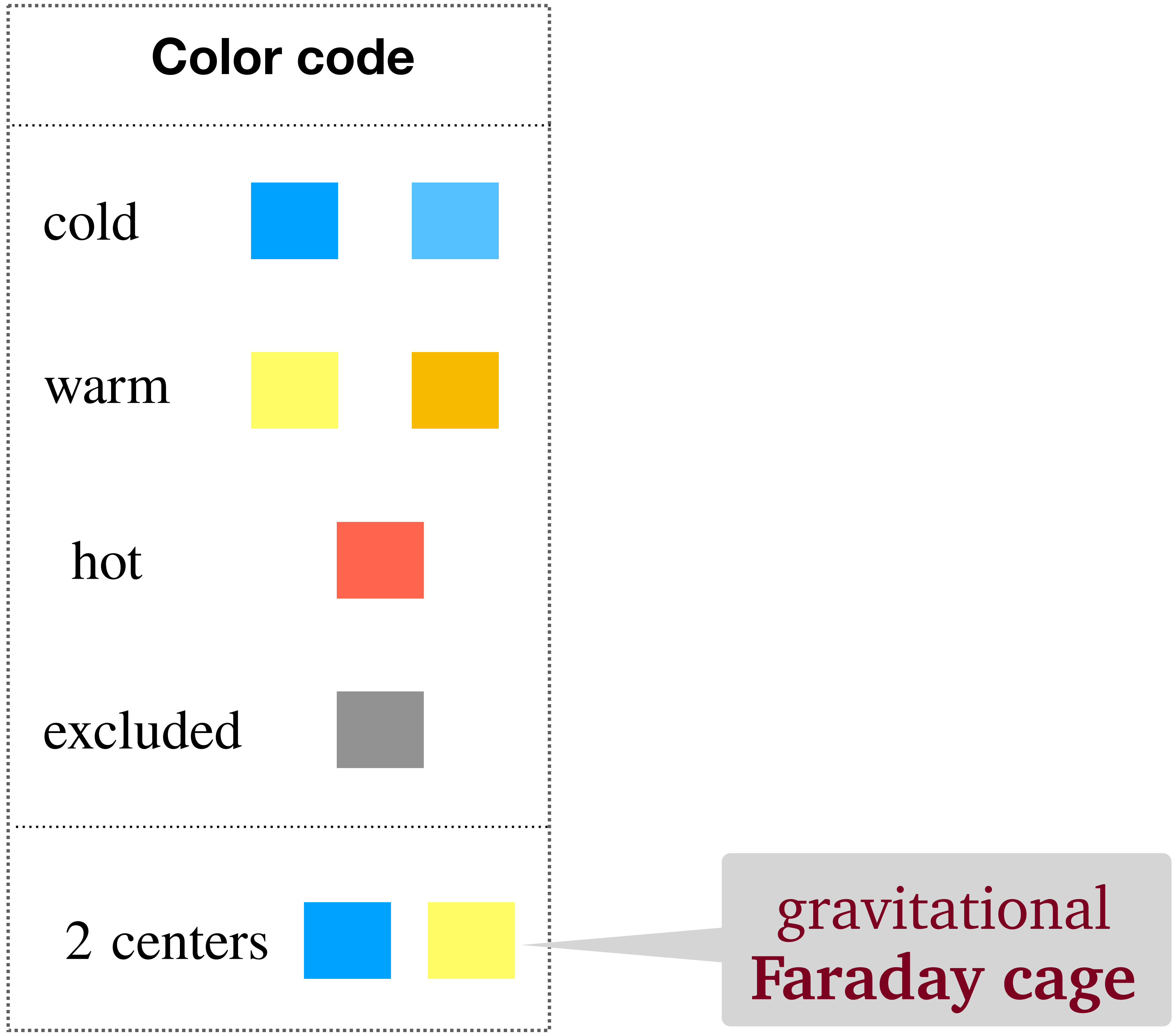
- ✗ 2 black holes $[\mathrm{H1},\ \mathrm{H1}]$ — *merger unavoidable*
- ✗ Centerless spacetimes $[\mathrm{E2}\ \text{or}\ \mathrm{E2}',\ \mathrm{E2}\ \text{or}\ \mathrm{E2}']$ — *No escape from wall*

  No traversable wormholes In Holo-ween
- ✗ $+$ ve vacuum energy $[\mathrm{E1}\ \text{or}\ \mathrm{E2},\ \mathrm{E2}']$ — *Casimir thms*

Kenneth, Klich; CB ’06

Possible phases:

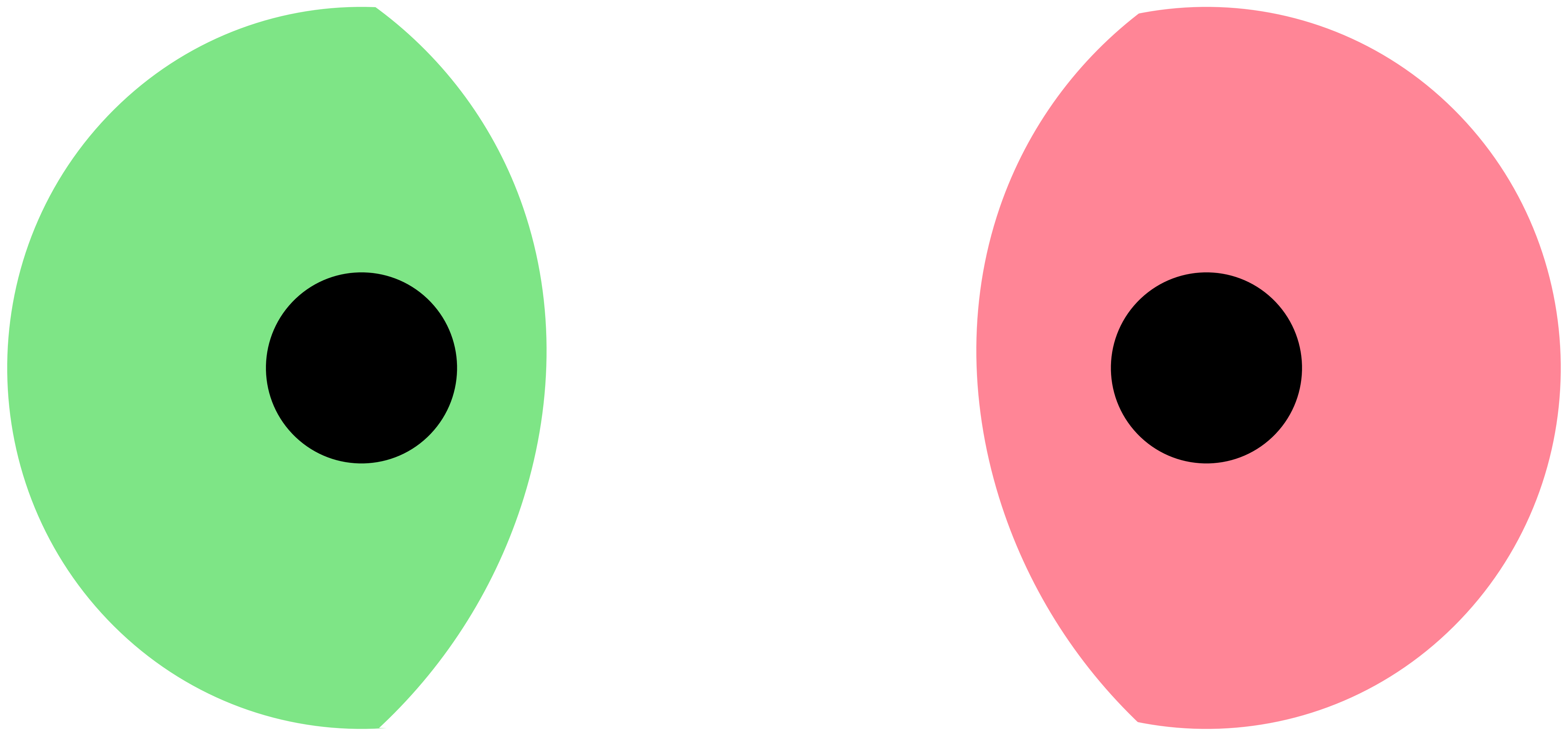




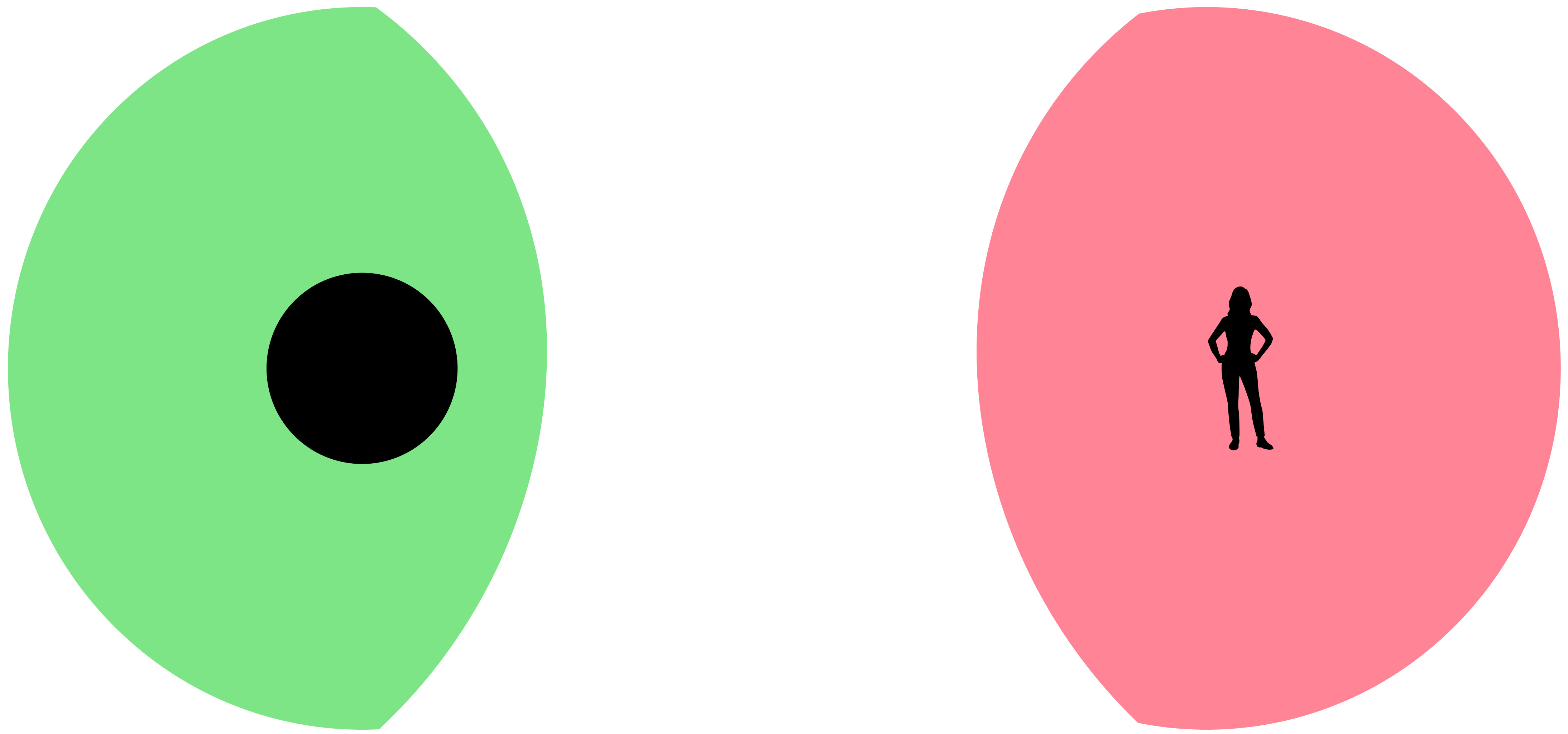
NB: *no warm phase in BCFT limit, End-of-the-World branes cannot avoid the BH*

Single horizon:

(matter thrown into system cannot form two BHs in separate traps)



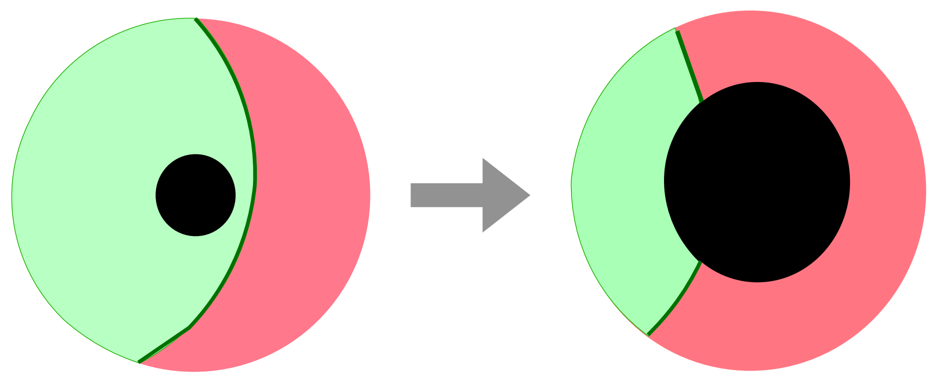
Gravitational **Faraday cage:**



Phase transitions:

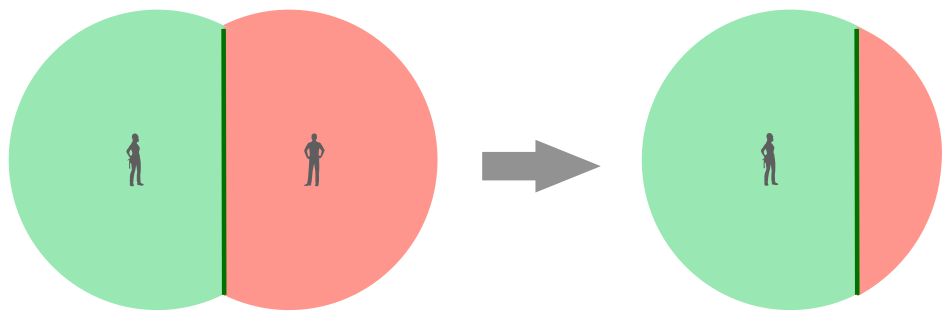
-- **Hawking-Page** (*formation of BH*) : 1rst order

-- **Warm-to-hot** (*wall enters horizon*) : 1rst order

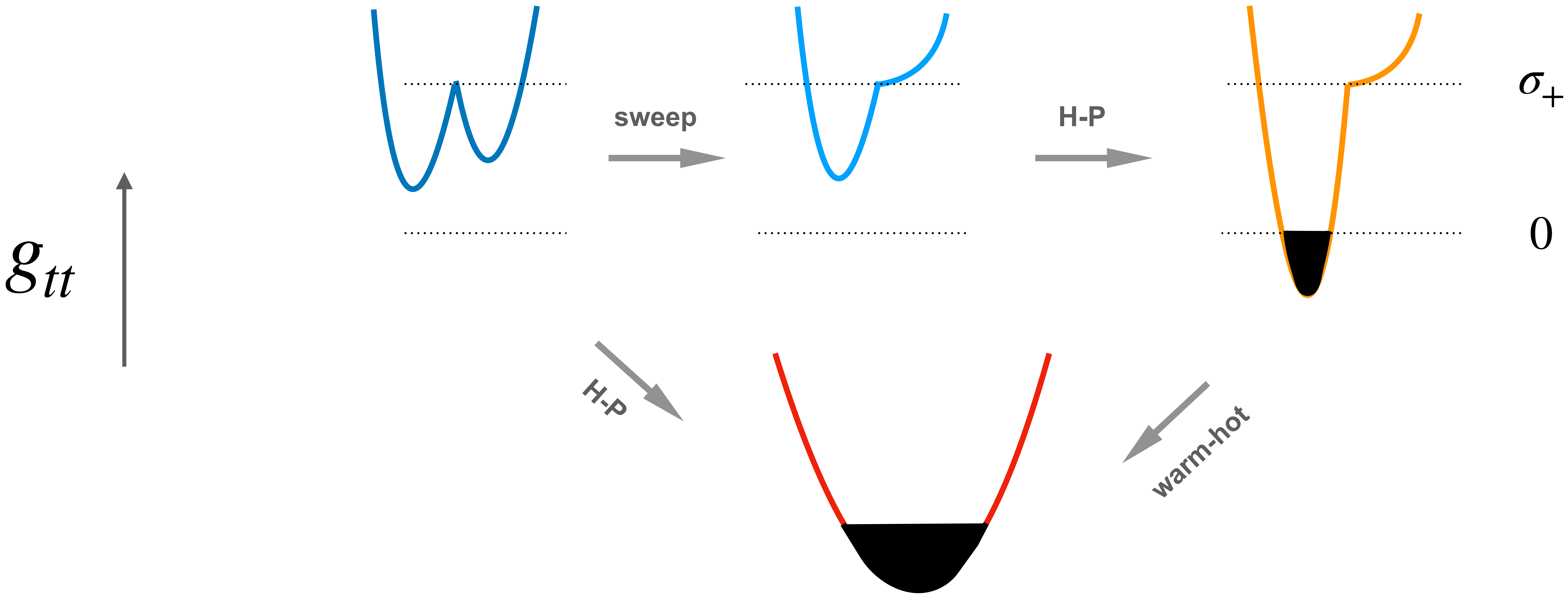


cannot be lowered continuously

-- **Sweeping** (*restpoint of inertial observer*) : continuous

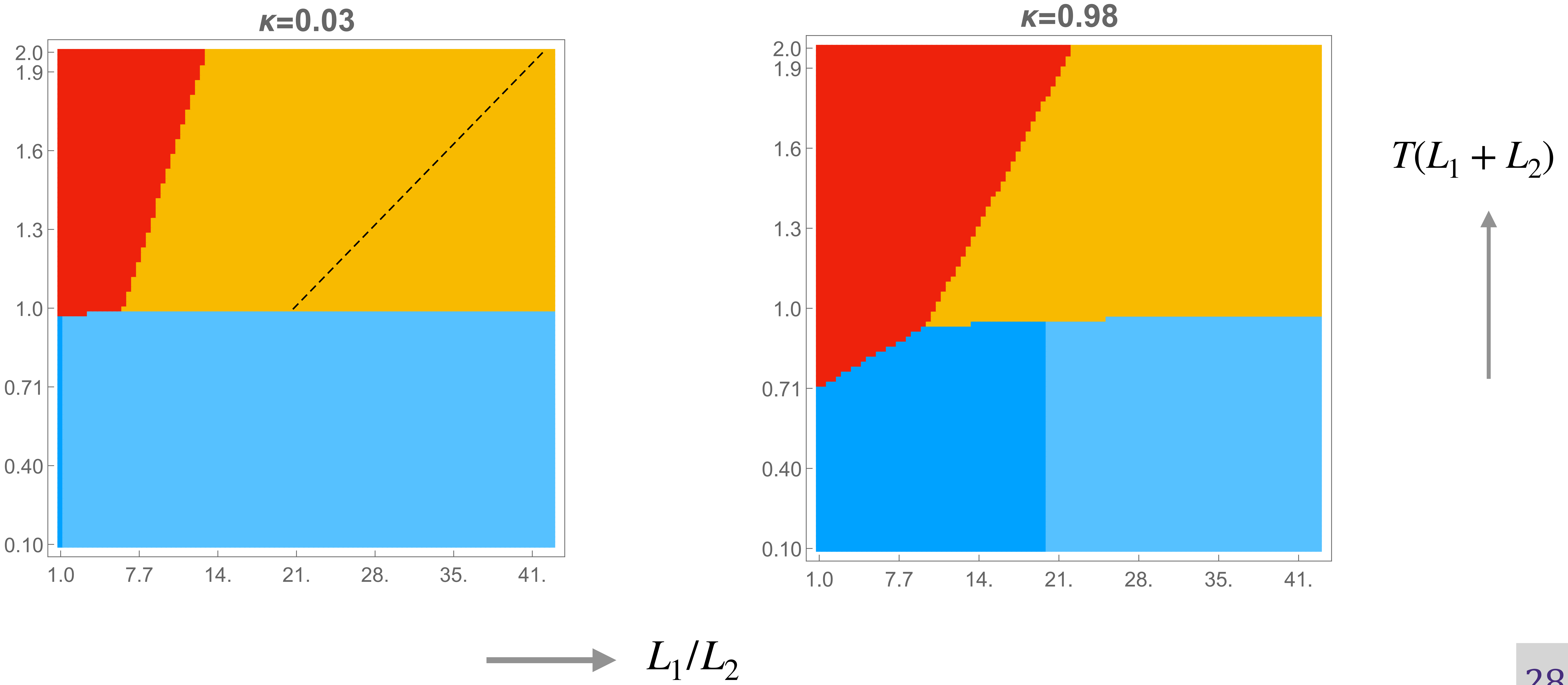


Blueshift along axis of reflection symmetry:

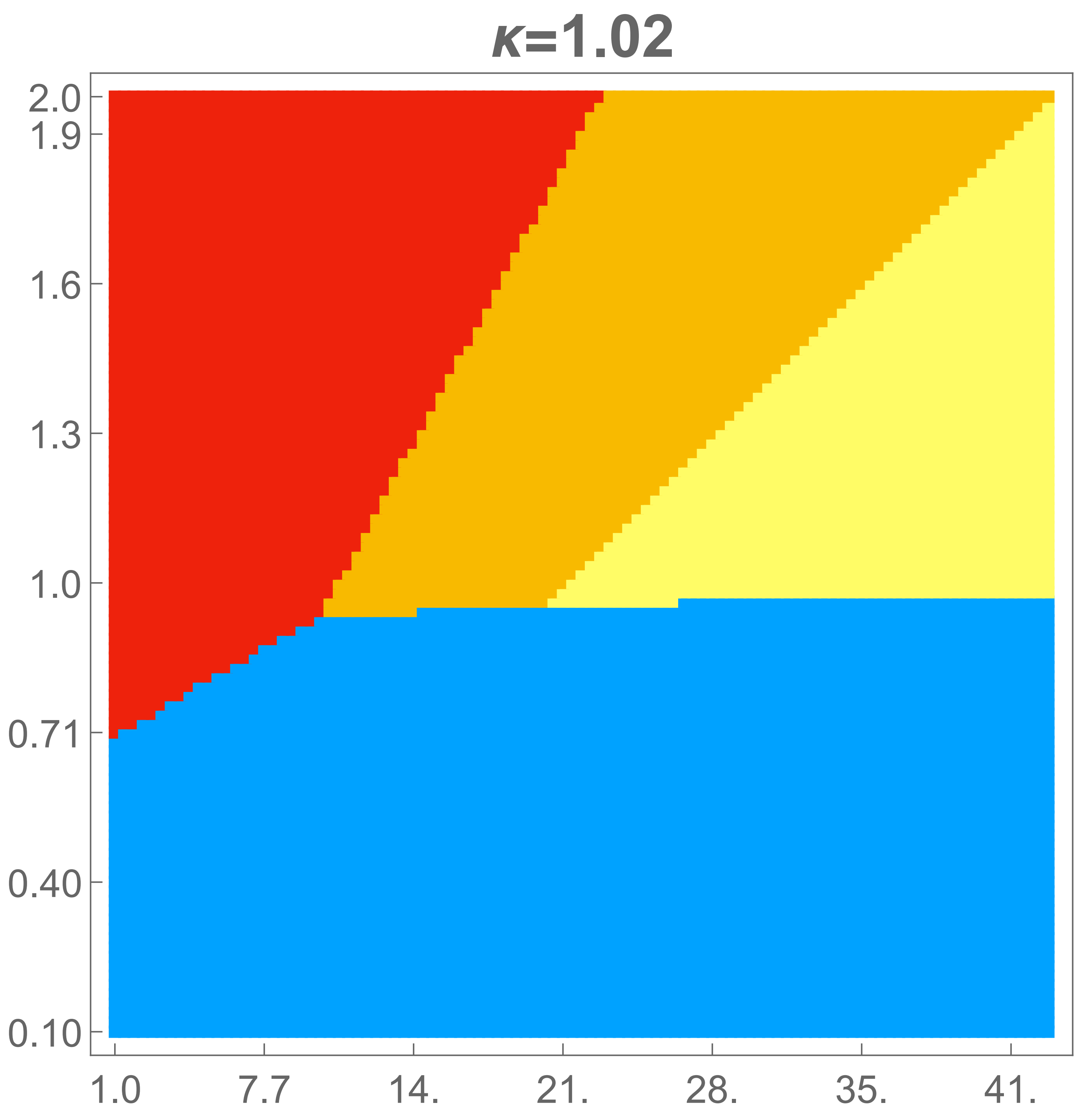


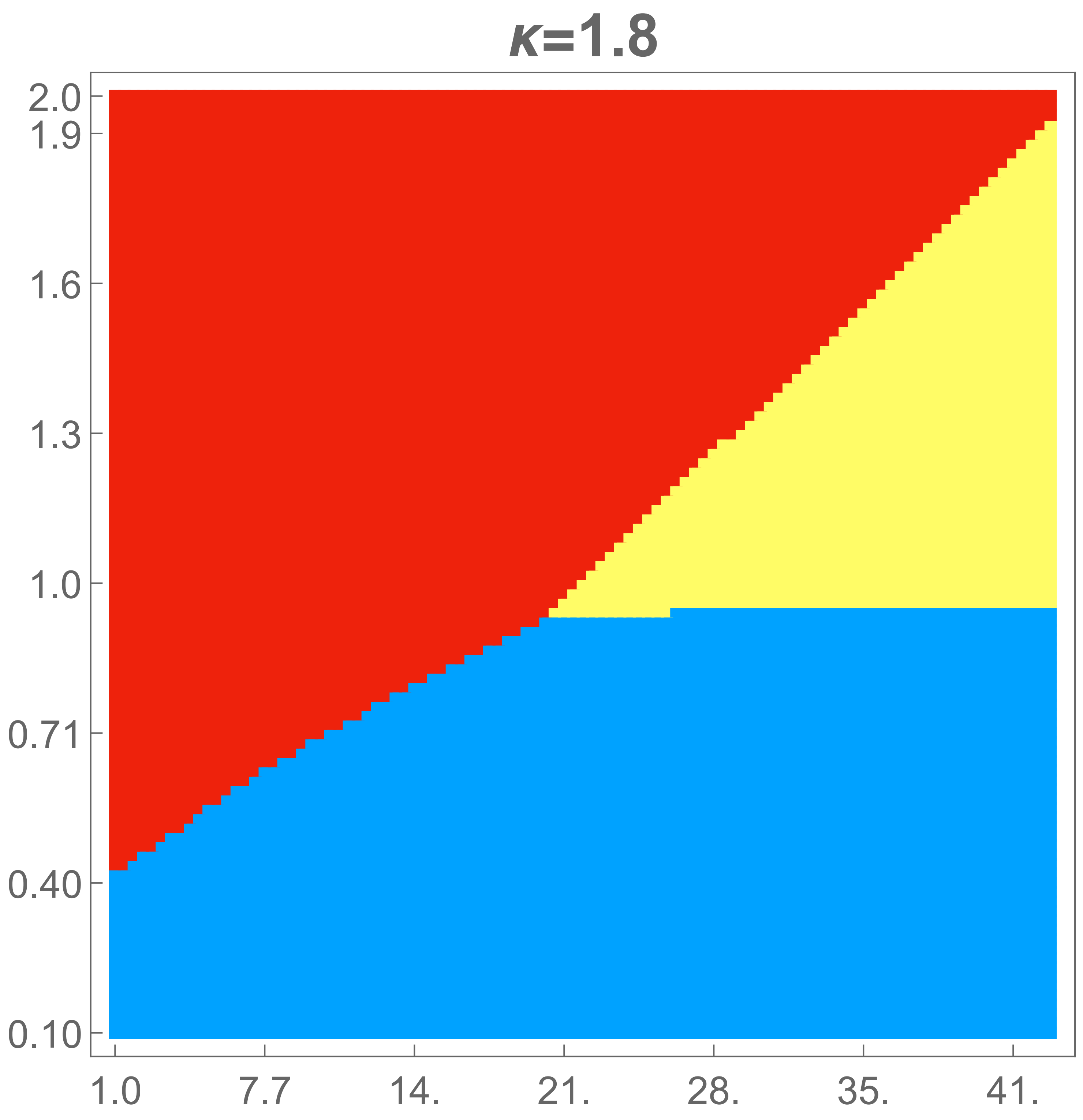
Phase diagrams for $\ell_1 = \ell_2$

$$\kappa = \lambda \ell_1 \in (0,2)$$



6. PHASES & TRANSITIONS





Heavier walls fall more readily into horizon, but protect better inertial observers

7. ICFT & 2 PUZZLES

Hawking-Page ⟷ Deconfinement transition

Witten '98

Aharony *et al* '05; Keller '11

Warm phase: coexistence of confined with deconfined CFT

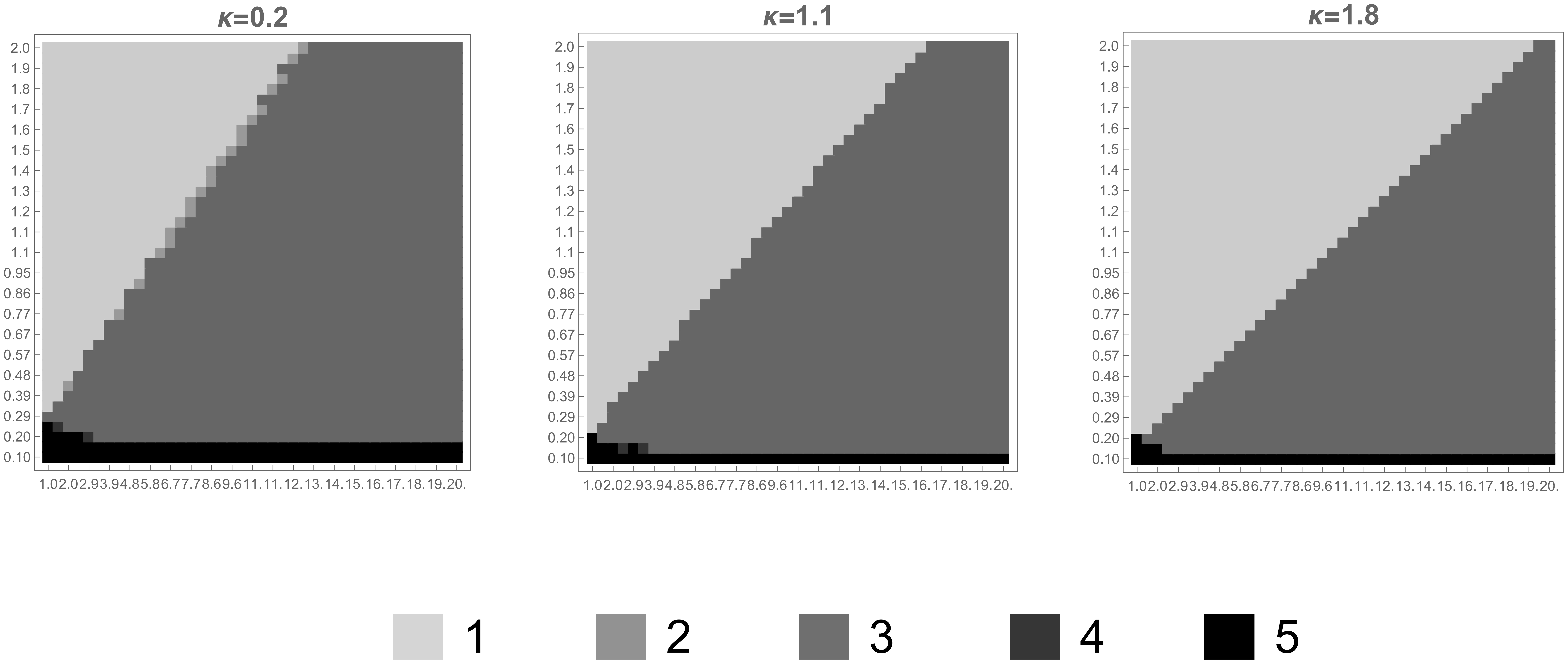
CFT order parameter for **warm-to-hot** transition = Polyakov loop

but what about gravity with thick walls ?

ICFT interpretation of **sweeping** transition ?

Entanglement of state? RT wedges ? *in progress*

Several black-hole solutions (some with -ve specific heat) can coexist:

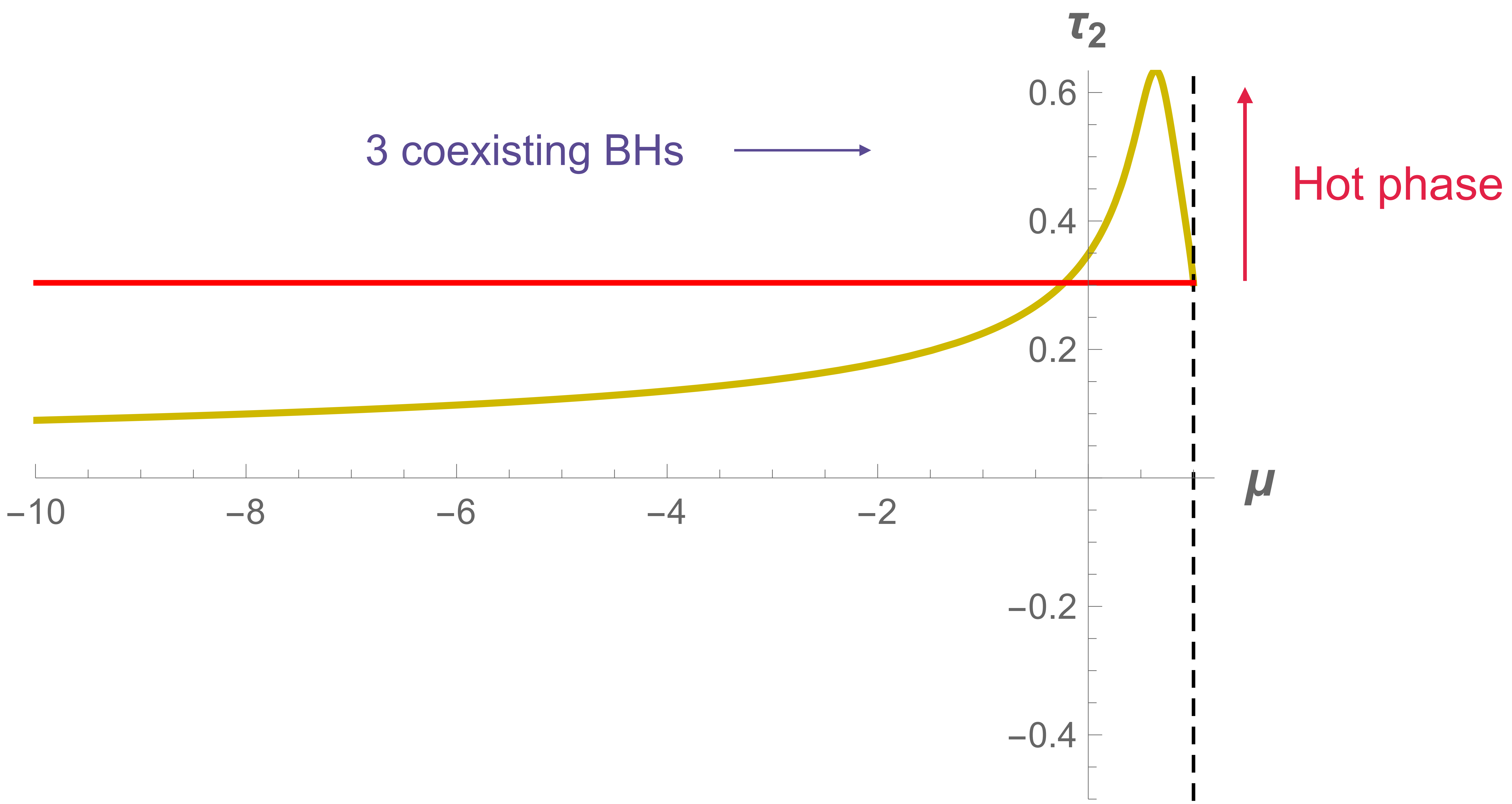


cf generalized JT gravity, Witten '20

$\tau_2 \equiv TL_2$

$\mu \equiv M_2/(2\pi T)^2$

Yellow curve: $\tau_2(\mu)$ for [H1,E2] or [H1,E2$'$] solutions



8. UNSTABLE BHs & EXOTIC FUSION

A surprise: in the limit $L_j \to 0$ the space of CFT_j shrinks to a point

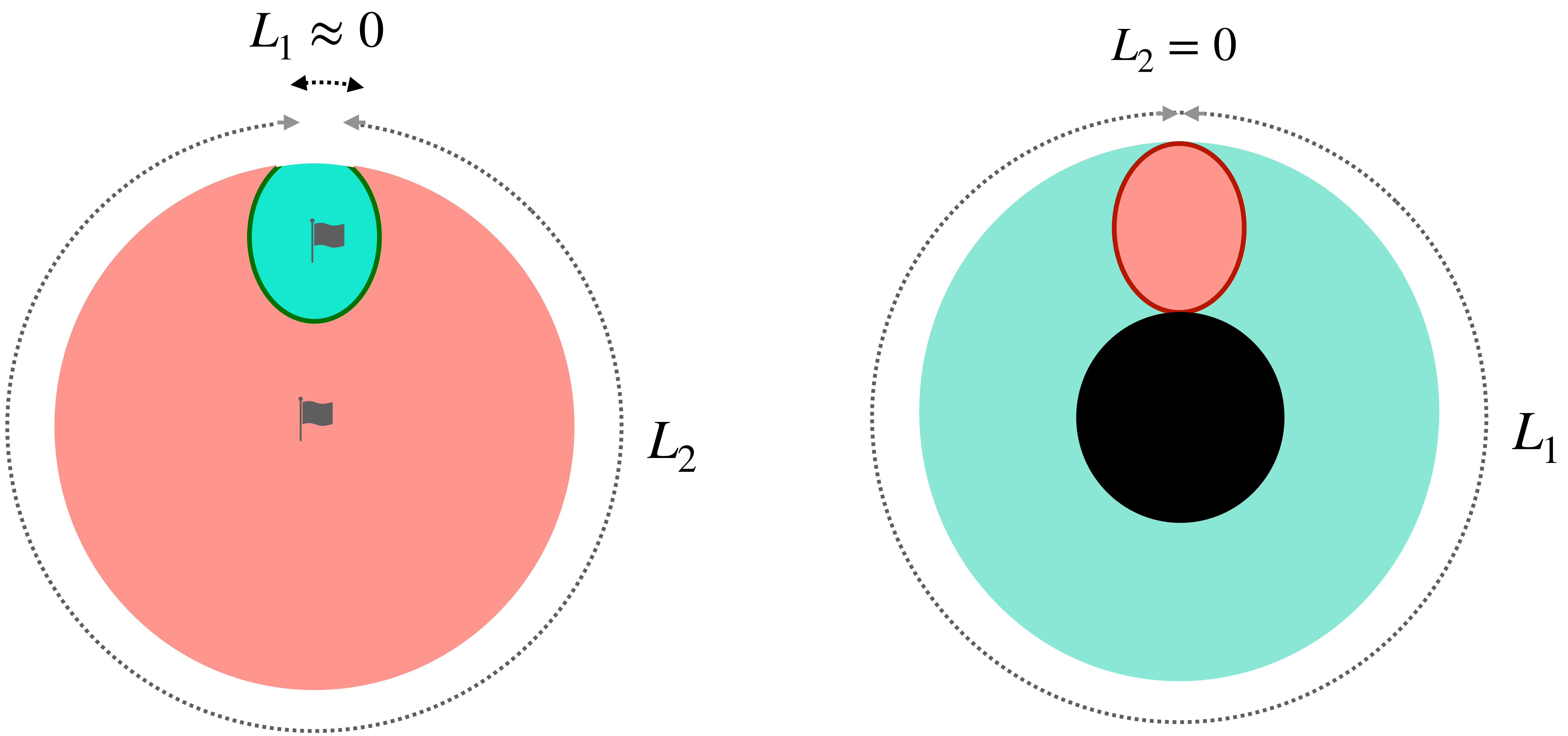
In free-field calculations, the fusion

$$\text{Interface} \otimes \text{Anti interface} = \mathbf{1} \oplus e^{-\#/TL_j} \cdots$$

CB, Brunner '07

Accordingly the corresponding slice should shrink away with $M_j \to -\infty$

But $\exists$ region in (ℓ_j, λ) parameter space where this expectation fails !



Suspended bubbles: asymptotically-free fusion ?

9. CONCLUSION

Rich phase diagram of Holographic Interfaces, resulting from competing forces :

wall repulsion of particles, wall tension, attraction of AdS trap and BH

-- 3 kinds of phase transition:
- HP and wall-entering-horizon **1rst order**
- sweeping **continuous**

-- Unstable Black Holes which evaporate/emigrate ?

-- Exotic phenomena *(fusion; wormholes)*

9. CONCLUSION

? What survives in top/down thick-wall models ?

Hard because no AdS fibers

? Entanglement RT wedges ?

? Extension to out-of-equilibrium states ?

? Decay of unstable BHs ?

Thank you