Towards a microscopic model of AdS fragmentation

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1. Background
2. An SYK-like model
3. Relations to string theory
4. Conclusions

Some (ancient) history

I will begin by reviewing some facts about ensembles with fixed charge Q in Einstein-Maxwell theory in 4d flat space.

These issues were discussed in [Maldacena, Michelson, Strominger, '98]

Some relevant history

A beautiful exact solution to Einstein-Maxwell theory is due to [Majumdar and Papapetrou '47]

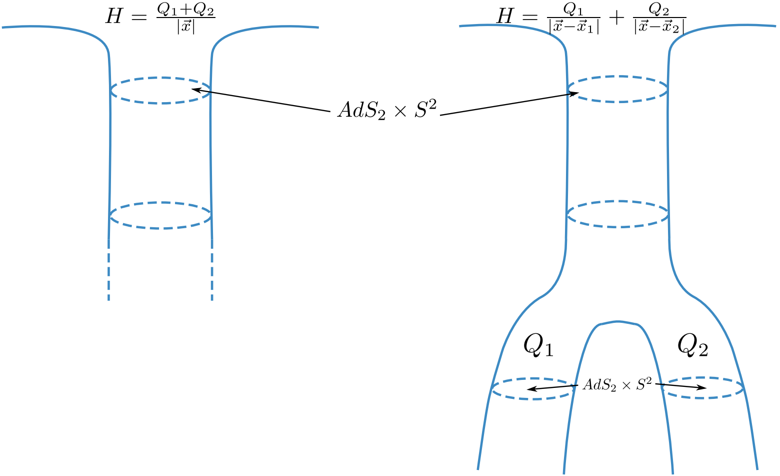
$$A = -\left(1 - H^{-1}\right) dt \qquad \text{and} \qquad ds^2 = -H^{-2}dt^2 + H^2 d\vec{x}^2$$

$$\text{EOM:} \qquad \nabla^2 H = 0 \longrightarrow H = 1 + \sum_a \frac{q_a}{|\vec{x} - \vec{x_a}|}$$

- q_a and $\vec{x}_a$ are free parameters
- The constant 1 in H leads to flat space at large $|\vec{x}|$, but can be omitted so that we have $\mathrm{AdS}_2 \times S^2$ at large distance.

Some relevant history

Consider the following two geometries:



For large $|\vec{x}|$ the geometries are equivalent, but the one on the right flows from a single $\mathrm{AdS}_2 \times S^2$ to a pair of separate ones as we approaches $\vec{x}_{1/2}$

Some relevant history

We can read off the difference between these two geometries from the multipole moments of the electrostatic field at infinity.

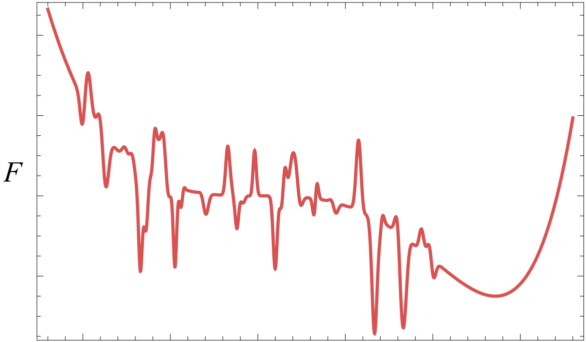
Some relevant history

We can read off the difference between these two geometries from the multipole moments of the electrostatic field at infinity.

Moreover, there exists a Euclidean instanton discovered by [Brill '92] that mediates between the two geometries.

Interpretation?

Generally, whenever there exists an instanton that tunnels between semiclassical states, we envision a (free) energy landscape of minima of some potential



Don't Ask

Open questions

- In the age of the SYK/Schwarzian paradigm for describing AdS_2, can we accommodate this fragmentation picture?
- What is the order parameter that distinguishes between the macroscopic states, dual to the nonzero dipole moment of the fragmented geometry?
- The separation $r \equiv |\vec{x}_1 - \vec{x}_2|$ has no potential, and changing r does not spoil the $SL(2,\mathbb{R})$ symmetries. Does this correspond to a marginal operator in the dual description?

And for the impatient/skeptical members of the audience, I will discuss some connections to string theoretic black holes at the end of the talk.

Disclaimer

I'm going to describe a model that has some features I would like to tie to the picture I've just described. But this picture has not been fully fleshed out.

The quantum p-spin model

Let us introduce a model of N bosons σ_i, studied by [Cugliandolo, Grempel, da Silva Santos, '01]:

$$Z = \int D\sigma_i \exp\left\{ - \int_0^\beta d\tau \, \left[\frac{M}{2} \dot{\sigma}_i(\tau) \dot{\sigma}_i(\tau) + J_{i_1 \dots i_p} \sigma_{i_1}(\tau) \dots \sigma_{i_p}(\tau) \right] \right\} \, ,$$

with a *spherical constraint*:

$$\sum_{i=1}^{N} \sigma_i(\tau) \sigma_i(\tau) = N$$

and disordered couplings sampled from:

$$P(J_{i_1 \dots i_p}) \propto \exp\left[- \frac{N^{p-1}}{p!} \frac{J^2_{i_1 \dots i_p}}{J^2} \right] \, .$$

The quantum p-spin model

- Since these are bosons, the spherical constraint prevents unstable runaways in the potential.
- This is a nonlinear sigma model where we have fixed the size of the sigma model manifold
- As a result, the the phases are labeled by the two dimensionless couplings M/β and βJ

Disorder average

As usual we want to compute

$$\beta\overline{F} = -\int dJ_{i_1\ldots i_p} P(J_{i_1\ldots i_p}) \log Z[J_{i_1\ldots i_n}] \ ,$$

but this requires us to compute $Z[J_{i_1\ldots i_p}]$ for arbitrary couplings. Use:

$$\log Z = \lim_{n\to 0} \partial_n Z^n$$

and take the average of Z^n for integer n:

$$\overline{Z^n} = \int dJ_{i_1\ldots i_p} P(J_{i_1\ldots i_p}) \int D\sigma_i^a Dz^a$$
$$\exp\left\{ -\int_0^\beta d\tau \ \left[\frac{M}{2}\dot{\sigma}_i^a(\tau)\dot{\sigma}_i^a(\tau) + J_{i_1\ldots i_p}\sigma_{i_1}^a(\tau)\ldots\sigma_{i_p}^a(\tau)\right] \right.$$
$$\left. + i\int_0^\beta d\tau \, z^a(\tau)\left(\sigma_i^a(\tau)\sigma_i^a(\tau) - N\right) \right\} \ .$$

Disorder average

Like in SYK, the disorder average couples replicas. To proceed, we introduce a bi-local variable:

$$N\, Q_{ab}(\tau,\tau') \equiv \sum_{i=1}^{N} \sigma_i^a(\tau)\sigma_i^b(\tau')$$

and the effective bi-local action is:

$$\frac{S_{\text{eff}}}{N} = -\frac{1}{2}\mathrm{Tr}\log\left[Q_{ab}(\tau,\tau')\right] - i\sum_{a=1}^{n}\int_0^\beta d\tau\, z^a(\tau)\left(Q_{aa}(\tau,\tau)-1\right)$$
$$-\sum_{a,b=1}^{n}\int_0^\beta\int_0^\beta d\tau\, d\tau' \left[\delta_{ab}\,\delta(\tau-\tau')\frac{M}{2}\partial_\tau^2 Q_{ab}(\tau,\tau') + \frac{J^2}{4}Q_{ab}(\tau,\tau')^p\right]\,.$$

where the z^a are Lagrange multipliers to impose the spherical constraint $Q_{aa}(0)=1$.

Schwinger-Dyson equations

Like in SYK, we can derive the Schwinger-Dyson equations.

$$
\begin{aligned}
&- \delta_{ab} \left[\frac{M}{2} \partial_\tau^2 + i z^a(\tau) \right] Q_{ab}(\tau, \tau') \\
&\qquad - \frac{pJ^2}{4} \int_0^\beta d\tau'' \, Q_{ac}^{p-1}(\tau, \tau'') Q_{cb}(\tau'', \tau') = \frac{1}{2} \delta_{ab} \, \delta(\tau - \tau') \ .
\end{aligned}
$$

Dropping the first line and taking $Q_{ab} = Q(\tau)\delta_{ab}$ gives us the SD equations of the SYK model, with the known late time conformal solution.

Important Caveat

Going back to the original definition, for $a \neq b$:

$$Q_{a\neq b}(\tau,\tau') = \frac{1}{N}\sum_{i=1}^{N}\overline{\langle\sigma_i^a(\tau)\sigma_i^b(\tau')\rangle} = \frac{1}{N}\sum_{i=1}^{N}\overline{\langle\sigma_i^a(\tau)\rangle\langle\sigma_i^b(\tau')\rangle}$$

where $\langle\cdot\rangle$ denotes a thermodynamic average in a single replica and $\overline{A}$ denotes a disorder average. In SYK the fundamental d.o.f were fermions and could not obtain vevs. Here they can and do.

In fact the low temperature thermodynamics requires that we consider the following 1-RSB ansatz for consistency

$$\mathbf{Q} = \begin{pmatrix} q(\tau,\tau') & u & u & & & & \\ u & q(\tau,\tau') & u & & 0 & & \cdots \\ u & u & q(\tau,\tau') & & & & \\ & & & q(\tau,\tau') & u & u & \\ & 0 & & u & q(\tau,\tau') & u & \\ & & & u & u & q(\tau,\tau') & \\ & \vdots & & & & & \ddots \end{pmatrix}$$

where the off-diagonal component u measures the overlap between replicas. There is an additional parameter m which determines the size of the diagonal blocks.

SD equations revisited

On this subspace we have:

$$-\frac{1}{2}\delta(\tau-\tau') = \left[\frac{M}{2}\partial_\tau^2 + iz(\tau)\right] q(\tau,\tau') \\ + \frac{pJ^2}{4}\int_0^\beta d\tau''\left(u^{p-1} - q(\tau,\tau'')^{p-1}\right)\left(u - q(\tau'',\tau')\right) \ ,$$

and boundary condition $q(0) = 1$.

The parameters u and m are determined by extremizing S_{eff}.

SD equations revisited

$$-\frac{1}{2}\delta(\tau-\tau') = \left[\frac{M}{2}\partial_\tau^2 + iz(\tau)\right] q(\tau,\tau')$$
$$+\frac{pJ^2}{4}\int_0^\beta d\tau'' \left(u^{p-1} - q(\tau,\tau'')^{p-1}\right)\left(u - q(\tau'',\tau')\right) \ ,$$

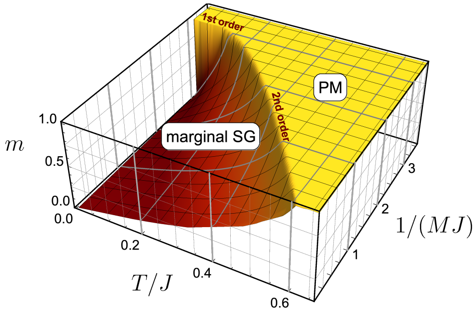
Depending on the relative sizes of βJ and M/β, for couplings where $u = 0$ we can have conventional SYK behavior with

$$q(\tau) \propto |\tau|^{-2/p}$$

Caveat, this conformal solution is not the global minimum of the F, in this phase$\longrightarrow$the state is gapped.

Phase transition

There is a line critical T and M line where $u \neq 0$ and $m \neq 1$ signaling a spin glass transition



Expanding:

$$q(\tau, \tau') = q_r(\tau, \tau') + u$$

and carefully analyzing the SD equations, we find:

$$q_r \propto (\tau - \tau')^{-2} \tag{1}$$

suggesting a conformal operator with $\Delta = 1$.

A new marginal operator has appeared.

Sublety with m

m does not satisfy the SD equations. So the conformal solution is not an equilibrium solution of this ensemble.

Can deal with this by changing to an ensemble where replica symmetry is *explicitly* broken [Mezard, '99]. This gives:

$$\frac{S_{\text{eff}}(Q^*)}{Nn} \equiv \beta m \Phi$$

m now acts as a fugacity for F much like β is a fugacity for E.

Thermodynamic analogy/interpretation

Usual case $\frac{S_{\text{eff}}(Q^*)}{Nn} \equiv \beta F$:

$$S = -\partial_T F \ , \qquad E = \partial_\beta(\beta F) \ .$$

SG phase $\frac{S_{\text{eff}}(Q^*)}{Nn} \equiv \beta m \Phi$:

$$\Sigma = -\partial_{1/m}(\beta\Phi) \ , \qquad F = \partial_m(m\Phi) \ .$$

The quantity Σ is an entropy-like quantity that counts metastable states.

Bulk interpretation?

Can we tie the appearance of $u \neq 0$ to the fact that we have developed a nonzero *averaged* dipole moment over the space of low temperature bulk configurations?

Is the appearance of a $\Delta = 1$ mode in the model also be tied to the moduli-space mechanics of moving the separate AdS_2 throats?

Is Σ counting a regularized moduli space volume for the throats?

Interpretation?

The picture I told about fragmentation is only part of the story. Each individual throat can also split up into its own set of fractal like throats.

So perhaps there should be an infinite number of order parameters (for each multipole moment) as well as a huge set of marginal operators, to accurately describe the bulk.

But in the spirit of being playful, let us look at the features this model has to offer.

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Quiver QM

There exist quantum mechanical models that arise from compactifying string theory on a Calabi-Yau threefold with D-branes wrapping certain cycles.

- These models have $\mathcal{N} = 4$ SUSY
- Open string modes between branes described by chiral multiplet d.o.f. (ϕ, ψ_α, F)

Low energy EFT

SUSY fixes the form of the interaction Lagrangian

$$L_{\text{int}} = \left(\frac{\partial W(\phi)}{\partial \phi_i^a} F_i^a + h.c. \right) + \left(\frac{\partial^2 W(\phi)}{\partial \phi_i^a \partial \phi_j^b} \psi_i^a \epsilon \psi_j^b + h.c. \right)$$

where a labels pairs of branes that intersect inside the CY and $i = 1, \ldots, N_a$ labels the number of light string states connecting the branes.

Generally W is an arbitrary polynomial of the ϕ^a, with coefficients carying data from the compactification.

Bosonic potential

The bosonic potential comes from integrating out F:

$$V = \sum_{i,a} \left| \frac{\partial W(\phi)}{\partial \phi_i^a} \right|^2 + \sum_a \left(-\theta_a + \sum_i |\phi_i^a|^2 \right)^2$$

- θ_a are called Fayet-Illiopoulos parameters
- They are permitted by SUSY and generally do appear
- θ_a impose something analogous to the spherical constraint!

p-spin model in ST

Can consider the first nontrivial example:

$$W = J_{ijk} \phi_i^1 \phi_i^2 \phi_k^3$$

where we treat J_{ijk} like a disorder, since it's related to some complicated CY_3 physics. Then we are getting very close to the p-spin model described earlier.

This quiver model is known to describe black holes, so we should map out exactly what features are black hole like!

Conclusions

- Today we identified a transition in GR that would be interesting to understand
- Looked at a toy model that shares some features
- I'm currently setting up to compute the OTOC in the spin glass phase. Many subtleties involved, will report back.
- Has the potential to connect to more string theoretic models.