A Wheeler DeWitt approach for Liouville quantum gravity

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Plan of the talk

Introduction

- Jackiw Teitelboim (JT) gravity and Random matrices
- $c = 1$ Liouville theory
- Correspondence between MQM in the double scaling limit and $c = 1$ Liouville theory

Main Part

- Liouville theory with boundaries
- Minisuperspace WdW wavefunctions
- Second quantised fermionic field theory (double scaled)
- Matrix model loop operators and correlation functions
- Single loops (Wavefunctions - density of states of dual theory)
- Loop correlator and spectral form factor (Euclidean wormholes)
- Cosmological regime

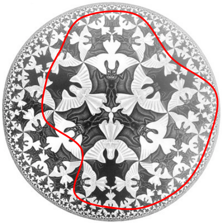
Summary and Future Directions

Jackiw - Teitelboim (JT) gravity

- JT gravity is a simple (almost topological) theory of gravity in 2d
- The action is (metric $g_{\mu\nu}$, dilaton ϕ, manifold $\mathcal{M}$)

$$S = S_0\,\chi(\mathcal{M}) + \int_{\mathcal{M}} \sqrt{g}\phi(R+2) + \phi_0 \int_{\partial\mathcal{M}} \sqrt{h}K$$

- The first term is (pure) 2d topological gravity, the second localises the action on $R = -2$ geometries



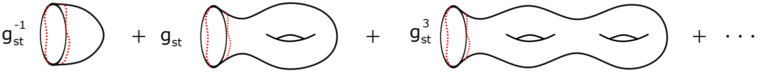
- The last term governs the dynamics of fluctuating boundary/ies
- e^{S_0} governs the topological expansion (like g_{st})
- $S_0 \sim N$ measures the microscopic degrees of freedom (e.x. SYK/random matrix model)

(JT) gravity and Random Matrices

[Saad-Shenker-Stanford, Maldacena, Johnson, Turiaci, Mertens,...]

- There is a connection between JT gravity and random matrices
- The JT path integral can be resummed using a model of random Hermitean matrices H_{ij}

$$\mathcal{Z} = \int_{N\times N} dH e^{-NV(H)}$$



- By inserting loop operators one can create (Multi)-boundary geometries
- JT gravity provides an interesting window to the physics of the quantum gravity path integral
- Euclidean Wormholes, "3rd quantisation", connection with Random Matrices ...
- Possible to study (p, q) minimal models and $c = 1$ Liouville theory

Liouville theory

[Polyakov, David, Distler, Kawai...], Reviews by: [Ginsparg, Nakayama]

Can one make sense of string theory in case the conformal anomaly is not canceled?

- Gauge fix only the worldsheet diffeos and keep the conformal mode of the metric dynamical.
- Note that the measure $\mathcal{D}g$ is not invariant under $g_{ab} \to e^{\rho(\sigma)} g_{ab}$.
- Exponentiating the conformal anomaly from the measure, the total action becomes ($\mu = 1, ...d$, conformal gauge $g_{ab} = \hat{g}_{ab} e^{\phi(\sigma)}$)

$$S_{CFT} = \frac{1}{4\pi} \int d^2\sigma \sqrt{\hat{g}} \left[\hat{g}^{ab} (\partial_a X^\mu \partial_b X_\mu + \partial_a \phi \partial_b \phi) + Q \hat{R} \phi + 4\pi \mu e^{2b\phi} \right] + ghosts$$

- This new theory is a "conformal theory" under the simultaneous transformation $g_{ab} \to e^{\rho(\sigma)} g_{ab}$, $\phi(\sigma) \to \phi(\sigma) - \rho(\sigma)$, iff

$$c_X + c_\phi + c_{gh} = 0\,, \quad c_\phi = 1 + 6Q^2\,, \quad Q = \sqrt{\frac{25-d}{6}}\,, \quad Q = b + 1/b$$

$\Rightarrow b$ is real up to $d = c_X = 1$

What is Matrix Quantum Mechanics

Reviews by: Ginsparg-Moore, Klebanov, Martinec,...

- MQM (gauged) is a $0+1$ dimensional quantum mechanical theory of $N \times N$ Hermitian matrices $M(t)$ and a non dynamical gauge field $A(t)$.
- The Path Integral is:

$$e^{-iW} = \int \mathcal{D}M\mathcal{D}A \exp\left[-iN \int_{t_{in}}^{t_f} dt \, \mathrm{Tr}\left(\frac{1}{2}\left(D_t M\right)^2 + \frac{1}{2}M^2 - \frac{\kappa}{3!}M^3 + ...\right)\right]$$

- One can diagonalise M by a unitary transformation $M(t) = U(t)\Lambda(t)U^\dagger(t)$ where $\Lambda(t)$ is diagonal and $U(t)$ unitary.
- One then picks up a Jacobian from the path integral measure ($\forall t$)

$$\mathcal{D}M = \mathcal{D}U \prod_{i=1}^{N} d\lambda_i \Delta^2(\Lambda), \quad \Delta(\Lambda) = \prod_{i<j}^{N}(\lambda_i - \lambda_j)$$

- This Vandermonde determinant is responsible for many interesting physical aspects of Matrix Models.

Fermionic description

- The Hamiltonian is

$$H = -\frac{1}{2\Delta^2(\lambda)} \frac{d}{d\lambda_i} \Delta^2(\lambda) \frac{d}{d\lambda_i} + \sum_{i<j} \frac{J_{ij} J_{ji}}{(\lambda_i - \lambda_j)^2} + V(\lambda_i) \ ,$$

- J_{ij} are "momenta" conjugate to $SU(N)$ rotations
- Set the gauge field to zero - impose the Gauss-law constraint $\delta S / \delta A = i[M, \dot{M}] \sim J = 0$ (*singlet sector projection*)
- Upon rescaling $\lambda \to \frac{\sqrt{N}}{\kappa} \lambda$ and redefining the wavefunction as $\tilde{\Psi}(\lambda) \equiv \Delta(\lambda) \Psi(\lambda)$, the Schrödinger equation now reads

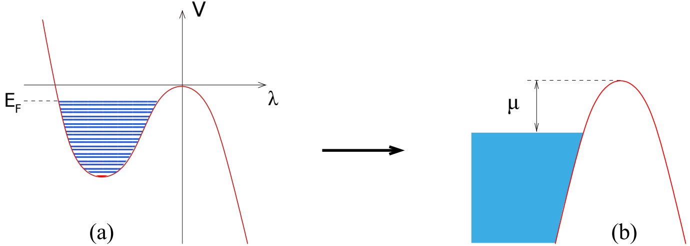
$$\left(-\frac{1}{2} \frac{d^2}{d\lambda_i^2} - \tfrac{1}{2} \lambda_i^2 + \frac{\sqrt{\hbar}}{3!} \lambda_i^3 + \dots \right) \tilde{\Psi}(\lambda) = \hbar^{-1} E \tilde{\Psi}(\lambda), \qquad \hbar^{-1} = \frac{N}{\kappa^2}$$

- This describes N non interacting fermions in a potential $V(\lambda)$.

Double scaling limit

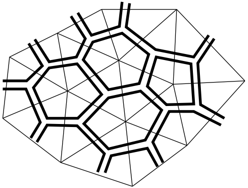
[Kazakov, Migdal...]

- Consider an initial state where the energy levels are populated up to some Fermi energy E_F below the top of the barrier, and send $\hbar \to 0$, $N \to \infty$, such that $E_F \to 0$.
- Enough to focus on the quadratic maximum of the potential. We hold $\mu = -E_F/\hbar$ fixed in the limit.
- The result is quantum mechanics of free fermions in an inverted harmonic oscillator potential, with states filled up to $-\mu < 0$.
- At this limit the model is perturbatively stable in $1/N \to 0$ expansion, since by WKB we can see that the tunneling probability goes to zero.



The connection with the QGR path integral is through this double scaling limit [Kazakov, Migdal...]. This is *not* the usual 't Hooft limit.

- The double scaling limit produces smooth surfaces out of the Matrix fat-graphs while at the same time keeping all higher genera. It is defined by $\hbar, E_F \to 0$ as we discussed, while keeping $\mu \sim g_{st}^{-1}$ fixed.



- The QGR theory is the $c = 1$ *Liouville theory*. (It can also be interpreted as a 2D critical string theory in a linear dilaton background with a time direction t and a space direction ϕ)

- MQM describes the dynamics of D_0 (ZZ)-branes whose excitations are a "Tachyon" and a 1-d gauge field (open-closed string duality). [McGreevy, Verlinde]

Main Part

Quantum Gravity Path Integral

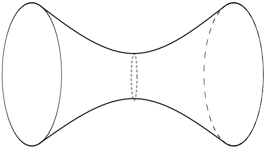
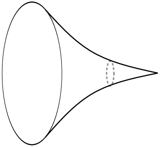
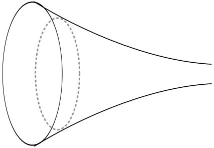
- In this talk we take the 2d quantum gravity path integral point of view of Liouville
- $\phi(z,\overline{z}), X(z,\overline{z})$ are the matter fields living on the two-dimensional space(-time)
- Our bulk space(-time) are the 2d surfaces $\Rightarrow$ We are describing the evolution of "universes embedded in superspace"
- Superspace is the target space in the string theory interpretation
- We can also have boundaries on our surfaces on which a usual Holographic dual (a la AdS/CFT) would be expected to reside
- MQM captures the dynamics of these simple 2d universes in superspace
- There is a second quantised double scaled fermionic field theory [Moore] with coordinates t, λ gives the best description to compute continuum observables

Classical Liouville solutions

- The Poincare disk is a solution of Liouville EOM's ($PSL(2,R)$ symmetry), the boundary is at $\phi \to \infty$
- More general solution; consider a quotient of the hyperbolic space H_2/Γ with Γ a discrete Fuchsian group

$$ds^2 = e^{2b\phi}(z,\overline{z})dzd\overline{z} = \frac{Q}{\pi\mu b}\frac{\partial A\overline{\partial}B}{\left(A(z) - B(\overline{z})\right)^2}dzd\overline{z}$$

- The properties of the geometry are governed by the monodromy of A, B around non-trivial cycles $\Rightarrow$ Three $SL(2,R)$ conjugacy classes
  - Hyperbolic: Describes handles (higher genera)
  - Elliptic: Punctures (local operators)
  - Parabolic: Macroscopic boundaries (ex: Poincare disk)

Boundaries and Boundary states

[Zamolodchikov, Zamolodchikov, Fateev, Teschner]

- The Liouville action with boundaries is

$$S = \int_{\mathcal{M}} d^2 z \sqrt{g} \left(\frac{1}{4\pi} g^{ab} \partial_a \phi \partial_b \phi + \frac{1}{4\pi} Q R \phi + \mu e^{2b\phi} \right) + \int_{\partial \mathcal{M}} ds g^{1/4} \left(\frac{QK\phi}{2\pi} + \mu_B e^{b\phi} \right)$$

- K is extrinsic curvature and μ, μ_B the bulk-boundary cosmological constants. For $c_{matter} = 1 \Rightarrow b = 1, Q = 2$ and $\mu_B = \sqrt{\mu} \cosh(\pi\sigma)$.
- Liouville theory has two types of boundary states
- The FZZT boundary state (FZZT are $D1$ branes with Neumann boundary conditions for the open strings $\Rightarrow$ extending along Liouville direction)

$$|B_\sigma\rangle = \int_{-\infty}^{\infty} d\nu e^{2\pi i \nu \sigma} \Psi_\nu(\sigma) |\nu\rangle ,$$

$$\Psi_\nu(\sigma, \mu) = (\mu)^{-i\nu/b} \frac{\Gamma(1 + 2i\nu b)\Gamma(1 + 2i\nu/b) \cos(2\pi\sigma\nu)}{2^{1/4}(-2i\pi\nu)}$$

Minisuperspace WdW wavefunctions

- Study the WdW equation at a minisuperspace level (zero modes)
- The bulk Liouville minisuperspace wavefunctions are ($\ell = e^{2\phi_0}$)

$$\left(-\frac{\partial^2}{\partial\phi_0^2} + 4\mu e^{2\phi_0} - q^2\right)\Psi_q(\phi_0) = 0$$

with solutions

$$\Psi_q^{macro}(\ell) = \frac{1}{\pi}\sqrt{q\sinh\pi q}\,K_{iq}(2\sqrt{\mu}\ell)$$

$$\int_0^\infty \frac{d\ell}{\ell}\Psi_q^{macro}(\ell)\Psi_{q'}^{macro}(\ell) = \delta(q-q')$$

For $c=1$, q is the momentum conjugate to X (matter boson)

- These are macroscopic states that correspond to the Laplace transform of the fixed μ_B FZZT wavefunctions

$$\Psi_q^{macro}(\ell) = \int_0^\infty d(\pi\sigma)e^{-2\ell\sqrt{\mu}\cosh(\pi\sigma)}\Psi_q(\sigma,\mu)$$

- The wavefunctions corresponding to local punctures/operators are non-normalisable

"Non-perturbative observables" - Fermionic field theory

- We wish to describe observables in the continuum double scaling limit $\Rightarrow$ second quantised fermionic field theory

$$S = \int dt\, d\lambda\, \hat{\psi}^\dagger(t,\lambda) \left(i\frac{\partial}{\partial t} + \frac{\partial^2}{\partial \lambda^2} + \frac{\lambda^2}{4} \right) \hat{\psi}(t,\lambda)$$

- Eigenmodes: normalised even/odd parabolic cylinder functions $\psi^s(\omega,\lambda)$

$$\hat{\psi}(t,\, \lambda) = \int d\omega\, e^{i\omega t}\, \hat{b}_s(\omega)\, \psi^s(\omega,\lambda)$$

with $\hat{b}_s(\omega)$ are continuum fermionic oscillators and the fermi-sea vacuum $|\mu\rangle$ (μ is a chemical potential), is defined by

$$\begin{aligned} \hat{b}_s(\omega)|\mu\rangle &= 0, \quad \omega < \mu \\ \hat{b}_s^\dagger(\omega)|\mu\rangle &= 0, \quad \omega > \mu \end{aligned}$$

- This is a "3rd quantised action" encapsulating topology changing processes (even though non-interacting)!
- Several choices for the non-perturbative vacuum: both sides filled - one side (wall), flux condition [Balthazar-Rodriguez-Yin]

Macroscopic loop operators

- The fixed length matrix model loop operator is

$$\hat{W}(L,x) \,=\, \frac{1}{N}\,\mathsf{Tr}\, e^{L\hat{M}(x)}$$

- The fixed boundary cc. loop operator is

$$\hat{W}(\mu_B,x) \,=\, -\frac{1}{N}\,\mathsf{Tr}\,\log[\mu_B-\hat{M}(x)] \,=\, \frac{1}{N}\sum_{l=1}^{\infty}\frac{1}{l}\,\mathsf{Tr}\,\left[\hat{M}(x)/\mu_B\right]^l - \log\mu_B$$

- In terms of the fermions, the most basic operator is the density operator

$$\hat{\rho}(x,\lambda) = \hat{\psi}^\dagger(x,\lambda)\hat{\psi}(x,\lambda)$$

- In the double scaling continuum limit we can rewrite Matrix operators in terms of the basic density operator
- In particular the continuum macroscopic loop operators with length ℓ

$$\hat{\mathcal{W}}_{cont}(\ell,x) = \int d\lambda\, e^{-\ell\lambda}\,\hat{\rho}(x,\lambda)$$

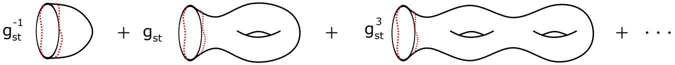
- Since the field theory is free we can compute any correlation function of loop operators in terms of multiple nested integrals [Moore]

One loop - WdW wavefunction

- With a single loop insertion, we are computing the WdW wavefunction of the bulk cc. operator keeping the boundary size ℓ fixed

$$\Psi_{WdW}(\ell,\mu,q=0) = \frac{\partial \mathcal{W}}{\partial \mu}(\ell,\mu,q=0) = \Re\left(i\int_0^\infty d\xi\, e^{i\xi}\, \frac{e^{-i\coth(\xi/2\mu)\frac{\ell^2}{2}}}{\sinh(\xi/2\mu)} \right)$$

- This expression takes into account the effects of topologies ($g_{st} \sim 1/\mu$ governs the genus expansion)



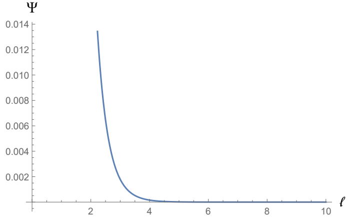
- It can be written in terms of Whittaker functions that obey

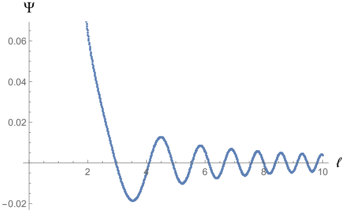
$$\left(-\left[\ell \frac{\partial}{\partial \ell}\right]^2 + 4\mu\ell^2 + 4q^2 - \ell^4 \right) \frac{W_{i\mu,q}(i\ell^2)}{\ell} = 0$$

- The last term is inducing wormhole-like effects that involve the square of the cosmological constant operator $\sim (\int e^{2\phi})^2$
- We would like to interpret this wave-function as the thermal partition function of the boundary dual: $\Psi_{WdW}(\ell) = Z_{dual}(\beta = \ell)$

One loop - WdW wavefunction

- The genus zero wavefunction vs. the non-perturbative answer (double well)





- Exponential vs slowly decaying envelope $\sim 1/\sqrt{\ell}\log\ell$
- There is a non-perturbative ambiguity due to the inverted oscillator potential
- If we put on a wall $\lambda > 0$, the partition function is

$$Z^{(+)}_{dual}(\ell) = \Re\left(\frac{i}{2}\int_0^\infty d\xi e^{i\mu\xi}\frac{e^{-\frac{i}{2}\ell^2\coth(\xi/2)}}{\sinh\xi/2}\,\textit{Erfc}\left[\frac{\ell}{\sqrt{2i\tanh\xi/2}}\right]\right)$$

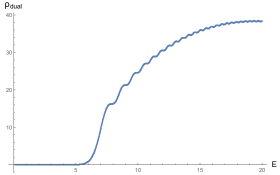
and is positive definite even at the non-perturbative level

One boundary - Density of states

- The Laplace transform of the wavefunction gives the density of states of the boundary dual ($\lambda = E$ plays the role of the energy of the boundary dual!)

$$\rho_{dual}(E) = \int dx \langle \mu | \hat{\psi}^\dagger(x,E) \hat{\psi}(x,E) | \mu \rangle$$

- Exponential growth $\sim \frac{1}{2\pi} \frac{e^{-\pi\mu + 2\sqrt{\mu}E}}{(\pi\mu)}$ for small energies
- Transitions to a Wigner semi-circle with oscillations $\sim \frac{1}{2\pi}\sqrt{E^2 - 4\mu^2} + Osc.$



- The chemical potential μ plays the role of a mass gap
- The DOS can have support on both positive and negative energies (non-perturbative definition of the model), but is always positive definite
- It is not clear whether it admits a Hilbert space interpretation

Comparison with minimal strings and JT gravity

[Johnson - Mertens - Turiaci]

- We can compare the genus zero wavefunction with the analogous results for minimal models and JT gravity
- Laplace transform for the DOS ($(2,p)$ minimal strings $\Rightarrow$ use $\nu = p/2$)

$$\int_{c-i\infty}^{c+i\infty} \frac{d\ell}{2\pi i}\, e^{\ell E}\, \frac{1}{\ell} K_\nu(2\sqrt{\mu}\ell) \;=\; \frac{1}{\nu} \sinh\left(\nu \cosh^{-1}(E/2\sqrt{\mu})\right)\;, \quad E > 2\sqrt{\mu}$$

- In a further limit $p \to \infty$, $E \sim E_{JT}/p$ one can obtain the density of states of JT gravity

$$\rho_{J.T.}^{Sch.}(E_{JT}, \gamma) = \frac{\gamma}{2\pi^2} \sinh\left(2\pi\sqrt{2\gamma E_{JT}}\right)$$

- Non-perturbative computations of the DOS for minimal models exhibit similar oscillatory behaviour as in $c = 1$ Liouville

Euclidean wormholes - Loop correlator

- Another quantity of interest is the correlator between two macroscopic loops $\langle \hat{\mathcal{W}}(\ell_1, q)\hat{\mathcal{W}}(\ell_2, q)\rangle$ given by

$$\int_0^\infty \frac{d\xi}{\xi \sinh(\xi/2)} e^{i\mu\xi - \frac{1}{2}i(\ell_1^2+\ell_2^2)\coth(\xi/2)} \int_0^\infty ds e^{-|q|s} \left(e^{i\ell_1\ell_2 \frac{\cosh(s-\xi/2)}{\sinh(\xi/2)}} - e^{i\ell_1\ell_2 \frac{\cosh(s+\xi/2)}{\sinh(\xi/2)}} \right)$$

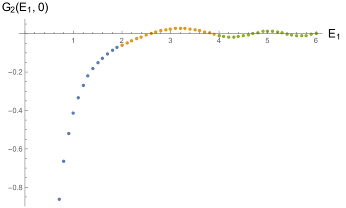
- The geometries that contribute to it are Euclidean wormholes connecting two boundaries
- The genus-zero answer is a propagator between two macroscopic wavefunctions

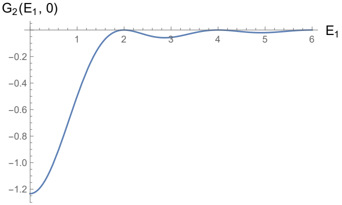
$$\int_{-\infty}^\infty dp \frac{1}{q^2+p^2} \frac{p}{\sinh(\pi p)} \Psi_p^{(macro)}(\ell_1) \Psi_p^{(macro)}(\ell_2)$$

- The result does not factorise even at the non-perturbative level $\Rightarrow$ What couples the (two) boundary theories?

Density two point function

- The density two point function $\langle\mu|\hat{\rho}_{dual}(E_1)\hat{\rho}_{dual}(E_2)|\mu\rangle_c$ can be computed and its behaviour resembles that of the *Sine-Kernel*





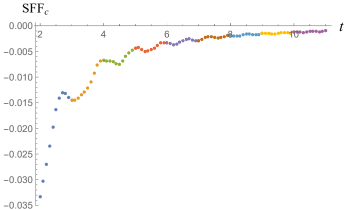
- The Sine-Kernel is indicative of level repulsion - a universal feature of quantum chaotic systems whose energy statistics coincide with random matrix statistics
- The differences though are indicative of a small deviation from the GUE/sine kernel results, that survives in the double scaling limit
- This is also pronounced in the *spectral form factor* (SFF)

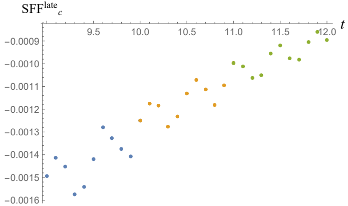
Spectral form factor

- Another observable of interest is the SFF: $\langle Z(\beta+it)Z(\beta-it)\rangle$
- Both disconnected and connected geometries contribute to it

$$SFF(\beta,t) = \int dE_1 dE_2 e^{-\beta(E_1+E_2)+it(E_1-E_2)} \langle\mu|\hat{\rho}_{dual}(E_1)\hat{\rho}_{dual}(E_2)|\mu\rangle$$

- The disconnected part is positive definite and decays as $\sim 1/t\, log^2 t$
- The connected part leads to an approximate ramp-plateau behaviour with persistent oscillations



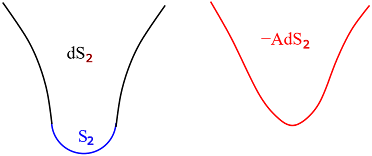


- Similar to the JT results but has features of the expected oscillatory behaviour of higher dimensional Unitary theories

Cosmological regime

[Maldacena-Turiaci-Yang, Cotler-Jensen-Maloney, Hertog]

- To compute a cosmological wavefunction, there is an alternative to Hartle-Hawking contour that passes through "$-AdS_2$ geometries"



- The dS_2 geometry and $-AdS_2$ metrics are

$$ds^2_{dS_2} = -d\tau^2 + \cosh^2\tau d\theta^2\,, \quad ds^2_{-AdS_2} = -d\tilde{\tau}^2 - \sinh^2\tilde{\tau} d\theta^2$$

where $\tau = i\frac{\pi}{2} + \tilde{\tau}$

- In the matrix model presciption this corresponds to an analytic continuation $\ell = -iz$
- The boundary "Hamiltonian" is now generating space translations

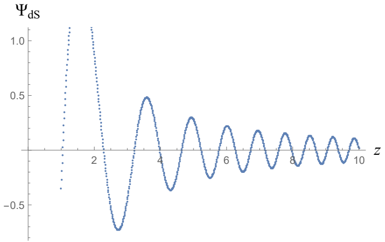
Cosmological regime in Liouville

[DaCunha-Martinec...]

- There have been studies of the cosmological regime of Liouville theory in the past, giving a plethora of wavefunctions at genus zero
- $\Psi_{n.b.} \sim J_\nu(z)$, is the no-boundary wavefunction (Hartle-Hawking contour). It is real, goes to zero at small z and oscillates at large z
- $\Psi_{t.} \sim H_\nu^{(2)}(z)$ corresponds to the *tunneling proposal*. It is complex and increasing at small z
- $\Psi_{s.c.} \sim H_{iq}^{(1)}(z)$ corresponds to the continuation $b \to ib$, $\phi \to i\phi$. This is the wavefunction for the supercritical Liouville $c > 25$
- A common property is that they are oscillatory at large $z \Rightarrow$ Large Lorentzian semiclassical geometries

Perturbative & non perturbative comments

- The non-perturbative analytically continued wavefunction



- The analytic continuation of our genus zero result is

$$\Psi_{cosm.}(z) \; = \; -i\frac{\pi\sqrt{\mu}}{z}H_1^{(1)}(2\sqrt{\mu}z)$$

- the asymptotic genus expansion and the analytic continuation do not commute ⇒ Stokes phenomena
- Our non-perturbative computation encodes several types of Bessel wavefunctions (as different asymptotic expansions of Whittaker functions)
- Our study shows that the natural non-perturbative models for AdS_2 and dS_2 are different

Summary and Future

Summary

- We performed an analysis $c = 1$ Liouville theory in the spirit of JT gravity
- This is a *richer* 2d QG theory admitting a (non-perturbative) MQM description
- MQM and the fermionic field theory provide a powerful "superspace description"
- Euclidean wormholes/topologies are automatically included in this 2d QG path integral
- We do not know if there is an AdS/CFT type of boundary dual, but it inevitably has to exhibit approximate *chaotic* properties (sine-kernel, ramp-plateau ...)
- Various non-perturbative completions (AdS_2 and dS_2 seem be dual to different such completions)

The dictionary

Quantum gravity	Matrix model	Boundary dual
Liouville potential $\mu\, e^{2\phi}$	Inverted oscillator potential	-
Cosmological constant μ	Chemical potential $-\mu$	IR mass gap μ
$D0$ particle (ϕ: D, X: N)	Matrix eigenvalue λ_i	Energy eigenvalue E_i
Boundary: $S_{bdy} = \mu_B \oint e^{\phi}$	Loop operator: $\langle \text{Tr} \log[\mathrm{z} - \lambda] \rangle$	Microcanonical $\langle \rho_{dual}(E) \rangle$
Bdy. cosm. const. μ_B	Loop parameter z	Energy E
fixed size bdy $\ell = e^{\phi_0}$	Loop length ℓ	Inv. temperature β
WdW wavefunction $\Psi(\ell)$	Fixed size loop oper. $\langle M_1(\ell) \rangle$	Partition func. $Z_{dual}(\beta)$
Third quantised vacuum	Fermi sea of eigenvalues	-
Closed surfaces	Fermionic density quanta	-
S-matrix of universes	S-matrix of density quanta	-
Two boundaries: $\ell_{1,2}$	Loop correlator $\langle M_2(\ell_1, \ell_2) \rangle$	SFF: $\ell_{1,2} = \beta \pm it$
Two boundaries: $\mu_B^{1,2}$	Density corr. $\langle \rho(\lambda_1)\rho(\lambda_2) \rangle$	DOS. correlator

- The missing entries either do not have an interpretation or we do not understand it yet
- For example a single boundary dual does not seem to capture the complete "3rd quantised Hilbert space", where we can have multiboundary configurations

Higher Dimensions ?

- It is hard to extrapolate our findings and comments in higher dimensions ($d > 3$)
- All the models so far are in 2d where they can also be interpreted as string theories of a certain kind and gravity is rather trivial
- The boundaries are introduced by studying correlators of the matrix theories
- Define analogous operators that create higher dimensional surface boundaries?
- BUConjecture by [McNamara - Vafa] (based on WG - Swampland conjectures and work by [Maxfield - Marolf]) $\Rightarrow$ Higher dimensional theories ($d > 3$) are fundamentally different: $\mathcal{H}_{BU}$ is trivial
- But still: What about AdS wormhole solutions in higher d? [Maldacena-Maoz]
  Discard them? [Arkani-Hamed - Orgera - Polchinski]
  Interacting QFT's? [Betzios - Kiritsis - O. P.]
  Refinement of Holography?

Future

- Study more refined observables (correlators of boundary operators)
- Study the non-singlet sector of MQM [Maldacena, Gaiotto, Betzios-O.P]
- Study other models such as WZW cosets

- Compute amplitudes for processes in superspace, (universes branching off etc...)
- It has been argued that the non-perturbative effects in JT gravity are doubly non-perturbative i.e. $e^{-c/g_s} \sim e^{-ce^{S_0}}$
- Here this expansion is governed by $e^{-c\mu}$ effects that do not seem to have such a doubly non-perturbative structure from the MQM point of view
- A doubly non-perturbative structure might arise again if μ admits a more microscopic description in terms of the putative boundary dual theory...
- What is the bulk origin of doubly layered expansions?
- Idea: Geometries inside geometries $\Rightarrow$ two different genus expansions: both in target space and on the worldsheet

Thank you!