Quantum BTZ

Roberto Emparan
ICREA & UBarcelona

Quantum Gravity, Holography and Quantum Information
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Based on 2007.15999 [hep-th]

Antonia Frassino

Benson Way

What’s a Quantum Black Hole?

A large-N matrix, with

fastly scrambling entries?

quantum black hole

Simpler goal:

Classical geometry of black hole modified

(possibly a lot) by quantum fields

Much insight gained this way

Quantum backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

classical Einstein tensor & metric

quantum matter renorm stress tensor
(many fields)

Coupled system: metric + ⟨QFT⟩

Very hard to solve simultaneously

Perturbative backreaction: limited insight

Quantum backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G\langle T_{\mu\nu}(g_{\alpha\beta})\rangle$$

Exact backreaction:

2D models: CGHS/RST, JT+CFT

Holographic reformulation

RE+Fabbri+Kaloper 2002
Tanaka 2002

Quantum backreaction — What for?

Radiating/Evaporating Black hole

Quantum (generalized) entropy $S_{gen} = \frac{A}{4G} + S_{out}$

Page curve turns when $S_{out} \sim \frac{A}{4G}$

Penington 2019
Almheiri+al 2019

Holographic approach

$\langle T_{\mu\nu} \rangle$: CFT on boundary geometry of bulk dual

classical bulk $\Leftrightarrow$ planar CFT ($N \to \infty$)

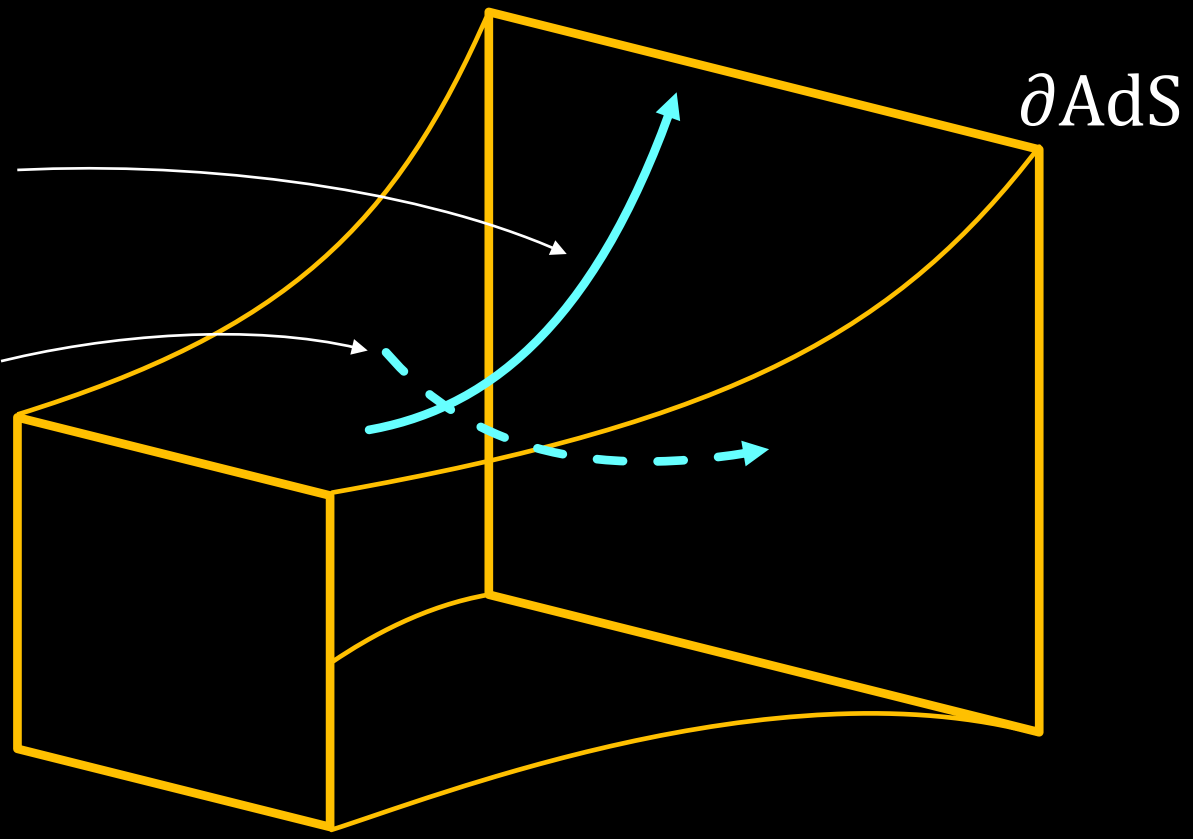
Conventional AdS/CFT has *fixed boundary* geometry

Make boundary geometry dynamical

Braneworld gravity

Randall+Sundrum 1999

Gravitational fluctuations



Braneworld gravity

Randall+Sundrum 1999

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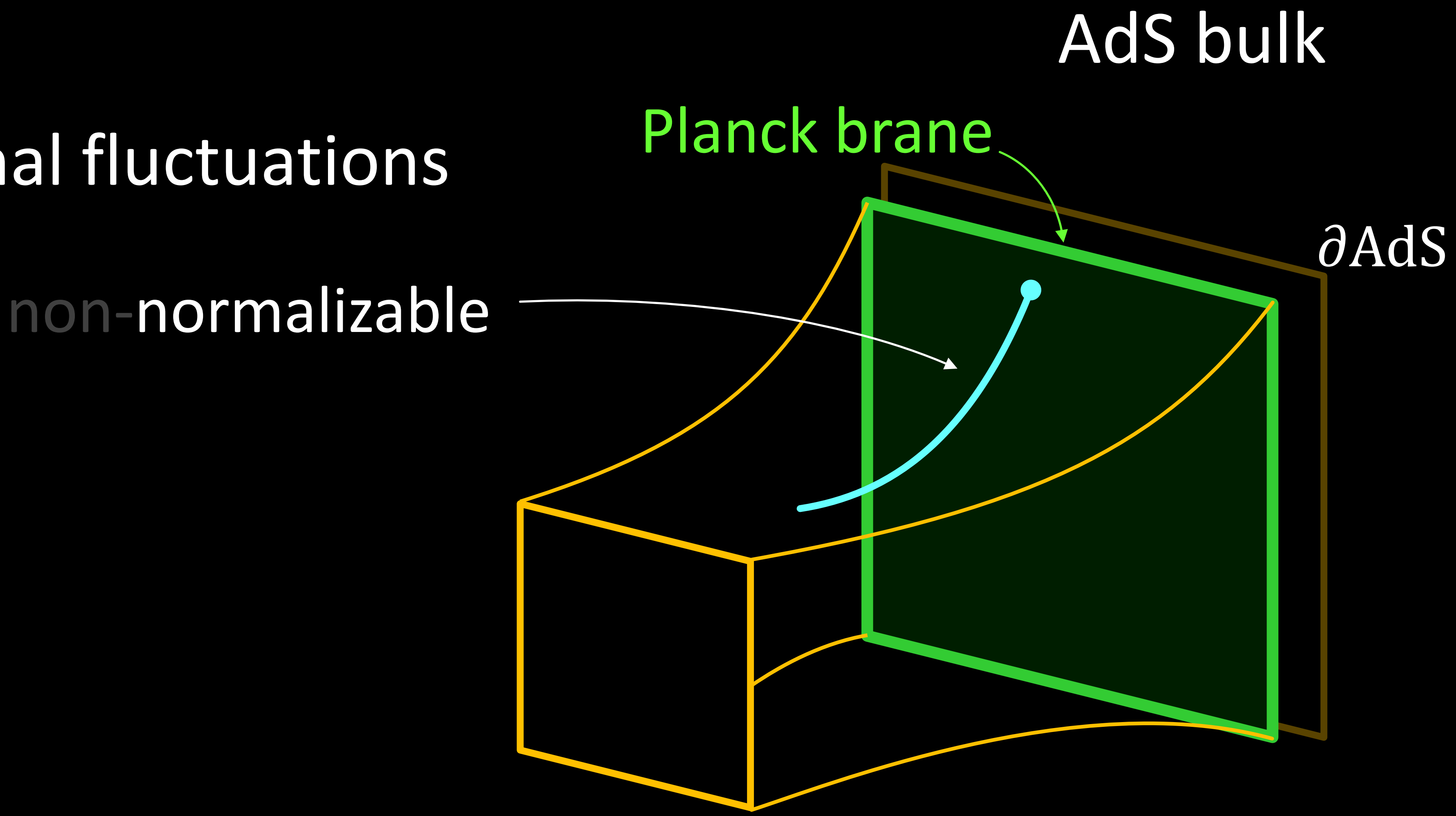


Braneworld dynamics

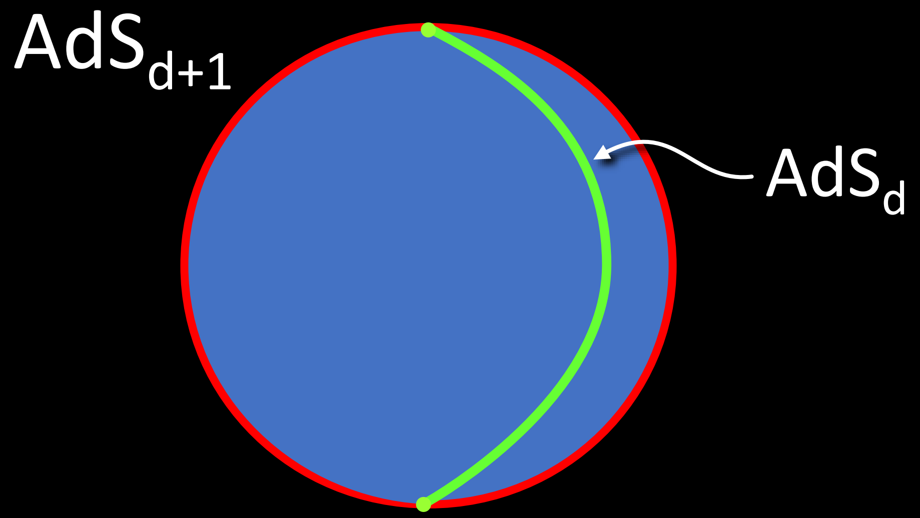
Graviton on the brane

Remaining grav bulk dynamics

is dual to CFT (cutoff & planar)

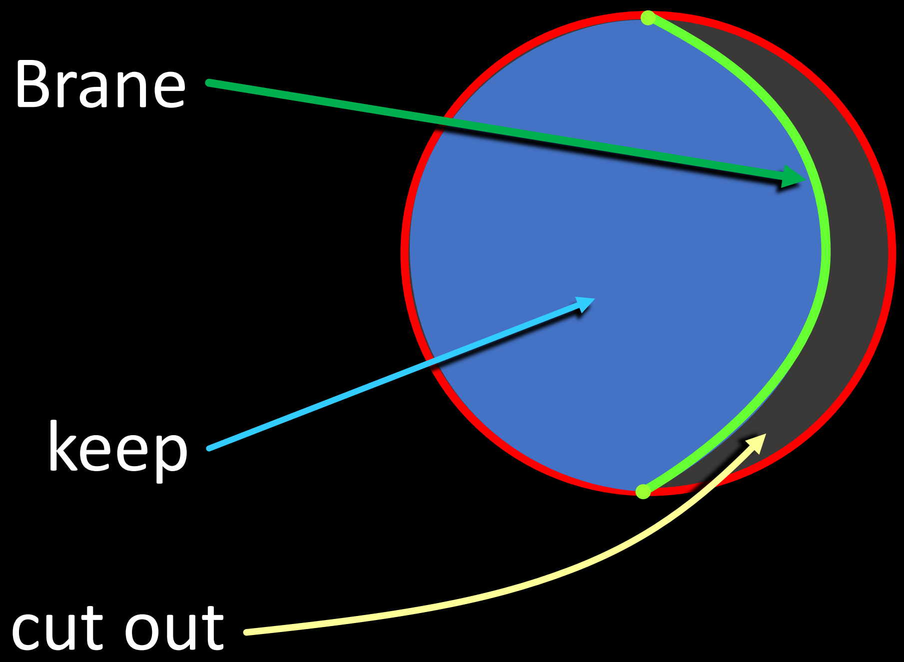
Slicing AdS

AdS slicing



Braneworlds

Karch-Randall: AdS branes

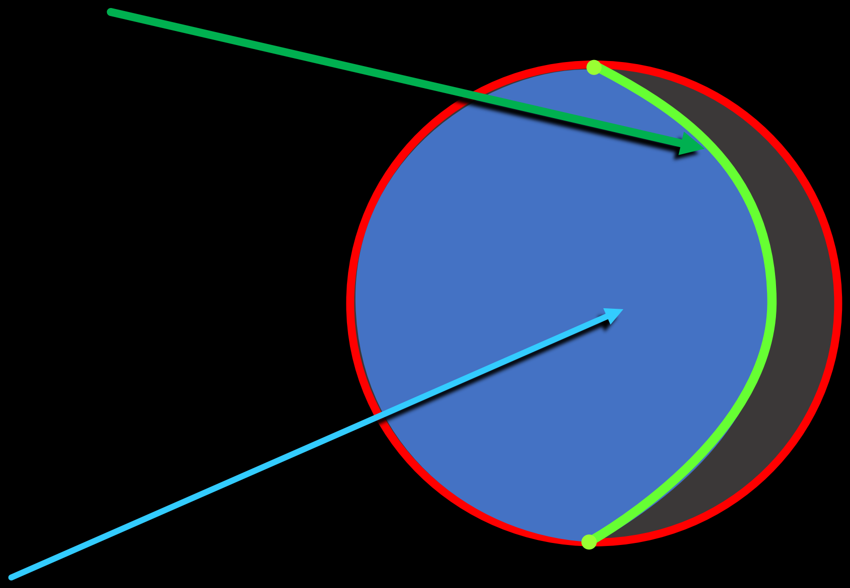


& paste to copy: 2-sided brane, $\mathbb{Z}_2$ orbifold

Randall+Sundrum 1999
Karch+Randall 2000

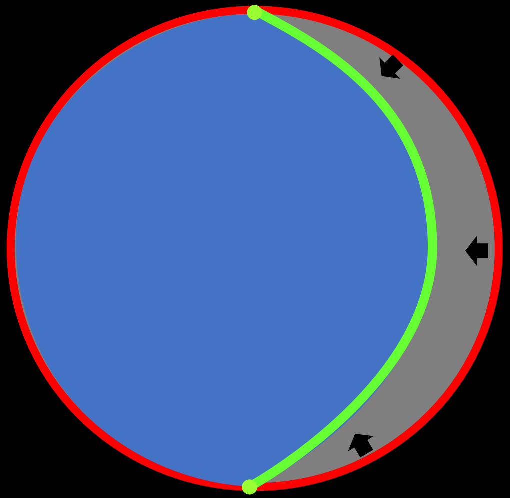
Braneworld holographies

Gravity effectively induced on brane



Holographic CFT describes bulk

Effective gravity + CFT

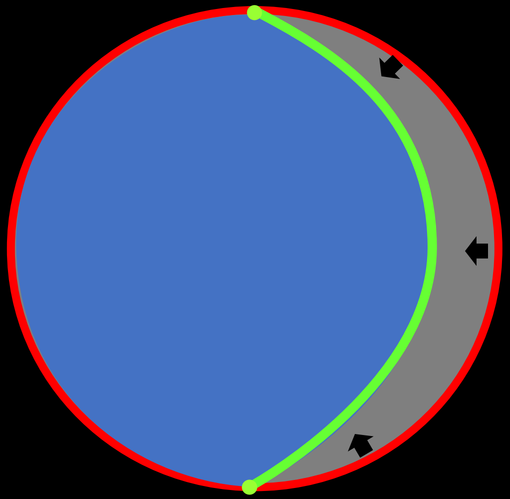


deHaro+Skenderis+Solodukhin 2000

Integrate from boundary to brane
à la Fefferman-Graham
(holographic renormalization)

i.e. solve bulk Einstein eqs with prescribed boundary metric, perturbatively away from the bdry

Effective gravity + CFT



deHaro+Skenderis+Solodukhin 2000

Do not introduce counterterms!
Keep finite cutoff: brane position

This integrates the CFT UV degrees of freedom and generates the effective action

Braneworld holography

$$\mathcal{G}_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle_{planar}$$

Einstein+higher curvature

large-N CFT stress tensor

Effective theory w/ cutoff (brane position)

Einstein+higher-curvature terms: induced by *CFT above cutoff*

$\langle T_{\mu\nu} \rangle$: holographic *CFT below cutoff*

Integrate bulk 4D gravity action → Effective action 3D gravity+CFT

$$I = \frac{1}{16\pi G_3}\int d^3x\,\sqrt{-h}\,\left[\frac{2}{L_3^2} + R + \ell^2\left(\frac{3}{8}R^2 - R_{ab}R^{ab}\right) + \cdots\right]$$

$$+ I_{CFT}$$

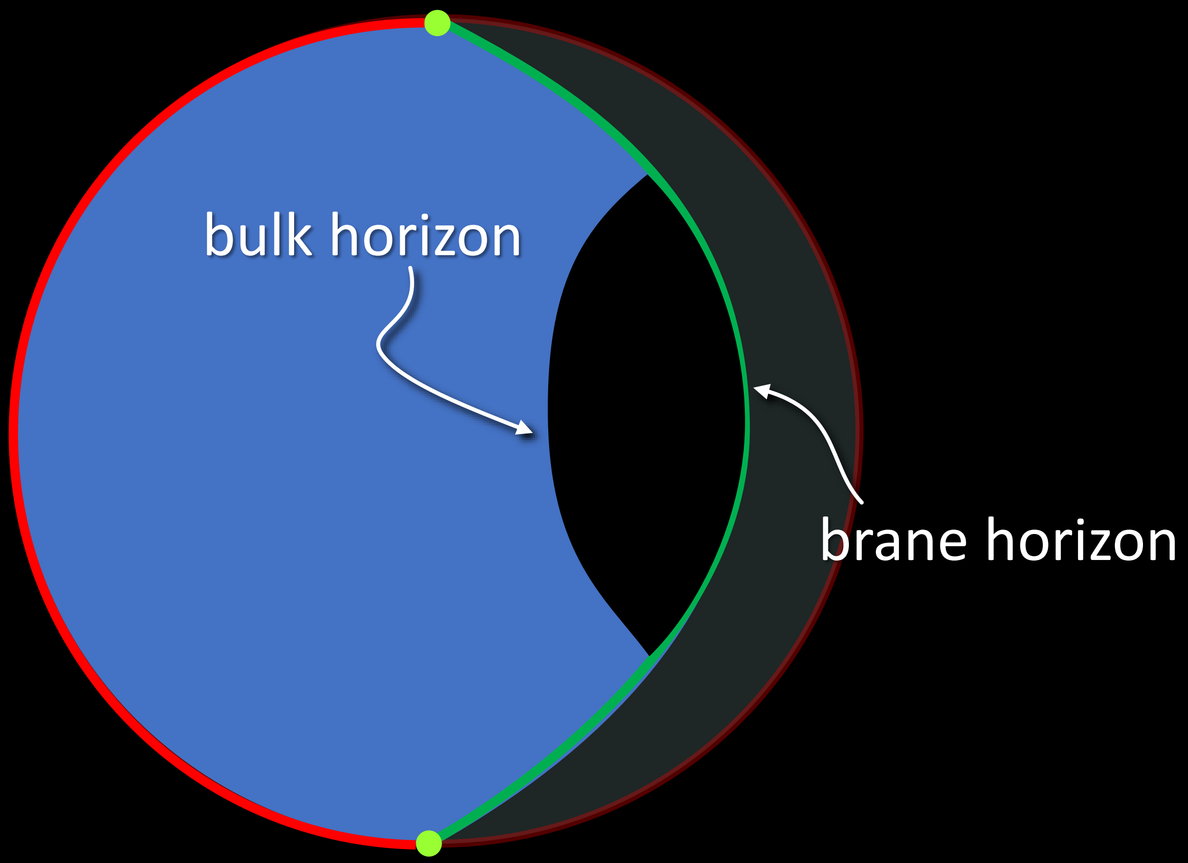
An exact 4D bulk solution is a 3D effective theory solution

exact CFT stress tensor

exact backreaction of the CFT (planar)

exact in *all* higher curvature corrections

Black hole on brane

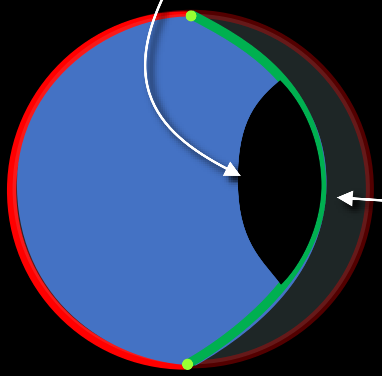


Quantum BH Entropy in bw holography

$$S_{gen} = S_{Wald} + S_{out}$$

Bulk bh entropy

$$S_{gen} = \frac{A_{d+1}}{4G_{d+1}}$$



Brane bh entropy

$$S_{Wald} = \frac{A_d}{4G_d} + \cdots$$

induced by entanglement of CFT above cutoff

CFT entanglement entropy

$$S_{out} = S_{gen} - S_{Wald}$$

(CFT below cutoff)

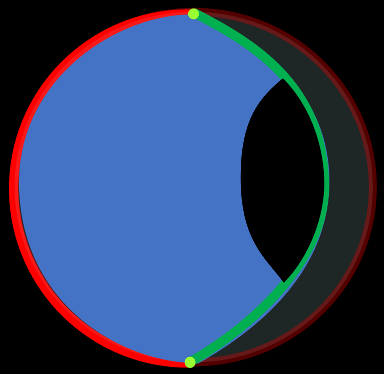
RT/HRT on brane
RE 2006

Quantum BH Entropy

If the holographic interpretation of braneworlds is consistent, then

$$T dS_{gen} = dM - \Omega\, dJ$$

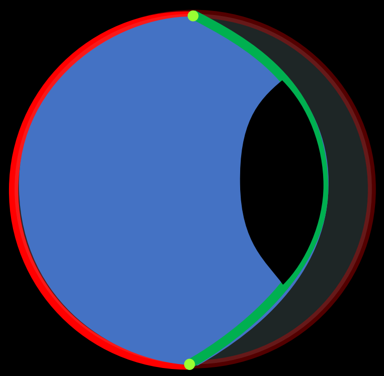
$$\Delta S_{gen} \geq 0$$



S_{BH}, S_{Wald} should not satisfy these

Quantum BH Entropy: Second law

$\Delta S_{gen} \geq 0$ bulk 2nd law



S_{BH}, S_{Wald} can decrease

for perturbative backreaction (non-holo): *Wall 2011*

Quantum BH Entropy: First law

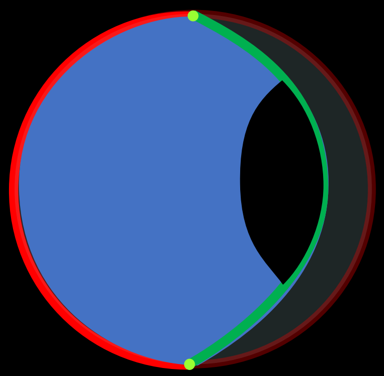
$$T dS_{gen} = dM - \Omega\, dJ$$

Not trivial!

In $d+1$ dim bulk

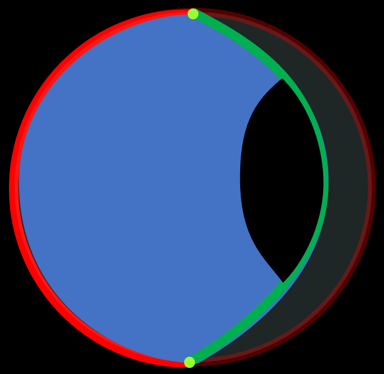
On d dim brane
w/ higher curvature gravity

Bulk 1st law?



Quantum BH Entropy: First law

$$T dS_{gen} = dM - \Omega\, dJ$$



We will test this in explicit exact solutions

Black hole on the brane: AdS C-metric

Plebański + Demiański 1976

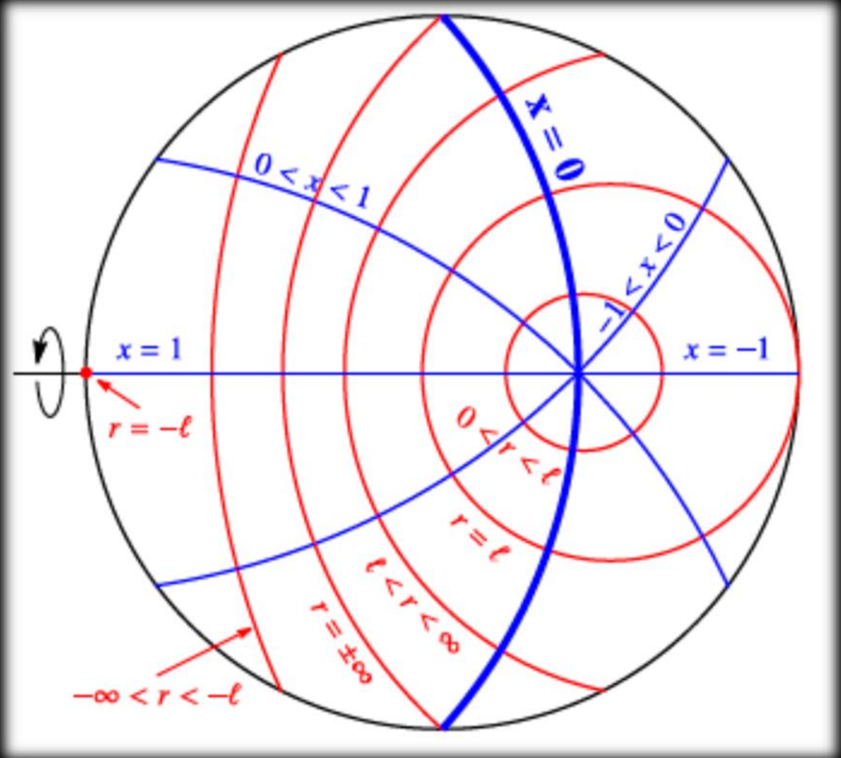
$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left(-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right)$$

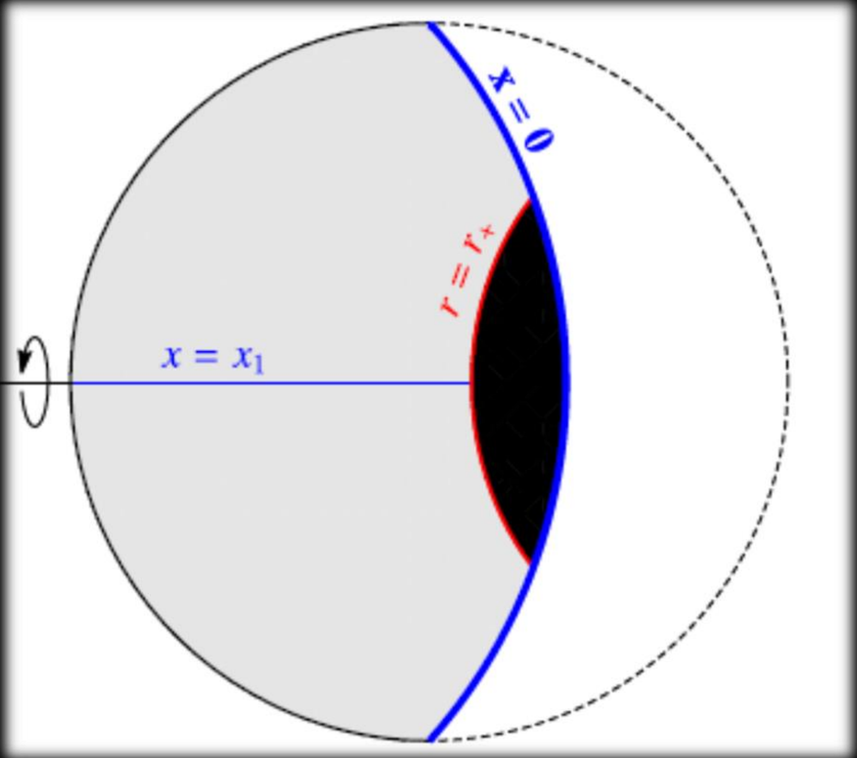
$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}$$

$$G(x) = 1 - \kappa x^2 - \mu x^3$$

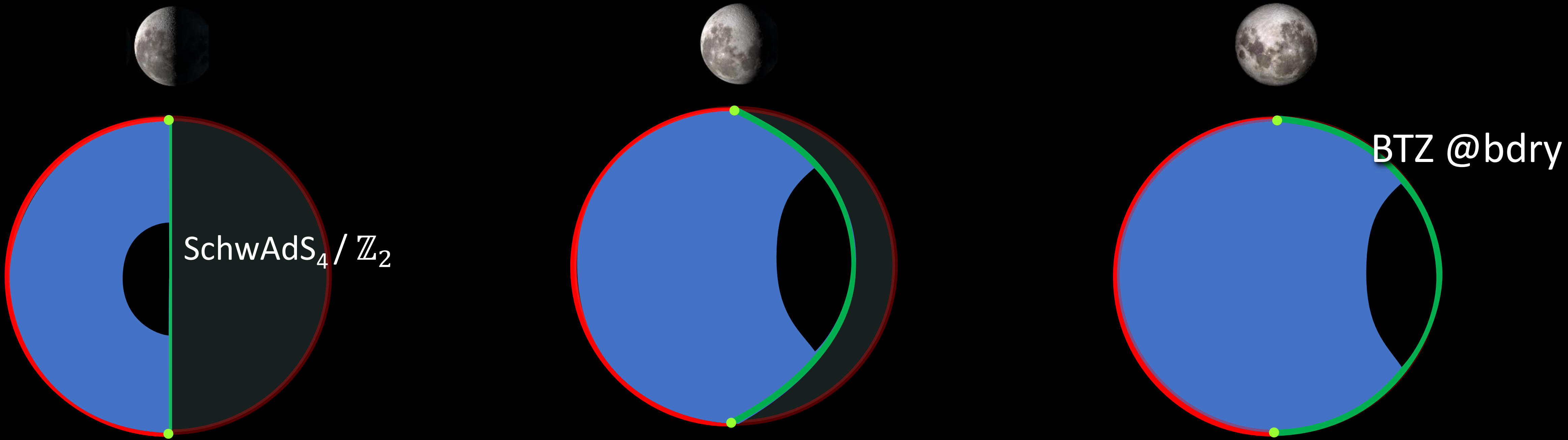
Adapted coordinates

Brane at $\mathbf{x = 0}$: $K_{ab} = -\frac{1}{\ell} h_{ab}$





Strength of backreaction: ℓ



$\ell \to \infty$

zero tension brane

"maximal backreaction"

no 3D gravity

$0 < \ell < \infty$

brane tension $\propto 1/\ell$

backreaction $\propto \ell$

cutoff energy $\propto 1/\ell$

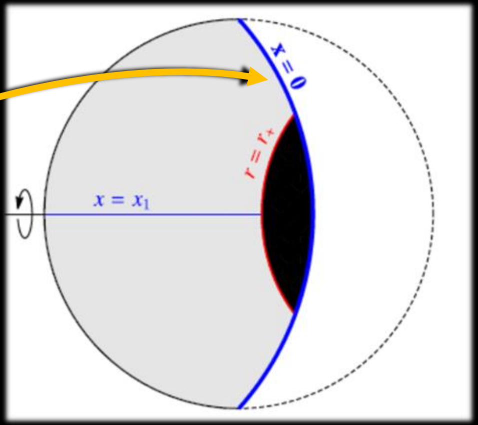
$\ell \to 0$

brane tension $\to \infty$

backreaction $\to 0$

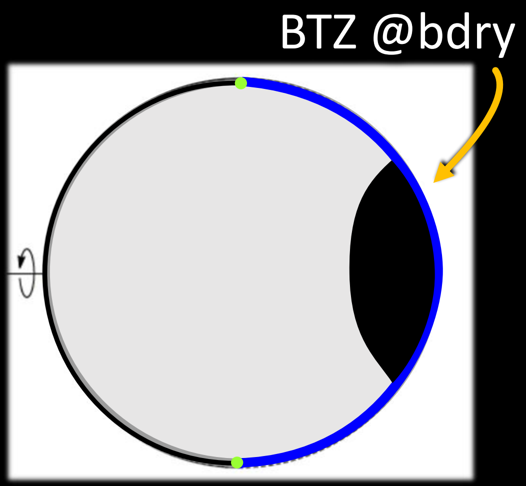
quBTZ metric

3D metric induced on brane at $x = 0$



$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}} + r^2 d\phi^2$$

quBTZ metric



3D metric induced on brane at $x = 0$

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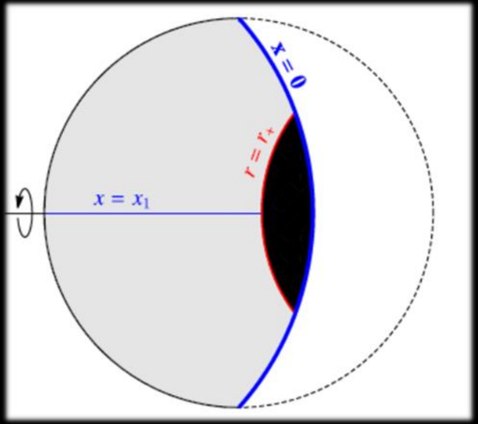
$\ell = 0$: BTZ black hole

quBTZ metric



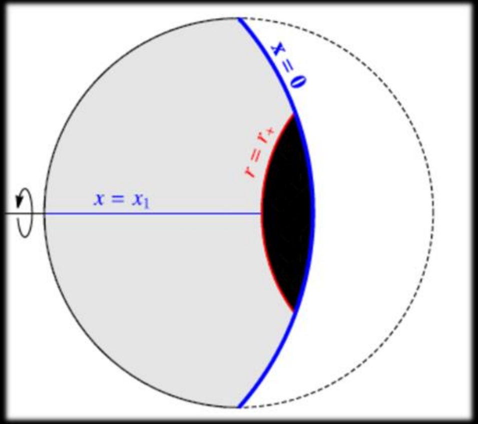
3D metric induced on brane at $x = 0$

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}} + r^2 d\phi^2$$

$\ell = 0$: BTZ black hole

$\ell > 0$: quantum-corrected BTZ $\neq \mathrm{AdS}_3/\Gamma$

quBTZ metric



quantum correction: $\ell = 2\,\hbar\, c\, G_3 + \cdots$

CFT central charge

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F(M)}{r}} + r^2 d\phi^2$$

$F(M)$ determined by bulk regularity

Small AdS_3 quantum bhs

RE+Fabbri+Kaloper 2002

$$-\frac{1}{8G_3} < M < 0$$

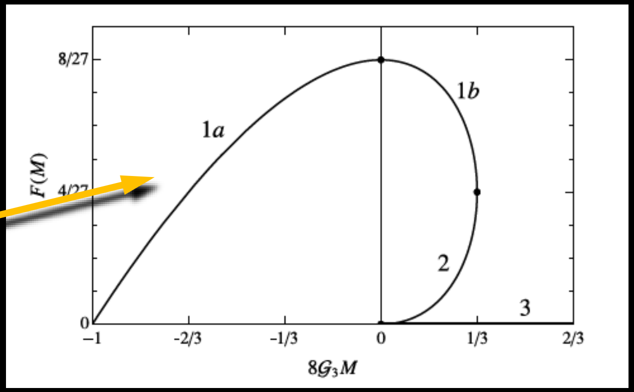
Conical defect → Black hole

Casimir on a cone

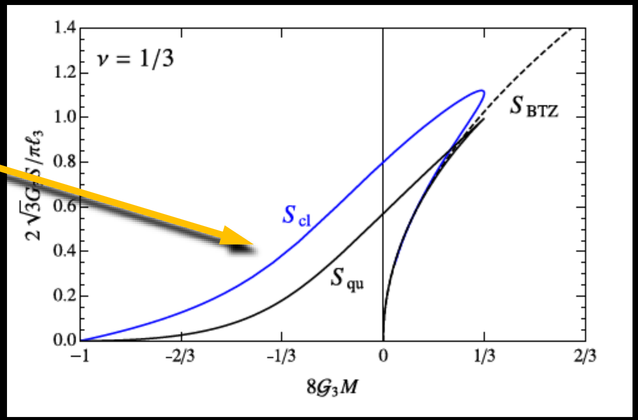
+ backreaction → quantum horizon

Quantum Cosmic Censorship

Stress tensor



Entropy

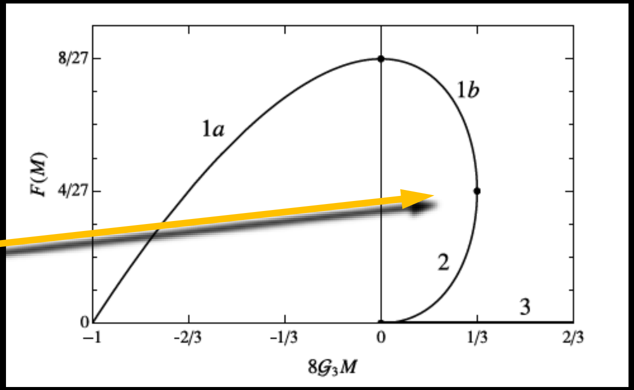


quantum BTZ

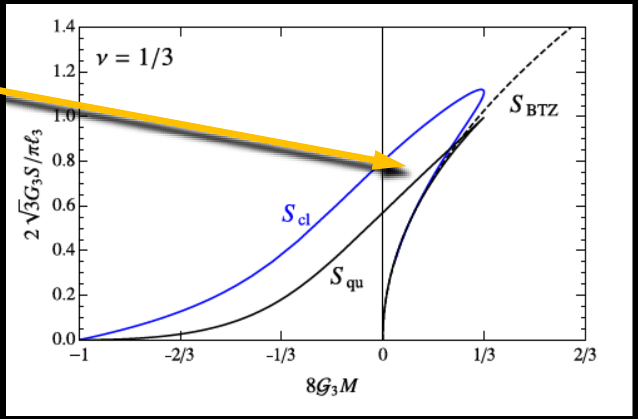
Quantum-corrected BTZ

$$0 < M < \frac{1}{24G_3}$$

Stress tensor



Entropy



Rotating quBTZ

Can have $M < J/\ell_3$

$\langle T_{ab} \rangle_{CFT}$ smooth at inner Cauchy horizon

but see *RE+Tomašević 2020 "Strong Cosmic Censorship in BTZ"*

Quantum BH Entropy: First law

$M \quad J$ measured in 3D eff theory

$T \quad \Omega$ (leengthy expressions)

$$S_{gen} = \frac{A_{bulk}}{4G_4}$$

$$TdS_{gen} = dM - \Omega dJ$$

So

Fully quantum-backreacted BTZ and CFT stress tensor
can be described *exactly* and *in detail*

Quantum entropy from braneworld holography: *consistent*

Efficient use of holography to *solve a hard quantum problem*

Going further

General holographic proof of $TdS_{gen} = dM - \Omega dJ$

More classical proofs of quantum theorems?

Holographic duals of massive gravities – 3D and higher d

Extensions: charge, higher-d?

Exact entanglement islands

Dual Hawking evaporation? *RE+Kaloper+Tanaka+al 2002-20??*

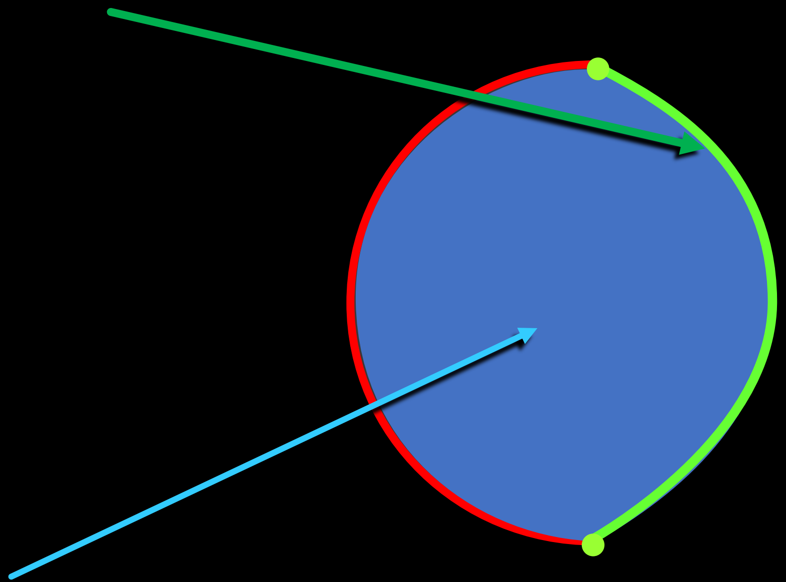
Thank you

Antonia Frassino

Benson Way

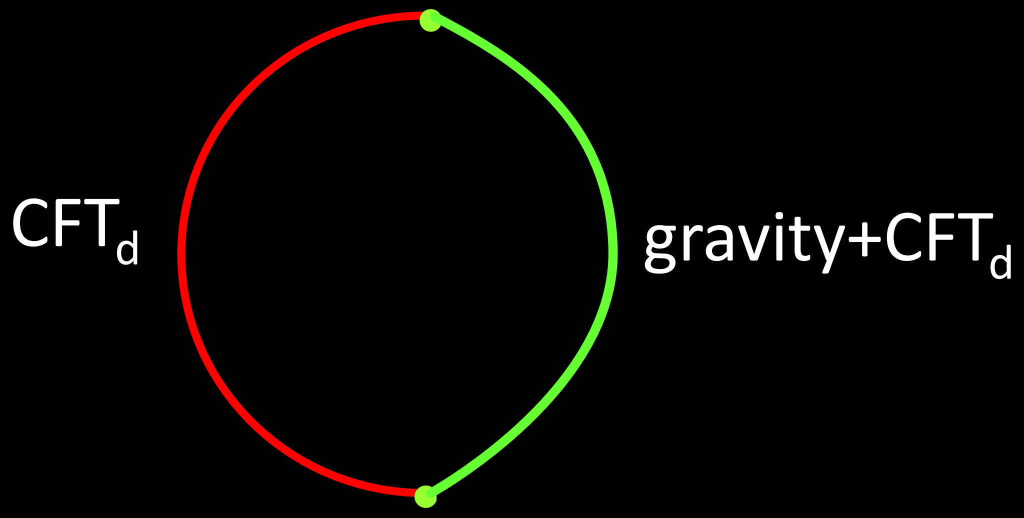
Braneworld holographies

Gravity effectively induced on brane

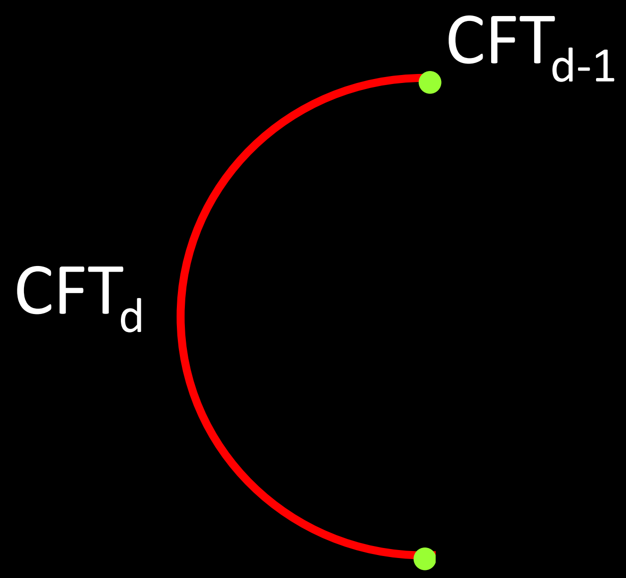


Holographic CFT describes bulk

Braneworld holographies



Other holographies

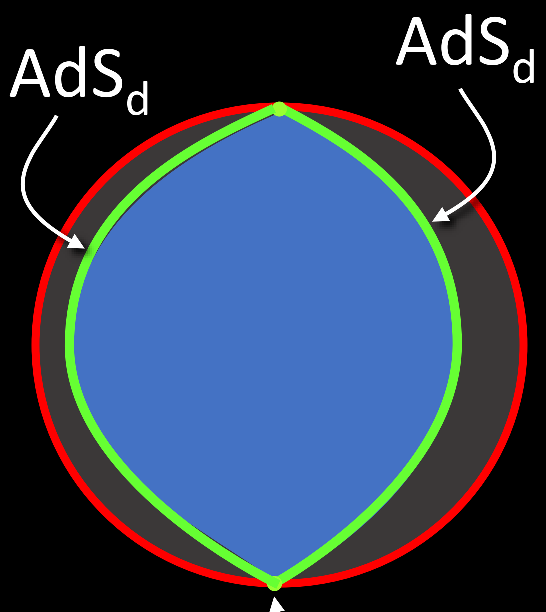


Boundary CFT

Karch+Randall 2001
Takayanagi 2011

RE+Horowitz+Myers 1999

Two gravitating branes



AdS_d x I = Holographic CFT_{d-1}

Geng et al 2020

Effective action gravity + CFT

Integrate 4D bulk action

→ 3D eff action

RE+Johnson+Myers 1999

3D h_{ab}, R_{ab}

$\frac{1}{L_3^2}$: 4D $\frac{1}{\ell_4^2}$ and brane tension $\frac{1}{\ell}$

ℓ: brane position ~ tension^{-1} ~ cutoff $^{-1}$

$$I = \frac{1}{16\pi G_3}\int d^3x\,\sqrt{-h}\left[\frac{2}{L_3^2} + R + \ell^2\left(\frac{3}{8}R^2 - R_{ab}R^{ab}\right) + \cdots\right]$$

$$+ I_{CFT}$$

$G_3 = \frac{G_4}{2\ell_4}$

holographic CFT

same as "New 3D massive gravity"

Bergshoeff+Hohm+Townsend 2009

G_3 “renormalized” by higher curvatures

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\,\frac{F(M)}{r}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\,\frac{F(M)}{r}} + r^2 d\phi^2$$

quantum correction: $\ell = 2\,\hbar\, c\, G_3 + \cdots$

CFT central charge

$F(M)$ determined by bulk regularity outside horizon

$$\langle T^a{}_b\rangle = \frac{\ell}{16\pi G_3}\frac{F(M)}{r^3}\,\mathrm{diag}\{1,1,-2\} + \cdots$$

not thermal plasma $\mathrm{diag}\{-2,1,1\}$

$\langle T^a{}_b \rangle$ for free fields

Steif, Shiraishi+Maki, Lifschytz+Ortiz 1993

Casals+Fabbri+Martínez+Zanelli 2016,2019

Free conformal scalar in BTZ

Method of images: BTZ = AdS_3/Γ

Satisfy KMS, Hartle-Hawking at horizon

Same structure: $\langle T^a{}_b \rangle = \frac{c}{8\pi}\frac{F(M)}{r^3}\,\mathrm{diag}\{1,1,-2\}$

different $F(M) = \sum_{n=1}^{\infty} F_n(M)$ (images)

Quantum BH Entropy: First law

$$M = \frac{1}{2\mathcal{G}_3} \frac{z^2(1 - \nu z^3)(1 + \nu z)}{(1 + 3z^2 + 2\nu z^3)^2}$$

(measured in 3D eff theory)

ν: backreaction parameter ℓ/ℓ_3

$$T = \frac{1}{2\pi\ell_3} \frac{z(2 + 3\nu z + \nu z^3)}{1 + 3z^2 + 2\nu z^3}$$

z: mass parameter

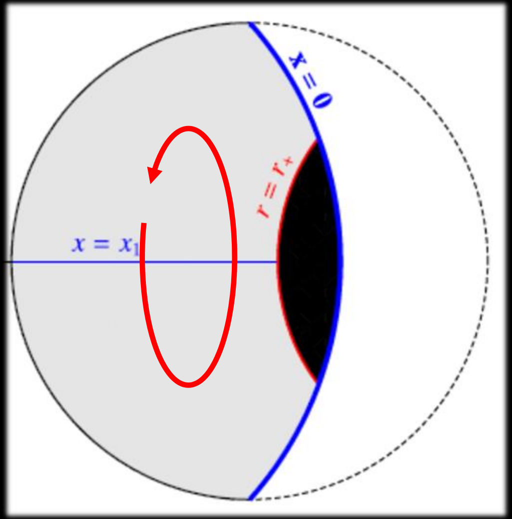
$$S_{gen} = \frac{A_{bulk}}{4G_4} = \frac{\pi\ell_3}{\mathcal{G}_3} \frac{z}{1 + 3z^2 + 2\nu z^3}$$

$$T\partial_z S_{gen} = \partial_z M$$

$$\Rightarrow TdS_{gen} = dM$$

Rotating quBTZ

Rotating AdS C-metric



Bulk structure similar to Kerr(-AdS$_4$)

inner&outer horizons, ring singularity

$\ell = 0$: rot BTZ black hole

$\ell > 0$: quantum-corrected rot BTZ $\neq$ AdS$_3/\Gamma$

Rotating quBTZ metric

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M - \ell\frac{F_1(M,J)}{r}\right)dt^2 + \left(r^2 + \ell\frac{F_2(M,J)}{r}\right)d\phi^2$$

$$- 8\mathcal{G}_3 J\left(1 + \ell\frac{F_3(M,J)}{r}\right)dt\,d\phi + \left(\frac{r^2}{\ell_3^2} - 8\mathcal{G}_3 M + \frac{(4\mathcal{G}_3 J)^2}{r^2} + \ell\frac{F_4(M,J)}{r}\right)^{-1} dr^2$$

$\ell = 0$: rot BTZ black hole

$\ell > 0$: quantum-corrected rot BTZ $\neq$ AdS_3/Γ