Multi-loop calculations and computing for precision physics at the LHC



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The Big Picture

Fundamental Large Hadron Collider Processes Strong Corrections to Gluon-Fusion Higgs Production Strong Corrections to Drell-Yan Lepton Production

Efficient Computer Algebra for Multi-Loop Calculations Efficient Integration by Parts Reduction Phenomenological Ramifications

Computing, Scientific Software, and Machine Learning Computing Scientific Software Machine Learning

Summary

Outline

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The Standard Model of Particle Physics

Microscopic model, a renormalizable quantum field theory.





Consider the 2012 Higgs Boson Discovery @ LHC $\,$

Decades of experimental and theoretical work to produce $\overline{e.g.}$



On the theory side, predict (as a function of s and M_h):

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- 1. How many Higgs bosons are produced?
- 2. What are the measurable decay modes of the Higgs?
- 3. What other interactions mimic these Higgs events?

For simplicity, focus on Higgs production. How do I estimate the total number of Higgs bosons I expect to produce after running the LHC for a fixed amount of time?

- 1. Calculate the cross section, σ , or the probability that the colliding protons produce a Higgs boson per unit time, divided by the particle flux density of the LHC.
- 2. The *integrated luminosity*, $\int Ldt$, is the particle flux density integrated over the time of data acquisition.
- 3. The number of events in the sample is then $\sigma \int L dt$.

Cross section calculations in the Standard Model are key!



Technical Complications of Perturbative Calculations

Beyond leading order, severe technical complications for ${\mathcal M}$

$$\mathcal{M}(q(p_1)\bar{q}(p_2) \rightarrow \mu(p_3)\bar{\mu}(p_4)) = \qquad \qquad + \qquad g \qquad + \qquad \cdots$$

- ▶ Feynman diagrams which contain loops lead to ill-defined integrals over the four-momenta of virtual quanta.
- Only ultraviolet divergences coming from large values of the loop momenta are subject to renormalization.
- ► For remaining infrared divergences, add higher-multiplicity scattering processes at the same relative order in α_s , *e.g.*

$$\mathcal{M}(q(p_1)\bar{q}(p_2) \to \mu(p_3)\bar{\mu}(p_4)g(k)) = \bigvee_{\gamma} \psi_{\gamma} \psi_{\gamma}$$

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Gluon-Fusion Higgs and Drell-Yan Lepton Production



Surprisingly, the most important source of Higgs bosons @ LHC. Georgi et. al., Phys. Rev. Lett. 40 (1978) 692



Clean experimentally, very useful to constrain the probability of finding a quark or gluon of a specified energy inside the proton. Drell and Yan, Phys. Rev. Lett. **25** (1970) 316

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<u>First Higgs and</u> Drell-Yan Predictions At $\mathcal{O}\left(\alpha_s^3\right)$

C. Anastasiou et. al., Phys. Lett. B737 (2014) 325;

Y. Li, A. von Manteuffel, RMS, and H. X. Zhu, Phys. Rev. D91 (2015) 036008

$$\lim_{\hat{s} \to M_i^2} \left\{ \Delta \hat{\sigma}_i^{\mathrm{N}^3 \mathrm{LO}} \right\} = H_i S_i, \text{ where } i = h \text{ or } \gamma$$



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$$H_i \text{ hard, effects due to virtual quanta:}$$



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 S_i soft, effects due to real radiation, soft to first approximation:



By studying the $\mathcal{O}\left(\alpha_s^3\right)$ virtual amplitudes, graphs like *e.g.*



it became clear that a new line of attack would be needed for the explicit evaluation of the basis integrals order-by-order in ϵ .



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- 2. Generic basis integrals at $\mathcal{O}(\alpha_s^4)$ can have, at worst, ϵ^{-8} poles; challenging to extract in an efficient way.



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3. After making all ϵ poles explicit, no obvious way to evaluate the resulting convergent integrals.

Andreas von Manteuffel, Erik Panzer, and RMS, JHEP 1502 (2015) 120;
 Phys. Rev. D93 (2016), 125014; RMS Phys. Rev. D99 (2019), 105010

We proved it is always possible to pick finite basis integrals!



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Finiteness can be achieved by adjusting the spacetime dimension and the denominator exponents:

$$\int \frac{d^{4-2\epsilon}k}{(p_1-k)^2(p_2+k)^2} \longrightarrow \int \frac{d^{6-2\epsilon}k}{[(p_1-k)^2]^2 [(p_2+k)^2]^2}$$



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- ▶ Fully-systematic and effective way to resolve ϵ poles.
- ▶ In many cases, a generic integration algorithm even exists!



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- ▶ Fully-systematic and effective way to resolve ϵ poles.
- ▶ In many cases, a generic integration algorithm even exists!
- ▶ I constructed first non-trivial examples of finite integrals:

$$\frac{1}{\mathcal{N}} \int \frac{d^{6-2\epsilon}k}{\left[(p_1-k)^2\right]^2 \left[(p_2+k)^2\right]^2} = \left[-(p_1+p_2)^2\right]^{-1-\epsilon} \left(1+\epsilon+2\epsilon^2+\cdots\right)$$

<u>Towards Virtual</u> Higgs and Drell-Yan @ $\mathcal{O}\left(\alpha_s^4\right)$

S.-O. Moch et. al., JHEP 10 (2017) 041; Phys. Lett. B782 (2018) 627

J. M. Henn et. al., JHEP 04 (2020) 018

A. von Manteuffel, E. Panzer, and RMS, Phys. Rev. Lett. 124 (2020), 162001

We have taken the first decisive steps towards the calculation of the virtual Drell-Yan and Higgs cross sections at $\mathcal{O}(\alpha_s^4)$!

$$\begin{split} &\Gamma_4^r = N_f^3 C_R \Big(\frac{64}{27} \zeta_3 - \frac{32}{81}\Big) + N_f^2 C_A C_R \Big(-\frac{224}{15} \zeta_2^2 + \frac{2240}{27} \zeta_3 - \frac{608}{81} \zeta_2 + \frac{923}{81}\Big) + N_f^2 C_F C_R \Big(\frac{64}{5} \zeta_2^2 - \frac{640}{9} \zeta_3 + \frac{2392}{81}\Big) \\ &+ N_f C_A^2 C_R \Big(\frac{2096}{9} \zeta_5 + \frac{448}{3} \zeta_3 \zeta_2 - \frac{352}{15} \zeta_2^2 - \frac{23104}{27} \zeta_3 + \frac{20320}{81} \zeta_2 - \frac{24137}{81}\Big) \\ &+ N_f C_A C_F C_R \Big(160 \zeta_5 - 128 \zeta_3 \zeta_2 - \frac{352}{5} \zeta_2^2 + \frac{3712}{9} \zeta_3 + \frac{440}{3} \zeta_2 - \frac{34066}{81}\Big) + N_f C_F^2 C_R \Big(-320 \zeta_5 + \frac{592}{3} \zeta_3 + \frac{572}{9}\Big) \\ &+ N_f \frac{d_F^{abcd} d_R^{abcd}}{N_R} \Big(-\frac{1280}{3} \zeta_5 - \frac{256}{3} \zeta_3 + 256 \zeta_2\Big) + \frac{d_A^{abcd} d_R^{abcd}}{N_R} \Big(-384 \zeta_3^2 - \frac{7936}{35} \zeta_2^3 + \frac{3520}{3} \zeta_5 + \frac{128}{3} \zeta_3 - 128 \zeta_2\Big) \\ &+ C_A^3 C_R \Big(-16 \zeta_3^2 - \frac{20032}{105} \zeta_2^3 - \frac{3608}{9} \zeta_5 - \frac{352}{3} \zeta_3 \zeta_2 + \frac{3608}{5} \zeta_2^2 + \frac{20944}{27} \zeta_3 - \frac{88400}{81} \zeta_2 + \frac{84278}{81}\Big) \end{split}$$

The quark and gluon cusp anomalous dimensions improve the $\mathcal{O}(\alpha_s^3)$ predictions for Higgs and Drell-Yan by allowing for full N³LL resummation, and we have already obtained further new analytic results for the $\mathcal{O}(\alpha_s^4)$ virtual corrections.

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Too many hard Feynman integrals are produced!



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F. Tkachov, Phys. Lett. **B100** (1981) 65; K. Chetyrkin and F. Tkachov, Nucl. Phys. **B192** (1981) 159 Integration by parts reduction to a basis of Feynman integrals



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 - ► The number of Feynman diagrams gets large at $\mathcal{O}(\alpha_s^3)$ and introduces a large number of complicated integrals.
 - Proceed by systematically applying Stokes's theorem to generate all possible linear relations between integrals.
 Subsequently, reduce to a basis set using linear algebra.
 - The typical size of the linear systems at O (α³_s) causes problems in practice, due to intermediate expression swell.
 S. Laporta, Int. J. Mod. Phys. A15 (2000) 5087



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Running the best program available at the time, Reduze 2, took me of the order of weeks and required GBs of RAM.



Solution: Finite Fields and Rational Reconstruction

A. von Manteuffel and ${\bf RMS},$ Phys. Lett. ${\bf B744}$ (2015) 101

By discussing with mathematicians, I developed a new integration by parts reduction algorithm, with polynomial run-time complexity in the size of the linear system to be solved!



Impact: Full Higgs and Drell-Yan Predictions at $\mathcal{O}\left(\alpha_{s}^{3}\right)$



My integration by parts algorithm enabled this progress!

<u>Towards Tri-Jet</u> Production at $\mathcal{O}\left(\alpha_s^2\right)$

$$\mathcal{M}(g(p_1)g(p_2) \to g(p_3)g(p_4)g(p_5)) = \underbrace{\alpha_{\text{WIN}}}_{\text{WINO}} + \underbrace{\alpha_{\text{WINO}}}_{\text{WINO}} + \underbrace{\alpha_{$$

- Once available, an important constraint on α_s .
- ▶ Depends on 5 kinematical invariants, $s_{ii+1} = (p_i + p_{i+1})^2$.
- Virtual cross section very hard, leading groups rely on finite fields and rational reconstruction for the integral reduction!

S. Badger et. al., Phys. Rev. Lett. 123 (2019) 071601; S. Abreu et. al., JHEP 1905 (2019) 084



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Machinery implemented in FiniteFlow, Fire 6, and Kira 2!



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Andreas von Manteuffel and RMS, JHEP **1704** (2017) 129 M. Heller, A. von Manteuffel, and RMS, Phys. Rev. **D102** (2020), 016025 M. Heller, A. von Manteuffel, **RMS**, and Hubert Spiesberger, arXiv:2012.05918



Mixed Electroweak-QCD Drell-Yan Amplitudes



Andreas von Manteuffel and RMS, JHEP **1704** (2017) 129 M. Heller, A. von Manteuffel, and RMS, Phys. Rev. **D102** (2020), 016025 M. Heller, A. von Manteuffel, **RMS**, and Hubert Spiesberger, arXiv:2012.05918

• Besides the concrete results we obtained in terms of well-understood mathematical functions, I achieved a major conceptual breakthrough by understanding for the first time the precise relationship between Kreimer's γ_5 scheme and the standard HVBM γ_5 scheme.







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- Besides the concrete results we obtained in terms of well-understood mathematical functions, I achieved a major conceptual breakthrough by understanding for the first time the precise relationship between Kreimer's γ_5 scheme and the standard HVBM γ_5 scheme.
- ► Due to the fact that there are no finite counterterms required for the restoration of chiral symmetry in Kreimer's γ_5 scheme, our work suggests a far simpler automation of $\mathcal{O}(\alpha_s \alpha)$ or $\mathcal{O}(\alpha^2)$ Feynman diagram calculations!

How Does This Work at the Practical Level?



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 At each order in perturbation theory, I envisaged to generate the Standard Model diagrams of interest with QGraf and then pipe them through Reduze 2 in order to match them onto suitable integral families.



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- At each order in perturbation theory, I envisaged to generate the Standard Model diagrams of interest with QGraf and then pipe them through Reduze 2 in order to match them onto suitable integral families.
- ▶ To accommodate the general case, I modified Reduze 2, adding my own fully-general C++ routine to partial fraction power products of propagator denominators with the same momentum but different masses, e.g. $(k^2 m_z^2)^{-1}(k^2)^{-1}$:





High-Performance Computing



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- ▶ Performed numerator algebra in Mathematica for ~ 50,000 multi-loop diagrams for projects using high-performance computing clusters available at JGU and MSU.
- At various times, needed to do scripting in Bash and become familiar with scheduling in both LSF and Slurm.
- Despite the infinite licenses available at many research institutions, Mathematica is closed-source, so it is reasonable to wonder whether it makes sense to employ it.



Turn a setback into an opportunity to improve the code!



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In Mathematica v10.3.1, Apart[#,x₁]& would fail on "complicated" inputs such as:

 $\frac{\mathtt{x}_2(\mathtt{d}\,\mathtt{x}_1+\mathtt{t})^3((\mathtt{x}_1+\mathtt{t})\mathtt{x}_2+\mathtt{x}_2^2(\mathtt{x}_1+\mathtt{d}+\mathtt{x}_1^2))^2}{\mathtt{x}_3(\mathtt{t}\,\mathtt{x}_1+(\mathtt{x}_2+\mathtt{x}_3+1))^4((-\mathtt{s}\,\mathtt{x}_3)\mathtt{x}_1-\mathtt{t}\,\mathtt{x}_2)^5(\mathtt{x}_3+\mathtt{x}_2+\mathtt{x}_1\mathtt{x}_2)^3}$



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▶ With a little effort, I was able to generalize the approach to univariate partial fractioning discussed by Erik Panzer in his thesis and arrive at an implementation far superior to the one available in even the latest version of Mathematica.



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- With a little effort, I was able to generalize the approach to univariate partial fractioning discussed by Erik Panzer in his thesis and arrive at an implementation far superior to the one available in even the latest version of Mathematica.
- Principally, this was possible by making a more intelligent choice about how to organize the output; in v12.0.0, the output of Apart has a ByteCount of 3613880, whereas my replacement has a ByteCount of only 660776.

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- ▶ Will eventually cover Spark, SQL, TensorFlow, and details of the most widely-used machine learning techniques.
- Besides the well-known applications of machine learning to particle physics phenomenology, I would like to see if I can develop a custom scheduler to serve the pheno group's high-performance computing needs in an optimal way.



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The Bottom Line

- ► Groundbreaking work on fundamental LHC processes requires new techniques and dedicated effort.
- ▶ First *ab initio* analytic calculation of the cusp anomalous dimensions and first analytic calculation of the collinear anomalous dimensions done; finite parts of four-loop quark and gluon form factors calculable in current setup with known methods and modest computing resources.
- ▶ Important conceptual and technical progress on EW-QCD Drell-Yan; virtual cross sections now within reach.
- Massively-parallel integration by parts algorithm I pioneered applicable to many hard calculations of current interest. In fact, for most processes discussed in this talk, no other methods have produced comparable results.
- Machine learning algorithms will become ever more important to particle physics. Exciting to see what is next!

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