Fast Pre-scramblers

Oleg Kaikov

MPP & LMU, Munich

IMPRS Colloquium, April 29th 2021

Idea

Information contained in a system diffuses before it scrambles

Appropriate definition?

Lower bound?

Models with enhanced memory capacity?

Black holes?

Scrambling

Prepare a quantum system with K d.o.f.s in a pure state $|in\rangle$

Evolve it unitarily in time $\hat{U}\,|in\rangle = |\psi(t)\rangle$

Thermalization: $|\psi(t)\rangle$ spreads from $|in\rangle$ to other states

Scrambling: in a time t_s, information within system is sufficiently distributed over *entire* Hilbert space w.r.t. a chosen measure

Lower bound: $\boxed{t_s \gtrsim \ln(K)}$ $\Rightarrow$ fast scramblers

Hayden, Preskill '07
Sekino, Susskind '08

Enhanced memory capacity

Want: system with high capacity to store information

$$\left[\hat{a}_j, \hat{a}_k^\dagger\right] = \delta_{jk}$$

Memory pattern: microstate $|n_1, \ldots, n_K\rangle$

$\mathcal{N}$-many microstates degenerate in energy $\Rightarrow$ entropy $S = \ln(\mathcal{N})$

$$\hat{H} = \qquad\qquad \varepsilon \sum_{k=1}^{K} \hat{n}_k$$

Dvali [1810.02336]

Enhanced memory capacity

Want: system with high capacity to store information

$$\left[\hat{a}_j, \hat{a}_k^\dagger\right] = \delta_{jk}$$

Memory pattern: microstate $|n_1, \ldots, n_K\rangle$

$\mathcal{N}$-many microstates degenerate in energy $\Rightarrow$ entropy $S = \ln(\mathcal{N})$

$$\hat{H} = \left(1 - \frac{\hat{n}_a}{N}\right) \varepsilon \sum_{k=1}^{K} \hat{n}_k$$

Assisted gaplessness: highly occupied master mode $\hat{n}_a$ interacts attractively with memory modes $\hat{n}_k$, lowers their energy gaps

Dvali [1810.02336]

Memory burden

$$\hat{H} = \left(1 - \frac{\hat{n}_a}{N}\right) \varepsilon \sum_{k=1}^{K} \hat{n}_k + C_b \left(\hat{a}^\dagger \hat{b} + \text{H.c.}\right)$$

Memory burden: stored information stabilizes system

$$|in\rangle = |n_a, n_b, n_1, \ldots, n_K\rangle = |N, 0, n_1, \ldots, n_K\rangle$$

$$\mu = -\frac{1}{N}\varepsilon \sum_{k=1}^{K} \langle \hat{n}_k(t) \rangle$$

$$\langle \hat{n}_a(t) \rangle = N \left[1 - \frac{C_b^2}{C_b^2 + \left(\frac{\mu}{2}\right)^2} \sin^2\left(\sqrt{C_b^2 + \left(\frac{\mu}{2}\right)^2}\, t\right)\right]$$

Avoid memory burden: rewrite to another set of modes K'

Prototype model

$$|in\rangle = |\underbrace{N}_{a}, \underbrace{0}_{b}, \underbrace{\overbrace{1, 1}^{=N_m}, 0, \dots, 0}_{K}, \underbrace{0, \dots, 0}_{K'}\rangle$$

$$\hat{H} = \varepsilon \left(1 - \frac{\hat{n}_a}{N}\right) \sum_{k=1}^{K} \hat{n}_k + \varepsilon \left(1 - \frac{\hat{n}_a}{N - \Delta}\right) \sum_{k'=1}^{K'} \hat{n}_{k'} + C_b \left(\hat{a}^\dagger \hat{b} + \text{H.c.}\right) +$$
$$+ C_m \left\{ \sum_{k=1}^{K} \sum_{k'=1}^{K'} f_1(k, k') \left(\hat{a}_k^\dagger \hat{a}_{k'} + \text{H.c.}\right) + \sum_{k=1}^{K} \sum_{l=k+1}^{K} f_2(k, l) \left(\hat{a}_k^\dagger \hat{a}_l + \text{H.c.}\right) + \right.$$
$$\left. + \sum_{k'=1}^{K'} \sum_{l'=k'+1}^{K'} f_3(k', l') \left(\hat{a}_{k'}^\dagger \hat{a}_{l'} + \text{H.c.}\right) \right\}$$

with $|f_i(k, l)| \in [0.5, 1]$ essentially random

Dvali, Eisemann, Michel, Zell '20

Constraints

Spherical symmetry, $Y_l^m(\theta, \phi)$ $\qquad \varepsilon = \sqrt{K}$

K' sector not gapless in vicinity of $|in\rangle$ $\qquad \Delta \gg \frac{N}{1 + \sqrt{N_m}\sqrt{K}}$

Memory modes remain gapless $\qquad C_b \lesssim \frac{1}{\sqrt{N}}$

$$C_m \lesssim \frac{1}{\sqrt{N_m}\sqrt{K}}$$

Set $K = K'$

6 parameters: $N, K, N_m, \Delta, C_b, C_m$

Black hole as a bound state of soft gravitons
"Quantum N-portrait": $N = S, \;\; K = S, \;\; N_m = S/2, \;\; S \gg 1$

Dvali, Gomez '13

2 effects

Scrambling $\supset$ state is distributed over *entire* Hilbert space

(1) $|\psi(t)\rangle$ spreads from $|in\rangle$ over $\{|v_i\rangle\}$ entirely with a minimum probability $p < 1/\mathcal{N}$

(2) Most probable state changes from $|in\rangle$ to $|v_i\rangle \neq |in\rangle$

2 effects

Scrambling $\supset$ state is distributed over *entire* Hilbert space

(1) $|\psi(t)\rangle$ spreads from $|in\rangle$ over $\{|v_i\rangle\}$ entirely with a minimum probability $p < 1/\mathcal{N}$

(2) Most probable state changes from $|in\rangle$ to $|v_i\rangle \neq |in\rangle$

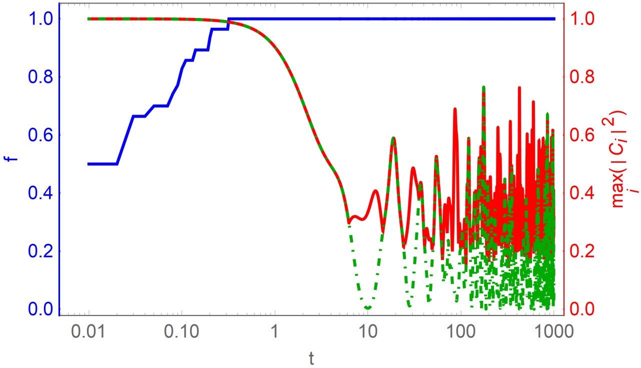
$$f \equiv \frac{1}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} H(|C_i|^2 - p) \;, \quad H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

$$|C_i|^2 = |\langle \psi(t)|v_i\rangle|^2 \;, \quad \mathcal{N} \equiv \dim\left(\{|v_i\rangle\}\right)$$

$$\boxed{t_f \;\widehat{=}\; t_{min} \text{ s.t. } f = 1}$$

$$t_c \;\widehat{=}\; t_{min} \text{ s.t. } \max_{i \in \{1,\ldots,\mathcal{N}\}} (|C_i|^2) \neq |C_{in}|^2$$

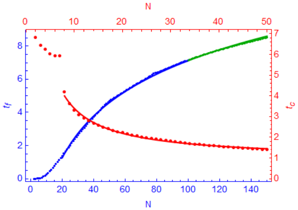
Example



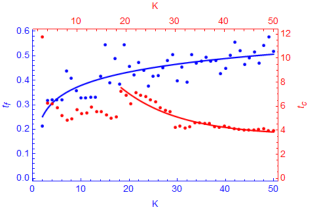
$$p = 5 \cdot 10^{-21}, \quad N = 4, \quad K = 4, \quad N_m = 2, \quad \Delta = 1, \quad C_b = 0.1, \quad C_m = 0.1$$

$$t_f = 0.32, \qquad t_c = 6.25$$

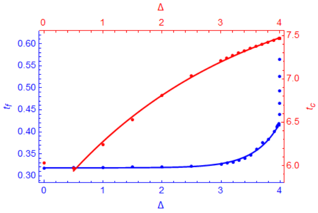
Scans



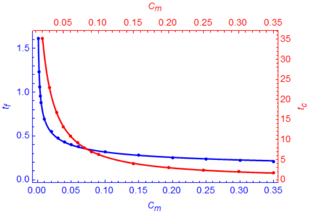
(a) $t_f \sim \ln(N)$ for $N \geq 20$



(b) $t_f \sim \ln(K)$



(c) $t_f \sim \mathrm{e}^{(2.373\pm0.087)\Delta}$ for $\Delta \leq 3.986$



(d) $t_f \sim C_m^{-0.3847\pm0.0051}$

Altered parameters: (a) $p = 5 \cdot 10^{-41}$

Fast pre-scramblers

Main result: $t_f \sim \ln(K)$

Model above possesses all-to-all couplings

We therefore define a system to be a *fast pre-scrambler*, if for an arbitrarily large but fixed range of number of degrees of freedom K there exists a sufficiently small minimum state probability threshold $p < 1/\dim(\{|v_i\rangle\})$, such that the state of the system $|\psi(t)\rangle$, initially in one basis state $|in\rangle$, spreads over the entire Hilbert space $\{|v_i\rangle\}$ into a superposition of all $\dim(\{|v_i\rangle\})$ basis states each with a probability $|C_i|^2 = |\langle\psi(t)|v_i\rangle|^2 > p$, in a time logarithmic in K.

Conjectures

(1) The fastest pre-scramblers spread the initial state of the system over its entire Hilbert space in a time logarithmic in the number of degrees of freedom, given a minimum state probability threshold

(2) The enhanced memory capacity model above is a fast pre-scrambler

(3) The time of (fast) pre-scrambling is $\leq$ than that of (fast) scrambling, irrespective of the choice of a specific measure of the uniformity of the state distribution for the latter

(4) Consequently, fast scramblers are fast pre-scramblers

(5) In particular, black holes are fast pre-scramblers

Outlook

Models with enhanced memory capacity in the context of:

Scrambling in terms of OTOCs:

$$F_{OTO}(t) \equiv \mathrm{tr}\left[\hat{\rho}^{1/4}\hat{A}(t)\hat{\rho}^{1/4}\hat{B}(0)\hat{\rho}^{1/4}\hat{A}(t)\hat{\rho}^{1/4}\hat{B}(0)\right]$$

$$\hat{\rho} \equiv \frac{1}{Z}\mathrm{e}^{-\beta\hat{H}}, \quad Z \equiv \mathrm{tr}\left[\mathrm{e}^{-\beta\hat{H}}\right]$$

Maldacena, Shenker, Stanford '16

Kitaev '15 KITP talks 1, 2

ETH $\Leftrightarrow$ fast scrambling?

Deutsch '91

Srednicki '94