Vacuum Bubble Stabilization

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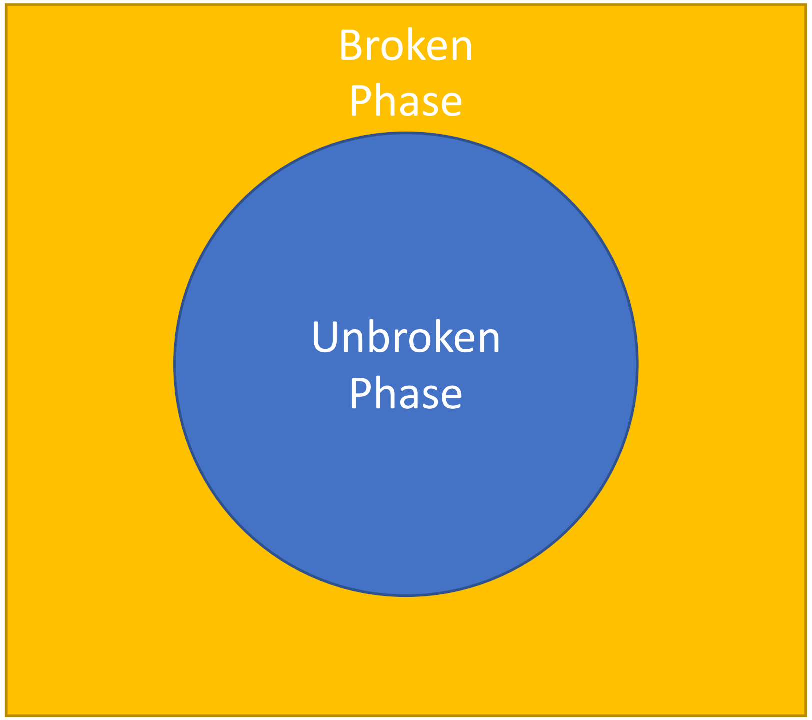
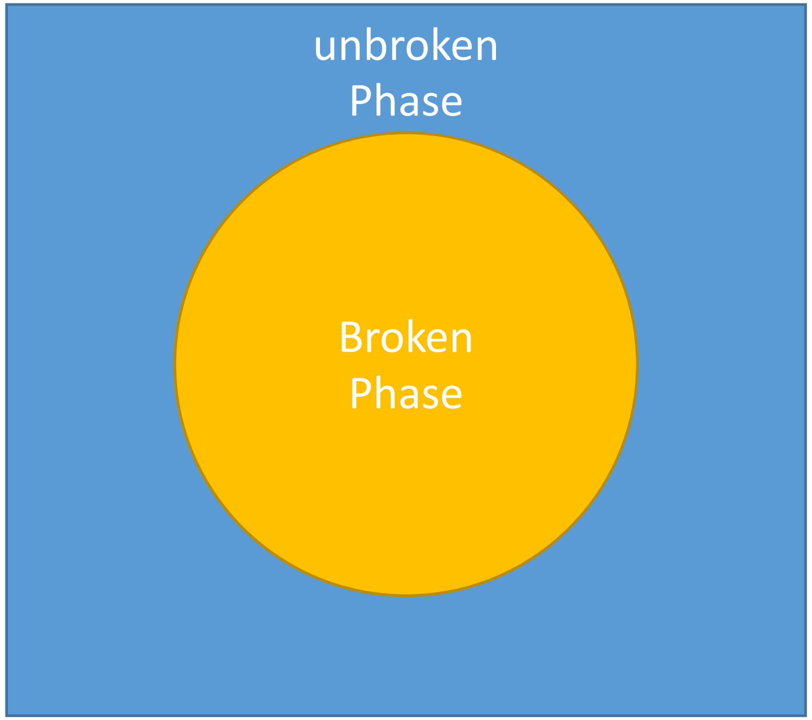
Vaccum Bubbles





Model of a saturon as a vacuum bubble

- Lets consider a theory of a scalar field ϕ in the adjoint representation of $SU(N)$.
- ϕ_α^β is a $N\times N$ traceless Hermitian matrix with $\alpha, \beta = 1, \dots, N$.
- The Lagrangian density is given by

$$L = \frac{1}{2}\,\mathrm{tr}\left[(\partial_\mu \phi)(\partial^\mu \phi)\right] - V[\phi],$$

with potential

$$V[\phi] = \frac{\alpha}{2}\mathrm{tr}\left[\left(f\phi - \phi^2 + \frac{I}{N}\mathrm{tr}\,[\phi^2]\right)^2\right],$$

where I is the unit $N\times N$ matrix.

Model of a saturon as a vacuum bubble

The vacuum equations

$$f\phi_{\alpha}^{\beta} - (\phi^2)_{\alpha}^{\beta} + \frac{\delta_{\alpha}^{\beta}}{N} Tr\phi^2 = 0,$$

have many degenerate solutions. They correspond to the breaking

$$SU(N) \to SU(N-K)\times SU(K)\times U(1)$$

with $0 < K < N$.

We consider the breaking with $K = 1$.

Model of a saturon as a vacuum bubble

Lets consider the ansatz

$$\phi^{\alpha}_{\beta} = \frac{\varphi(x)}{\sqrt{N(N-1)}} diag\left((N-1), -1, \dots, -1\right).$$

The potential becomes

$$V[\phi] = \frac{\alpha}{2} \mathrm{tr}\left[\left(f\phi - \phi^2 + \frac{I}{N} \mathrm{tr}\,[\phi^2]\right)^2\right]$$

$$V[\varphi] = \frac{\tilde{\alpha}}{2} \varphi^2 \left(\tilde{f} - \varphi\right)^2$$

Model of a saturon as a vacuum bubble

$$V[\varphi] = \frac{\tilde{\alpha}}{2}\varphi^2\left(\tilde{f} - \varphi\right)^2$$

where

$$\tilde{\alpha} = \frac{(N-2)^2}{N(N-1)} \xrightarrow{N\to\infty} \alpha$$

$$\tilde{f} = \frac{f\sqrt{\{N(N-1)\}}}{(N-2)} \xrightarrow{N\to\infty} f$$

Model of a saturon as a vacuum bubble

$$V[\varphi] = \frac{\tilde{\alpha}}{2}\varphi^2\left(\tilde{f} - \varphi\right)^2$$

is minimimized by

$$\phi = \begin{cases} 0. \\ \tilde{f} \end{cases}$$

Model of a saturon as a vacuum bubble

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is minimimized by

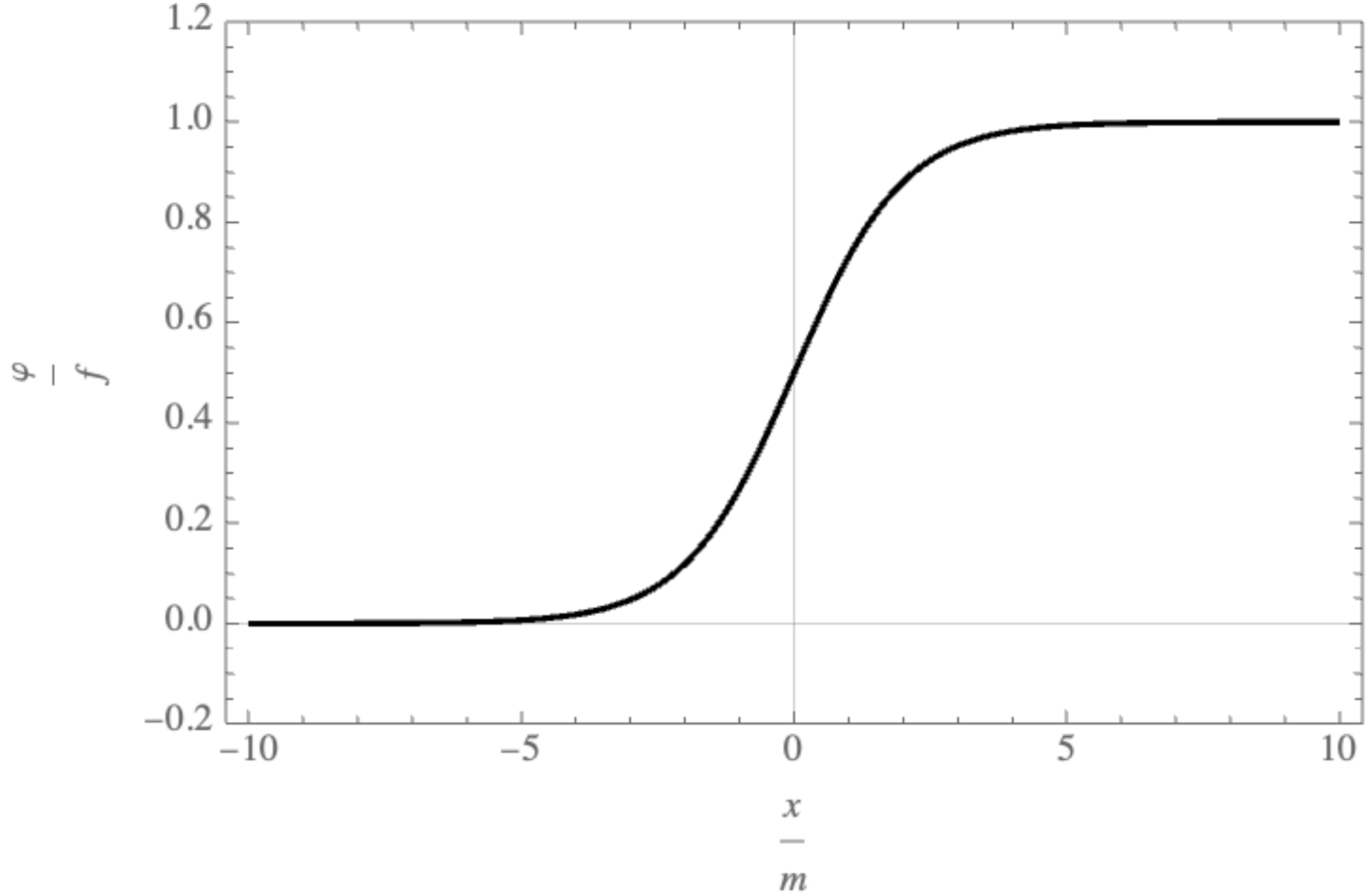
$$\phi = \begin{cases} 0. \\ \tilde{f} \end{cases}$$

$0.$ → Unbroken Phase

$\tilde{f}$ → Broken Phase

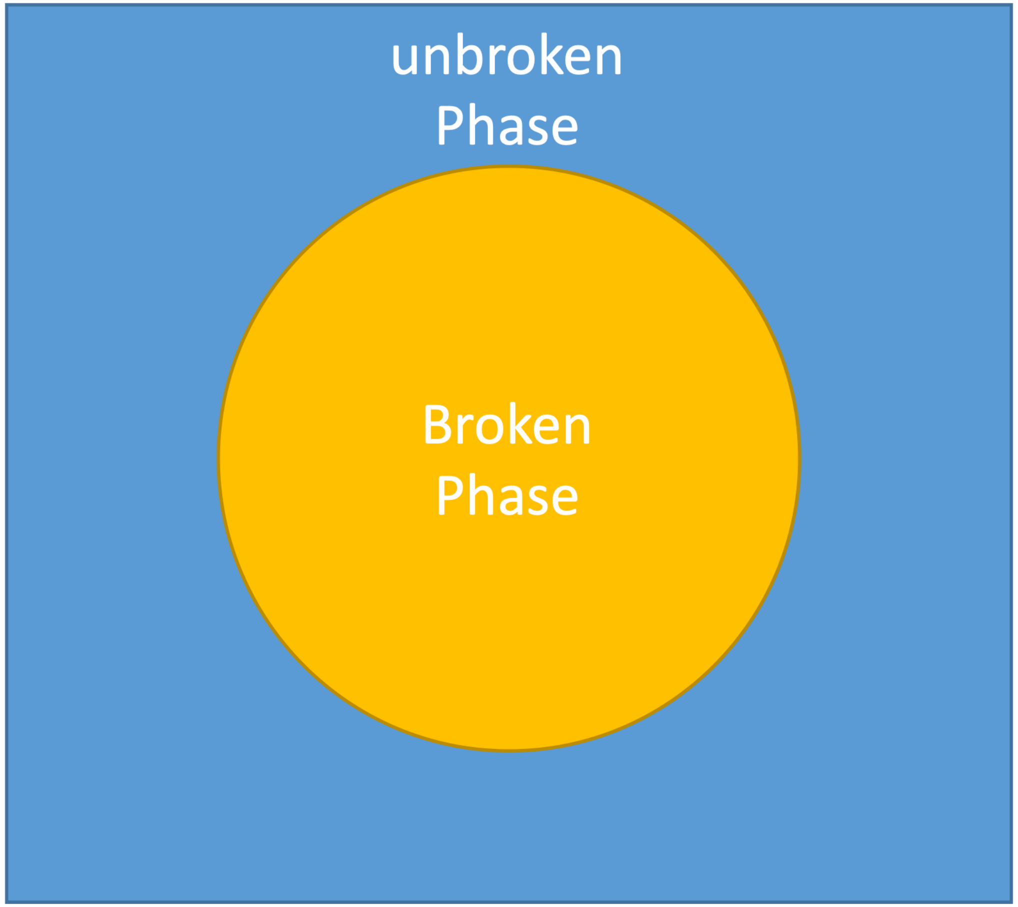
Domain wall profile

$$\varphi(x) = \frac{\tilde{f}}{2}\left(1 + \tanh\left(\frac{mx}{2}\right)\right)$$



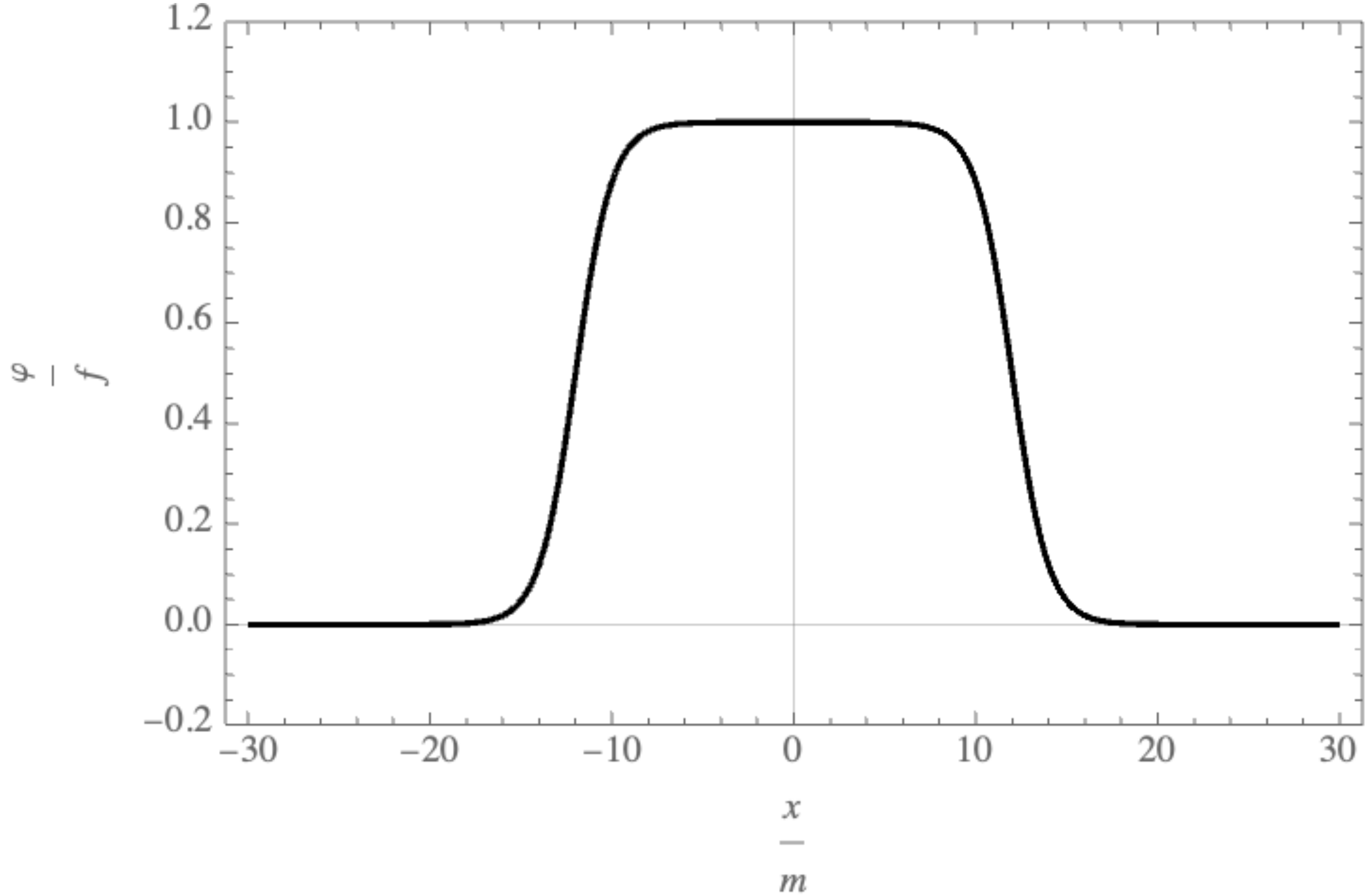
$m = \sqrt{\tilde{\alpha}}\,\tilde{f} = \sqrt{\alpha} f$

Vaccum Bubbles

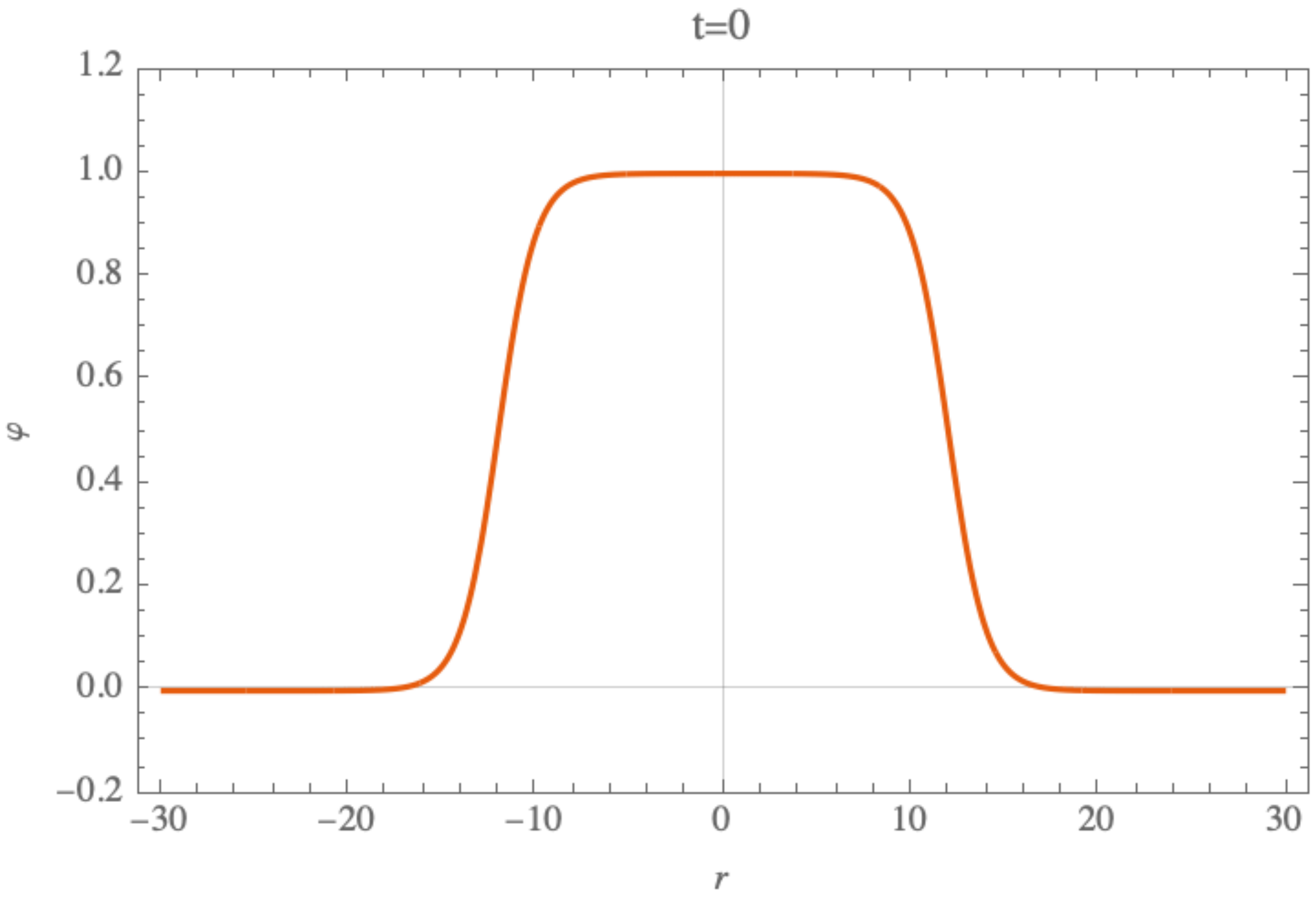


Vaccum Bubbles

$$\varphi(\boldsymbol{x}) = \frac{\tilde{f}}{2}(1 + \tanh\left(\frac{m(r-\boldsymbol{x})}{2}\right))$$



Vaccum Bubbles Decay



ρ

$m = 1$
$r = 12$

Vaccum Bubbles Micro-states

Due to $SU(N) \rightarrow SU(N-1)\times U(1)$ SSB, the broken vacuum contains

$$N_{Gold} = 2(N-1)$$

massless Goldstone bosons Localized within the bubble.

These gapless Goldstone modes give rise to an exponential number of bubble micro-states n_{st}, which scales as[1]

$$n_{st}(r) \sim \left(1 + \frac{2N}{s(r)}\right)^{s(r)} \left(1 + \frac{s(r)}{2N}\right)^{2N}$$

[1]G. Dvali, Entropy Bound and Unitarity of Scattering Amplitudes, JHEP03(2021) 126, arXiv:2003.05546.

Vaccum Bubbles Micro-states

$$n_{st}(r) \sim \left(1 + \frac{2N}{s(r)}\right)^{s(r)} \left(1 + \frac{s(r)}{2N}\right)^{2N}$$

Here $s(r)$ is the time-averaged space integral of $\varphi^2(x)$.

For large bubbles $(r \gg m)$, making use of the thin wall approximation, it is given by

$$s(r) \approx \frac{4\pi}{3} \frac{(rm)^3}{\alpha}$$

whereas for small ($r \sim m^{-1}$) bubbles it scales as

$$s(r) \sim \frac{1}{\alpha}$$

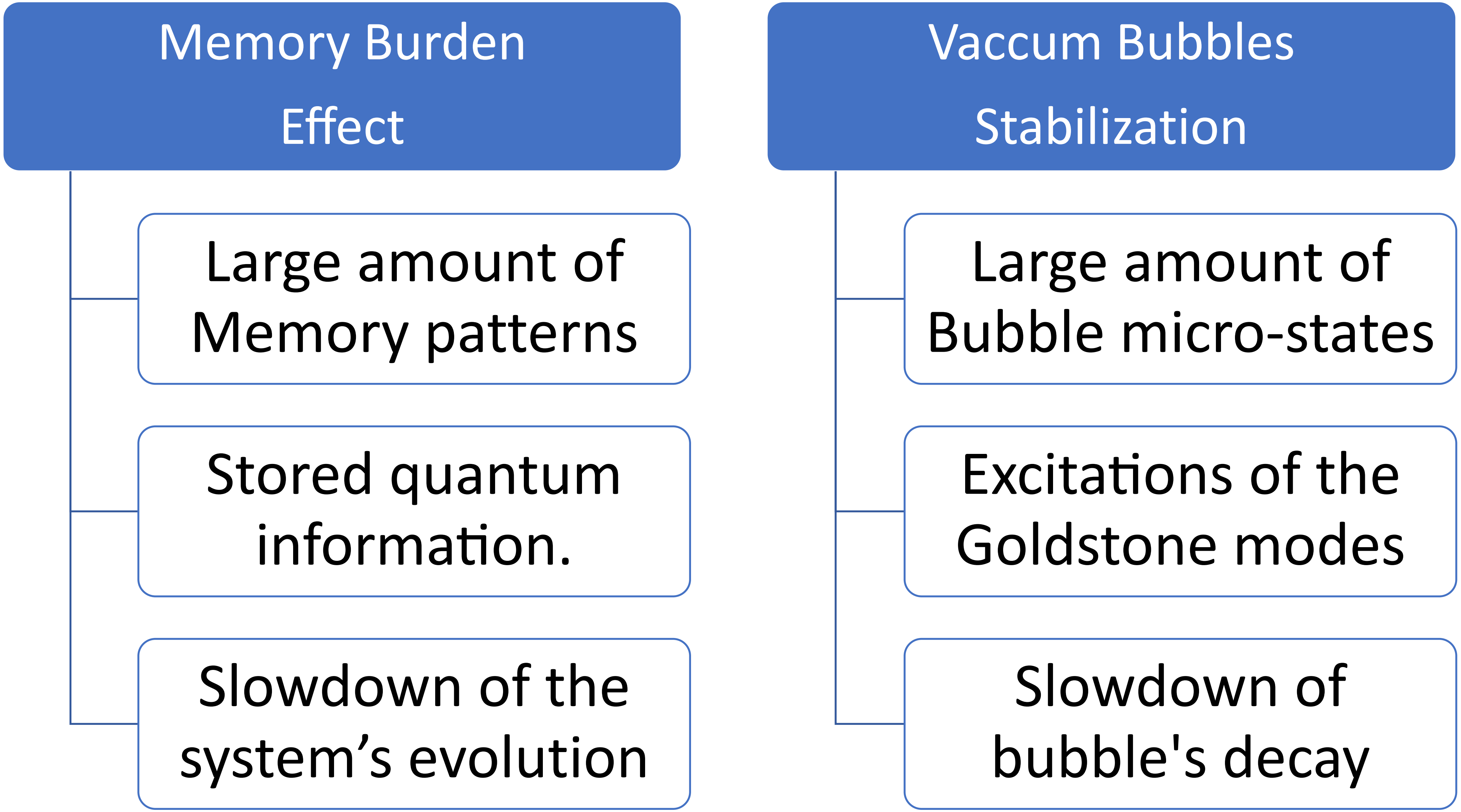
Vaccum Bubbles Stabilization

These micro-states, in their turn, contribute to the corresponding micro-state entropy ofthe bubble

$$S = \ln(n_{st})\,.$$

In [1] it was suggested that the high entropy of the bubble stabilizes it. That is, the occupied Goldstone modes prevent the bubble from collapsing or spreading out. This is argued to occur due *to the memory burden effect* [2,3].

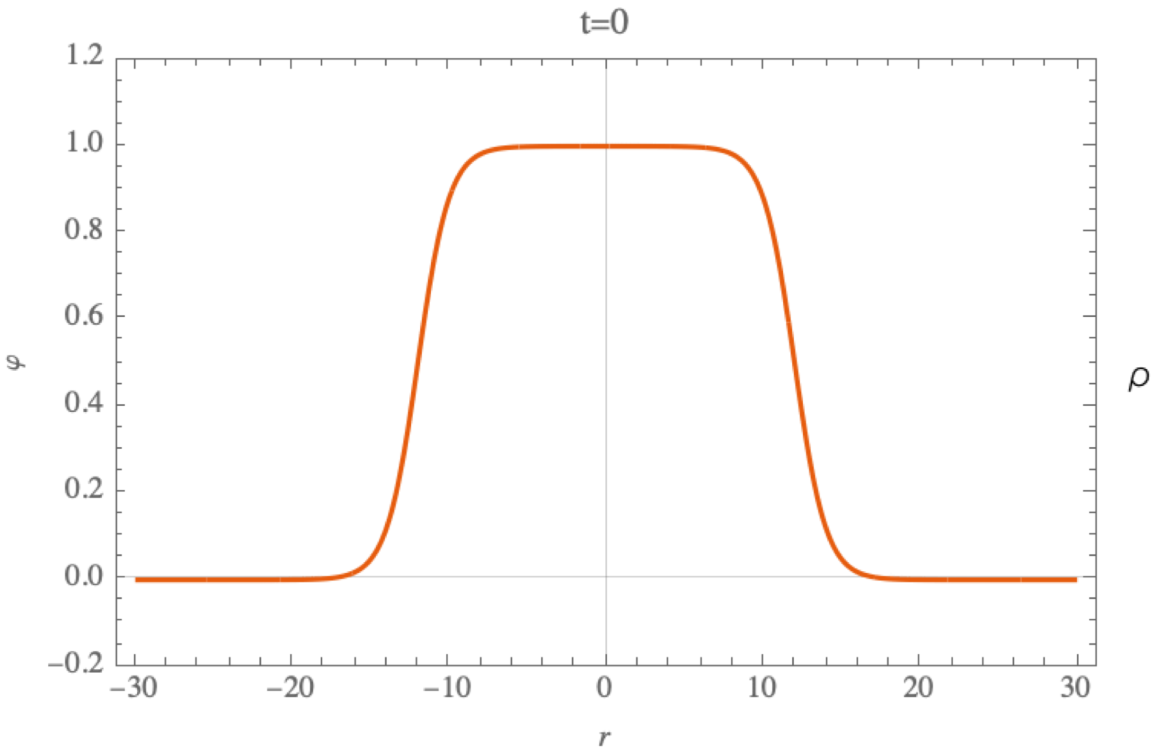
[2] G. Dvali,A Microscopic Model of Holography: Survival by the Burden of Memory, arXiv:1810.02336.
[3] G. Dvali, L. Eisemann, M. Michel, and S. Zell,Universe's Primordial Quantum Memories, JCAP03 (2019) 010, arXiv:1812.08749.

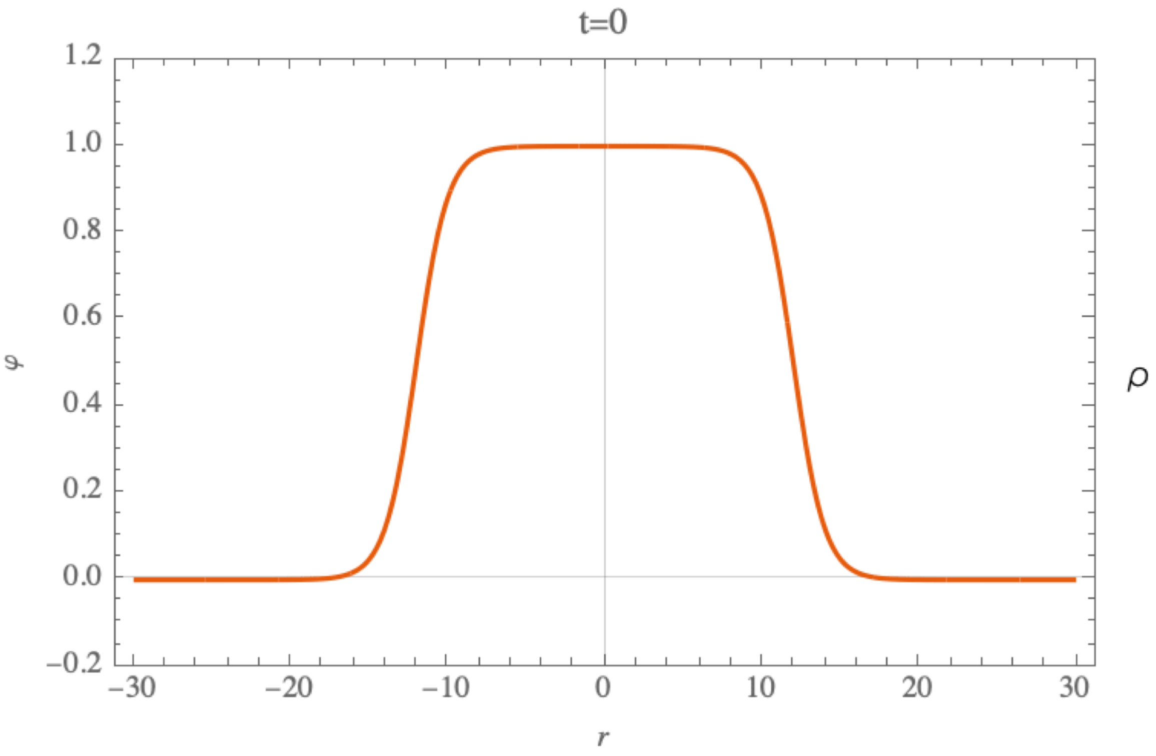

Memory Burden Effect
Large amount of Memory patterns
Stored quantum information.
Slowdown of the system’s evolution
Vaccum Bubbles Stabilization
Large amount of Bubble micro-states
Excitations of the Goldstone modes
Slowdown of bubble's decay

Vaccum Bubbles Stabilization

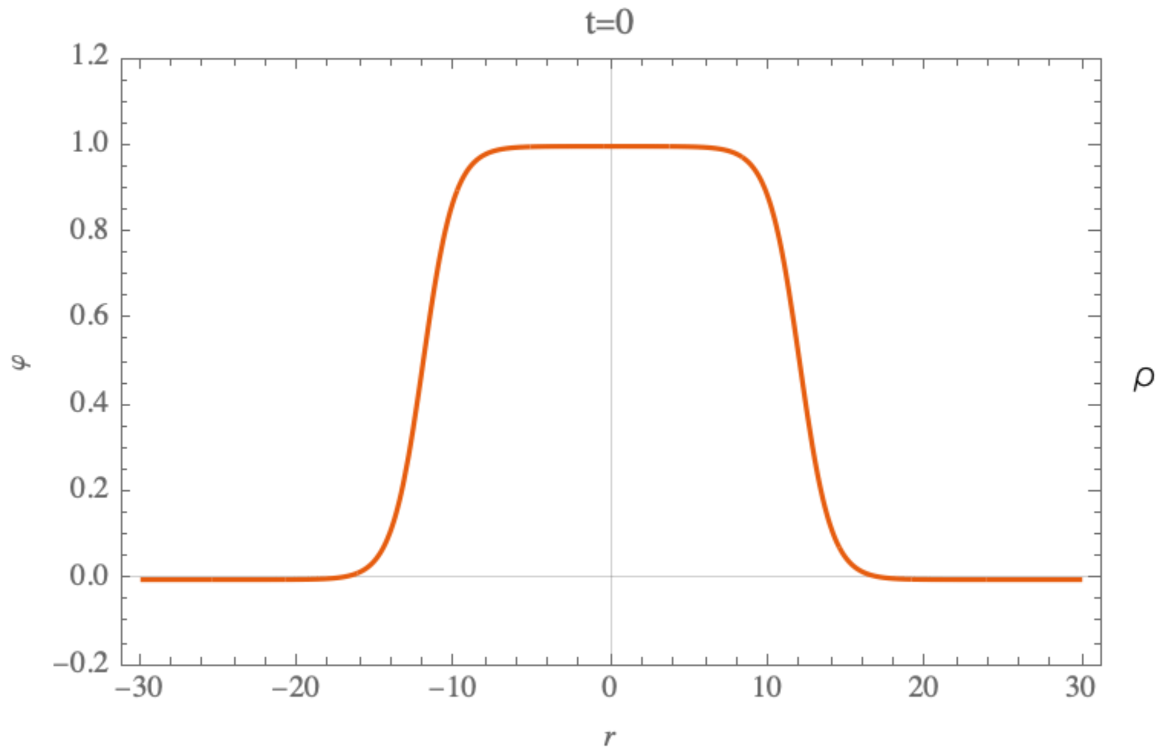
- Below we test this proposition and consider a classical analogue of the memory burden effect where the Goldstone mode occupation number is macroscopic.

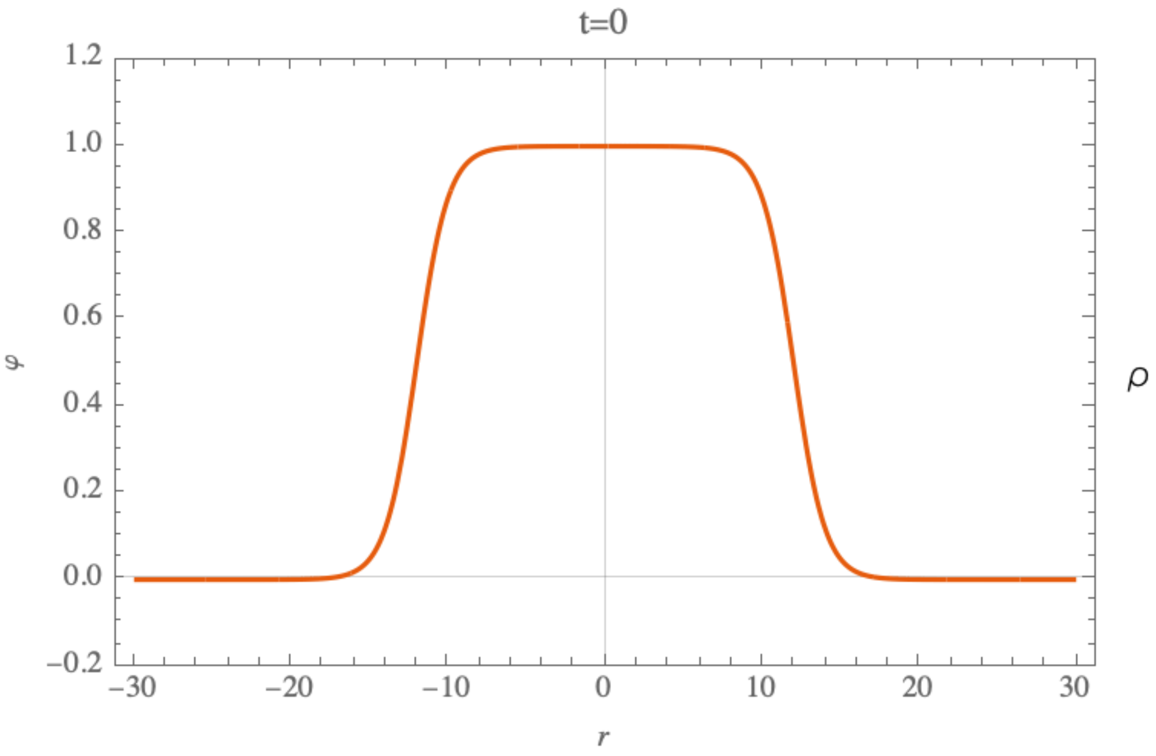
Vaccum bubble Stabilization



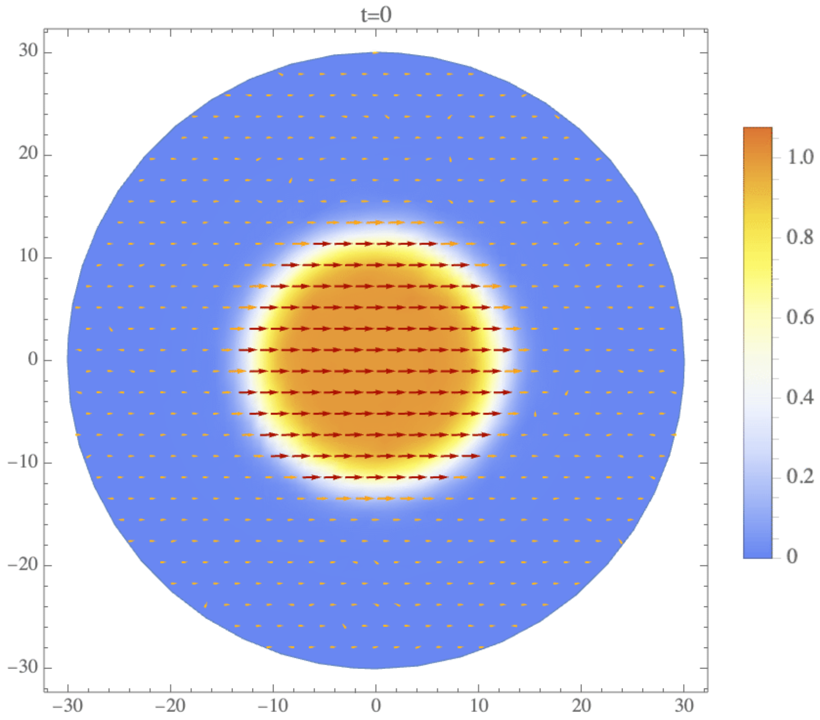


Vaccum bubble Stabilization





Vaccum bubble Stabilization



Vaccum Bubbles Stabilization

Lets consider the following ansatz

$$\Phi^{\alpha}_{\beta} = \frac{\varphi(x)}{\sqrt{N(N-1)}} diag\left((N-1), -1, \dots, -1\right).$$

$$\phi^{\alpha}_{\beta} = \left(U^{\dagger}\Phi U\right)^{\alpha}_{\beta},$$

$$U = \exp[-i\theta^{a}T^{a}]$$

and choose (for simplicity) $\theta^{a} = \theta(t,x)\delta^{a1}$

$$U = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) & & & \\ -i\sin(\theta/2) & \cos(\theta/2) & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix}.$$

Vaccum Bubbles Stabilization

- The Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \varphi\right)\left(\partial^\mu \varphi\right) + \frac{N}{4(N-1)}\varphi^2\left(\partial_\mu \theta\right)\left(\partial^\mu \theta\right) - \frac{\tilde{\alpha}}{2}\varphi^2\left(\tilde{f} - \varphi\right)^2 ,$$

- We solve the field equations considering the following initial conditions

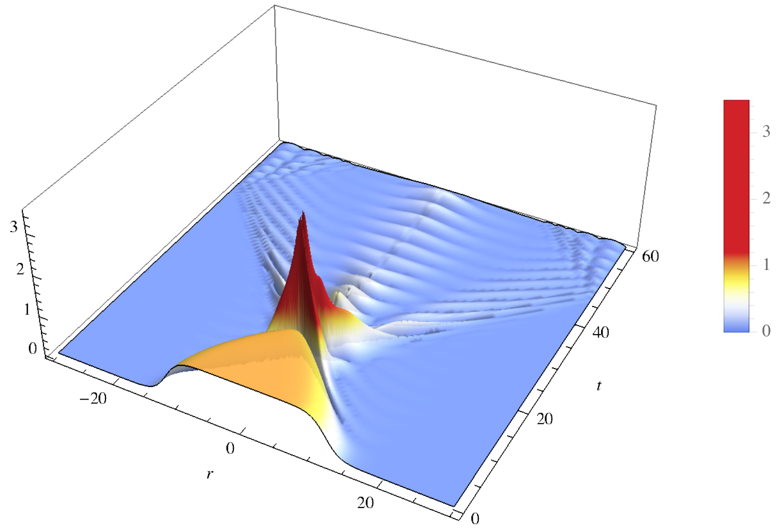
$$\varphi(t,r)|_{t=0} = \frac{\tilde{f}}{2}\left[1 \pm \tanh\left(\frac{m(R_0 - r)}{2}\right)\right] ,$$

$$\dot{\theta} \equiv \partial_t \theta(t,r)|_{t=0} = \sqrt{\frac{2(N-1)}{N}}\frac{\tilde{\omega}}{\tilde{f}} = \frac{\sqrt{2}(N-2)}{N}\frac{\tilde{\omega}}{f},$$

Vaccum Bubble

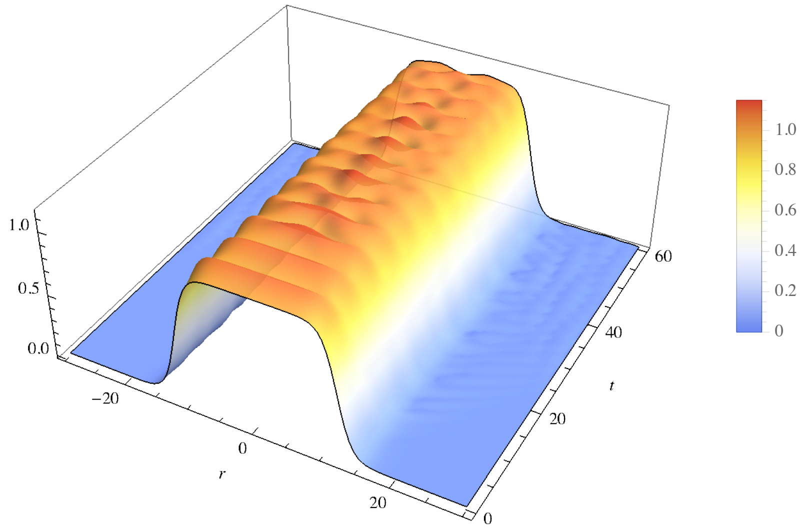
Decay

$(\dot{\theta} = 0)$



Stabilization

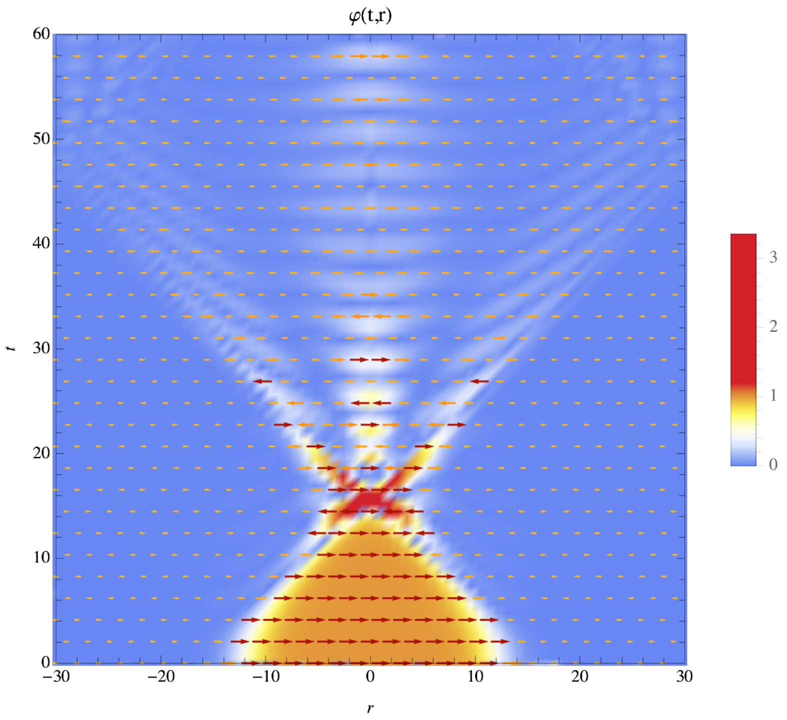
$(\dot{\theta} \neq 0)$



Vaccum Bubble

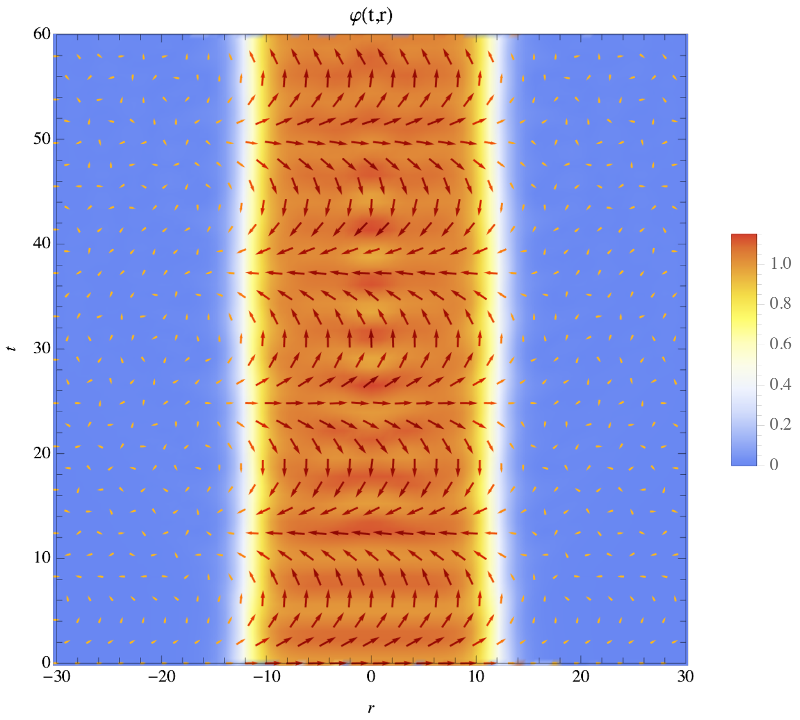
Decay

$$(\dot{\theta} = 0)$$



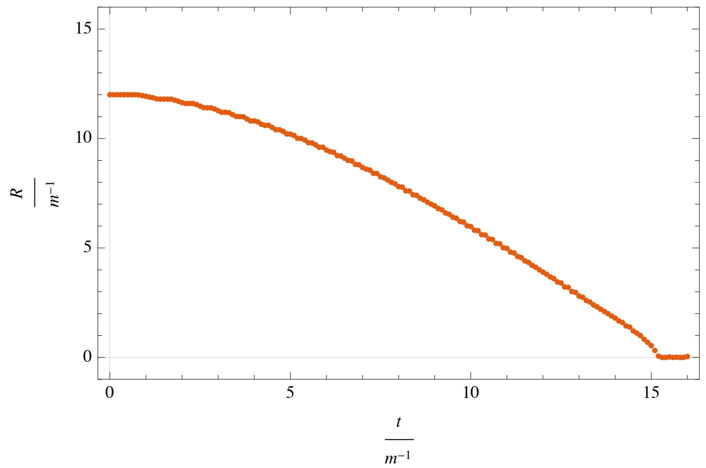
Stabilization

$$(\dot{\theta} \neq 0)$$

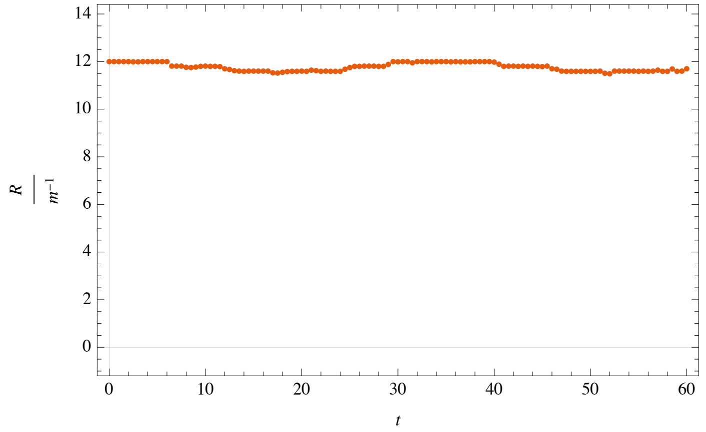


Vaccum Bubble

Decay
$(\dot{\boldsymbol{\theta}} = \mathbf{0})$



Stabilization
$(\dot{\boldsymbol{\theta}} \neq \mathbf{0})$



Critical Frecuency $\dot{\theta}_c$

- We observed in the numerical simulations that at a certain critical frecuency we get the bubble to stabilice
- Bellow and above $\dot{\theta}_c$, the bubble oscilates respect to a smaller or bigger radius, repectively, but does not decay during the simulated time.
- We estimated $\dot{\theta}_c$ using the thin wall approximation, for a bubble of initial radius R_0, to be

$$\frac{2(N-2)}{\sqrt{3N(N-1)}}\sqrt{\frac{\alpha}{mR_0}}$$

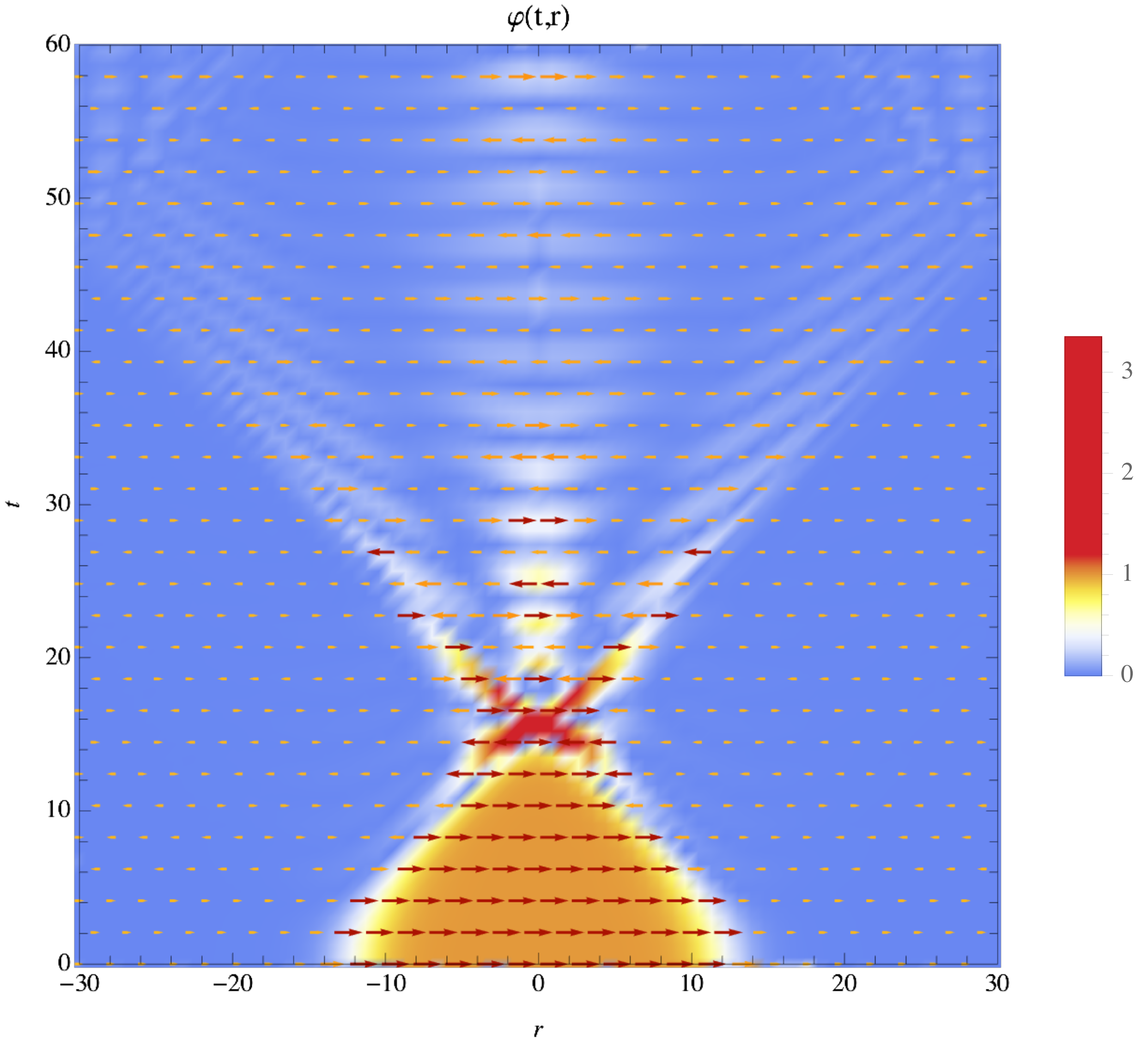
Conclusions and outlook

- We have shown and explicit classical ananlog of the memory burden effect
- The macroscopic ocupation number of the Goldstone modes, parametrized by $\dot{\theta}$, not only slows down the vaccum bubble decay as it increases, but there is a critical frecuency $\dot{\theta}_c$ at which the bubble stabilizes.
- We are now cosidering frecuencies above and bellow $\dot{\theta}_c$, and studying the bubble dynamics.
- What about small bubbles? In this case quantum effects become important, reason why we need to include them in our analysis.

Thank you

Appendix

Vaccum bubble Decay



Vaccum bubble Decay