Renormalization and low-energy limit of Lorentz violating field theories

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Why to break Lorentz invariance?

Unitarity, locality, causality + Poincaré $\Rightarrow$ limited number of renormalizable field theories.
There is a possibility to modify the concept of renormalizability, breaking the Lorentz invariance and enlarging the set of consistent QFTs.

Enormous precision of Lorentz invariance observed in nature:

- the symmetry could be broken at very high energies;
- in the low-energy limit we must recover the covariant theory.

Several applications of Lorentz violating field theories: high-energy extensions of the Standard Model, theories of gravitation, effective theories in nuclear physics or condensed matter physics.

Breaking the Lorentz symmetry

Space-time is a d-dimensional manifold M with the Lorentz group $SO(1, d-1)$ acting on it.
We split M into "time" and "space" submanifolds,

$$M = \widehat{M} \times \overline{M},$$

where $\widehat{M}$ has a residual invariance $SO(1, \widehat{d}-1)$, while in $\overline{M}$ we have invariance under spatial rotations $SO(\overline{d})$.

Any tensor is split consequently:

$$T_{\mu_1 \cdots \mu_n} = T_{(\widehat{\mu}_1, \overline{\mu}_1) \cdots (\widehat{\mu}_n, \overline{\mu}_n)}.$$

Example

$$x_\mu = (\widehat{x}, \overline{x}), \quad \partial_\mu = (\widehat{\partial}, \overline{\partial}), \quad A_\mu = (\widehat{A}, \overline{A}).$$

Weighted power counting

Definition

$P_{n,k}(x, y)$ is a **weighted polynomial** of weight k if $P_{n,k}(\xi^n x, \xi y)$ is a polynomial in ξ of degree nk.

Weighted scale transformations: time and space variables behave differently under a scale transformation:

$$\widehat{x} \to e^{-\Omega}\widehat{x}, \quad \overline{x} \to e^{-\Omega/n}\overline{x}.$$

The power counting criterion has to be modified, assigning different weights to the variables (Anselmi):

$$[\widehat{x}] = -1, \quad [\overline{x}] = -\frac{1}{n}, \quad [\widehat{\partial}] = 1, \quad [\overline{\partial}] = \frac{1}{n}.$$

Lagrangians are weighted polynomials in the momenta and the fields.

Free scalar field

The homogeneous Lagrangian for a free scalar field is

$$\mathcal{L}_0 = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^{2n-2}}(\overline{\partial}^n\varphi)^2,$$

and it's invariant under weighted scale transformations if

$$[\varphi] = (đ - 2)/2,$$

where $đ \equiv \widehat{d} + \overline{d}/n$. $\mathcal{L}$ has weight $[\mathcal{L}] = đ$.
We can add non homogeneous quadratical terms

$$\mathcal{L} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{m^2}{2}\varphi^2 + \frac{c^2}{2}(\overline{\partial}\varphi)^2 + \sum_{i<n}\frac{a_i}{2\Lambda_L^{2i-2}}(\overline{\partial}^i\varphi)^2 + \frac{1}{2\Lambda_L^{2n-2}}(\overline{\partial}^n\varphi)^2.$$

The free propagator is

$$G_0 = \frac{1}{\widehat{p}^2 + \overline{p}^{2n}/\Lambda_L^{2n-2}}.$$

Renormalizable theories

$$\mathcal{L}_{(\widehat{d},\overline{d})} = \mathcal{L}_0 + \sum_{N,\alpha,q_1,q_2} \frac{\lambda(N,q_1,q_2,\alpha)}{N!\Lambda_L^{q_1+q_2+N(d/2-1)-d}} \left[\widehat{\partial}^{q_1}\overline{\partial}^{q_2}\varphi^N\right]_\alpha .$$

Vertexes are renormalizable if the coupling constant has weight $[\lambda] \geq 0$.

- Time derivatives of order higher than 2 are not introduced by renormalization and perturbative unitarity is conserved.
- The counterterms are polynomial in m, c and all the a_i
- The maximal number of external scalar legs that a renormalizable vertex can have is

$$N_{\max} = \mathrm{int}\left[\frac{2đ}{đ - 2}\right].$$

Polynomiality in the fields, for strictly renormalizable theories, requires $đ > 2$.

Fermions

The free Lagrangian for fermions is

$$\mathcal{L}_f = \bar{\psi}\widehat{\partial\!\!\!/}\psi + \frac{1}{\Lambda_L^{n-1}}\bar{\psi}\overline{\partial\!\!\!/}^{\,n}\psi.$$

The field has weight $[\psi] = [\bar{\psi}] = (\text{đ} - 1)/2$. The propagator is

$$G_f = \frac{-i\widehat{p\!\!\!/} + (-i)^n \overline{p\!\!\!/}^{\,n}/\Lambda_L^{n-1}}{\widehat{p}^2 + \overline{p}^{2n}/\Lambda_L^{2n-2}}.$$

We can add interaction terms with a number of $\bar{\psi} - \psi$ legs less than

$$N_{\max} = \mathrm{int}\left[\frac{\text{đ}}{\text{đ} - 1}\right].$$

Polynomiality for strictly renormalizable theories requires $\text{đ} \geq 1$.

Regularization and renormalization

Dimensional regularization: continuation to complex dimension is made independently in $\widehat{M}$ and $\overline{M}$: $\widehat{D} = \widehat{d} - \widehat{\epsilon}$ and $\overline{D} = \overline{d} - \overline{\epsilon}$.

- One loop: divergences are poles $\propto 1/\epsilon$, where $\epsilon = \widehat{\epsilon} + \overline{\epsilon}/n$.
- More than one loop: there is a subtraction algorithm for subdivergences similar to the Lorentz invariant one.

$$\mathcal{L} = \frac{1}{2}(\widehat{\partial}\varphi_B)^2 + \frac{1}{\Lambda_{L\,B}^{2n-2}}(\overline{\partial}^n\varphi_B)^2 + \sum_N \frac{\lambda_{N\,B}}{N!\Lambda_{L\,B}^{K_N}}\overline{\partial}^q\varphi_B^N.$$

The bare quantities are

$$\varphi_B = Z_\varphi^{1/2}\varphi, \quad \Lambda_{L\,B} = Z_{\Lambda_L}\Lambda_L, \quad \lambda_{N\,B} = \mu^{\epsilon(N/2-1)}(\lambda_N + \Delta_N).$$

The renormalization constants are power series in the coupling constants λ_N, without explicite dependences on μ/Λ_L (Anselmi).

The usual Callan-Symanzik equation holds, with $\widehat{\beta}_\lambda = \mu\,\mathrm{d}\lambda/\mathrm{d}\mu$.

Example: renormalization of a scalar model in $(2,2)$ dimensions, $n=2$

$$\mathcal{L}_{(2,2)} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{c^2}{2}(\overline{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^2}(\overline{\partial}^2\varphi)^2 + \frac{m^2}{2}\varphi^2 + \frac{\lambda_4}{4!\Lambda_L^2}\varphi^2(\overline{\partial}\varphi)^2 + \frac{\lambda}{4!}\varphi^4 + \frac{\lambda_6}{6!\Lambda_L^2}\varphi^6.$$

$$\beta_4 = \frac{5\lambda_4^2}{2(12\pi)^2}, \qquad \beta_6 = \frac{5\lambda_4}{(8\pi)^2}\left(\lambda_6 - \frac{\lambda_4^2}{36}\right), \qquad \beta_{c^2} = \frac{\lambda_4 c^2}{3(8\pi)^2}.$$

The model is IR-free, and for $t = \ln(\mu|x|) \to \infty$

$$\lambda_4(t) \sim \frac{2(12\pi)^2}{5t}, \qquad \lambda_6(t) \sim \frac{1}{20}\lambda_4^2, \qquad \lambda(t) \sim \left(\frac{\lambda_4(t)}{\lambda_4(0)}\right)^{13/10},$$

$$c^2(t) = c_0^2\left(\frac{\lambda_4(t)}{\lambda_4(0)}\right)^{3/10}.$$

Theories at low energies

In order to find the low-energy limit, taking $t \to \infty$ is not sufficient.

At great distances the physical quantities follow **ordinary power counting**, thus we must perform also the limit $\Lambda_L \to \infty$.

Lorentz violating theories become of the form

$$\mathcal{L}_{\text{l.e.}} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{c^2}{2}(\overline{\partial}\varphi)^2 + \frac{m^2}{2}\varphi^2 + \frac{\lambda_3}{3!}\varphi^3 + \frac{\lambda}{4!}\varphi^4 + j\varphi$$
$$+ \eta\bar{\psi}\widehat{\not{\partial}}\psi + v\bar{\psi}\overline{\not{\partial}}\psi + M\bar{\psi}\psi + g\varphi\bar{\psi}\psi$$

(Colladay-Kostelecký)

Now we can calculate the IR limit, using the low-energy RG.

Decoupling for $\Lambda_L \to \infty$

It can be shown that renormalizing a theory at high energies, and then performing the limit $\Lambda_L \to \infty$, is equivalent to renormalize directly the effective theory at low energies, up to a scheme change.

Theorem

- Let $\mathcal{L}^{(H)}$ be a Lagrangian renormalizable with weighted power counting, and let $\mathcal{L}^{(L)} = \lim_{\Lambda_L \to \infty} \mathcal{L}^{(H)}$ be renormalizable with ordinary power counting.
- Let $G^{(H)}(k, m; \Lambda_L)$ be a Green function of the theory (H), and $G^{(L)}(k, m)$ the corresponding Green function in the theory (L).

There exist two renormalization schemes $\Gamma^{(H)}$ and $\Gamma^{(L)}$ such that

$$\lim_{\Lambda_L \to \infty} G_R^{(H)}(k, m; \Lambda_L) = G_R^{(L)}(k, m).$$

Identification $\Lambda_L \sim \Lambda$

When $\Lambda_L \to \infty$ there are divergences in Λ_L summed to the poles $1/\epsilon$ (or to the Λ divergences). To regularize the low-energy theory all divergences have to be subtracted.

We have to identify $\Lambda_L \propto \Lambda$, with an arbitrary proportionality constant.

$$\log \Lambda_L = \log \Lambda + const. \sim \frac{1}{\epsilon_{\text{l.e.}}},$$

where $\epsilon_{\text{l.e.}} = \widehat{\epsilon} + \overline{\epsilon}$ at low energies.

MS scheme: $\widehat{\epsilon} = 0$, $\overline{\epsilon} = n\epsilon$.
$\log \Lambda_L \sim 1/n\epsilon = 1/\overline{\epsilon}$ in the logarithmic divergences makes it possible to recover the right behavior at low energies.
The coefficients in front of the quadratic divergences remain undetermined.

Recovery of covariance

The recovery of Lorentz invariance at low energies is not automatic: in the limit $t \to \infty$ c has to be 1 for all fields.
If $c_i = 1 + \delta_i$, where δ_i are small, then

$$\frac{\mathrm{d}\delta_i}{\mathrm{d}t} = -C_{ij}\delta_j.$$

- If C is positive definite, covariance is automatically restored in the IR,
- otherwise it will be necessary to set $\delta_i \equiv 0$.

There are results indicating that the Lorentz invariant surface in parameter space is stable under renormalization group for all CPT-even theories (Colladay & MacDonald).

Yukawa theory with N fermions, I

$$\mathcal{L} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{c^2}{2}(\overline{\partial}\varphi)^2 + \frac{\lambda}{4!}\varphi^4 + \sum_{\alpha=1}^{N}\left(\bar{\psi}_\alpha \widehat{\not{\partial}}\psi_\alpha + v_\alpha \bar{\psi}_\alpha \overline{\not{\partial}}\psi_\alpha + g_\alpha \varphi \bar{\psi}_\alpha \psi_\alpha\right).$$

Let $c^2 = 1 + \delta_{c^2}$ and $v_\alpha = 1 + \delta_\alpha$.
At zero order in δ the Lorentz invariant result holds

$$g_\alpha^2(t) = \frac{g_\alpha^2(0)}{1 + \frac{5g_\alpha^2(0)}{8\pi^2}t}.$$

At first order we have $\frac{\mathrm{d}\delta}{\mathrm{d}t} \equiv -\beta_\delta = -\mathbf{C}\cdot\delta$, where

$$\delta \equiv \begin{pmatrix} \delta_{c^2} \\ \delta_1 \\ \vdots \\ \delta_N \end{pmatrix}, \quad \mathbf{C} \equiv \frac{1}{3(4\pi)^2}\begin{pmatrix} 12\sum_\alpha g_\alpha^2 & -24g_1^2 & \cdots & -24g_N^2 \\ -g_1^2 & 2g_1^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -g_N^2 & 0 & \cdots & 2g_N^2 \end{pmatrix}.$$

Yukawa theory with N fermions, II

Case $g_\alpha = g$, $\forall\alpha$: eigenvalues and eigenvectors

$$\tilde\lambda_0 = 0, \qquad \tilde\delta^{(0)} = 2\delta_{c^2} + \sum_\alpha \delta_\alpha,$$

$$\tilde\lambda_i = 2, \qquad \tilde\delta^{(i)} = \delta_{i+1} - \delta_1, \quad \text{for } i = 1, \cdots, N-1,$$

$$\tilde\lambda_N = (12N+2), \qquad \tilde\delta^{(N)} = -12N\delta_{c^2} + \sum_\alpha \delta_\alpha.$$

We find the solutions for $\tilde\delta^{(i)}(t)$

$$\tilde\delta^{(i)}(t) = \tilde\delta^{(i)}(0)\left(1 + \frac{5g_0^2}{8\pi^2}t\right)^{-\tilde\lambda_i/30},$$

with a logarithmic dependence on the energy (powers of $t = \ln(\mu/E)$).

Experimental data

Problem: there are many different estimates for Λ_L.

- Neutrino mass from terms like $\varphi^2\bar{\psi}\psi$: $\Lambda_L \sim 10^{14}$ GeV;
- Quantum gravity, etc.: $\Lambda_L \sim 10^{18}$ GeV (Planck's mass);
- Lower bounds on Λ_L from measures of $\delta_c = c - 1$ for different particles.

Several data about precision of Lorentz invariance, mostly from astrophysical measures (Kostelecký & Russell).

	δ_c	E (GeV)
e	10^{-15}	$10^{3\div 5}$
μ	10^{-11}	$10^{4\div 5}$
τ	10^{-8}	$10^{4\div 5}$
ν	10^{-21}	10^{2}

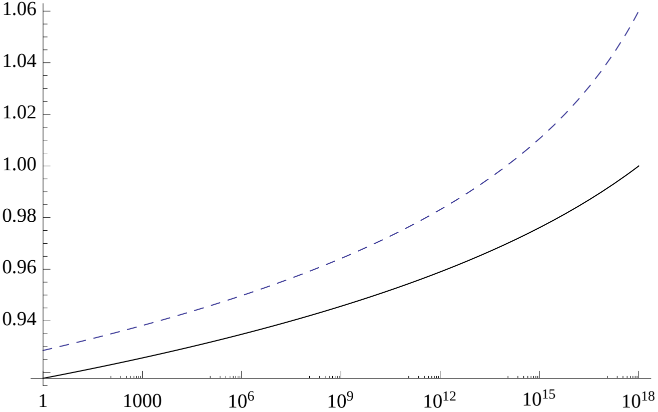
	δ_c	E (GeV)
p	10^{-9}	$10^{4\div 5}$
π	10^{-10}	$10^{4\div 5}$
K	10^{-9}	$10^{4\div 5}$
D	10^{-8}	$10^{4\div 5}$
B	10^{-7}	$10^{4\div 5}$

Decay time of high-energy cosmic rays (10^{11} GeV) $\Rightarrow \delta_c \lesssim 10^{-21}$.

δ_c running

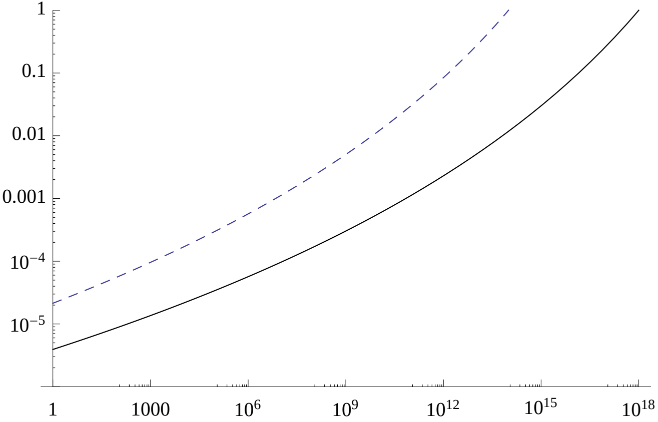
Can the running of δ_c alone explain the smallness of these parameters at low energies?

- The differences $\delta_\alpha - \delta_\beta$ have no appreciable running over many orders of magnitude: they are small after fine-tuning.



δ_c running

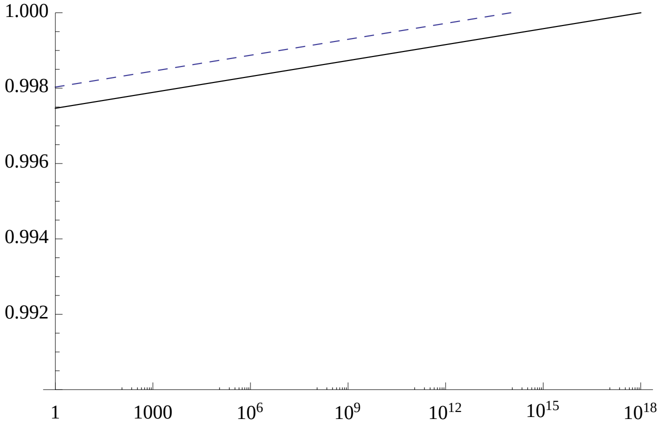
- The running of $\tilde{\delta}_N \simeq (6N+1)\delta_c$ depends strongly on the number of fermions N and the coupling constant g.



(a) $g_0 = 1, N = 24$

δ_c running

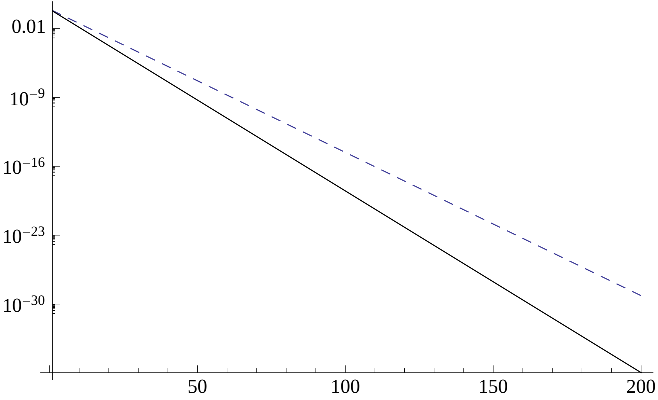
- The running of $\tilde{\delta}_N \simeq (6N+1)\delta_c$ depends strongly on the number of fermions N and the coupling constant g.



(b) $g_0 = 0.01, N = 24$

Dependence on the number of fermions N

The rate at witch δ_c becomes small in the IR grows with N.



$\delta_c(E = 10^5\mathrm{GeV})$, with $\delta_c(\Lambda_L) = 1$, as a function of N.

Operators of dimension > 4

Including higher derivative terms the dispersion relations are modified:

$$E_s = \sqrt{c^2 \overline{p}^2 + \sum_k b_k^2 \frac{\overline{p}^{2k}}{\Lambda_L^{2k-2}} + \frac{\overline{p}^{2n}}{\Lambda_L^{2n-2}} + m^2},$$

$$E_f = \sqrt{\overline{p}^2 \left(c - \sum_{k\,\mathrm{odd}} (-1)^{\frac{k+1}{2}} \frac{b_k}{\Lambda_L^{k-1}} \overline{p}^{k-1} \right)^2 + \left(m + \sum_{k\,\mathrm{even}} (-1)^{\frac{k}{2}} \frac{b_k}{\Lambda_L^{k-1}} \overline{p}^{k} \right)^2}.$$

For $n = 2$:

$$E \simeq c\overline{p} + \frac{\overline{p}^3}{2\Lambda_L^2} \quad \Rightarrow \quad v = c + \frac{\overline{p}^2}{2\Lambda_L^2}.$$

- $\delta_c \lesssim 10^{-15}$ at 10^5 GeV $\Rightarrow \Lambda_L \gtrsim 10^{12\div 13}$ GeV.
- $\delta_c \lesssim 10^{-23}$ at 10^{11} GeV $\Rightarrow \Lambda_L \gtrsim 10^{22}$ GeV (Gagnon & Moore).

Operators of dimension > 4 (n=3)

For $n = 3$ (Standard Extended Model, Anselmi):

$$E \simeq c\overline{p} + \frac{b^2}{2}\frac{\overline{p}^3}{\Lambda_L^2} + \frac{4-b^4}{8}\frac{\overline{p}^5}{\Lambda_L^4} \Rightarrow v = c + \frac{b^2}{2}\frac{\overline{p}^2}{\Lambda_L^2} + \frac{4-b^4}{8}\frac{\overline{p}^4}{\Lambda_L^4}.$$

- $\delta_c \lesssim 10^{-15}$ at 10^5 GeV $\Rightarrow$ $\Lambda_L \gtrsim 10^9$ GeV, and no fine-tuning is needed on b.
- $\delta_c \lesssim 10^{-23}$ at 10^{11} GeV $\Rightarrow$ $\Lambda_L \gtrsim 10^{16\div 17}$ GeV, and $b \lesssim 10^{-8}$.

For fermions:

$$E \simeq c\overline{p} + \frac{b^2-2}{2}\frac{\overline{p}^3}{\Lambda_L^2} + \frac{4b^2-b^4}{8}\frac{\overline{p}^5}{\Lambda_L^4},$$

so $b^2 \simeq 2$; all the precedent results hold.

Conclusions

- Low-energy limits of Lorentz violating theories, renormalizable respect to weighted power counting, are theories that are renormalizable in the usual sense.
- The recovery of Lorentz invariance in the infrared limit is triggered by the behavior of some parameters. In the considered cases Lorentz invariance is automatically restored. Anyway, to obtain the required precision we must advocate a fine tuning at low energies.
- Experimental limits are mostly in agreement with the estimates coming from the neutrino masses, that predict a value for Λ_L of about 10^{14} GeV. However, the scale of Lorentz violations could be lower than the Planck scale.
- It could be interesting to review the experimental data considering modifications to the high-energy physics potentially coming from the Lorentz violating interactions.

Standard Extended Model

The Lorentz violating extension of the Standard Model has $\widehat{d} = 1$ and $n = 3$ (Anselmi). A simplified Lagrangian, containing only terms that are generated by renormalization, is

$$\mathcal{L} = \frac{1}{4} \sum_G \left(2\widetilde{F}^G \eta^G(\overline{\Upsilon}) \widetilde{F}^G + \overline{F}^G \tau^G(\overline{\Upsilon}) \overline{F}^G \right)$$

$$+ |\widehat{D}\varphi|^2 + \frac{a_0}{\Lambda_L^4} |\overline{D}^3 \varphi|^2 + \frac{a_1}{\Lambda_L^2} |\overline{D}^2 \varphi|^2 + a_2 |\overline{D}\varphi|^2 + m_H^2 \varphi^2 + \frac{g^2 \lambda_4}{4} |\varphi|^4$$

$$+ \sum_{a,b=1}^{3} \sum_I \bar{\psi}_I^a \left(\delta^{ab} \widehat{\not{D}} + \frac{b_I^{ab}}{\Lambda_L^2} \overline{\not{D}}^3 + c_I^{ab} \overline{\not{D}} \right) \psi_I^b + \sum_\alpha \frac{Y_{4f}}{\Lambda_L^2} [\bar{\psi}\psi\bar{\psi}\psi]_\alpha$$

$$+ g \sum_{a,b=1}^{3} \left(Y_1^{ab} \bar{L}^{ai} \ell_R^b + Y_2^{ab} \bar{u}_R^a Q_L^{bj} \varepsilon^{ij} + Y_3^{ab} \bar{Q}_L^{ai} d_R^b \right) \varphi^i + \text{h.c.}$$

$$+ \sum_{a,b=1}^{3} \frac{\bar{g}^2}{4\Lambda_L} Y^{ab} (L^{ai} \varphi_i)(L^{bj} \varphi_j) + \sum_I \sum_\alpha \frac{g}{\Lambda_L^2} [\overline{DF} (\bar{\psi}_I \overline{\Gamma} \psi_I)]_\alpha.$$

Some scalar models

The homogeneous Lagrangians (invariant under weighted scale transformations) have the general form

$$\mathcal{L} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^{2n-2}}(\overline{\partial}^n\varphi)^2 + \sum_{N,\alpha} \frac{\lambda(N,\alpha)}{N!\Lambda_L^{(n-1)(N+\widehat{d}-\widehat{d}N/2)}} \left[\overline{\partial}^{nd(N)}\varphi^N\right]_\alpha ,$$

with $d(N) \equiv \text{đ}(1 - N/2) + N$.

Two models in $d = 4$ $(n = 2)$

$$\mathcal{L}_{(1,3)} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^2}(\overline{\partial}^2\varphi)^2 + \frac{\lambda_6}{6!\Lambda_L^4}\varphi^4(\overline{\partial}\varphi)^2 + \frac{\lambda_{10}}{10!\Lambda_L^6}\varphi^{10},$$

$$\mathcal{L}_{(2,2)} = \frac{1}{2}(\widehat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^2}(\overline{\partial}^2\varphi)^2 + \frac{\lambda_4}{4!\Lambda_L^2}\varphi^2(\overline{\partial}\varphi)^2 + \frac{\lambda_6}{6!\Lambda_L^2}\varphi^6.$$

All the super-renormalizable terms that are compatible with the symmetries can be added.

Some models of fermions and scalars

A four fermion model $(d = 4, n = 3)$

$$\mathcal{L}_{(1,3)} = \bar\psi\widehat{\partial\!\!\!/}\psi + \frac{1}{\Lambda_L^2}\bar\psi\overline{\partial\!\!\!/}^3\psi + \sum_\alpha \frac{\lambda_\alpha}{\Lambda_L^2}\left[\bar\psi^2\psi^2\right]_\alpha.$$

Two homogeneous models $(d = 4, n = 2)$

$$\mathcal{L}_{(1,3)} = \mathcal{L}_0(\varphi) + \mathcal{L}_f(\bar\psi,\psi) + \frac{\lambda_2}{2\Lambda_L^2}\varphi^2(\bar\psi\overline{\partial\!\!\!/}\psi) + \frac{\lambda_2'}{2\Lambda_L^2}\varphi^2\overline{\partial}(\bar\psi\overline{\gamma}\psi)$$
$$+ \frac{\lambda_4}{4!\Lambda_L^3}\varphi^4\bar\psi\psi + \frac{\lambda_6}{6!\Lambda_L^4}\varphi^4(\overline{\partial}\varphi)^2 + \frac{\lambda_{10}}{10!\Lambda_L^6}\varphi^{10}.$$

$$\mathcal{L}_{(2,2)} = \mathcal{L}_0 + \mathcal{L}_f + \frac{\lambda_2}{2\Lambda_L}\varphi^2\bar\psi\psi + \frac{\lambda_4}{4!\Lambda_L^2}\varphi^2(\overline{\partial}\varphi)^2 + \frac{\lambda_6}{6!\Lambda_L^2}\varphi^6.$$

Gauge theories

Interactions with gauge fields are built through minimal coupling with $D = (\widehat{D}, \overline{D}) = (\widehat{\partial} + g\widehat{A}, \overline{\partial} + g\overline{A})$. The weights of g and the fields are

$$[g] = 2 - \frac{đ}{2}, \qquad [\widehat{A}] = \frac{đ}{2} - 1, \qquad [\overline{A}] = \frac{đ}{2} - \frac{3}{2}.$$

The gauge Lagrangian is

$$\mathcal{L}_g = \frac{1}{4}\left\{\widehat{F}^2 + 2\widetilde{F}\eta(\overline{\Upsilon})\widetilde{F} + \overline{F}\tau(\overline{\Upsilon})\overline{F} + \frac{1}{\Lambda_L^2}(\widehat{D}\overline{F})\xi(\overline{\Upsilon})(\widehat{D}\overline{F})\right\},$$

where $\widehat{F} = F_{\widehat{\mu}\widehat{\nu}}$, $\widetilde{F} = F_{\widehat{\mu}\overline{\nu}}$, $\overline{F} = F_{\overline{\mu}\overline{\nu}}$ and $\overline{\Upsilon} = \overline{D}^2/\Lambda_L^2$.
The regularity of the propagators requires:

$$\widehat{d} = 1, \quad đ < 2 + \frac{2}{n}, \quad d\,\text{even}, \quad n\,\text{odd}.$$

Example: scalar tadpole (n=2)

$$= \int \frac{\mathrm{d}^{\widehat{D}}\widehat{p}}{(2\pi)^{\widehat{D}}} \frac{\mathrm{d}^{\overline{D}}\overline{p}}{(2\pi)^{\overline{D}}} \frac{-\lambda - \frac{\lambda_4}{6\Lambda_L^2}(\overline{k}^2 + \overline{p}^2)}{\widehat{p}^2 + c^2\overline{p}^2 + \frac{\overline{p}^4}{\Lambda_L^2}} \equiv \mathcal{T}_0 + \mathcal{T}_2\overline{k}^2.$$

Using dimensional regularization we find

$$\mathcal{T}_2 = \frac{\lambda_4 c^2}{6(4\pi)^2}\Big(\frac{1}{\epsilon} - 2\log\Lambda_L + 1 - \gamma_E - \log\big(\frac{c^2}{4\pi}\big)\Big),$$

$$\mathcal{T}_0 = \frac{\Lambda_L^2}{(4\pi)^2}\Big[\Big(\frac{1}{2\epsilon} - \log\Lambda_L\Big)\Big(2\lambda c^2 - \frac{\lambda_4 c^4}{6}\Big) + const.\Big].$$

To eliminate all the divergences $\propto \overline{k}^2$ we set

$$\log\Lambda_L = \frac{1}{2\epsilon} + const. = \frac{1}{\overline{\epsilon}} + const.$$

Yukawa theory with N fermions

First order correction to g_α:

$$g_\alpha^2(t) = g_\alpha^2(0)\Bigg\{1 + \frac{5g_\alpha^2(0)}{8\pi^2}t + \frac{g_\alpha^2(0)}{g_0^2}\sum_{i=0}^{N}\frac{A_i^{(\alpha)}\tilde{\delta}^{(i)}(0)}{1-\tilde{\lambda}_i/30}\left[\left(1+\frac{5g_0^2}{8\pi^2}t\right)^{1-\tilde{\lambda}_i/30} - 1\right]\Bigg\}^{-1}.$$