Black Holes-Saturons Correspondence

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MPP & LMU

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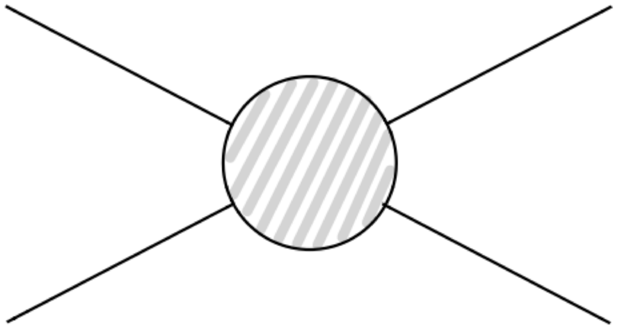
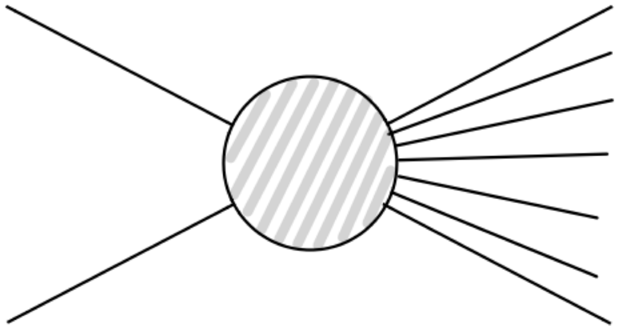
Recap: Entropy Bound

Unitarity

Cross-section:
$\boldsymbol{\sigma_{2\to 2}}$

Cross−section:
$\boldsymbol{\sigma_{2\to n}\, n_{st}}$





[1] G. Dvali, Entropy Bound and Unitarity of Scattering Amplitudes, JHEP03(2021) 126, arXiv:2003.05546.
[2] G. Dvali, Bounds on Quantum Information Storage and Retrieval, Philosophical Transactions of Royal Society A, arXiv:2107.10616

Entropy Bound

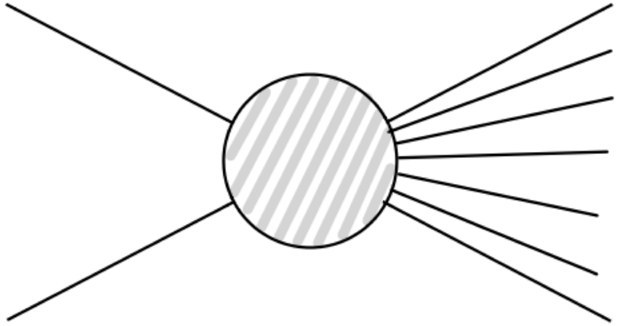
Unitarity

$$\sigma_{2\to n}\, n_{st} \sim e^{-1/\alpha} e^{S}$$

imposes the following bound on the entropy

$$S_{max} = \frac{1}{\alpha} = \frac{Area}{G_{Gold}}$$

$\boldsymbol{\sigma_{2\to n}\, n_{st}}$



Entropy Bound Imposed by Unitarity

- Consider self-sustained object of size R in d space-time dimensions,

$$S \leqslant \frac{Area}{G_{Gold}}$$

- $Area \sim R^{d-2}$
- G_{Gold} is the Goldstone coupling

- Alternatively, the *bound* is

$$S \leqslant \frac{1}{\alpha}$$

- α is as an effective running coupling of the theory evaluated at the scale $1/R$.

Entropy Bound Imposed by Unitarity

- The maximal entropy compatible with unitarity is

$$S_{\mathrm{Max}} = \frac{1}{\alpha} = \frac{Area}{f^{-2}},$$

where $f \equiv G_{Gold}^{-1/2}$ is the canonically normalized *Goldstone decay constant*.

Saturons

- We refer to the objects saturating the entropy bounds as ***Saturons***.
- Different saturons are discussed in [1-3]. These include:
  - Certain solitons, instantons, baryons, oscillons,
  - ***Black Holes***,
  - Lumps of classical fields,
  - ***Vacuum Bubbles.***

[1] G. Dvali, Entropy Bound and Unitarity of Scattering Amplitudes, JHEP03(2021) 126, arXiv:2003.05546.
[2] G. Dvali, Bounds on Quantum Information Storage and Retrieval, PTRS A, arXiv:2107.10616
[3] G. Dvali and O. Sakhelashvili, Black-Hole-Like Saturons in Gross-Neveu, arXiv:2111.03620.

Saturons

Saturons share the following universal properties [1-2]

- Their entropy satisfies the *area law*:

$$S_{\text{Max}} = \frac{1}{\alpha} = \frac{Area}{f^{-2}}$$

- If unstable, they decay with a rate which gives the thermal rate of the *temperature*

$$T \sim \frac{1}{R},$$

  up to $1/S$ corrections.

- In semi-classical treatment, they exhibit an *information horizon.*

- The minimal time-scale required for the start of the *information retrieval* is

$$t_{min} = \frac{Volume}{G_{Gold}} = \frac{R}{\alpha} = S_{\max}R$$

  where $Volume \sim R^d$.

[1] G. Dvali, Entropy Bound and Unitarity of Scattering Amplitudes, JHEP03(2021) 126, arXiv:2003.05546.
[2] G. Dvali, Bounds on Quantum Information Storage and Retrieval, PTRS A, arXiv:2107.10616

Saturons as Black Holes

Black Holes properties

- Their entropy satisfies the *area law*: [1]

$$S \sim \frac{Area}{M_p^{-2}} \sim \frac{Area}{f^{-2}}$$

- $f \sim M_p$

- Decay rate thermal rate of *temperature*

$$T \sim \frac{1}{R},$$

  up to $1/S$ corrections.

- In semi-classical treatment, they exhibit an *information horizon.*

- The minimal time-scale required for the start of the *information retrieval* is [2]

$$t_{min} = SR \sim \frac{R^3}{M_p^{-2}} \sim \frac{Volume}{M_p^{-2}}$$

[1] S. W. Hawking, Particle creation by black holes, Commun. Math. Phys. 43, 199 (1975).
[2] D. N. Page, Information in Black Hole Radiation, Phys. Rev. Lett. 71, 3743 (1993).

Saturons

- Saturons share the following universal properties [1-2]
  - **Their entropy satisfies the *area law*:** **TODAY**

$$\boldsymbol{S_{\mathrm{Max}} = \frac{1}{\alpha} = \frac{Area}{f^{-2}}}$$

  - If unstable, they decay with a rate which gives the thermal rate of the *temperature*

$$T \sim \frac{1}{R},$$

    up to $1/S$ corrections.

  - **In semi-classical treatment, they exhibit an *information horizon*.** **TODAY**

  - The minimal time-scale required for the start of the information retrieval is bounded from below by,

$$t_{min} = \frac{Volume}{G_{Gold}} = \frac{R}{\alpha} = S_{\max}R$$

    where $Volume \sim R^d$.

[1] G. Dvali, Entropy Bound and Unitarity of Scattering Amplitudes, JHEP03(2021) 126, arXiv:2003.05546.
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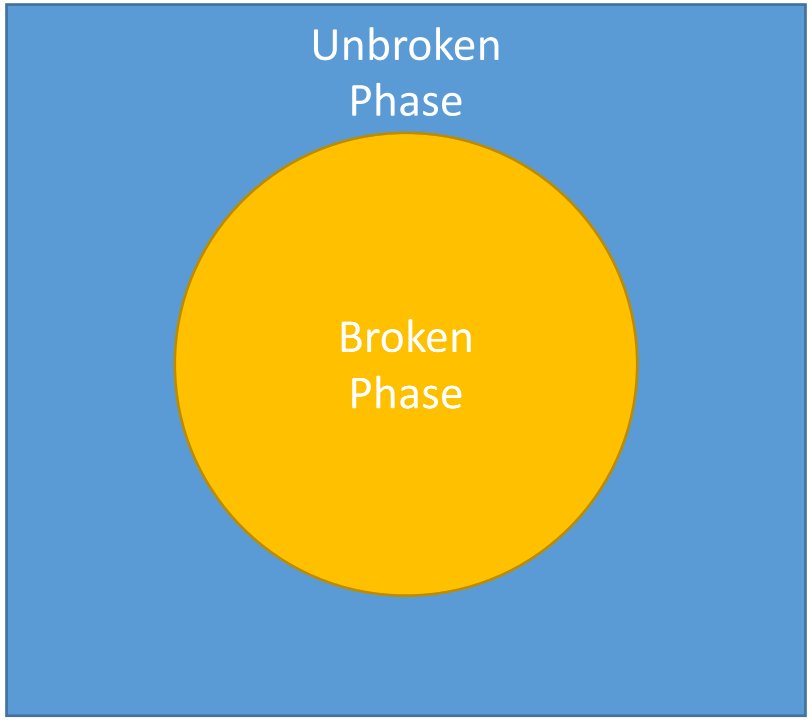
Saturon as a Vacuum Bubble

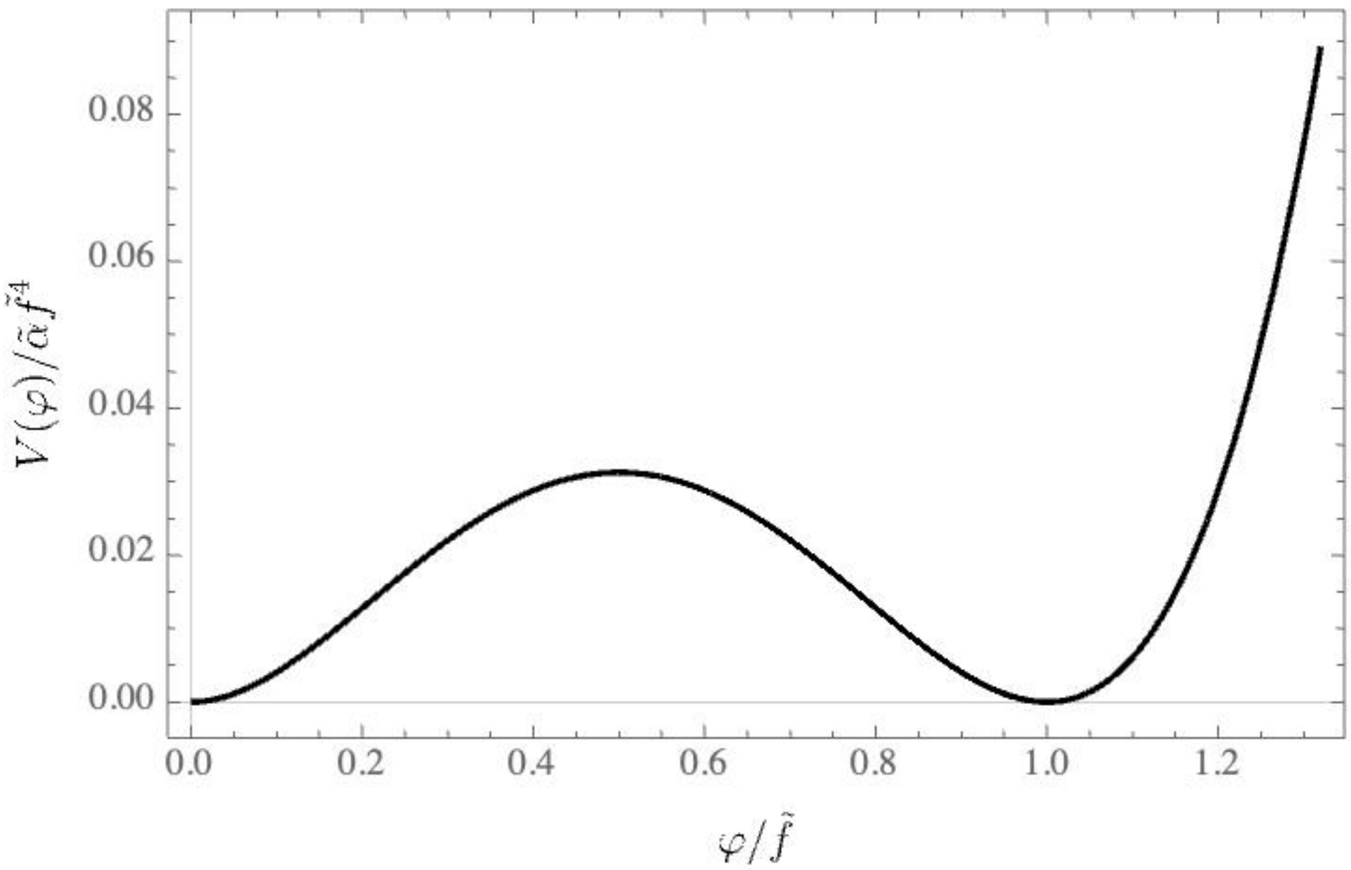
A Model

Vacuum Bubbles



$$SU(N) \to SU(N-1)\times U(1)$$

Model of a Saturon as a Vacuum Bubble



$$V(\varphi) = \frac{\tilde{\alpha}}{2}\varphi^2\left(\tilde{f} - \varphi\right)^2$$

Model of a Saturon as a Vacuum Bubble

$$V(\varphi) = \frac{\tilde{\alpha}}{2}\varphi^2\left(\tilde{f} - \varphi\right)^2$$

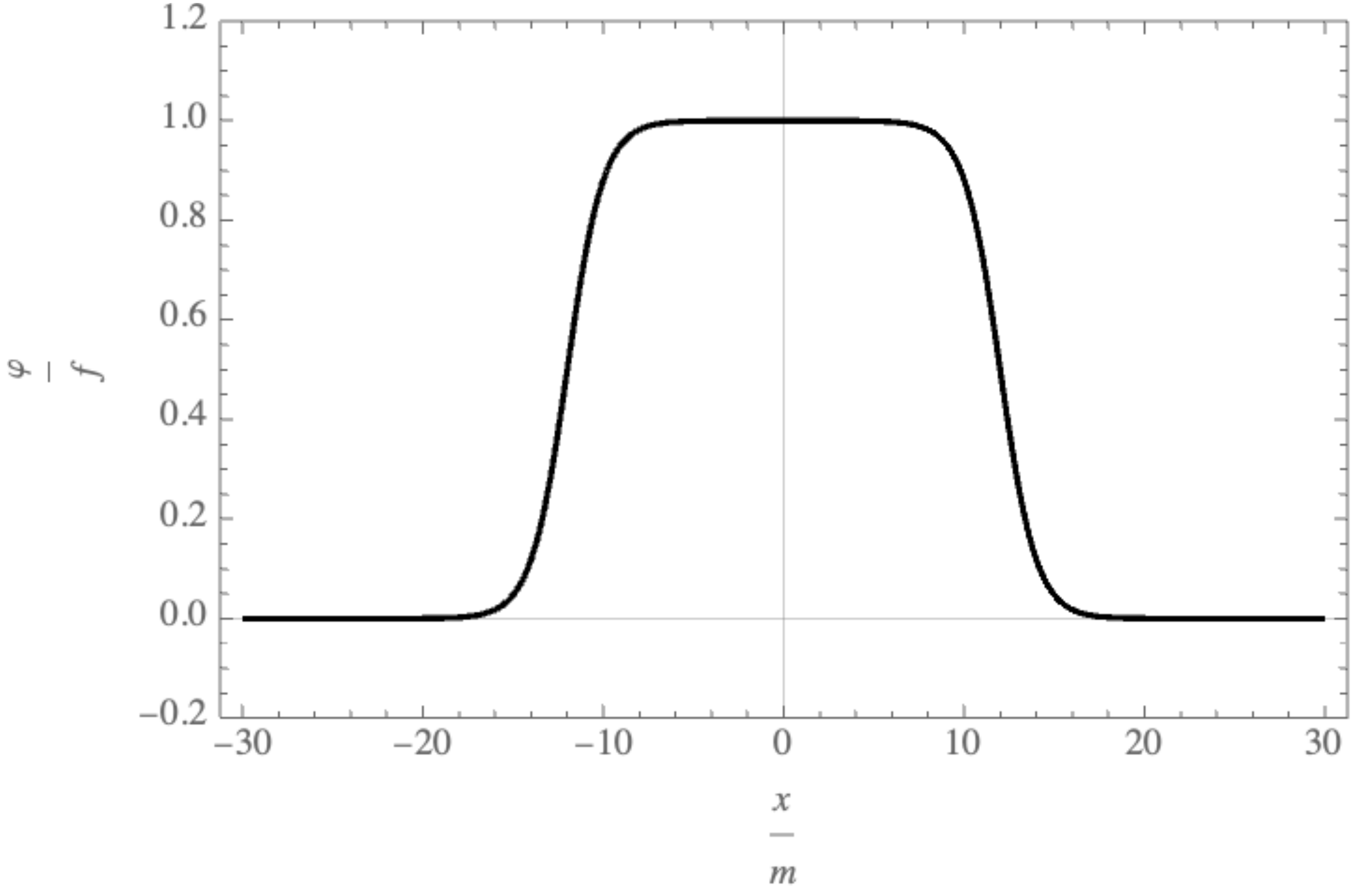
is minimized by

$$\langle\varphi\rangle = \begin{cases} 0. \\ \tilde{f} \end{cases}$$

Unbroken Phase: $SU(N)$

Broken Phase: $SU(N-1)\times U(1)$

Vacuum Bubbles:
Thin Wall Approximation ($R \gg m^{-1}$)



$$\varphi(r) = \frac{\tilde{f}}{2}\left(1 + \tanh\left(\frac{m(R-r)}{2}\right)\right)$$

$m = 1$
$R = 12$

Vacuum Bubbles Stabilization

A Memory Burden Effect

Vacuum Bubbles Stabilization

- Consider the ansatz

$$\phi^{\alpha}_{\beta} = \left(U^{\dagger}\Phi_{\mathrm{D}}U\right)^{\alpha}_{\beta},$$

where

$$\Phi_{\mathrm{D}} = \frac{\varphi(r)}{\sqrt{N(N-1)}}\,\mathrm{diag}\left((N-1), -1, \dots, -1\right)$$

$$U = \exp[-i\theta^{a}T^{a}].$$

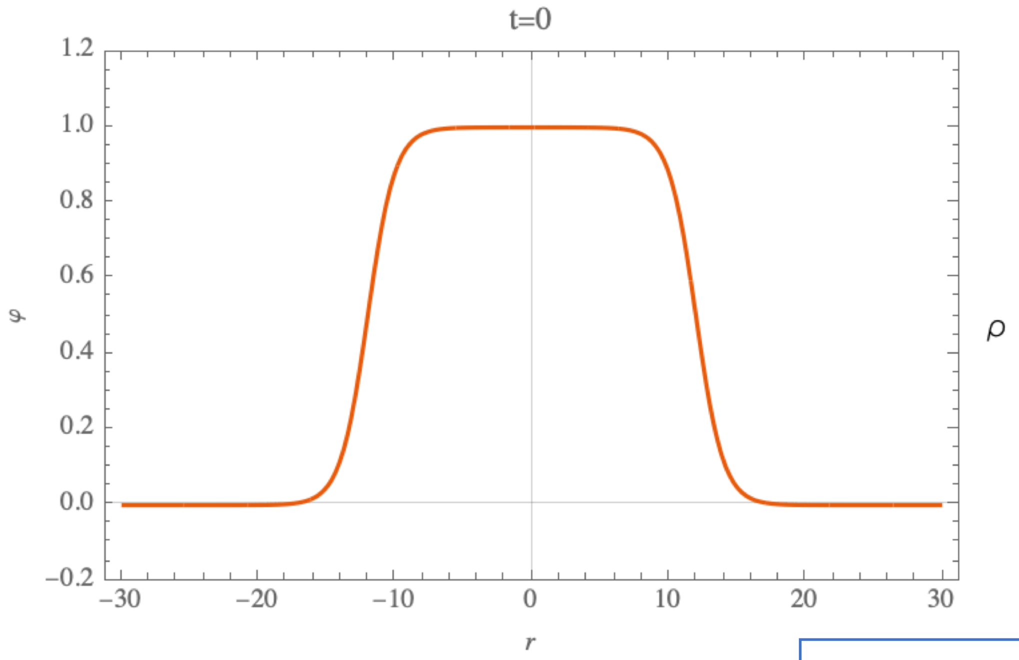
- Here T^{a} corresponds to the respective broken generators of $SU(N)$ [6].

$$\theta^{a} = \delta^{a1}\omega t$$

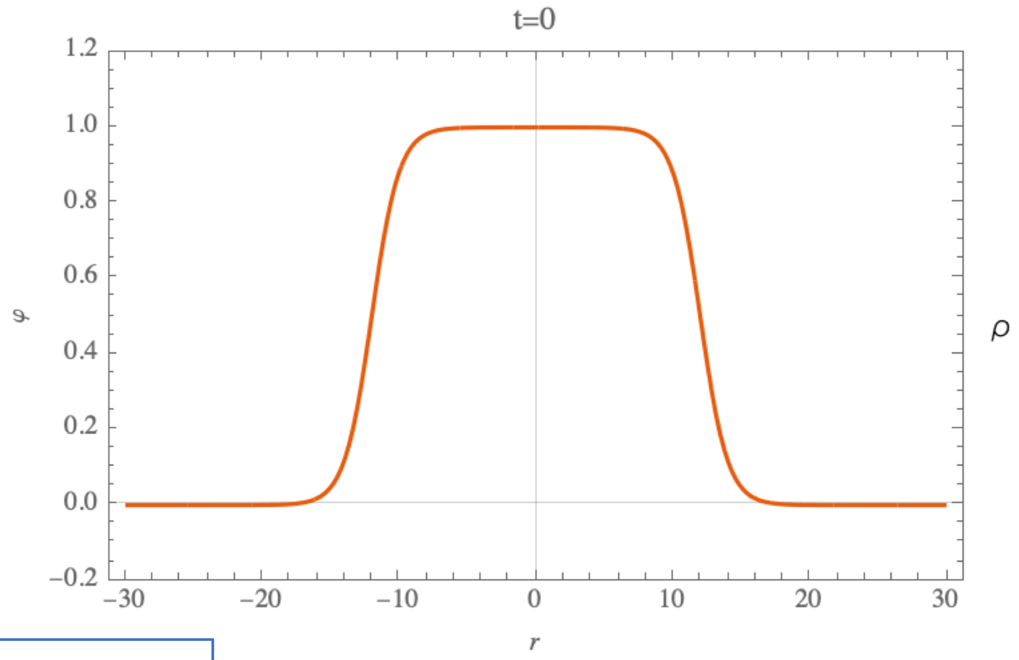
[6] G. Dvali, O. Kaikov, and J. S. V. Bermudez, (2021), arXiv:2112.00551 [hep-th]

Vacuum Bubbles Stabilization:

$\dot{\boldsymbol{\theta}} = \mathbf{0}$



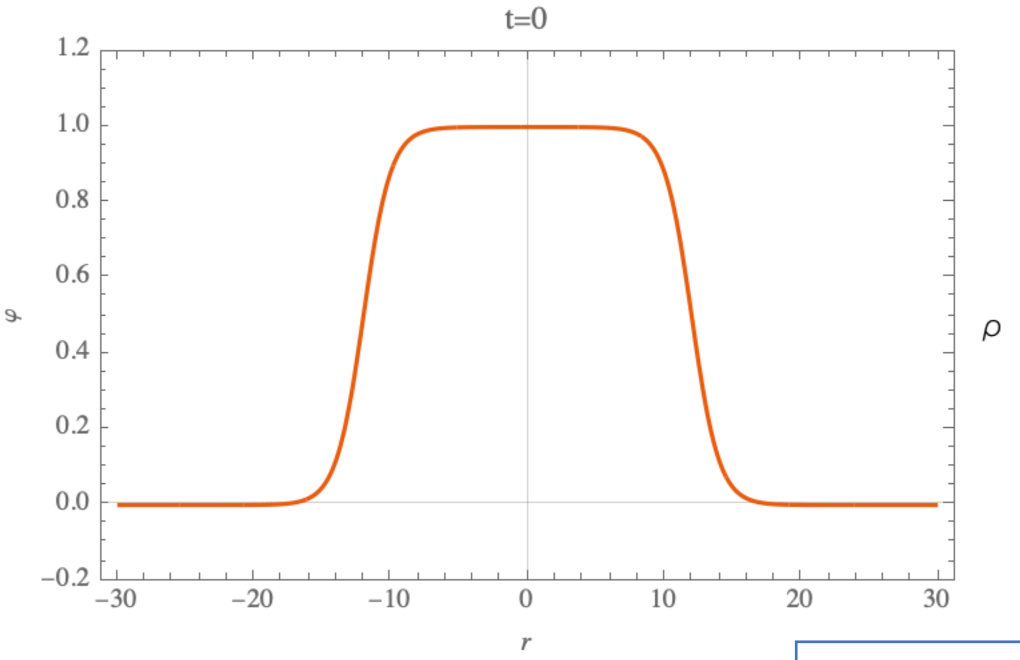
$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega}$



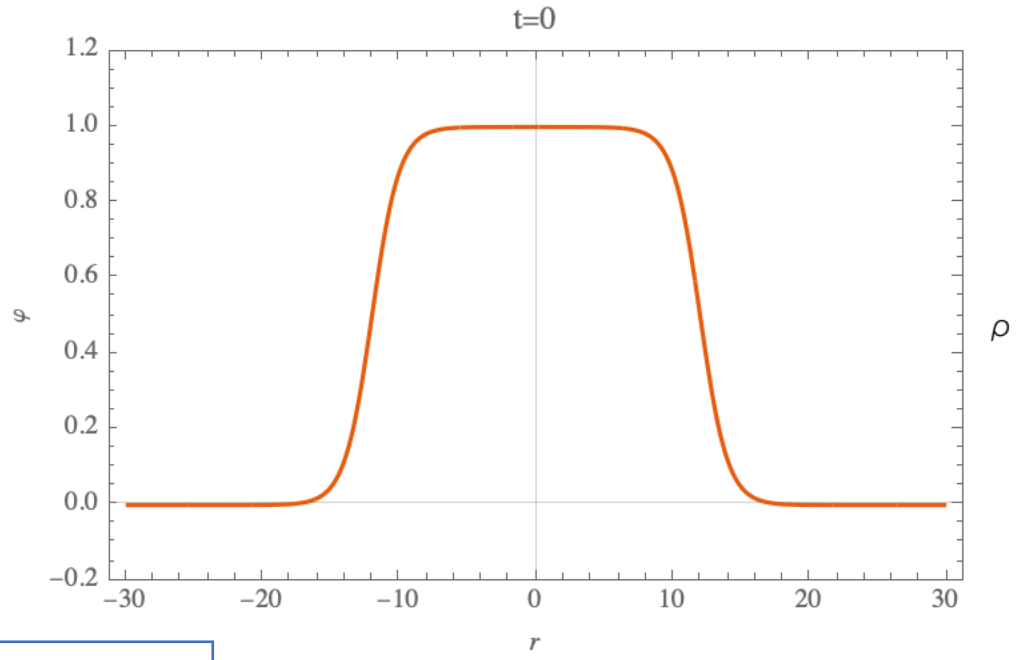
Thin wall approximation for:
$\omega \approx 0.24\, m,$
$R_\omega = \frac{12}{m}$

Vacuum Bubbles Stabilization:

$\dot{\theta} = 0$



$\dot{\theta} = \omega$



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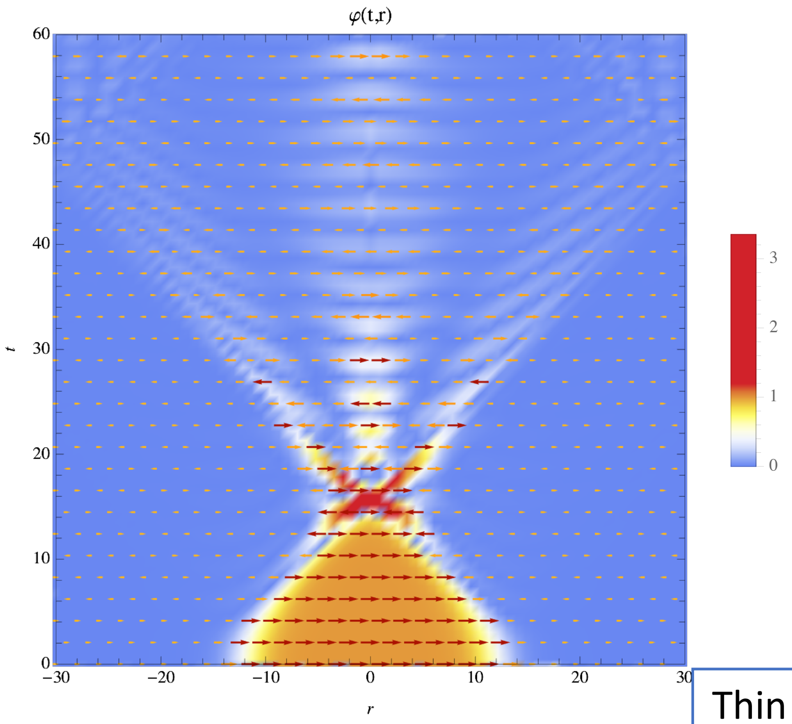

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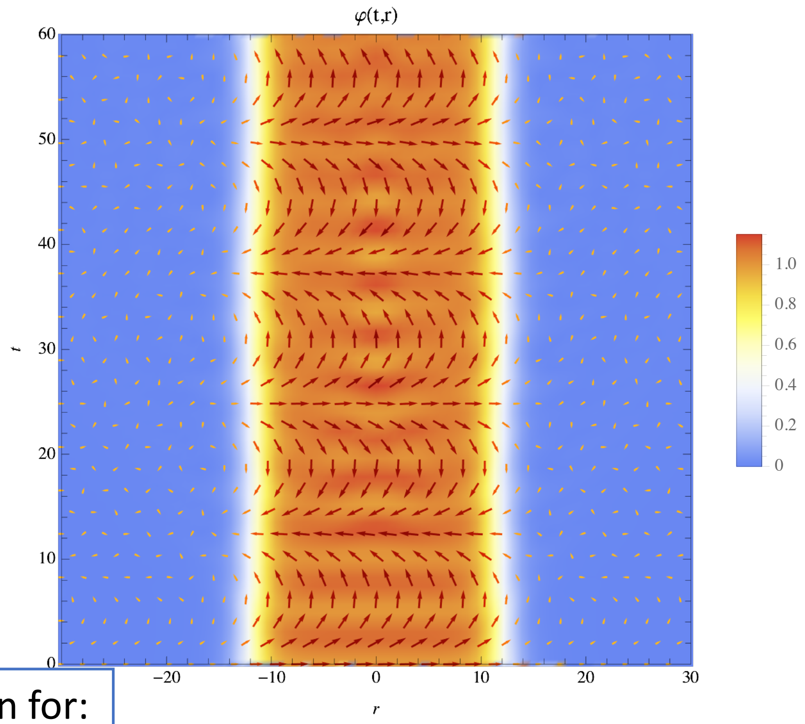

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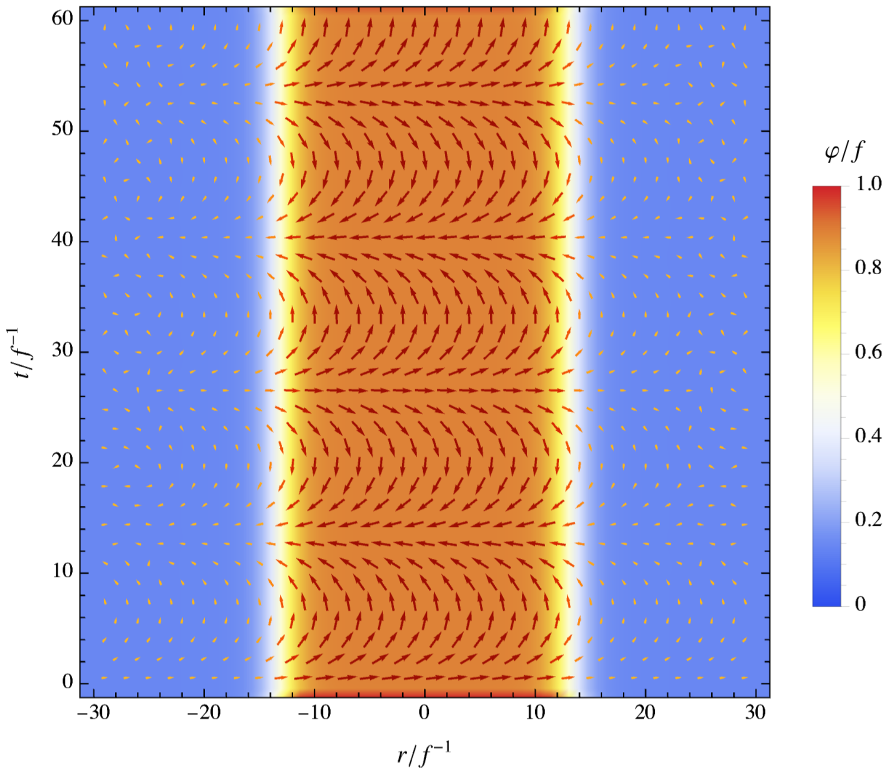
Vacuum Bubbles Stabilization

- *A stationary bubble is obtained thanks to the excitations of the) Goldstone mode(s).*

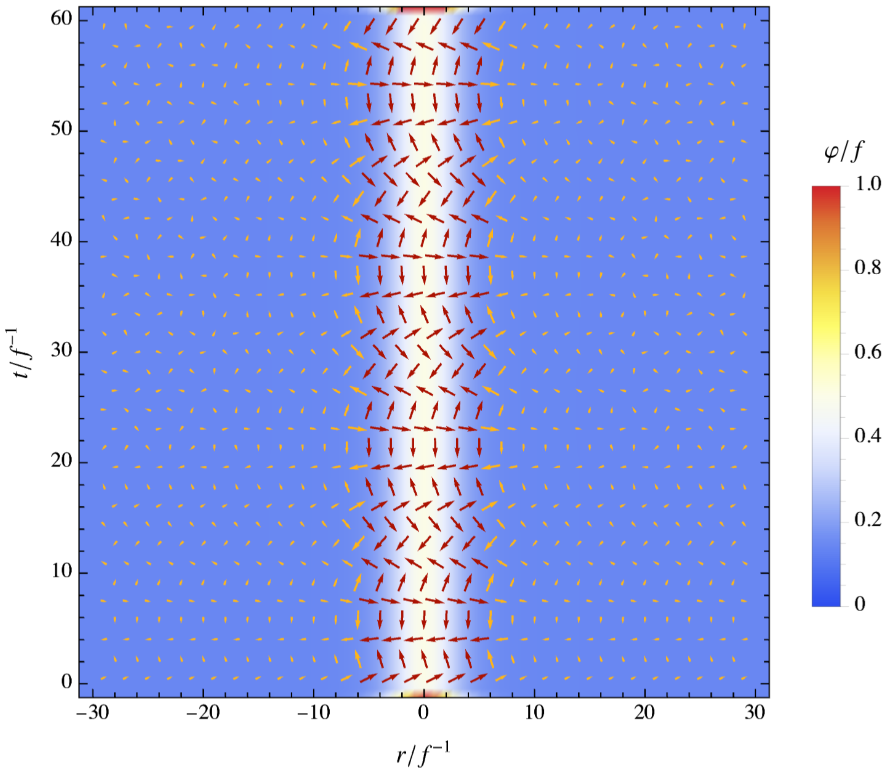
- The bubble is stable because of two factors:
  1) The Goldstone $SU(N)$ charge is conserved
  2) The same amount of charge in the exterior vacuum would cost much higher energy.

Vacuum Bubbles Stabilization

Large Bubbles
$R \sim 12m^{-1}$



Small Bubbles
$R \sim 1.02m^{-1}$



Vacuum Bubbles Micro-state Entropy

Large Bubbles

$$\omega \ll m$$
$$m^{-1} \ll R$$

- $N_G \gg \frac{1}{\alpha} \sim N \sim N_{Gold}$
- $1 \gg \lambda$
- $\mathrm{S} \approx 2N \ln\left(\frac{\mathrm{e}}{\lambda}\right) \sim \frac{1}{\alpha}\ln\left(\frac{m^{10}}{\omega^{10}}\right)$
- $S \ll S_{\max} \sim \frac{1}{\alpha}\frac{m^6}{\omega^6}$

Small Bubbles

$$\omega \sim m$$
$$m^{-1} \sim R$$

- $N_G \sim \frac{1}{\alpha} \sim N \sim N_{Gold}$
- $1 \sim \lambda$
- $\mathrm{S} \sim \frac{1}{\alpha} \sim E_{\mathrm{Bubble}} R$
- $\mathrm{S} \sim S_{max}$

[8] J. D. Bekenstein,Universal Upper Bound on the Entropy-to-Energy Ratio for BoundedSystems, Phys. Rev. D23no. 2 (1981), 287-298.47

Vacuum Bubbles Micro-state Entropy

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Small Bubbles -> Saturons

$$\omega \sim m$$

$$m^{-1} \sim R$$

- $N_G \sim \frac{1}{\alpha} \sim N \sim N_{Gold}$
- $1 \sim \lambda$
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Information Horizon

Saturons in semiclassical limit

Information Horizon

- A universal property of the saturons is that in semi-classical limit, they possess a strict *information horizon*.
- The information stored in the interior of the saturon can not be extracted in any form.
- The general physical reason is that in this limit the *memory modes*, that carry quantum information, *decouple.*

Semi-classical Limit

- The limit in which the classical bubble solution experiences no back reaction from quantum fluctuations

$$\alpha \to 0, \qquad R = \text{finite}, \qquad \omega = \text{finite}, \qquad \alpha N = \text{finite}.$$

- Simultaneously

$$f \to \infty, \qquad m = \text{finite}.$$

Semi-classical Limit

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- Simultaneously

$$f \to \infty, \qquad m = \text{finite}.$$

- *Recall:* *For BH* $f \sim M_p$

Semi-classical Limit

- The limit in which the classical bubble solution experiences no back reaction from quantum fluctuations

$$\alpha \to 0, \qquad R = \text{finite}, \qquad \omega = \text{finite}, \qquad \alpha N = \text{finite}.$$
$$f \to \infty, \qquad m = \text{finite}.$$

- In this limit, the effective coupling of a Goldstone mode of frequency ε

$$\alpha_G = \frac{\varepsilon^2}{f^2}$$

goes to zero for any finite ε.

Goldstone Horizon

- **At finite $\boldsymbol{f}$**, a Goldstone mode of frequency $\varepsilon \ll m$ cannot propagate outside the bubble, even though the coupling α_G is finite.
- Two cases
  - The energy of an internal perturbation ε is such

$$\varepsilon < m,$$

  propagation is impossible due to the finite energy gap.
  - $\varepsilon \ll m$, the perturbation energy can exceed the mass gap at the expense of a large occupation number n_ε of Goldstone quanta.

$$n_\varepsilon \to 1,$$

  such a process is exponentially suppressed by afactor $\mathrm{e}^{-n_\varepsilon}$

Goldstone Horizon: An Example

- Lets consider a perturbation on a stable vacuum bubble, ϕ_{VB},

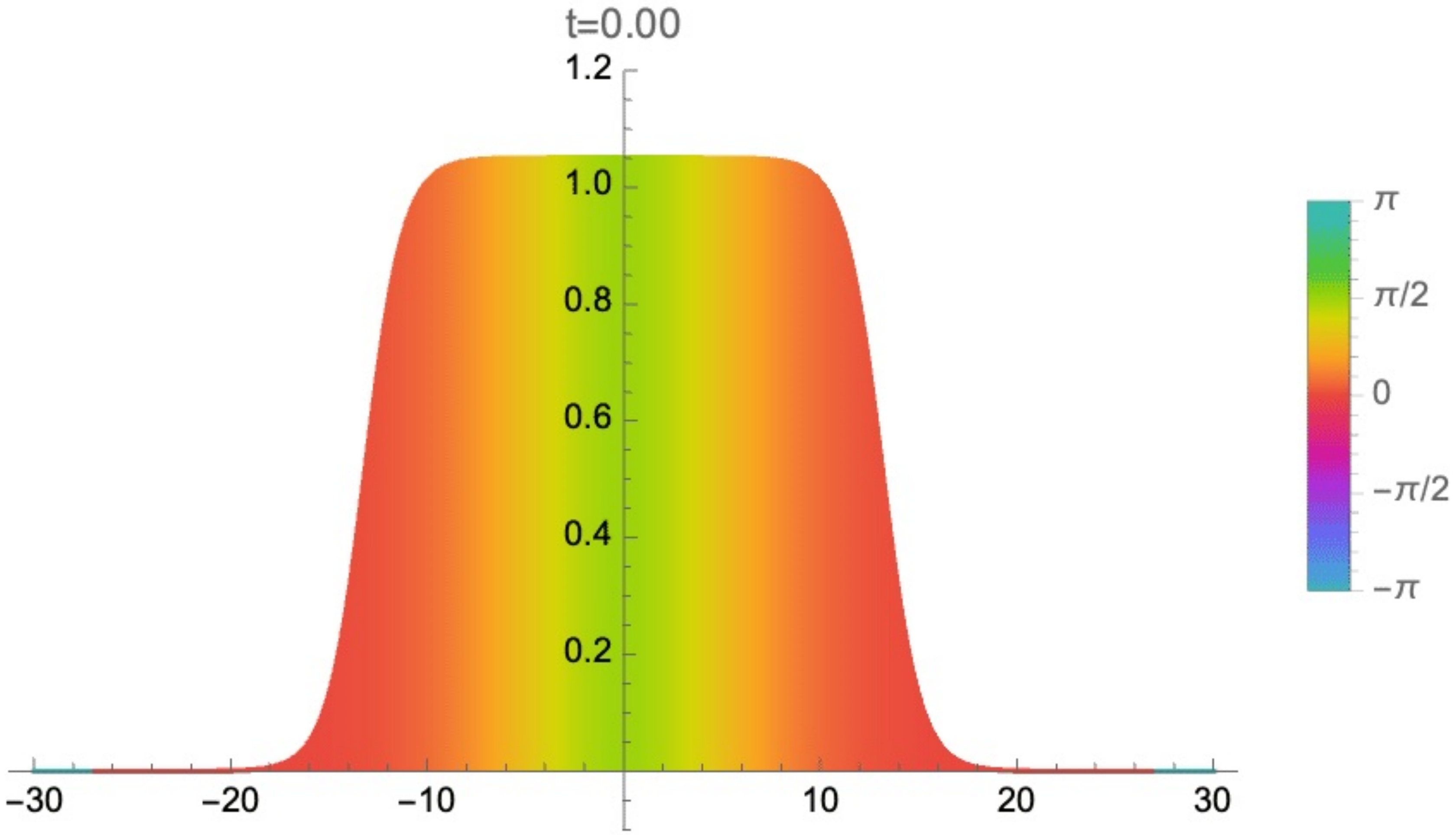
$$\phi^{\alpha}_{\beta}\Big|_{t=0} = p(r){\phi_{VB}}^{\alpha}_{\beta}(r)$$

$$p(r) = \exp\left[\frac{i\pi}{2}\frac{f(r)}{f(0)}T^{1}\right]$$

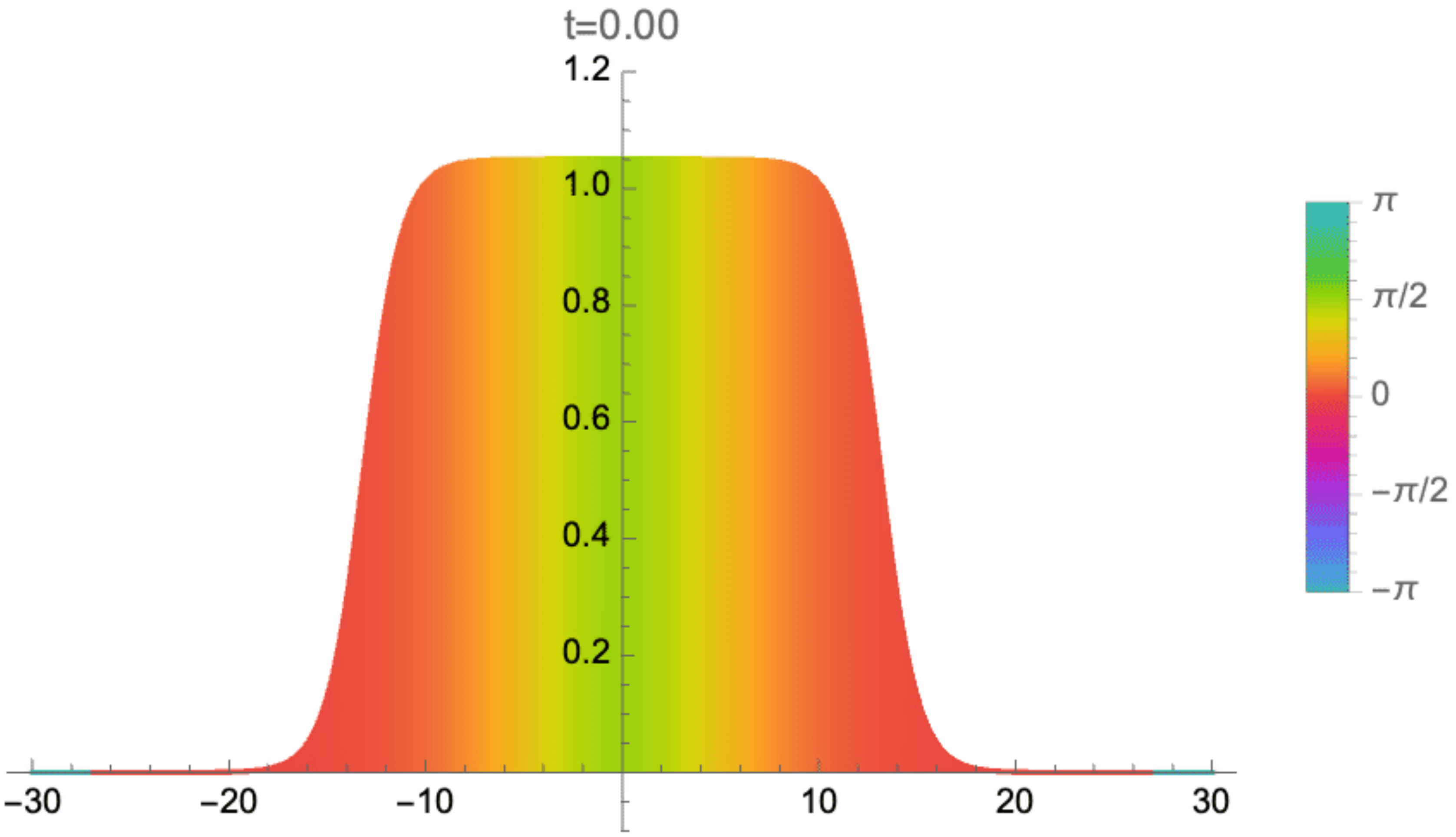
where

$$f(r) = \frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{r^2}{2\sigma^2}}$$

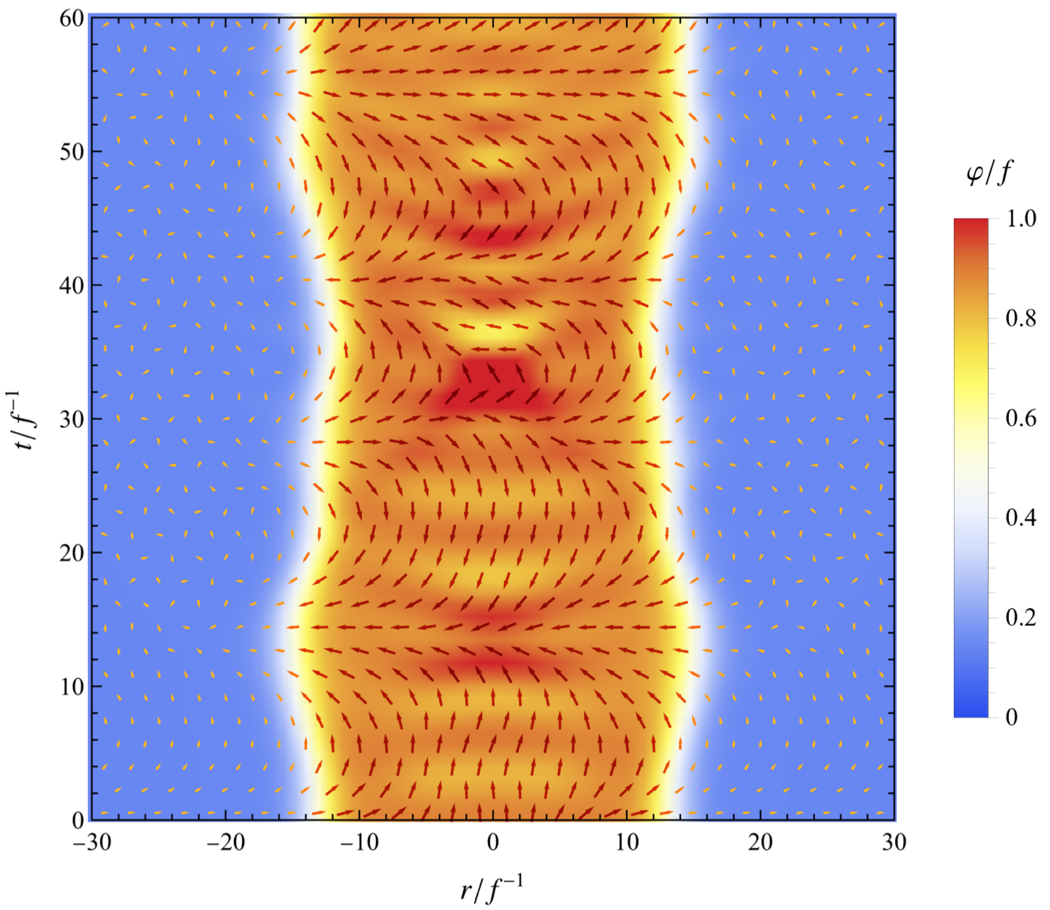
Goldstone Horizon: An Example

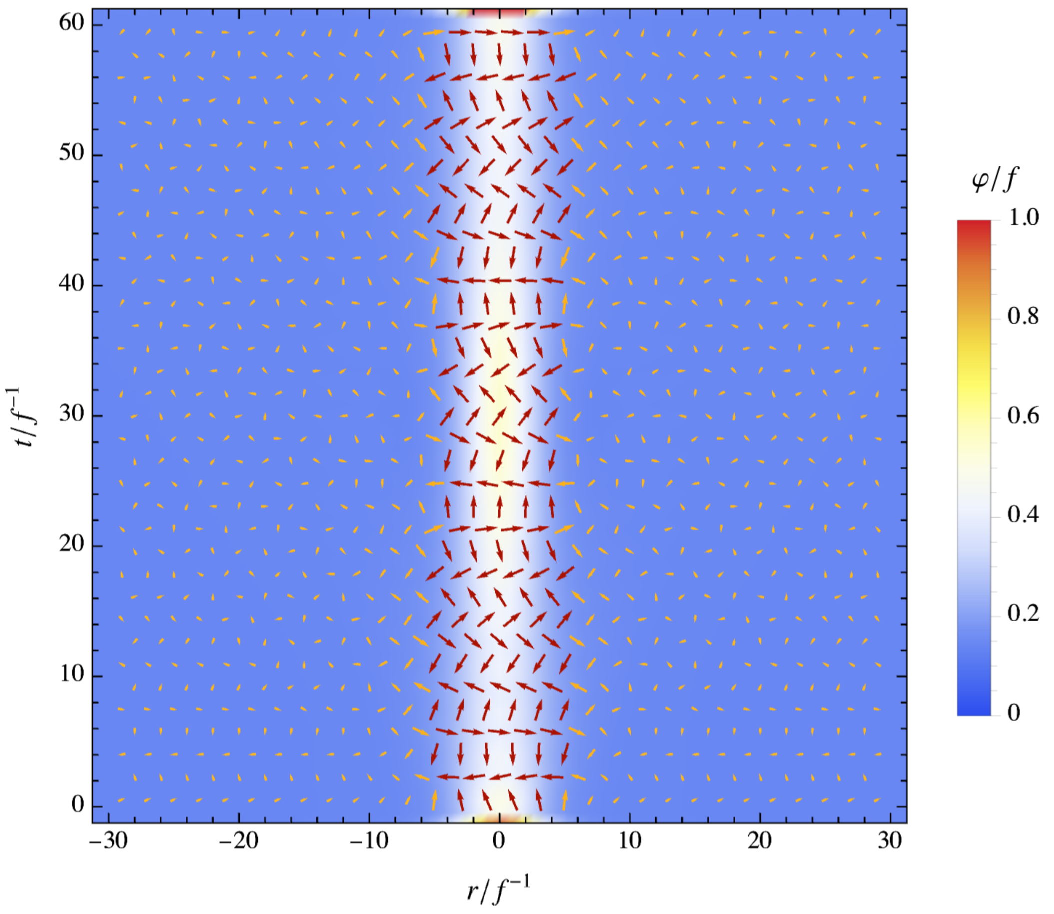


Goldstone Horizon: An Example



Goldstone Horizon





$\sigma \equiv 5/m$

Correspondence to Black Holes

Correspondence to Black Holes

Saturons

- $S = (fR)^2 = \alpha^{-1}$
- $T = R^{-1}$
- $t_{\min} = R^3\ f^2 = SR$
- Information/Goldstone Horizon

Black Holes

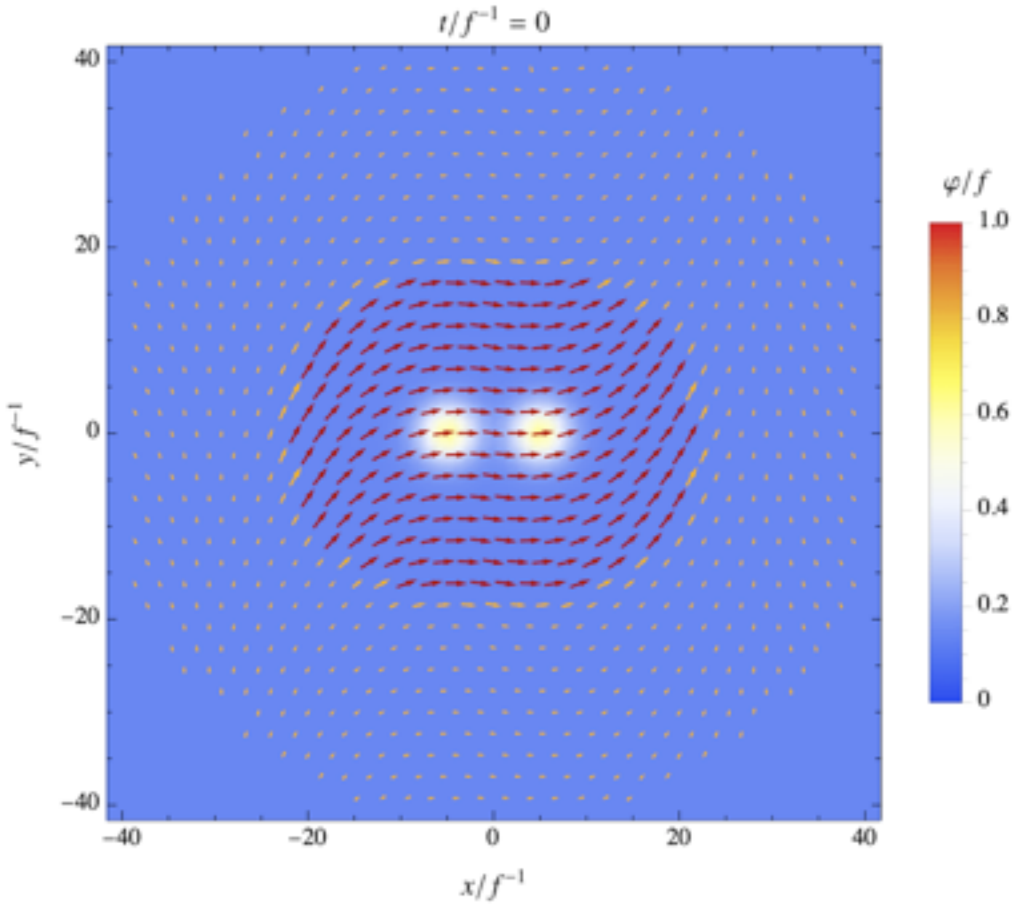
- $S = (M_P R)^2$
- $T = R^{-1}$
- $t_{\min} = R^3\ M_P^2 = SR$
- Information Horizon

Outlook

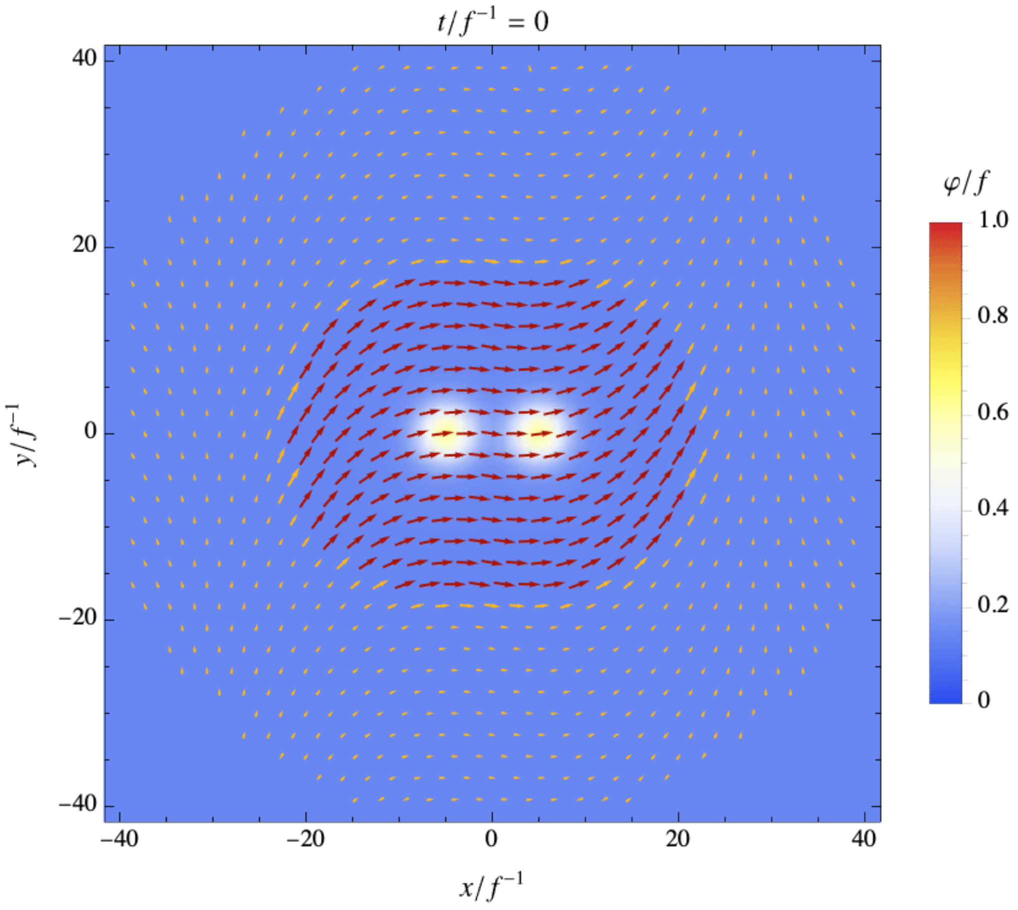
- Generalizations of our study are currently researched.
- Departures from semi-classical behavior can become observable for the black holes that are relatively old and close to their half-decay time.
- The light primordial black holes, provided they exist, can be within a potentially interesting window.
- Very recently some other possible observational consequences for rotating black holes were discussed in [7]

[7] G. Dvali, F. Kühnel and M. Zantedeschi, Vortexes in Black Holes, arXiv:hep-th/2112.08354.

Outlook



Outlook



Conclusions

- We have shown an explicit example of a Saturon as a Vacuum Bubble. Small bubbles saturate the entropy bound and correspond to the saturons of the theory.
- A large (macroscopic) occupation number of the Goldstone modes, parametrized by ω, stabilizes the Vacuum Bubbles. This phenomenon is due to the memory burden effect and could be relevant for PBH
- Saturons (Vacuum Bubbles) exhibit a goldstone horizon, analog to the information horizon of Black holes.

Conclusions

- *Area law*:

$$S_{\max} = \frac{1}{\alpha} = \frac{Area}{f^{-2}}$$

- *Temperature*

$$T \sim \frac{1}{R},$$

  up to $1/S$ corrections.

- In semi-classical treatment, they exhibit an *information horizon.*

- *Information retrieval* after

$$t_{min} = \frac{Volume}{G_{Gold}} = \frac{R}{\alpha} = S_{\max}R$$

Thank you

Appendix

Vacuum Bubbles Stabilization: Quantum picture of classical stability

- The energies in terms of the occupation numbers of the corresponding quanta are

$$E_{\mathrm{int}}=\omega N_G, \qquad \text{where,} \qquad N_G \equiv \frac{1}{\alpha}\frac{m^5}{\omega^5}\left(\frac{16\pi}{81}\right),$$

$$E_{\mathrm{wall}}=m N_\varphi, \qquad \text{where,} \qquad N_\varphi \equiv \frac{1}{\alpha}\,\frac{m^4}{\omega^4}\left(\frac{8\pi}{27}\right).$$

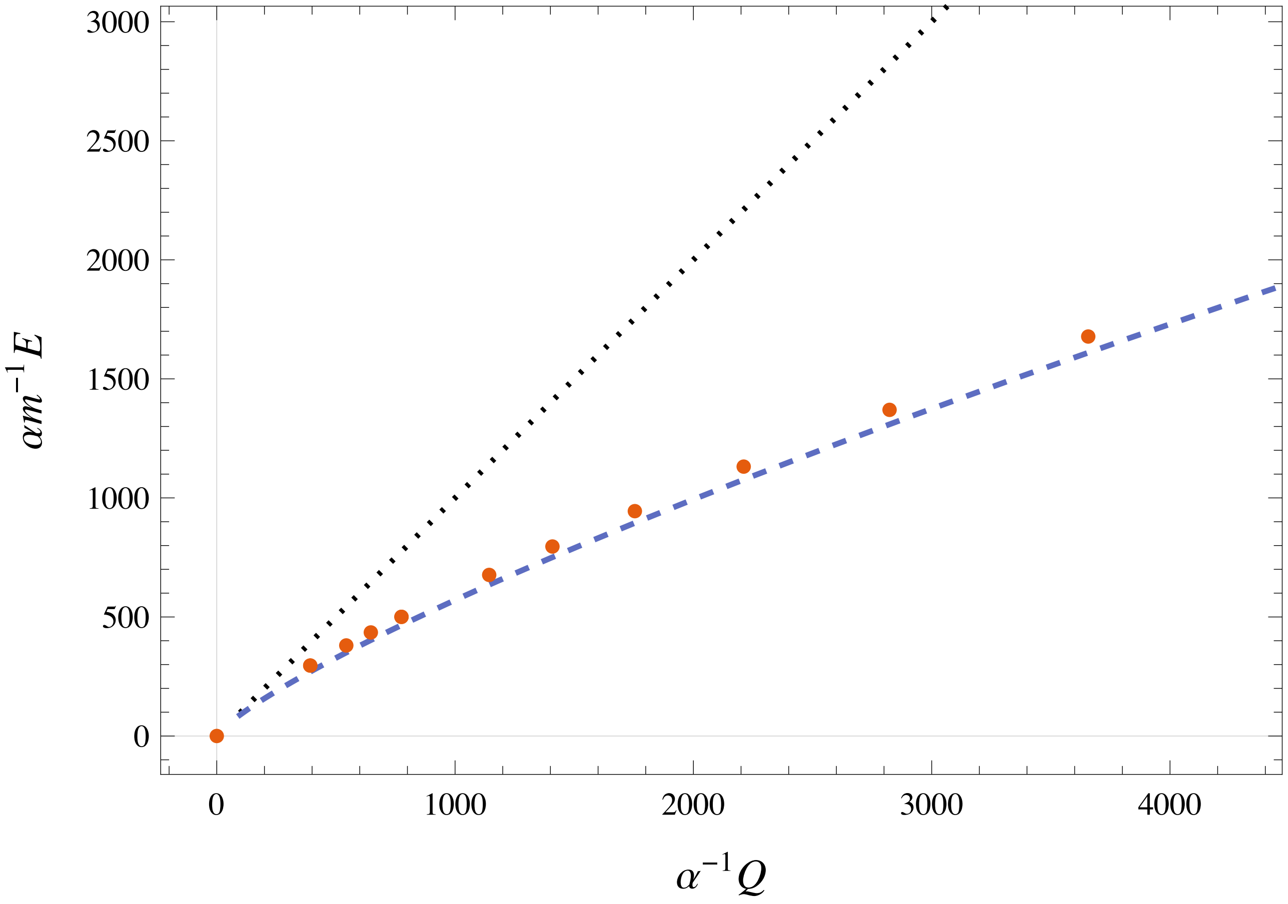
- Observe that $\frac{N_G}{N_\varphi} = \frac{2}{3}\frac{m}{\omega} \gg 1$ for the thin walls.

Vacuum Bubbles Stabilization: Quantum picture of classical stability

- Conclusion: *A stationary bubble is obtained thanks to the excitations of the) Goldstone mode(s).*

- The bubble is stable because of two factors:
  1) The fact that the Goldstone $SU(N)$ charge is conserved; and
  2) The fact that the same amount of charge in the exterior vacuum would cost much higher energy.

Spectrum



3000
2500
2000
1500
1000
500
0
0
1000
2000
3000
4000
$\alpha m^{-1} E$
$\alpha^{-1} Q$

Memory Burden Effect

- Large amount of Memory patterns
- Stored quantum information.
- Slowdown of the system’s evolution

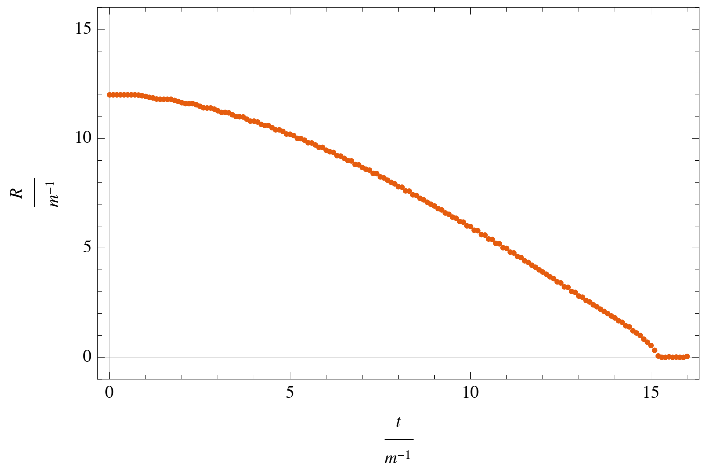
Vaccum Bubbles Stabilization

- Large amount of Bubble micro-states
- Excitations of the Goldstone modes
- Slowdown of bubble's decay

Vacuum Bubble

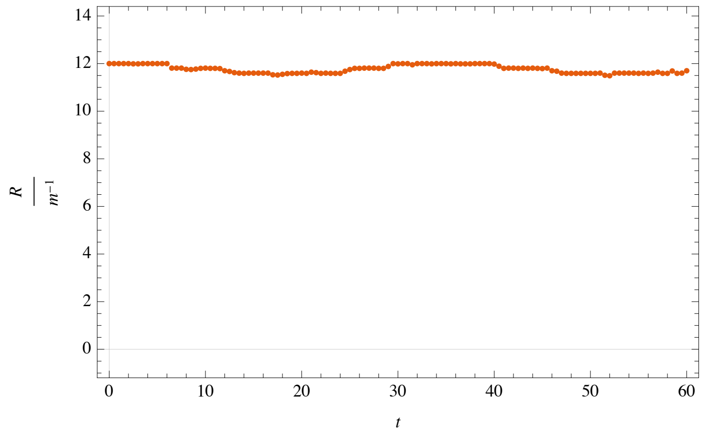
Decay

$$\left(\dot{\boldsymbol{\theta}} = \mathbf{0}\right)$$

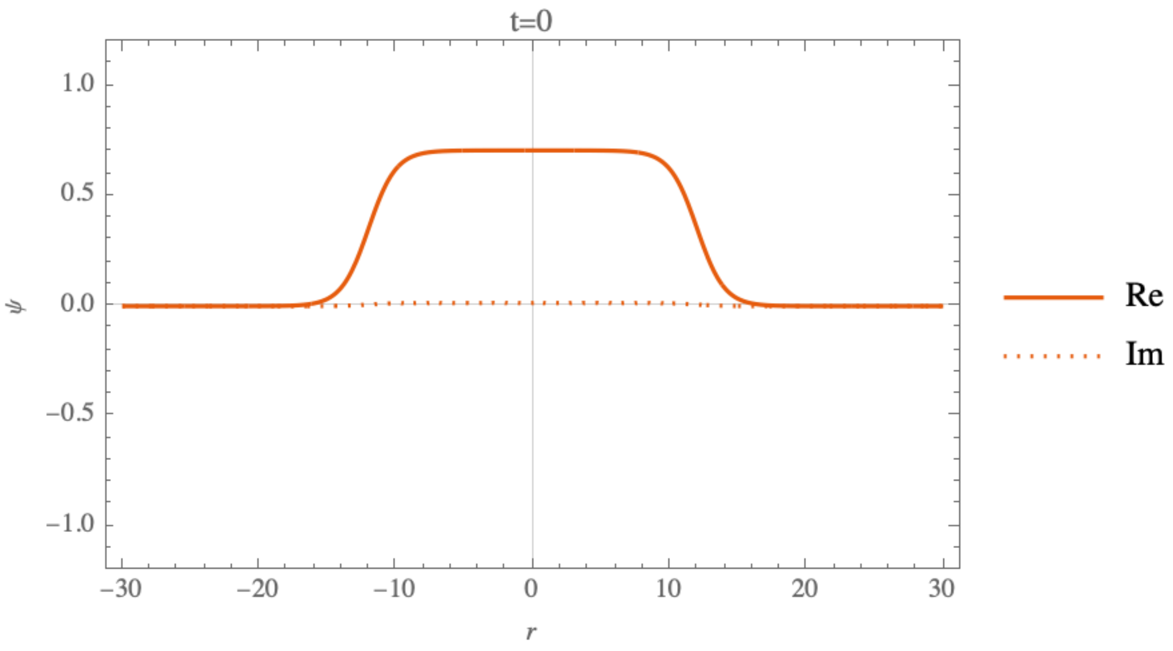


Stabilization

$$\left(\dot{\boldsymbol{\theta}} \neq \mathbf{0}\right)$$



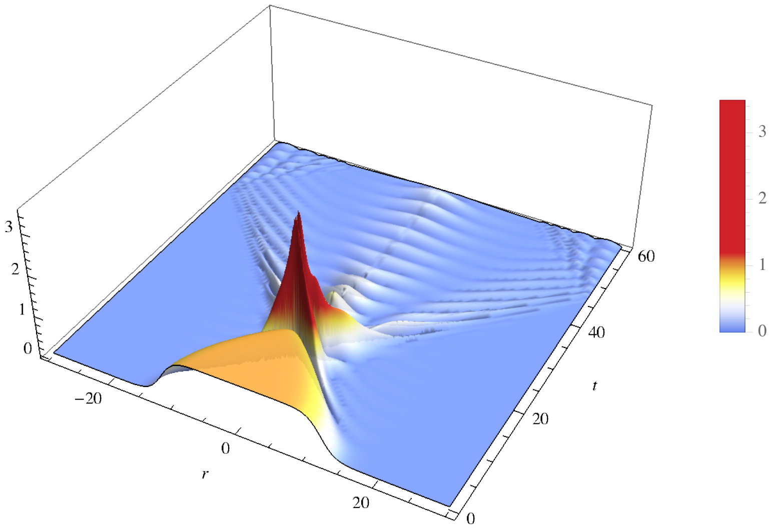
Stabilization



Vacuum Bubble Stabilization

Decay

$(\dot{\boldsymbol{\theta}} = \mathbf{0})$



Stabilization

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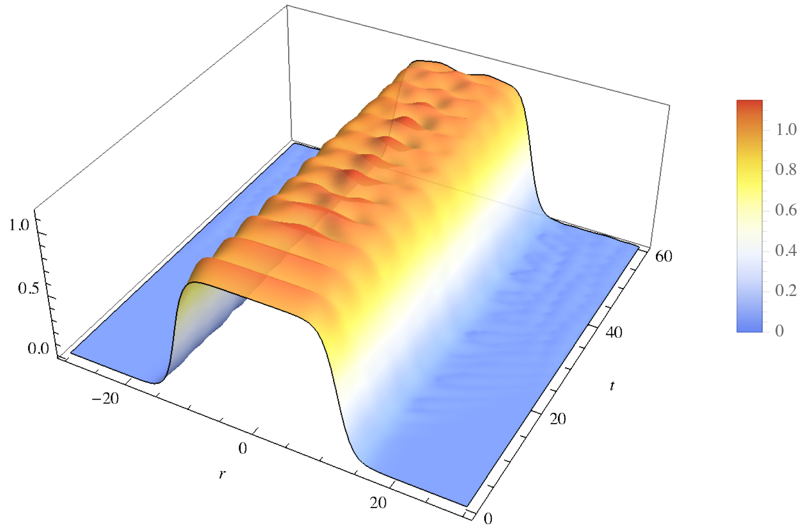
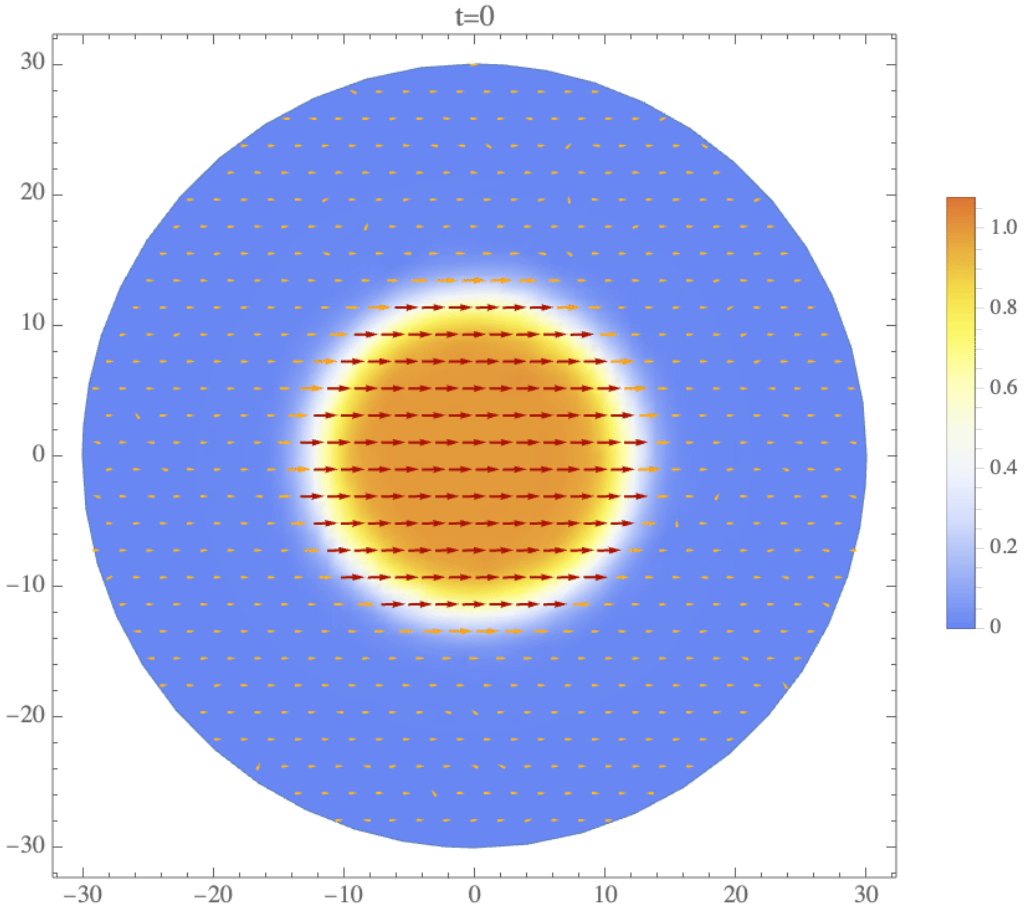


Vaccum Bubble Stabilization in d=3



Vaccum bubble Decay