False Vacuum Decay and Functional Determinants

The Fubini Lipatov instanton and the lifetime of the SM

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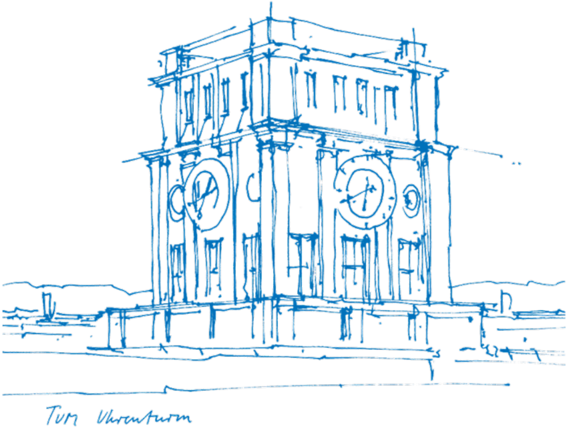

Outline

1. False Vacuum Decay
   - Metastability in the quantum world
   - The Higgs false vacuum decay
   - How do we evaluate the vacuum decay rate?
2. The Problem with Functional Determinants

Metastability in the quantum world

False vacua and quantum tunneling

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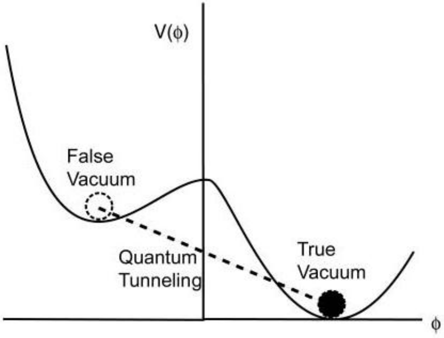
- The field in the false vacuum will eventually tunnel through the potential barrier and land on a more stable minimum. We call this phenomenon **false vacuum decay**.

Metastability in the quantum world

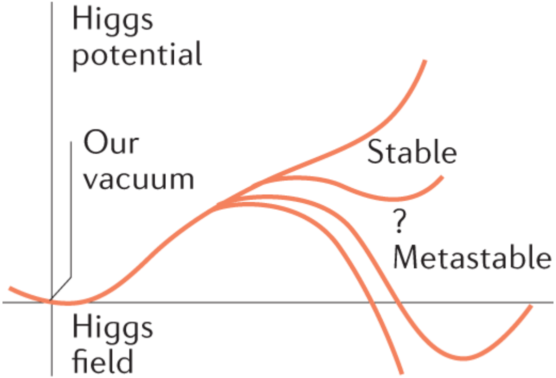
False vacua and quantum tunneling

Plot from https://arxiv.org/pdf/astro-ph/0005003.pdf



The Higgs potential

Plot from Bass, S.D., De Roeck, A. & Kado, M. The Higgs boson implications and prospects for future discoveries. Nat Rev Phys 3, 608–624 (2021). https://doi.org/10.1038/s42254-021-00341-2



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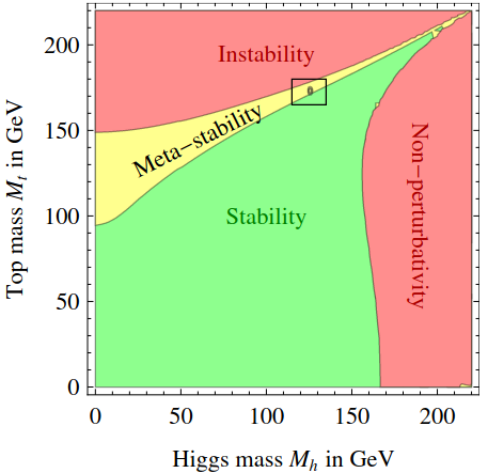

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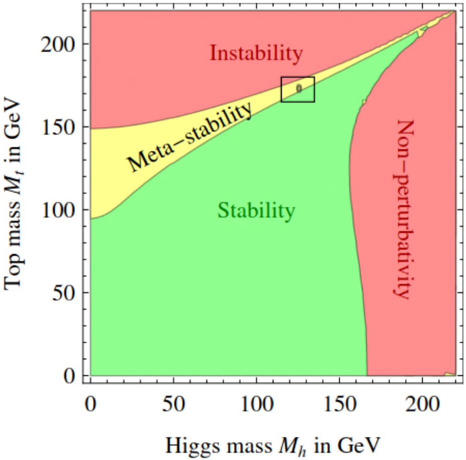

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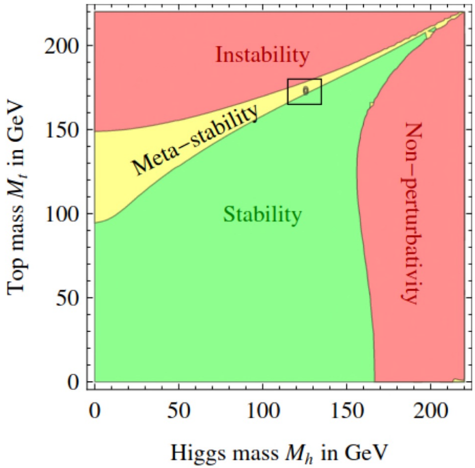

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Is the Standard Model unstable?

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- An estimate for the lifetime of the Standard Model, namely for the decay rate of the Higgs field, can be obtained through known methods.
- However, these rely on approximations not entirely valid in this case.

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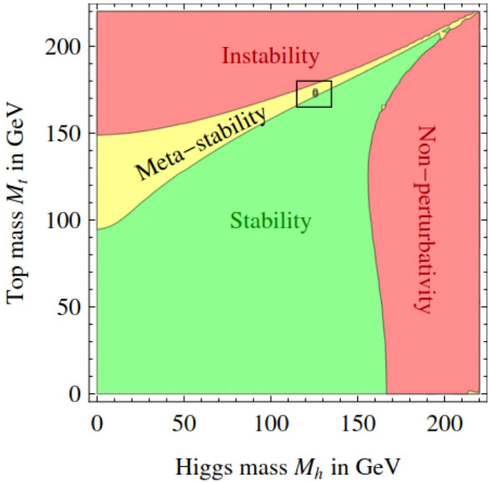

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$$\Gamma \propto \left(\frac{\det S''[\varphi_b]}{\det S''[\varphi_{FV}]}\right)^{-\frac{1}{2}} e^{-S[\varphi_b]+S[\varphi_{FV}]} \tag{3}$$

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The Fubini-Lipatov instanton

The issue with scale invariance

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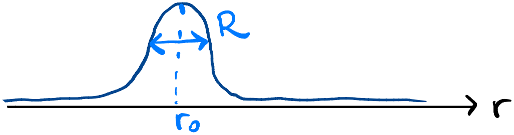

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  - Instanton size $R \to$ dilatation $\to$ dilatational zero mode $\varphi_{\mathrm{dil}} \propto \partial_R \varphi_{\mathrm{FL}}$

The zero and the negative modes

- The decay rate (3) is unfortunately divergent

$$(\det S''[\varphi_b])^{-1/2} \sim \left(\prod_n \lambda_n\right)^{-1/2} \xrightarrow{\text{zero modes}} \text{``}\left(0^{-1/2}\right)\text{''} \to \infty \tag{6}$$

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  2. Add 1-loop quantum corrections and renormalize. This will break scale invariance, eliminating the remaining dilatational zero mode.
- The operator $S''[\varphi_b]$ also contains a negative mode, which is responsible for the imaginary part of the ground state energy.

Resolvent Method

- Green function's problem

$$S''[\varphi]G(x,x') = \delta(x-x')\,, \quad \text{where} \quad S''[\varphi] = -\Box_x + V''(\varphi(x)) \tag{7}$$

- Spectral decomposition

$$G(x,x') = \int d\lambda \frac{f_\lambda(x) f_\lambda(x')}{\lambda} \tag{8}$$

- Define

$$(S''[\varphi] + s)G(s;x,x') = \delta(x-x') \tag{9}$$

then

$$\log \frac{\det S''[\varphi_b]}{\det S''[\varphi_{\text{FV}}]} = -\int dx \int_0^\infty ds\,[G_{\text{bounce}}(s;x,x) - G_{\text{FV}}(s;x,x)] \tag{10}$$

Subtracting the zero modes

- Define the subtracted Green's function as orthogonal to the space of zero modes

$$\int dx' G^{\perp}(x, x')\phi_0(x') = 0 \tag{11}$$

- Differential equation for the subtracted Green's function

$$S''[\varphi]G^{\perp}(x, x') = \delta(x - x') - \sum \phi_0(x)\phi_0^*(x') \tag{12}$$

Quantum corrections

- 1-loop quantum corrections to the bounce equation

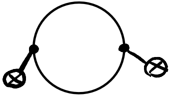
$$-\Box\varphi_b^{(1)}(x) + V'(\varphi_b^{(1)}(x)) + \varphi_b^{(1)}(x)\Pi^{(1)}_{\text{ren}}(x(\varphi_b^{(1)})) = 0 \tag{13}$$

- Quantum corrected equation for the 2-point function

$$\begin{aligned}&\left[-\Box + V''(\varphi_b^{(1)}(x)) + \Pi^{(1)}_{\text{ren}}(x(\varphi_b^{(1)}))\right] G_{1-\text{loop}}(x,x') \\ &+ \int dy \varphi_b^{(1)}(x)\Sigma(x,y)\varphi_b^{(1)}(y)G_{1-\text{loop}}(y,x') = \delta(x-x')\end{aligned} \tag{14}$$

and

$$\Sigma(x,y) = \frac{\delta\Pi^{(1)}_{\text{ren}}(x(\varphi_b^{(1)}))}{\delta\varphi_b^{(1)}(y)} \tag{15}$$



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  4. Eventually yield a more precise evaluation of the SM lifetime.

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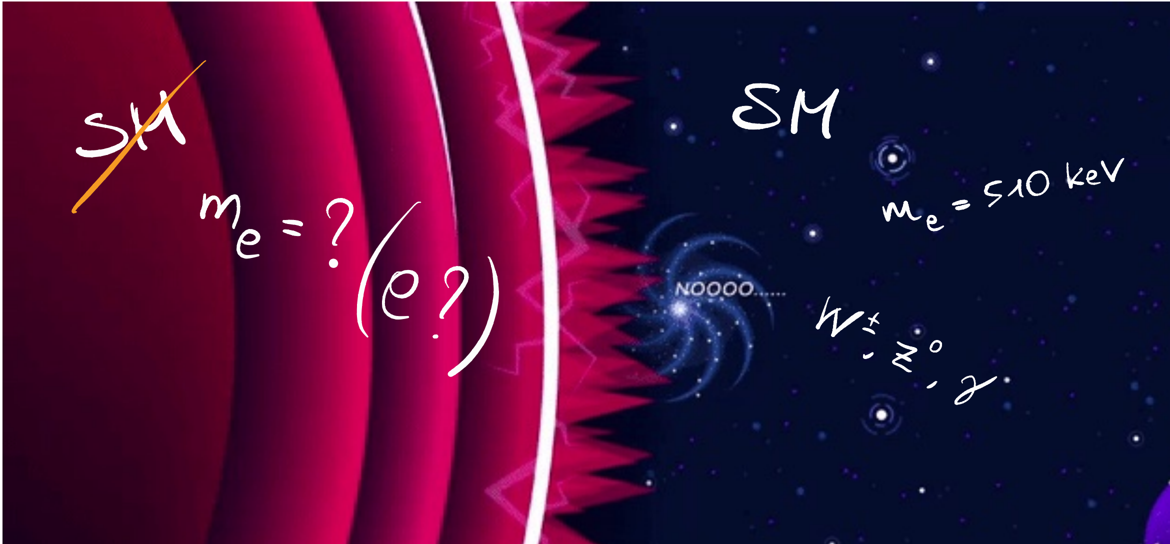

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