Lectures on Neutrino Mass Physics

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References

- "Unification and Supersymmetry", R. N. Mohapatra (Springer-Verlag, 3rd ed.)
- "Massive neutrinos in physics and astrophysics", R. N. Mohapatra and P. B. Pal (World Scientific, 3rd ed.)

Standard model and neutrino mass

- SM

  ➢ Gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$

  ➢ Matter: Doublets: $Q \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$; $\psi_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$;

  Singlets: u_R; d_R; e_R

  Higgs: $H \equiv \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$

  ➢ $\mathcal{L}_Y = h_u \bar{Q}_L H u_R + h_d \bar{Q}_L \tilde{H} d_R + h_e \bar{\psi}_L \tilde{H} e_R + h.c.$

- Neutrino massless to all orders because of two facts: No ν_R + exact B-L symmetry.
- Discovery of neutrino mass → first evidence for physics beyond the standard model.

Other reasons to go beyond SM

- **Some puzzles of SM:**

(i) Origin of Mass: two mass problems:

(a) quark masses <H>; Requires Higgs $m^2 < 0$ and why $m << M_{Planck}$

(b) neutrino masses (?); Requires different Higgs.

(ii) Origin of Flavor:

Fermion masses, mixings, CP and P-violation

(iii) Cosmological Issues:

Dark matter, Origin of matter, inflation

(iv) Origin of Parity violation

How does a neutrino mass look like ?

- Since neutrino is electrically neutral, conservation laws and relativity allows two possibilities for fermion masses:
- $\overline{\psi}_L \psi_R$ vs $\psi_L^T C^{-1} \psi_L$  or 

  Dirac Majorana
- **Dirac masses**:

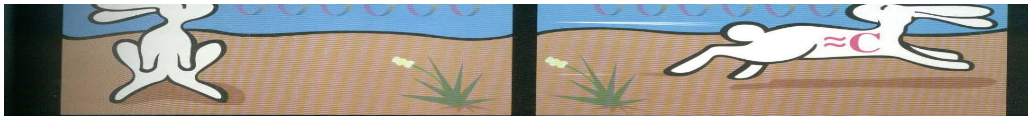
  (i) they requires an additional symmetry (**L**) – estblishing Dirac nature will reveal a new sym of nature.

  (ii) They imply the existence of new neutrinos (RH comp.)
- **Majorana mass: No symmetry required, nor any new neutrinos :: a more natural choice for neutrino mass.**

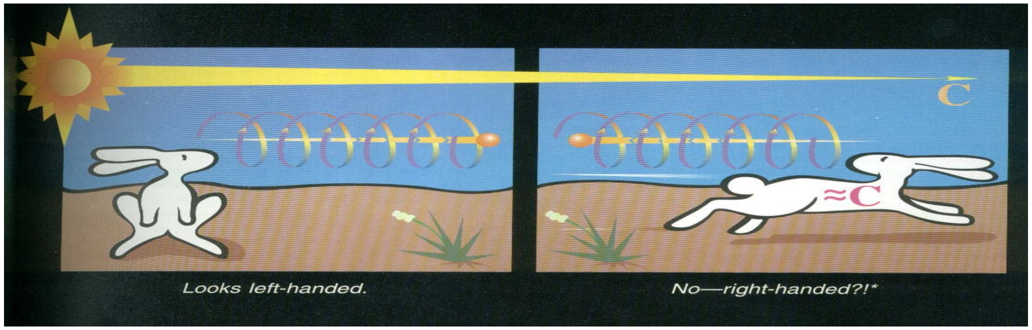
  As we will see, theories seem to prefer this.

Mass and helicity:

- S=0; relativistic transformation: no change in state:



- S=1/2



More about Dirac, Weyl, Majorana fermions

- Dirac equation from Q. Mech. Lecture:
- Notation: Write $\nu = \begin{pmatrix} \xi \\ i\sigma_2\chi^* \end{pmatrix}$;
  ξ, χ two-component objects;

  γ matrix convention: $\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$; $\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$;
  $\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$;
  $\nu_L = \begin{pmatrix} \xi \\ 0 \end{pmatrix}$ and $\nu_R = \begin{pmatrix} 0 \\ i\sigma_2\chi^* \end{pmatrix}$;

- ξ, χ are representations of SL(2,c) group.
- Mass term-4-comp:-L= m**1** $\overline{\psi}_L \psi_R$ + m**2** $\psi_L^T C^{-1} \psi_L$ + h.c.
  +m**3** L→R
- Two comp language**:** $L = m_1 \xi\chi + m_2 \xi\xi + m_3 \chi\chi + h.c.$

Weyl to Majorana

- m=0 spin half: Equation:
- $$i\bar{\sigma}^{\mu\dot{\alpha}\beta}\partial_\mu \xi_\beta = 0$$

or

$$\vec{\sigma}\cdot\vec{p}\,\xi = E\xi$$

This → particle: $\Leftarrow \Longrightarrow$ anti-particle: $\Rightarrow \Longrightarrow$

For a single 2-comp spinor, only way to have mass is to have the particle and anti-particle be same . It must be a Majorana fermion.

Field Theory for Weyl Case

- Consider particle moving in the z-direction:

$$\vec{\sigma}\cdot\vec{p}\xi = E\xi \qquad \sigma_3\xi = \frac{\pm|E|}{p}\xi$$

- +ve E → $\xi = \alpha = \begin{pmatrix}1\\0\end{pmatrix}$ -ve E → $\xi = \beta = \begin{pmatrix}0\\1\end{pmatrix}$

- Field expansion of $\xi \sim \sum_p [a_{\mathbf{P},+} e^{-ip.x}\alpha + a^\dagger_{\mathbf{P},-} e^{ip.x}\beta]$

Field Theory for Majorana

- Majorana-single 2-comp spinor: $\mathscr{L} = i\xi^\dagger\bar{\sigma}^\mu\partial_\mu\xi - \frac{1}{2}m(\xi\xi + \xi^\dagger\xi^\dagger)$

$$i\bar{\sigma}^{\mu\dot{\alpha}\beta}\partial_\mu\xi_\beta = m\xi^{\dagger\dot{\alpha}}$$

$$\begin{aligned}\xi &= \sum_p [a_{\mathbf{p},+}e^{-ip.x} - a^\dagger_{\mathbf{p},-}e^{ip.x}]\alpha\sqrt{E+p} \\ &+ \sum_p [a_{\mathbf{p},-}e^{-p.x} + a^\dagger_{\mathbf{p},+}e^{ip.x}]\beta\sqrt{E-p}.\end{aligned}$$

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

More on Majorana fermions

- In a beta decay, the particle produced is anti-neutrino. If it is Majorana, what is the meaning of an anti-neutrino and neutrino
- This characterization really comes from the m→ 0 limit where left-handed helicity + is the neutrino and RH - anti-neutrino.
- For small Majorana masses,

$$\xi = \sum_p [a_{\mathbf{p},+} e^{-ip.x} - a^\dagger_{\mathbf{p},-} e^{ip.x}] \alpha \sqrt{E+p}$$
$$+ \sum_p [a_{\mathbf{p},-} e^{-p.x} + a^\dagger_{\mathbf{p},+} e^{ip.x}] \beta \sqrt{E-p}.$$

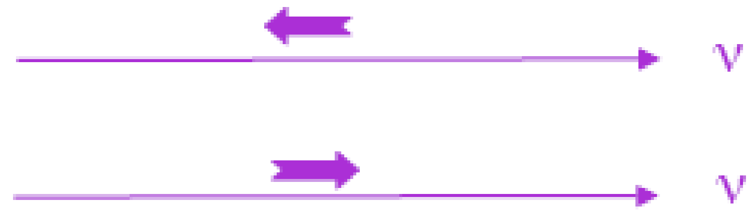
creates anti-nu and from the second line, a bit of nu with amplitude m/E.

Similarly in nu-less double beta decay → proportional to m

$$<\xi(x)\xi(0)> \to <0| a\_a\_^+ |0> \neq 0$$

Majorana neutrino in pictures:

- Two states:



- In SM

Left-handed

$$L_{SM} = -\frac{g}{\sqrt{2}}\left(\bar{\ell}_L \gamma^\lambda \nu_L W_\lambda^- + \bar{\nu}_L \gamma^\lambda \ell_L W_\lambda^+\right)$$

Absorbs right-handed $\bar{\nu} = \nu$

When $\bar{\nu} = \nu$

Dirac vs Majorana mass matrices

- One 2-comp spinor: $\mathscr{L}$ **mass =** $-\frac{1}{2}m(\xi\xi + \xi^\dagger\xi^\dagger)$
- **Two: Majorana M=** $\begin{pmatrix} m_1 & \mu \\ \mu & m_2 \end{pmatrix}$ **vs Dirac:** $\begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix}$
- Zeros reflect L symmetry.
- Majorana mass matrix is symmetric and can be diagonalized: In particular if $m_2 >> m_1, \mu$, it gives two hierarchical eigen values:(basis of seesaw mechanism)

$$m_{light} \cong -\frac{\mu^2}{m_2}; M_{heavy} \cong m_2$$

Neutrino magnetic moment:

- A matter of great interest for understanding the nature of interactions of the neutrino:
- 4-component language:

$$\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} \to (\chi^T \sigma_2 \sigma_{[\mu}\bar{\sigma}_{\nu]}\xi + \chi \leftrightarrow \xi)F^{\mu\nu}$$

- Need two 2-comp spinors to get mag mom.

i.e. one needs Dirac neutrino or two different neutrino families to get a nonzero mag mom. (transition mag mom.)

- Current limits from lab: < 10^{-10} Bohr mag.;
- Astrophysics two orders stronger.

Fermion masses and mixings:

If there are more fermions of the same kind, then
$\mathcal{L}_{mass} = M_{ab}\bar{\psi}_{a,L}\psi_{b,R}$

☞ Masses and mixings from the Lagrangian

- ➢ M_{ab}= Mass matrix
- ➢ Diagonalize the mass matrix
  $U^{\dagger}MV = diag(m_1, m_2, \cdot, \cdot)$
- ➢ U, V gives the mixings between different (L, R) fermions, ψ_a and m_i are the actual masses e.g. for quarks, U_{ab} contains the CKM mixings (e.g. $U_{CKM} = U_u^{\dagger}U_d$, where$U$ and V denote the rotations in the up and the down sector)

- **Key to understanding fermions is to study their mass matrices:**

Definitions:

- Flavor to mass basis

flavour mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix

In general: $U = U^+_e U_\nu$

- Mixing matrix U_{PMNS}:

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

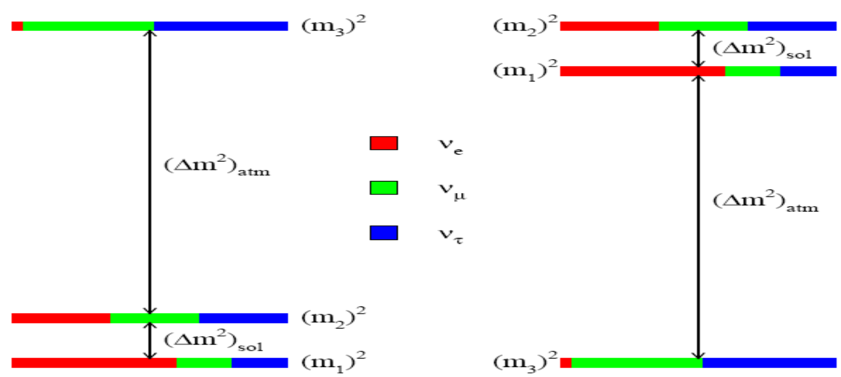
- Masses: $m_{1,2,3}$ def.

$$\Delta m^2_{atm} = m^2_3 - m^2_2 > \text{or} < 0$$

$$\Delta m^2_{sol} = m^2_2 - m^2_1 > 0$$

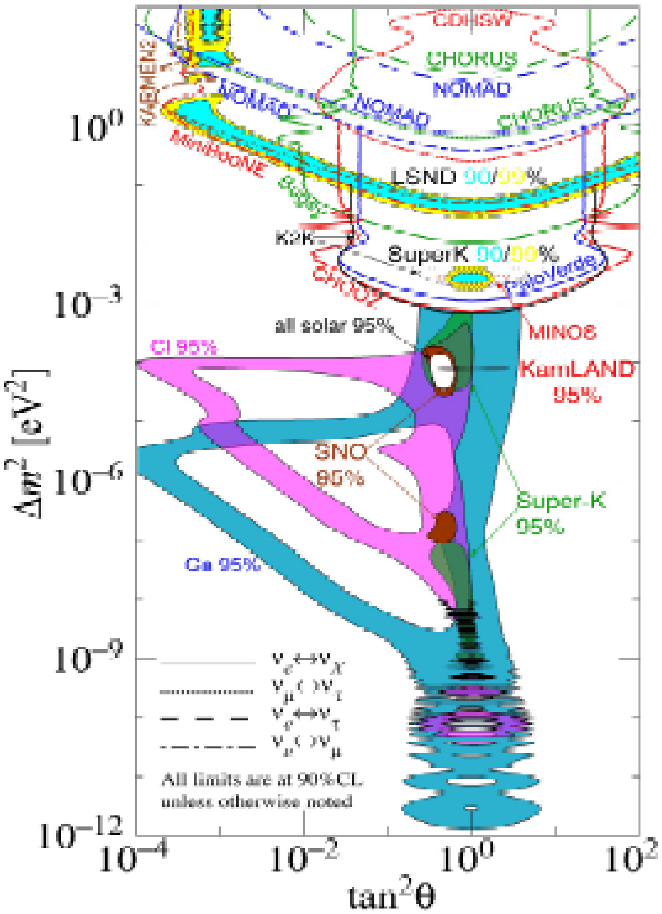
Present information:

- **Masses:** $\Delta m^2_{sol} \cong 7.67\times10^{-5} eV^2$ **;** $\Delta m^2_{Atm} \cong 2.39\times10^{-3} eV^2$
- **Mixings:** $\sin^2\theta_{12} \cong .312; \sin^2\theta_{23} \cong .466$ **;** $\sin^2\theta_{13} \leq .04$
- **Overall mass scale:** < .1- 1 eV (roughly) (WMAP,)
- **Mass ordering not known:**



$\Delta m^2_{atm} = m_3^2 - m_2^2 > \text{or} < 0$

PDG summary



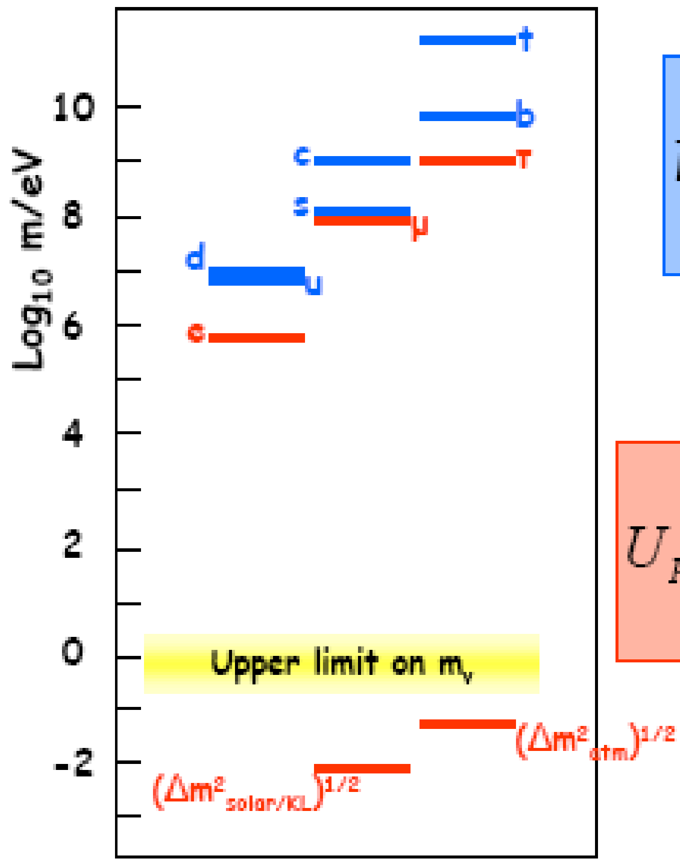
http://hitoshi.berkeley.edu/neutrino

parameter	best fit	2σ	3σ
Δm^2_{21} [10^{-5}eV^2]	$7.59^{+0.23}_{-0.18}$	7.22–8.03	7.03–8.27
$\lvert\Delta m^2_{31}\rvert$ [10^{-3}eV^2]	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.318^{+0.019}_{-0.016}$	0.29–0.36	0.27–0.38
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.013^{+0.013}_{-0.009}$	≤ 0.039	≤ 0.053

Thomas Schwetz†, Mariam Tórtola‡ and José W. F. Valle§

Quark mixings vs Lepton mixings:

- Compare what is already known:



$$V_{CKM} \cong \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 1.00 & 0.04 \\ 0.01 & 0.04 & 1.00 \end{pmatrix}$$

Adapted from PDG 2002

$$U_{PMNS} \cong \begin{pmatrix} 0.84 & 0.54 & 0.1 \\ -0.44 & 0.56 & 0.71 \\ 0.32 & -0.63 & 0.71 \end{pmatrix}$$

An Interesting mixing pattern ?

- **Tri-bi-maximal mixing for neutrinos:**

$$U = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

- **Is it exact ? If not how big are corrections ?**

Things we need to know:

- **Absolute mass scale:**
- **Mass hierarchy** $\Delta m^2_{atm} = m_3^2 - m_2^2 > \text{or} < 0$
- **Mixing angle** θ_{13}
- **CP violation**
- **Dirac or Majorana:**
- **Extra neutrinos : heavy as well as light**

- **What physics is implied by what we already know ?**

Absolute mass scale

- Expt.

1. ^3H Decay end point: $\Sigma_i m_i^2 |U_{ei}|^2 \leq 2.2$ eV^2 (KATRIN expected to improve it to 0.2 eV)

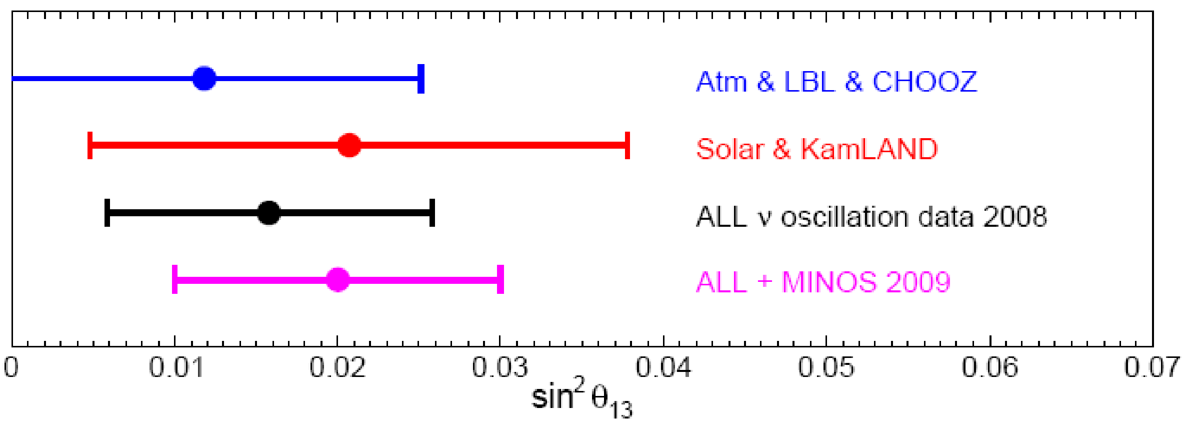
2. Cosmology: $\Sigma m_i \leq 0.4$ eV (WMAP, SDSS: will be improved by Planck)

3. If neutrino Majorana i.e. $\nu = \bar{\nu}$, $\beta\beta_{0\nu}$ results imply: $\Sigma_i U_{ei}^2 m_i \leq 0.3 - 0.5$ eV (Expected improvement to 0.03 eV)

Missing mixing angle info.

- θ_{13}

(Fogli, Lisi, Marrone, Pallazo and Rotunno'08)

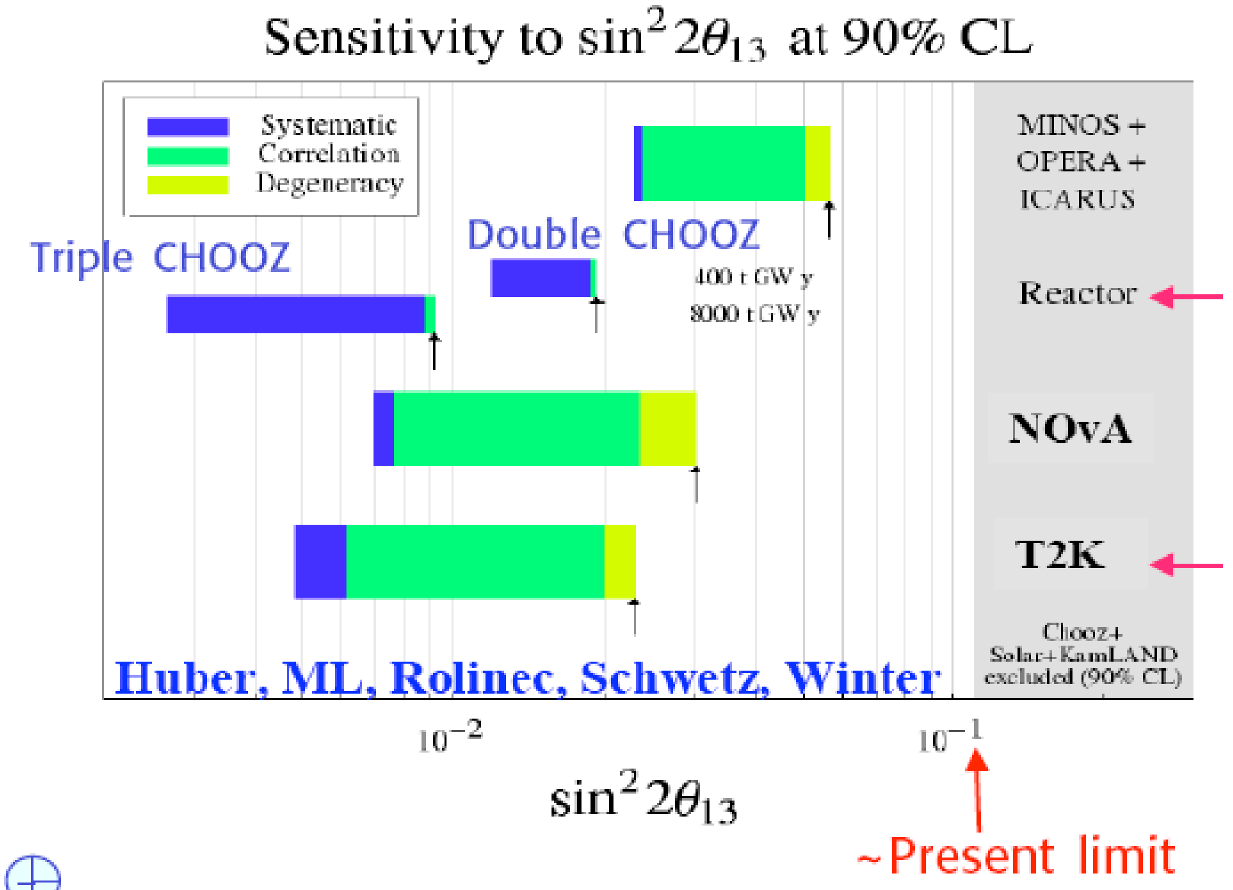


Too early for definite conclusion--However

Value of θ_{13} significant for new physics

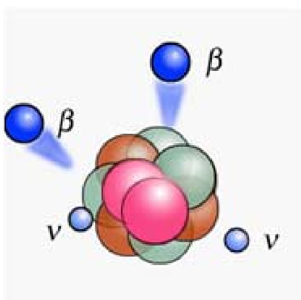
How far can we go ?

- **Expts:**

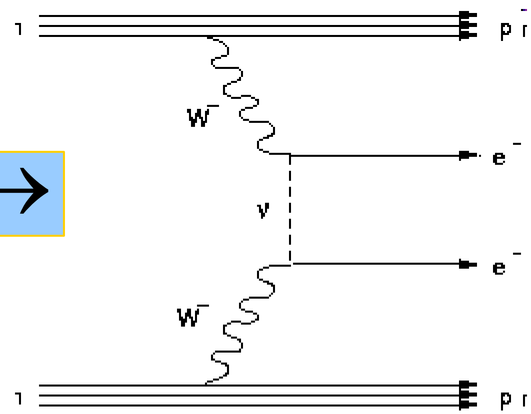


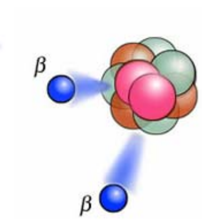
Majorana neutrino and neutrinoless double beta decay:

- Majorana implies that $\nu = \bar{\nu}$
- It can lead to $n + n \rightarrow p + p + 2e^{-}$

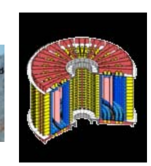


$\nu = \bar{\nu}$ $\longrightarrow$





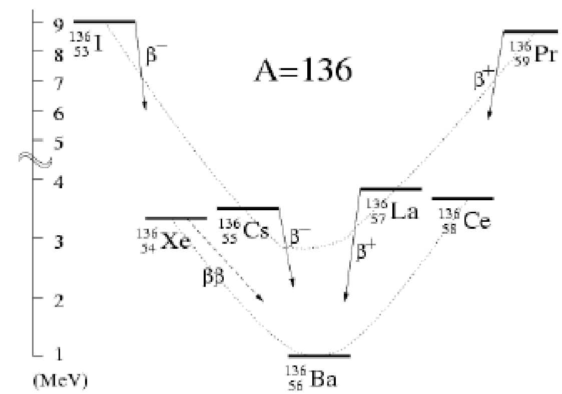
- Many expts searching for it:



Possible candidate nuclei

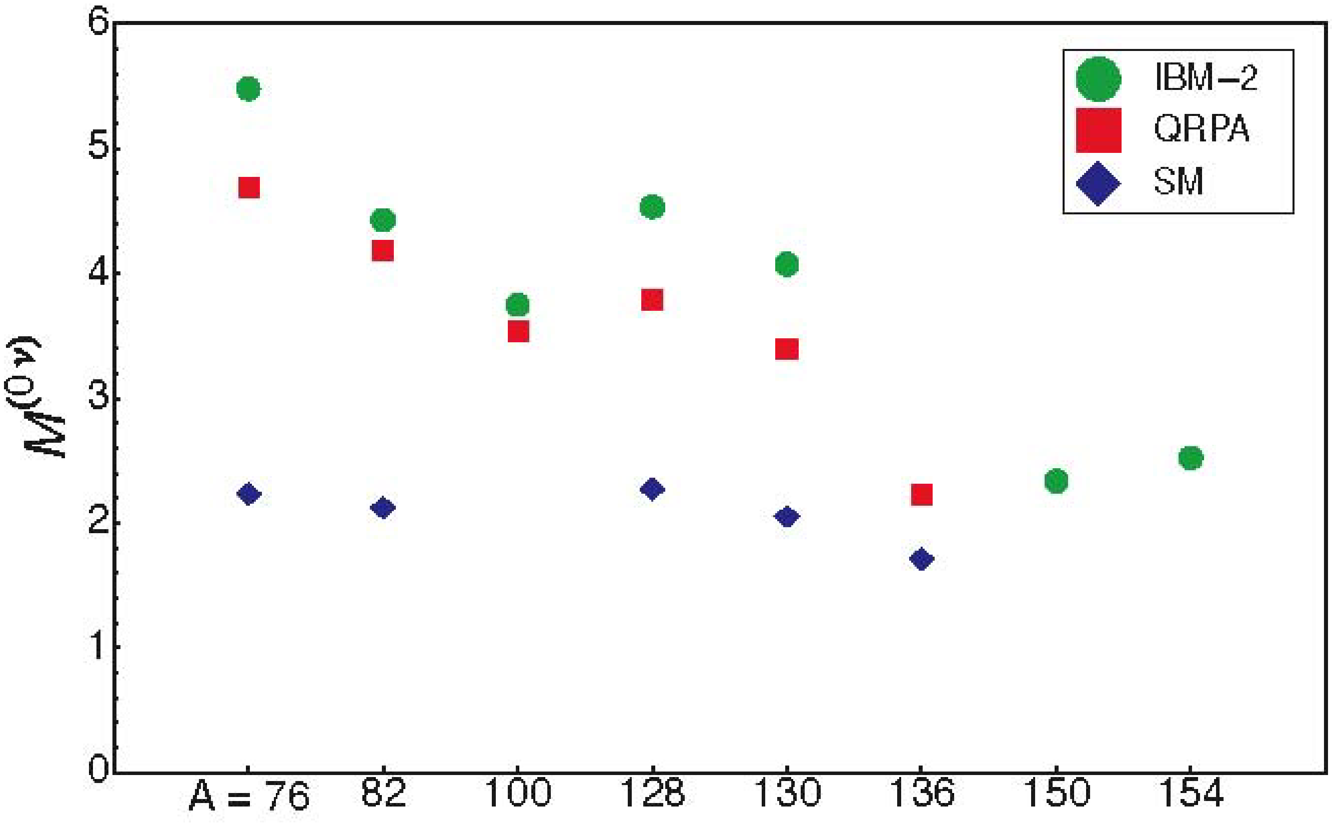
- Nuclei: Higher the Q-value the better.

	Q (MeV)	Abund.(%)
$^{48}Ca \rightarrow ^{48}Ti$	4.271	0.187
$^{76}Ge \rightarrow ^{76}Se$	2.040	7.8
$^{82}Se \rightarrow ^{82}Kr$	2.995	9.2
$^{96}Zr \rightarrow ^{96}Mo$	3.350	2.8
$^{100}Mo \rightarrow ^{100}Ru$	3.034	9.6
$^{110}Pd \rightarrow ^{110}Cd$	2.013	11.8
$^{116}Cd \rightarrow ^{116}Sn$	2.802	7.5
$^{124}Sn \rightarrow ^{124}Te$	2.228	5.64
$^{130}Te \rightarrow ^{130}Xe$	2.533	34.5
$^{136}Xe \rightarrow ^{136}Ba$	2.479	8.9
$^{150}Nd \rightarrow ^{150}Sm$	3.367	5.6



Matrix element uncertainty:

- P. Vogel- (factor of 2-3)



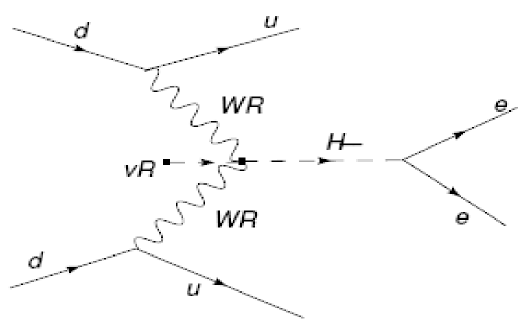
Predictions for nu-less double beta decay for normal and inverted hierarchy

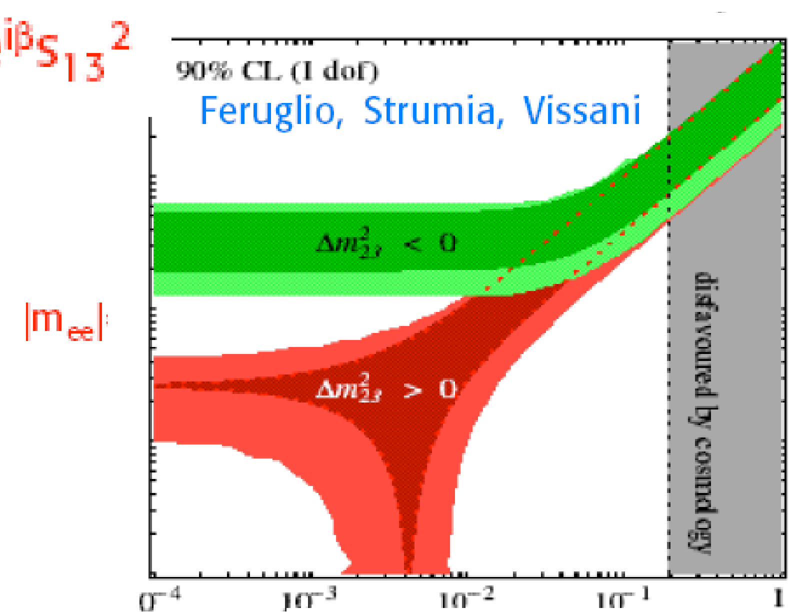
- Measures effective neutrino mass:

$$|m_{ee}|=c_{13}^2\ [m_1 c_{12}^2+e^{i\alpha} m_2 s_{12}^2]+m_3 e^{i\beta} s_{13}^2$$

- Should be observed if inverted mass H.

☞ **CAUTION!** even with normal hierarchy, there could be "large" effects from heavy particles such as sparticles, doubly charged Higgs bosons or RH Majorana neutrinos.





How can we tell Majorana from Dirac experimentally ?

- Some ongoing expts could even tell:

☞ **Sign of Δm^2, $\beta\beta_{0\nu}$ and KATRIN result can tell us a lot:**

$\beta\beta_{0\nu}$	Δm^2_{32}	KATRIN	Conclusion
yes	> 0	yes	Degenerate, Majorana
yes	> 0	No	Degenerate, Majorana or normal or heavy exchange
yes	< 0	no	Inverted, Majorana
yes	< 0	yes	Degenerate, Majorana / $W_R - exchange$
no	> 0	no	Normal, Dirac or Majorana
no	< 0	no	Dirac
no	< 0	yes	Dirac
no	> 0	yes	Dirac

Sterile neutrinos

- Mini-Boone did not confirm LSND- so no compelling need for sterile nu's.
- KeV steriles possible as dark matter candidates:(Abazajian, Fuller Kusenko; Shaposnikov et al.)
- Mixings constrained by solar and atmospheric data
- BBN allows at most one with large mixing angle.
- Peak searches,Beam dump

(Atre, Han, pascoli, Zhang)

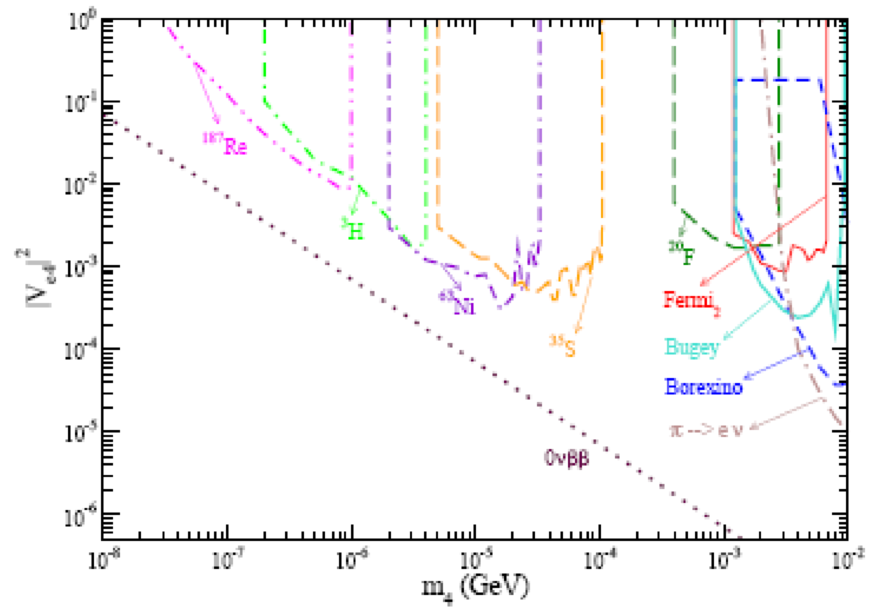
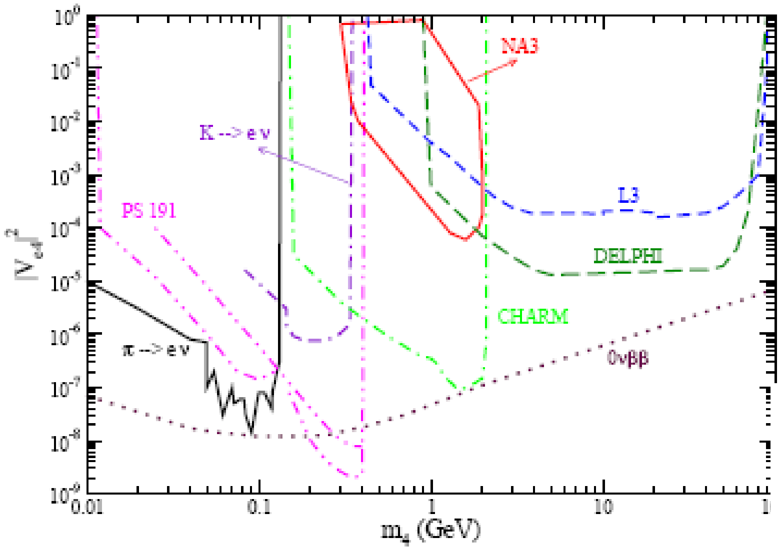
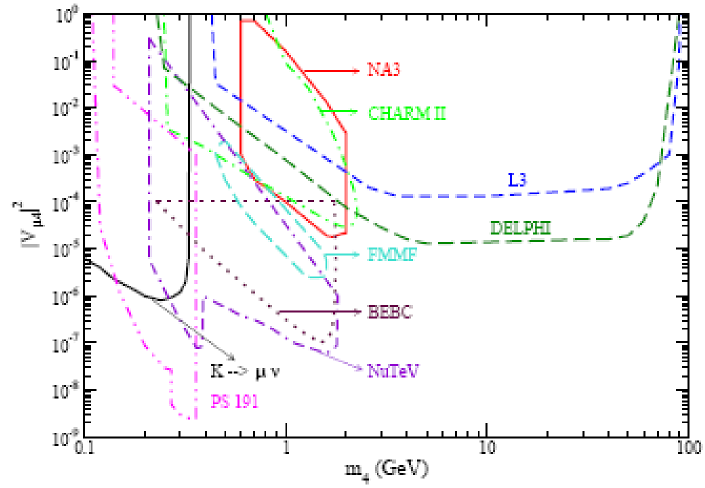


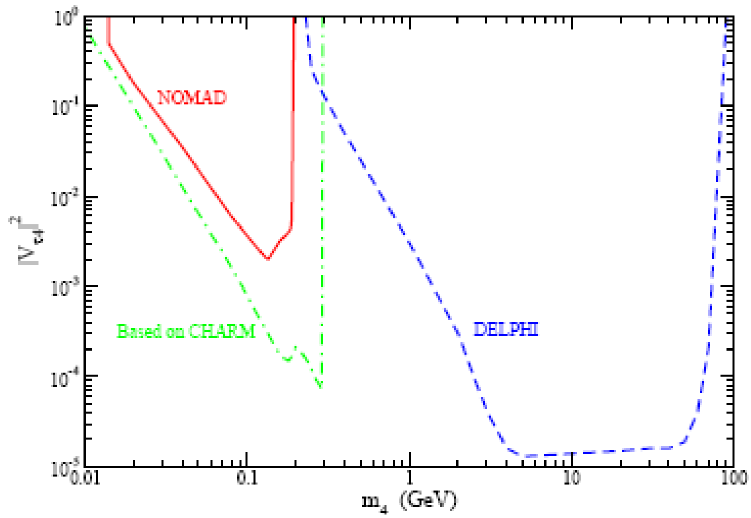
Figure 2: Bounds on $|V_{e4}|^2$ versus m_4 in the mass range 10 eV–10 MeV. The excluded regions with contours labeled ^{187}Re [76], ^{3}H [77] , ^{63}Ni [78] , ^{35}S [79] , ^{20}F and Fermi$_2$ [80] refer to the bounds from kink searches. All the limits are given at 95% C.L. except for the ones from Ref. [80] which are at 90% C.L.. The areas delimited by short dashed (blue) contour labeled Borexino and solid (cyan) contour labeled Bugey are excluded at 90% C.L. by searches of N_4 decays from the Borexino Counting Test facility [81] and Ref. [82] respectively. The region with long-dash-dotted (grey) contour, labelled $\pi \to e\nu$, is excluded by peak searches [83]. The dotted (maroon) line labeled $0\nu\beta\beta$ indicates the bound from searches of neutrinoless double beta-decay [84].

Constraints on other mixings

- E-s, Mu-s and tau-s







Now to theory: Primer on fermion masses and mixings:

☞ Look for bilinears of the form $\bar{\psi}_L \psi_R$ in the Lagrangian

If there are more fermions of the same kind, then $\mathcal{L}_{mass} = M_{ab} \bar{\psi}_{a,L} \psi_{b,R}$

☞ Masses and mixings from the Lagrangian

- M_{ab}= Mass matrix
- Diagonalize the mass matrix $U^\dagger M V = diag(m_1, m_2, \cdot, \cdot)$
- U, V gives the mixings between different (L, R) fermions, ψ_a and m_i are the actual masses e.g. for quarks, U_{ab} contains the CKM mixings (e.g. $U_{CKM} = U_u^\dagger U_d$, where$U$ and V denote the rotations in the up and the down sector)

- **Key to understanding fermions is to study their mass matrices:**

Goal of Theory

- **Determining and understanding the Neutrino mass matrix :**
- **Two parts to the story:**

$$M_\nu = m_\nu \times A_F$$

(i) Scale m_ν

(ii) Flavor structure A_F

(The neutrino matrix)

Specific Challenges

(i) Scale issue: Why $m_\nu << m_{q,l}$ **? :**

(ii) Flavor issues: A_F **?**

A. Milder mass hierarchy compared to quarks and charged leptons:

$$\frac{m_{sol}}{m_{atm}} \approx \theta_C >> \frac{m_\mu}{m_\tau}, \frac{m_s}{m_b}$$

B. Neutrino mixing angles much larger than quark mixings: e.g. $V^l_{23} \approx 0.7 >> V^{CKM}_{23} \approx 0.04$ **etc.**

C. Quarks and leptons so different- are they unifiable ?

SM + RH nu

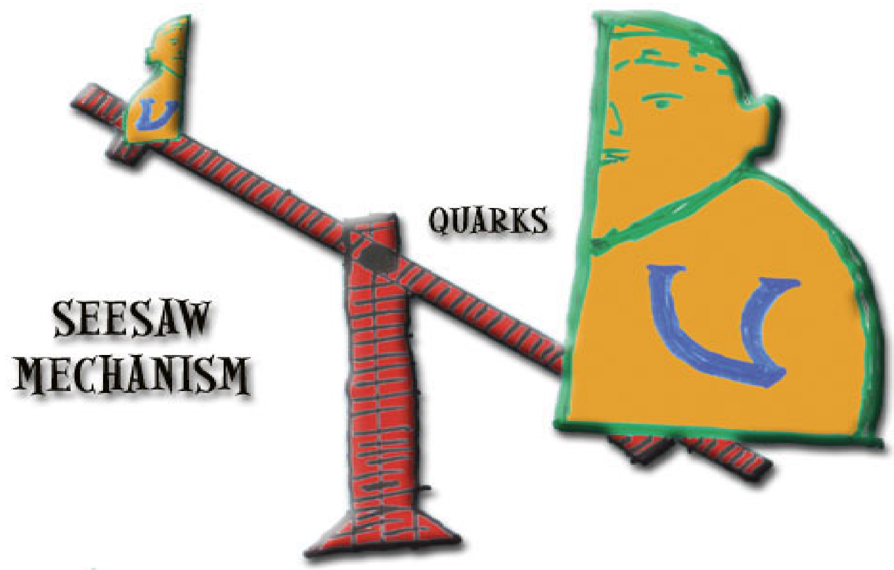
- Add RH nu to SM and tune h_nu=10^-12.
- Right order of magnitude.
- Radiatively stable due to chiral sym. $\nu \rightarrow \gamma_5 \nu$
- Why so small coupling ?
- No way to test this.

Neutrino mass as a high scale effect

- Neutrino mass vanishes in SM:
- SM is of course part of a bigger theory which manifests at a scale M;
- This new theory could induce operators that give nu mass. Form of effective operator:
- $$\frac{LHLH}{M} \rightarrow m_\nu = \frac{<H>^2}{M}$$ (Weinberg)
- **Could it be gravity ? Too small.**
- **What is the scale M and where it comes from ?**

Why $m_\nu << m_{q,l}$ and M?

- **Seesaw Paradigm:**
- **Add heavy right handed neutrinos N_R or heavy something to SM and play seesaw with them:**



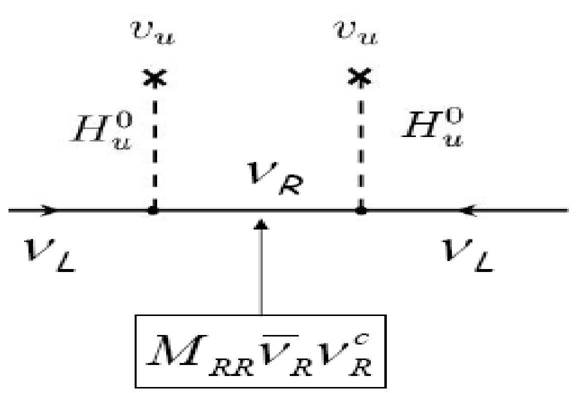
- **Two classes of seesaws depending on whether N is Majorana or Dirac.**

Type I seesaw

(Minimal extension)

- **Heavy Majorana** N_R

$$L_Y = h_\nu \overline{L} H N_R + M_R NN$$



- M_R Breaks B-L : New scale and new physics beyond SM.
- After EWSB

  -Neutrino majorana

$$m_\nu \cong -\frac{h_\nu^2 v_{wk}^2}{M_R}$$

Requires strong hierarchy: $\frac{m_D}{M_R} \equiv \frac{h_\nu v_{wk}}{M_R} \sim 10^{-7} - 10^{-10}$

- Minkowski,Gell-Mann, Ramond Slansky,Yanagida, Mohapatra,Senjanovic,Glashow

Inverse Seesaw

- **Mostly Dirac N_R** i.e. add another singlet S

$$L_Y = h_\nu \bar{L} H N_R + M N_R S + \mu SS \qquad \mu << M$$

$$(\nu_L, \nu_R, S)$$

$$\begin{pmatrix} 0 & hv_{wk} & 0 \\ hv_{wk} & 0 & M \\ 0 & M & \mu \end{pmatrix}$$

$$m_\nu \cong -m_D^T M^{-1} \mu M^{-1} m_D$$

- **Requires weaker hierarchy** $\frac{m_D}{M} \sim 10^{-3}$

(RNM,86;RNM, Valle'86)

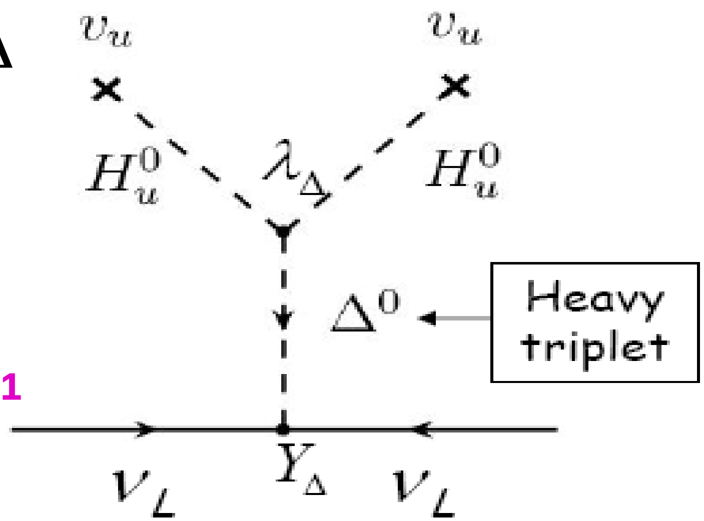
It is not the "largeness" of M but "smallness of mu"--

Other seesaws:

- **Type II seesaw:** a heavy **triplet Higgs:**
- $(\Delta^{++}, \Delta^{+}, \Delta^{0})$ → $fLL\Delta + \mu HH\Delta$

$$m_\nu = \frac{f\mu v_{wk}^2}{M_\Delta^2}$$

Lazaridis, Shafi, Wetterich; R.N.M.,Senjanovic; Schecter, Valle'81



- **Type III seesaw:** triplet fermion instead of NR in type I case: **(Foot, He, Lew, Joshi)**

Seesaw at LHC ?

- **Neutrino masses do not determine seesaw scale**
- $m_D \approx m_t$ → $M_R \approx 10^{14} GeV$; **M**$_U$~ 10^{16} GeV

  Both $m_D \approx m_t$ **and high seesaw scale i**ndication for SUSYGUTs; No collider signals !
- $m_D \approx m_e$ → B-L scale at **TeV**

LHC signals with only gauge forces; **No GUTs** with type I.

- Inverse seesaw → B-L at TeV and **GUTs** can co-exist:

  since $m_D \approx m_t$ possible

Seesaw phenomenology

- Seesaw matrix involves both light and heavy RH neutrinos: Diagonalization therefore leads to non-unitary PMNS:
- **Type I** case diagonalizing Unitary matrix

$$V = \begin{pmatrix} V_{3\times3} & V'_{3\times3} \\ V''_{3\times3} & V'''_{3\times3} \end{pmatrix}$$

PMNS→ $\mathcal{N} \equiv V_{3\times3} \simeq \left(1 - \frac{1}{2}FF^\dagger\right)U$

Typical departure from unitarity→ $\frac{m_\nu}{M_R} \leq 10^{-12}$

Situation different for Inverse seesaw (9x9 matrix): $V = \begin{pmatrix} V_{3\times3} & V_{3\times6} \\ & V_{6\times6} \end{pmatrix}$

- $\mathcal{N} \equiv V_{3\times3} \simeq \left(1 - \frac{1}{2}FF^\dagger\right)U$ departure from unitarity much bigger

Current bounds:

- Antusch, Biggio, Gavela, Fernandez-Martinez, Blenow, Lopez-Pavon, Ohlsson, Donini, Altarelli

$$|\eta| < \begin{pmatrix} 2.0\times10^{-3} & 3.5\times10^{-5} & 8.0\times10^{-3} \\ 3.5\times10^{-5} & 8.0\times10^{-4} & 5.1\times10^{-3} \\ 8.0\times10^{-3} & 5.1\times10^{-3} & 2.7\times10^{-3} \end{pmatrix} \qquad \eta = \frac{1}{2}FF^{+}$$

- **Search for departure from unitarity may be a hint for inverse seesaw or at least something beying beyond simple type I or type III seesaw.**
- **Type II seesaw: no departure from unitarity:**

Higher dimensional corrections to seesaw:

- Possible new operators from high scale physics:

Type I → $\delta\mathcal{L}^{d=6} = c^{d=6}_{\alpha\beta} \left(\overline{\ell_{L\alpha}}\tilde{\phi}\right) i\partial\!\!\!/ \left(\tilde{\phi}^{\dagger}\ell_{L\beta}\right)$ /M^2

→Leads to non-unitarity via neutrino KE.

Type II:

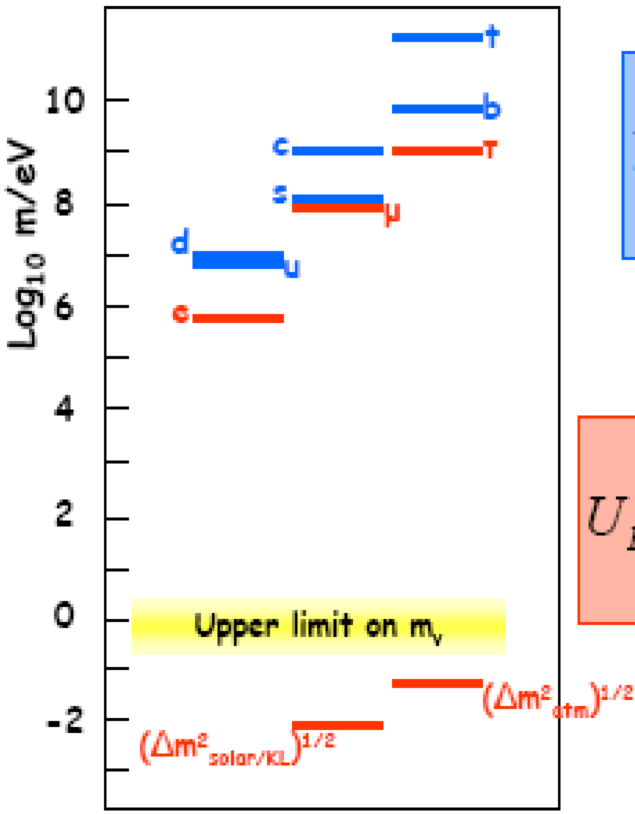
$$\begin{cases} \delta\mathcal{L}_{4F} = \frac{1}{M_{\Delta}{}^2}\left(\overline{\widetilde{\ell_L}}\, Y_{\Delta}\, \vec{\tau}\, \ell_L\right)\left(\overline{\ell_L}\vec{\tau}\, Y^{\dagger}_{\Delta}\, \widetilde{\ell_L}\right) \\ \delta\mathcal{L}_{6\phi} = -2\left(\lambda_3 + \lambda_5\right)\frac{|\mu_{\Delta}|^2}{M^4_{\Delta}}\left(\phi^{\dagger}\phi\right)^3 \\ \delta\mathcal{L}_{\phi D} = \frac{|\mu_{\Delta}|^2}{M^4_{\Delta}}\left(\phi^{\dagger}\vec{\tau}\tilde{\phi}\right)\left(\overleftarrow{D_{\mu}}\overrightarrow{D^{\mu}}\right)\left(\tilde{\phi}^{\dagger}\vec{\tau}\phi\right) \end{cases},$$

- Type III: $\delta\mathcal{L}^{d=6} = c^{d=6}_{\alpha\beta}\left(\overline{\ell_{L\alpha}}\vec{\tau}\tilde{\phi}\right) iD\!\!\!/\left(\tilde{\phi}^{\dagger}\vec{\tau}\ell_{L\beta}\right),$
- **A way to distinguish between seesaws** !

Abada, Biggio, Gavela,Bonnet, Hambye

Flavor pattern:

- Quarks vs leptons:



$$V_{CKM} \cong \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 1.00 & 0.04 \\ 0.01 & 0.04 & 1.00 \end{pmatrix}$$

Adapted from PDG 2002

$$U_{PMNS} \cong \begin{pmatrix} 0.84 & 0.54 & 0.1 \\ -0.44 & 0.56 & 0.71 \\ 0.32 & -0.63 & 0.71 \end{pmatrix}$$

Quarks vs leptons

- Hints from data on mass matrices for model bldg.
- Must give large mixings: $\varepsilon_i \sim \lambda =$ Cabibbo angle
- **Quarks**:-up quarks diagonal:

$$M_d = m_b \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

The Neutrino Matrix:

- **Flavor of the Neutrino flavor research**

Generic mass matrix (NH) $\varepsilon_i \approx \lambda_{Cabibbo} << 1$

θ_{23} **NEAR MAXIMAL** θ_{23} **maximal**

TBM

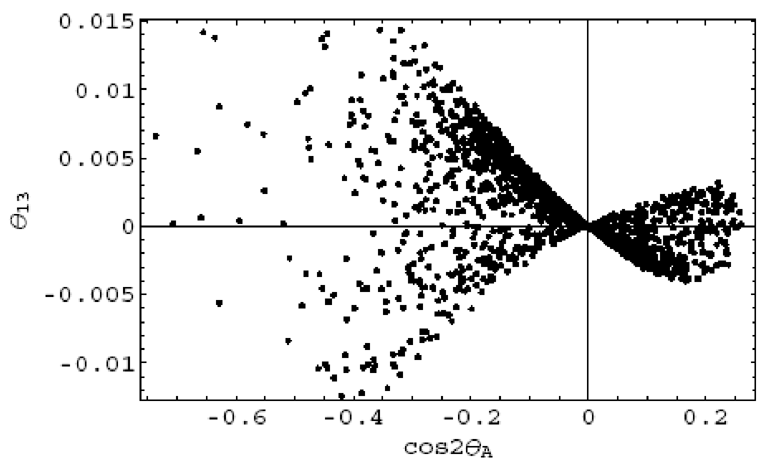
$$\begin{pmatrix} \varepsilon_5^{n\geq 1} & \varepsilon_4 & \varepsilon_3 \\ \varepsilon_4 & 1+\varepsilon_1 & -1 \\ \varepsilon_3 & -1 & 1+\varepsilon_2 \end{pmatrix} \rightarrow \begin{pmatrix} \varepsilon_5^{n\geq 1} & \varepsilon_3 & \varepsilon_3 \\ \varepsilon_3 & 1+\varepsilon_1 & -1 \\ \varepsilon_3 & -1 & 1+\varepsilon_1 \end{pmatrix} \rightarrow \begin{pmatrix} \varepsilon_1 & \varepsilon_3 & \varepsilon_3 \\ \varepsilon_3 & 1+\varepsilon_1 & -1+\varepsilon_3 \\ \varepsilon_3 & -1+\varepsilon_3 & 1+\varepsilon_1 \end{pmatrix}$$

$\rightarrow \mu \leftrightarrow \tau$ **sym.**

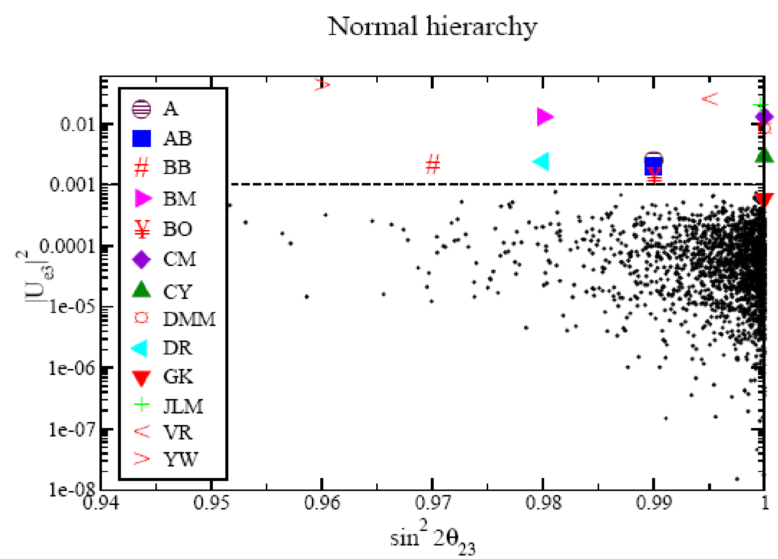
5 parameters **3 param.** **2 param.**

Testing mu-tau sym using θ_{13}

- Weak breaking of mu-tau sym. → correlation between θ_{13} and departure from maximality of atmospheric mixing angle:



mu-tau sym



GUT vs mu-tau

Inverted hierarchy:

- One possibility:
- $$\begin{pmatrix} 0 & m_1 & m_2 \\ m_1 & 0 & 0 \\ m_2 & 0 & 0 \end{pmatrix} + \delta m$$
- Approx sym. $L_e - L_\mu - L_\tau$ But sym breaking large.
- Another possibility: $\begin{pmatrix} 1+\alpha & \epsilon & \delta \\ \epsilon & 1+\beta & \eta \\ \delta & \eta & \gamma \end{pmatrix}$ or $\begin{pmatrix} 1+\alpha & \epsilon & \delta \\ \epsilon & -1+\beta & \eta \\ \delta & \eta & \gamma \end{pmatrix}$
- no sym.

Approximate mass matrices:

- δm can have 6 small parameters:
- There may be corrections to PMNS from the charged lepton sector, which are also constrained by symmetries that give TBM.
- These corrections can teach us a lot about physics beyond SM and throw light on the flavor puzzle.

Lecture II

Testing neutrino mass physics:

(i) Lepton flavor violation

Expts: MEG, PRISM/PRIME

(ii) Testing at LHC

Present status of lepton flavor violation

- MEGA: $B(\mu \rightarrow e+\gamma)$ < 2x10^{-11}
- BELLE,BABAR $B(\tau \rightarrow \mu\gamma) \leq 4.5 \times 10^{-8}$
- Future $B(\mu \rightarrow e+\gamma)$ 10^{-13} MEG

  10^{-18}, JPARC, PRISM

Since neutrino oscillationsa violate flavor by large amount, they could lead to other LFV effects !

Simple Dirac extension of SM

- Silent, stealth model—

- Even though nu mixings are large, hardly any lepton violation

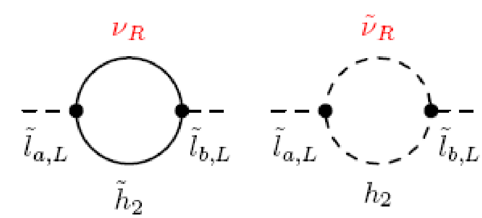
$$A(\mu \to e\gamma) \propto \frac{g^2 (m_\nu^+ m_\nu)_{21} m_\mu}{16\pi^2 M_W^4}$$

- Same story with Majorana seesaw nu without new TeV scale physics e.g. susy or LR.

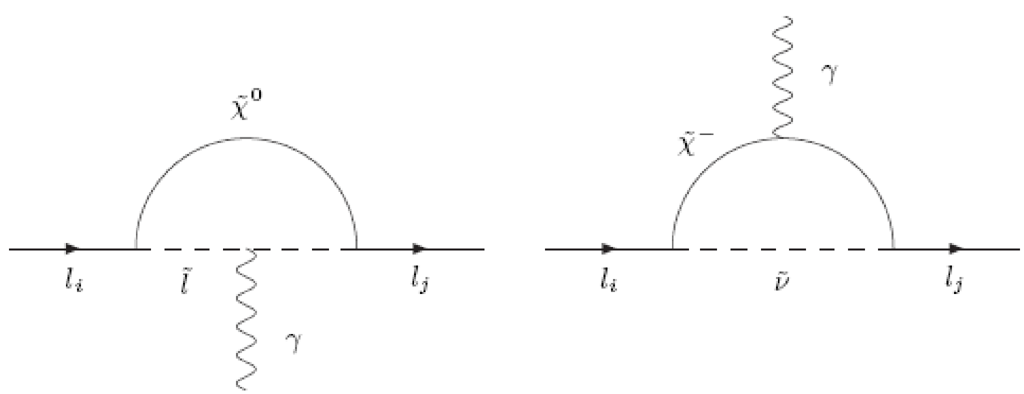
Seesaw with SUSY and LFV

- SUSY assume→ no flavor mixing for sleptons and degenerate mass at GUT scale.
- Extrapolate→ slepton flavors mix:
- Amount: $\delta m_L^2 = -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu^\dagger L Y_\nu)$ $\quad L_{ij} = \ln\left(\frac{M_X}{M_i}\right)\delta_{ij},$

virtual effects of heavy (s)neutrinos



- LFV:

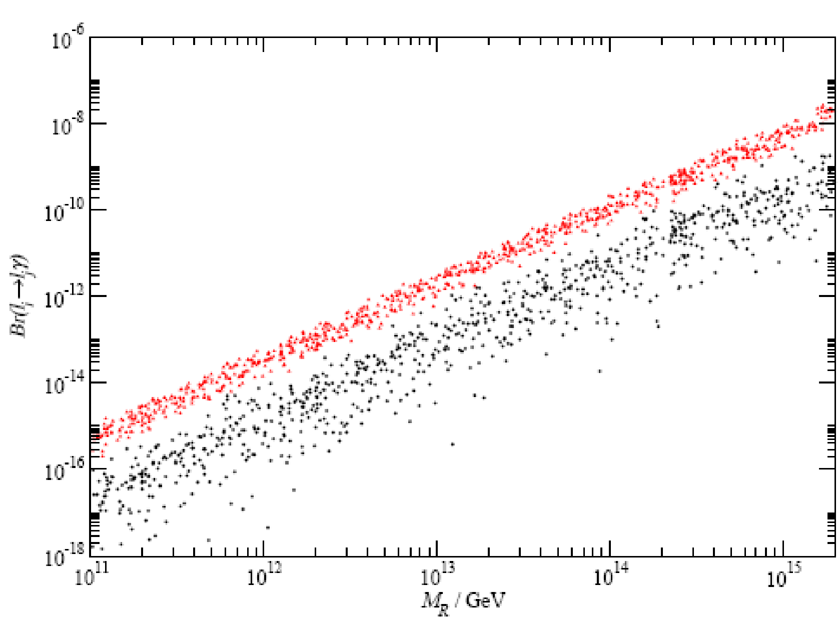


Magnitude:Type I

- Typical branching ratio

$$\Gamma(l_i \to l_j\gamma) \propto \alpha^3 m_{l_i}^5 \frac{|(\delta m_L)_{ij}^2|^2}{\tilde{m}^8} \tan^2\beta$$

$$\frac{Br(\mu \to 3e)}{Br(\mu \to e\gamma)} \approx \frac{\alpha}{8\pi}\frac{8}{3}\left(\ln\frac{m_\mu^2}{m_e^2} - \frac{11}{4}\right)$$



- h increases as MR does.

LFV in type II

- Type II superpotential:

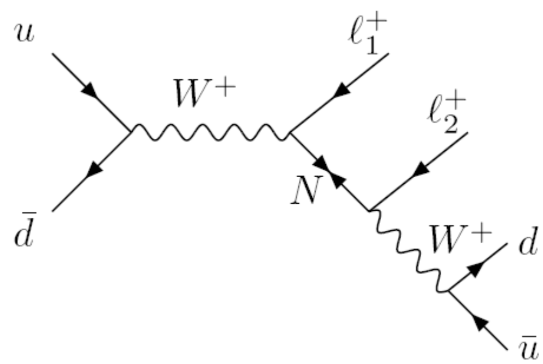
$$W_T = \frac{1}{\sqrt{2}} \mathbf{Y}_T^{ij} L_i T L_j + \frac{1}{\sqrt{2}} \lambda H_2 \bar{T} H_2 + M_T T \bar{T} \qquad \mathbf{m}_\nu^{ij} = \frac{v_2^2 \lambda}{M_T} \mathbf{Y}_T^{ij}$$

- Slepton mixings: $(\mathbf{m}_{\tilde{L}}^2)_{ij} \simeq -\frac{9\, m_0^2 + 3\, a_0^2}{8\pi^2} \left(\mathbf{Y}_T^\dagger \mathbf{Y}_T \right)_{ij} \ln \frac{\Lambda}{M_T}$
- LFV directly measures neutrino mass matrix

LHC signals of seesaw-Type I case

- Are there any observable signals ?
- Seesaw + only sm interactions:

$$L_Y = h_\nu \bar{L} H N_R + M_R N N + L_Y^{SM}$$

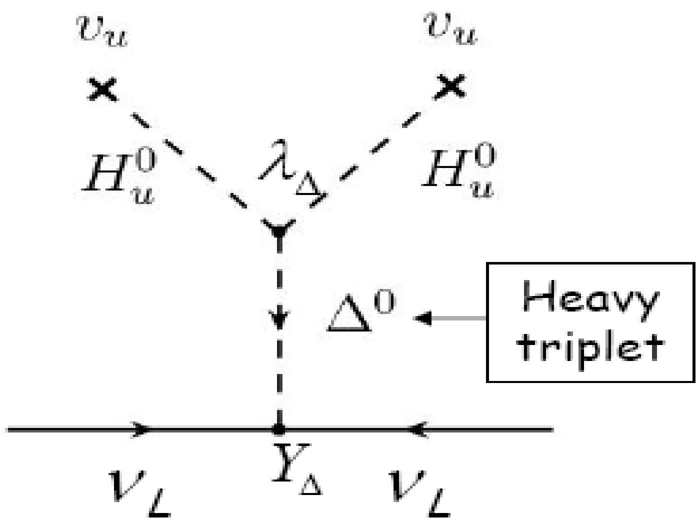


- Del Aguila et al.
- First condition: M_R ~ TeV or less; neutrino masses require $h_\nu \sim h_e \sim 10^{-5.5}$ not more fine tuning than SM.
- Production only through $\nu - N$ mixing. Observable signal requires mixing > 0.01. Typical mixing is $\theta^2 \sim \frac{m_\nu}{M_R} \sim 10^{-12}$; not observable. Situation will change with new forces. Type II, III situation different.

Type II seesaw at LHC

- A heavy **triplet Higgs: TeV mass**
- $(\Delta^{++}, \Delta^{+}, \Delta^{0})$ →
- **L=** $fLL\Delta + \mu HH\Delta$

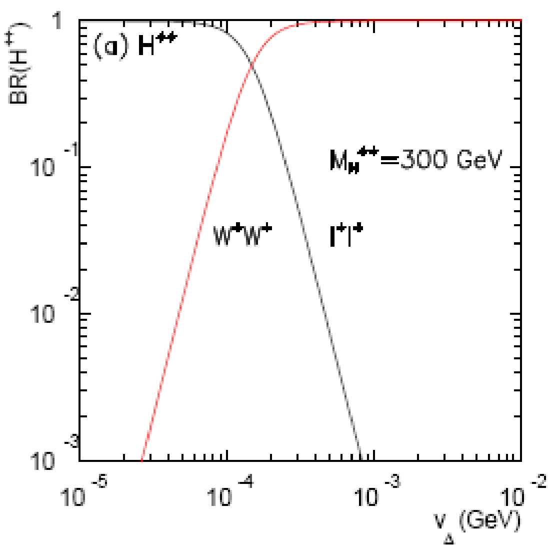
$$m_\nu = \frac{f\mu v_{wk}^2}{M_\Delta^2}$$

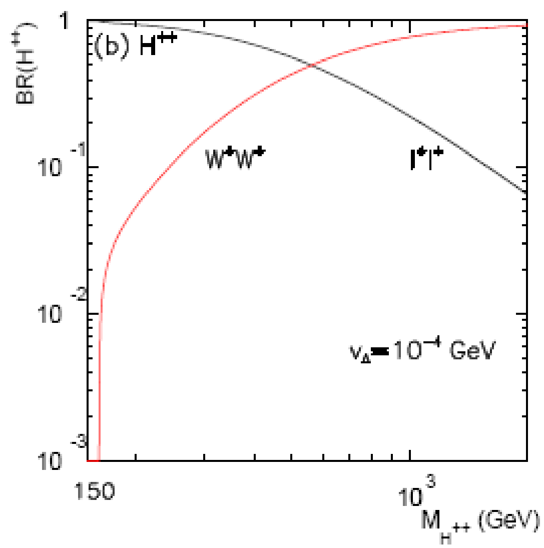


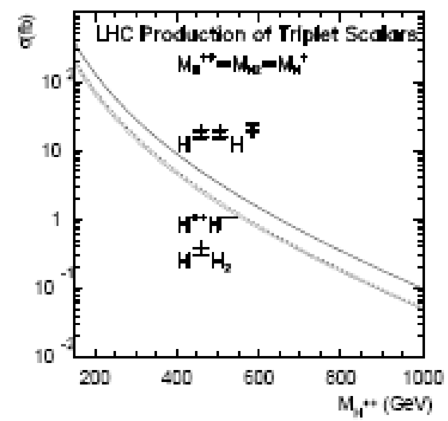
Type II case:

- Direct production of Delta fields: Decay channel

$$\Delta^{++} \rightarrow l^{+}l^{+}, W^{+}W^{+}$$







Type III case:

- Lagrangian:

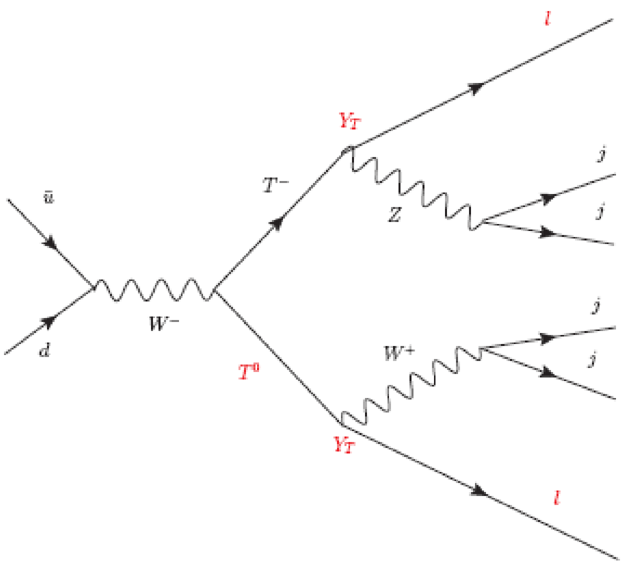
$$L = L_{SM} + h_\nu \bar{l}\,\vec{\tau} \cdot \vec{\Sigma} H + M_\Sigma \vec{\Sigma} \cdot \vec{\Sigma} + h.c.$$

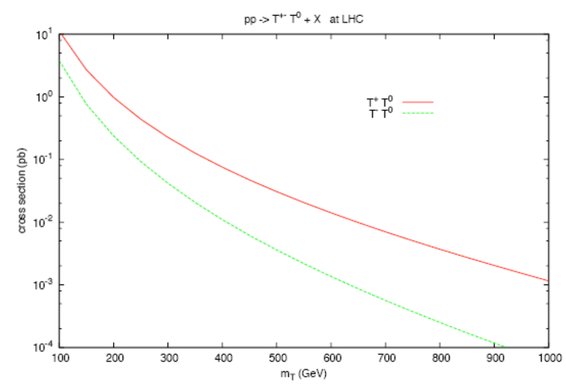
- Consequences:

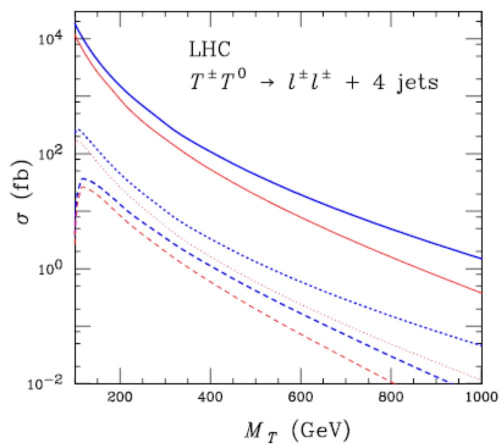
- e- Σ^- mix; as do $\nu - \Sigma^0$

Type III at LHC:

- W-exchange in pp- collision at LHC: Reach <TeV **(Nemevsek, Kamenik, Bajc, Senjanovic…)**







14 TeV 10 TeV

Dotted:
branching fraction.

Dashed:
selection cuts.

New gauge forces likely giving neutrino mass ?

- Seesaw requires new physics below Planck scale.
- A natural understanding of this comes when there is a gauge symmetry whose breaking gives seesaw scale.

- **With gauge forces, seesaw can be visible at LHC.**

- Obvious local symmetry is B-L. Could it be larger ?

Theoretical consistency of adding new particles:

- Adding scalars to SM does not raise new issues. But adding fermions does.
- Anomaly cancellation:

- Type I: it brings new anomaly free group B-L.
- Type III: Anomaly free group can be B-L or different.

Triangle anomaly: Type I

- Gauge theory must be anomaly free:

$$Tr[\theta_a\{\theta_{b,}\theta_c\}]=0$$

- SM satisfies them.

Apply to SM

- Only arbitrary quantum No. Y
- → $Y_l = -3Y_q; Y_u = -2Y_d; 2Y_l = Y_e$

$$Y_u + Y_d = 2Y_q$$

- Any extra U(1)_X is multiple of U(1)_Y.

Emergent Gauge degrees of freedom

- SM: TrU(1)_{B-L}[SU(2)]^2=0

However

$$Tr(B-L)^3 \neq 0$$

- Add nu_R→ Tr [(B-L)^3]=0
- New emergent gauge degree of freedom → B-L

Other reasons for Local B-L

- **Neutrino masses → seesaw scale much lower than Planck scale→ New symmetry (B-L).**
- **Gauged B-L eliminates R-parity problem of MSSM and ensures proton stability and dark matter: Another advantage of B-L** (RNM'86; Martin'92)
- **Extend SM gauge symmetry to include B-L-** many ways-

Inverse seesaw also more natural with gauge forces

- Inverse seesaw case:
- Why $\begin{pmatrix} 0 & hv_{wk} & 0 \\ hv_{wk} & 0 & M \\ 0 & M & \mu \end{pmatrix}$ why not $\begin{pmatrix} 0 & hv_{wk} & h'v_{wk} \\ hv_{wk} & M' & M \\ h'v_{wk} & M & \mu \end{pmatrix}$
- New Gauge symmetry can explain this !!

B-L as a part of left-right symmetry

- **SM:** $\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad u_R \quad d_R$ $\qquad \begin{pmatrix} u_L \\ d_L \end{pmatrix} \overset{P}{\longleftrightarrow} \begin{pmatrix} u_R \\ d_R \end{pmatrix}$
- $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad e_R$ $\qquad$ **add** $\nu_R \rightarrow \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \overset{P}{\longleftrightarrow} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$

$\rightarrow$

(Mohapatra, Pati, Senjanovic)

Left-Right (LR) details

- **New Gauge group**: $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$
- **New W' and Z'** $W_L^{\pm}$ $W_R^{\pm}$ Z, Z', γ
- **Fermion assignment**

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \overset{P}{\Leftrightarrow} \begin{pmatrix} u_R \\ d_R \end{pmatrix} \qquad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \overset{P}{\Leftrightarrow} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

- ***Two Avatars of LR:***
- ⇨ **type I** $\phi(2,2,0)\ ; \Delta_R(1,3,+2) \oplus \Delta_L(3,1,+2)$
  ⇨ **Inverse seesaw** $\phi(2,2,0)$ **+** $\chi_L(2,1,-1) + \chi_R(1,2,-1)$

Parity Violation out of Spontaneous Breaking and electric charge formula

- The weak Lagrangian of model:

$$L = \frac{g}{2}[\vec{J}_L^{\ \mu} \cdot \vec{W}_{\mu L} + \vec{J}_R^{\ \mu} \cdot \vec{W}_{\mu R}]$$

- **Weak Lagrangian Parity Inv.; Low energy parity violation due to** $M_{W_R, Z'} >> M_{W_L, Z}$
- **A more satisfactory formula for Q:**
- **SM:** $Q = I_{3L} + \frac{Y}{2}$ **Y is a free parameter.**
- **LR:** $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$ **All entries physical.**

SEESAW FOR NEUTRINOS:

Case (i)

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & fv_R \end{pmatrix} \qquad \downarrow \; <\Delta_R> \neq 0$$

$$SU(2)_L \otimes U(1)_Y$$

$$\begin{pmatrix} fv_L & h\kappa \\ h\kappa & fv_R \end{pmatrix} \qquad \downarrow \; <\phi> = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \qquad M_{W_L}, M_Z \neq 0; m_{q,l} \neq 0$$

$$U(1)_{em}$$

$$m_\nu \cong fv_L - M^T{}_D M_R^{-1} M_D$$

$m_\nu \to 0$ as $M_{W_R} \to \infty$

Explains small neutrino mass- relates smallness to weakness of V+A forces.

Similar for Inverse seesaw

SEESAW FOR NEUTRINOS:

Inverse Seesaw

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & fv_R \\ 0 & fv_R & \mu \end{pmatrix}$$

$$\downarrow$$

$$< \chi_R >= v_R$$

$$SU(2)_L \otimes U(1)_Y$$

$$\begin{pmatrix} 0 & h_\nu \kappa & 0 \\ h_\nu \kappa & 0 & fv \\ & fv & \mu \end{pmatrix}$$

$$\downarrow$$

$$< \phi >= \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$$

$$M_{W_L}, M_Z \neq 0; m_{q,l} \neq 0$$

$$U(1)_{em}$$

$$m_\nu \cong -m_D^T M^{-1} \mu M^{-1} m_D$$

$$M = fv_R$$

Quark and lepton masses:

- **SM:** $$L_Y = h_u \overline{Q} H u_R + h_d \overline{Q} \tilde{H} d_R + h_e \overline{L} \tilde{H} e_R$$
- **13 parameters;**
- **LR:** $$L_Y = h_{u,d} \overline{Q}_L \phi_{u,d} Q_R + h_{e,\nu} \overline{L} \phi_{d,u} R + f L L \Delta_L + L \leftrightarrow R$$
- **For u,d,e sector same 13 parameters except now Yukawa coupling matrices are hermitean due to LR symmetry.**

Bound on LR scale

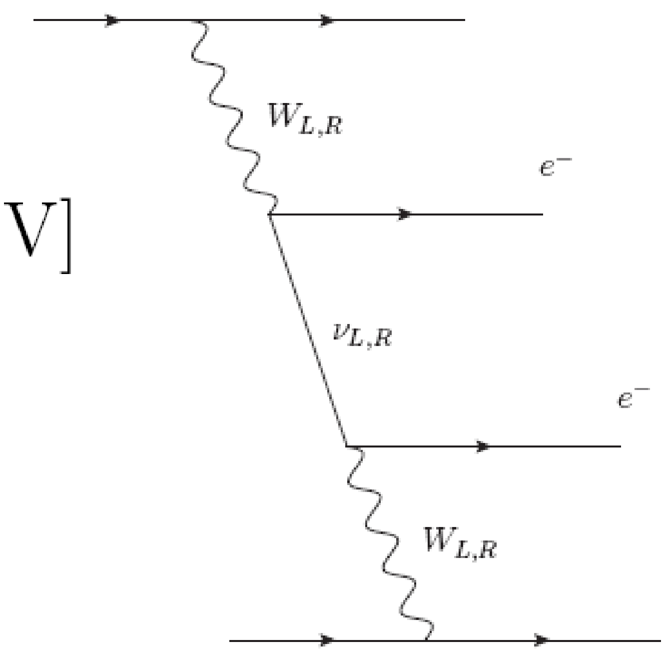
- Low energy observables: combination of KL-KS, epsilon, d_n together.(uncertainty from long distance contribution);
- Parity defined as usual:($\psi_L \leftrightarrow \psi_R$) minimal model:
- $$M_{W_R} \geq 4TeV$$ (An,Ji,Zhang,RNM '07)
- Parity as C (as in SUSY i.e. $\psi \leftrightarrow \psi^c$) (Maezza, Nesti Nemevsek,Senjanovic'10)

$$M_{W_R} \geq 2.5TeV$$

- With SUSY: bounds weaker: > 1 TeV (An, Ji, Zhang'08)
- Collider (CDF,D0) 640-750 GeV.

Bounds from Nu-less double beta decay

- New contributions from WR-N exchange **(only for Case I)** (RNM, 86; Hirsch, Klapdor, Panella 96)
- **Diagram:**



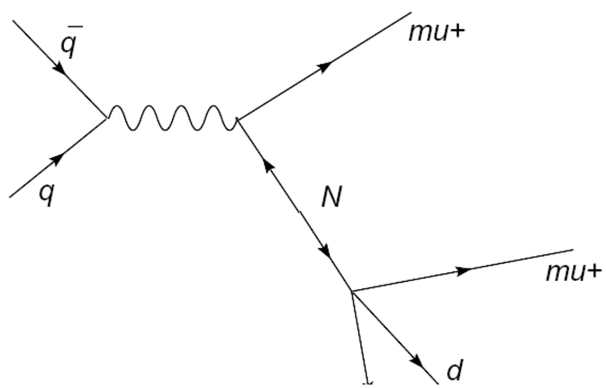
→ $$m_{W_R} \geq 1.1 \left(\frac{\langle m_N^{(V)} \rangle}{1\text{TeV}}\right)^{(-1/4)} [\text{TeV}]$$

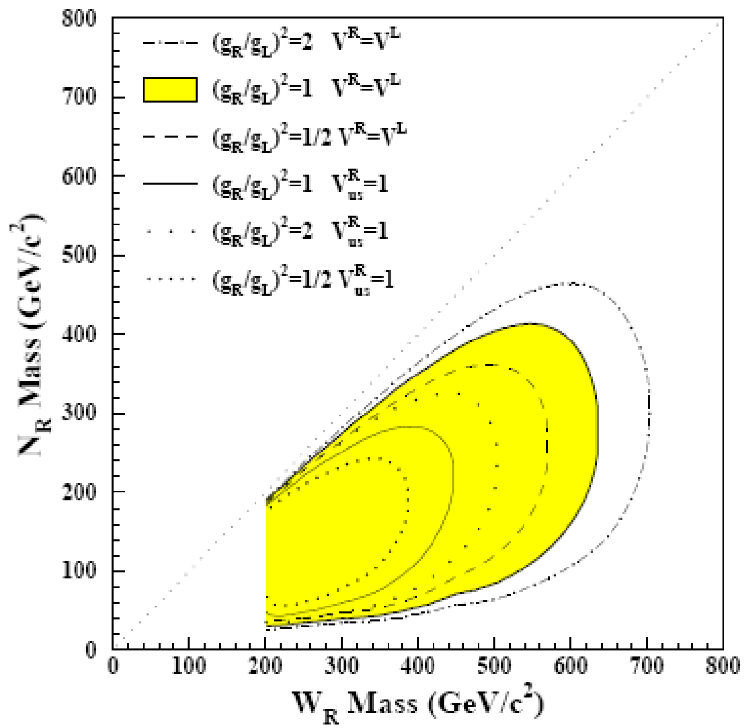
From Ge76:

- **Consistent with WR in the TeV range.**

D0 and CDF (Case I and II)

Collider signal mode:case I (Keung, Senjanovic)





W_R signal: pp->lljj; like sign; similar for Dirac N

(Abachi et al. ; **Phys.Rev.Lett.76:3271-3276,1996.** DO : **700 GeV**

Z’ Mass limit

Different sources for the limits:

- **LEP data, Atomic parity violation**
- **Roughly MZ’ > 800 GeV !** (Langacker,..)

- **WR and Z’ phenomenologically allowe above 2 TeV.**

LHC Signals

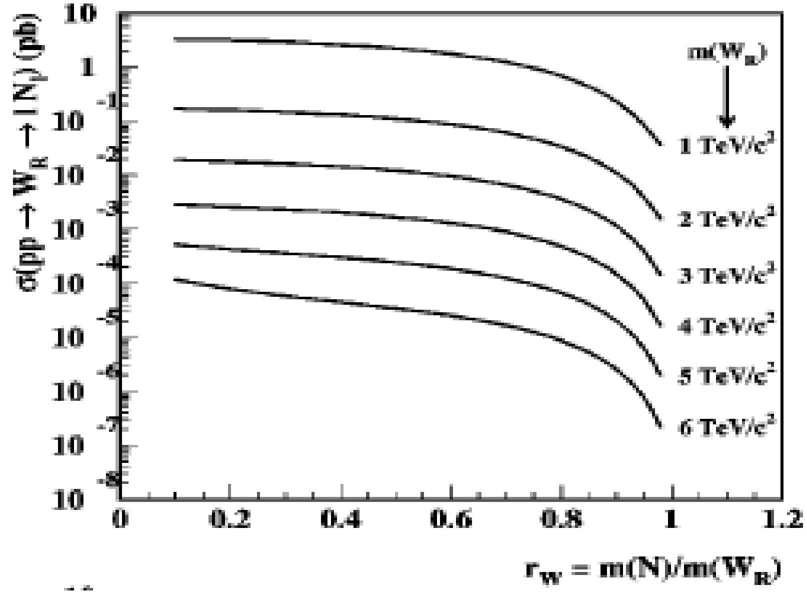
- LHC can access new particles of the model i.e. WR, Z', N
- What are the signatures ?
- (Azuelos et al; Del Aguila, Aguilar-Saavedra; Gnienko et al; Han, Perez et al....)
- Can we rule out GUTs by these observations ?

Collider signal with WR

- Depends on mass of WR; for WR in the few TeV range, N-decay profile changes:
- **No WR case:** $$N \rightarrow \frac{1}{3} l^- jj + \frac{1}{3} l^+ jj + \frac{1}{3} l^+ l^- \not{E}$$
- **With WR (TeV)** $$N \rightarrow \frac{3}{8} l^+ jj + \frac{3}{8} l^- jj + \frac{1}{4} llljj$$
- No missing E in second case;
- **Trilepton signal very sub-dominant.**

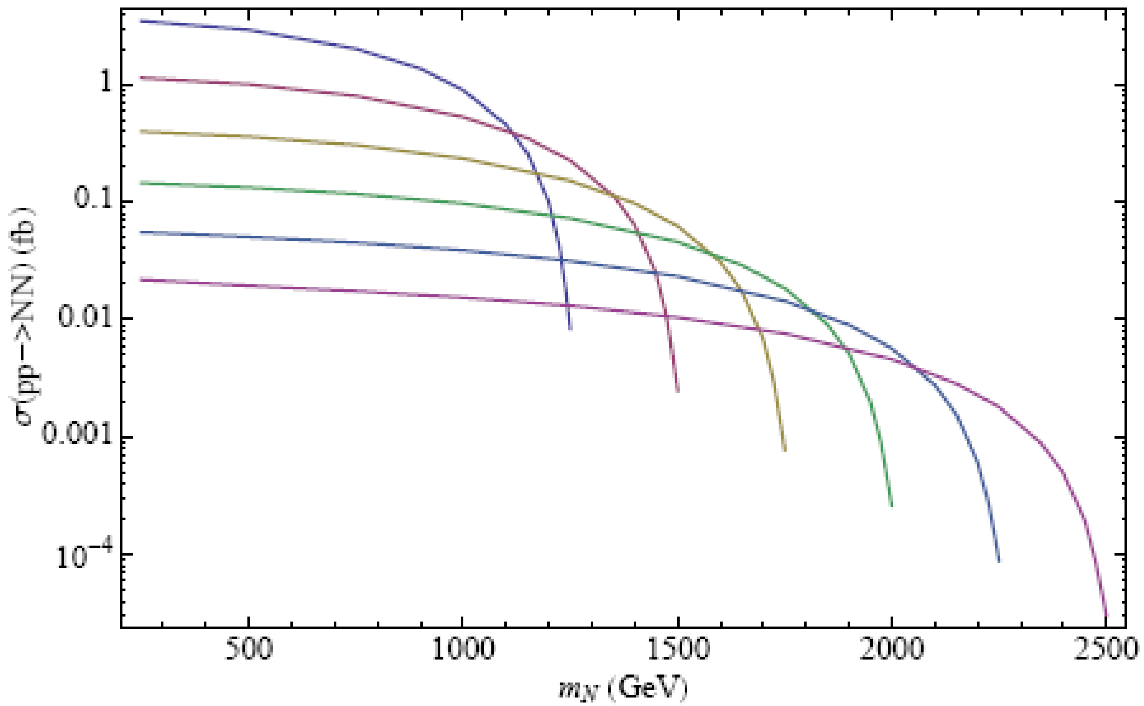
WR production at LHC

- Rates:



TeV Z' cross section at LHC

- **LHC Z' reach - 4 TeV** (Hewett, Rizzo; Petriello, Quackenbush;....)
- Cross section for pp→Z'→NN (Z'→NN branching ratio ~20%)



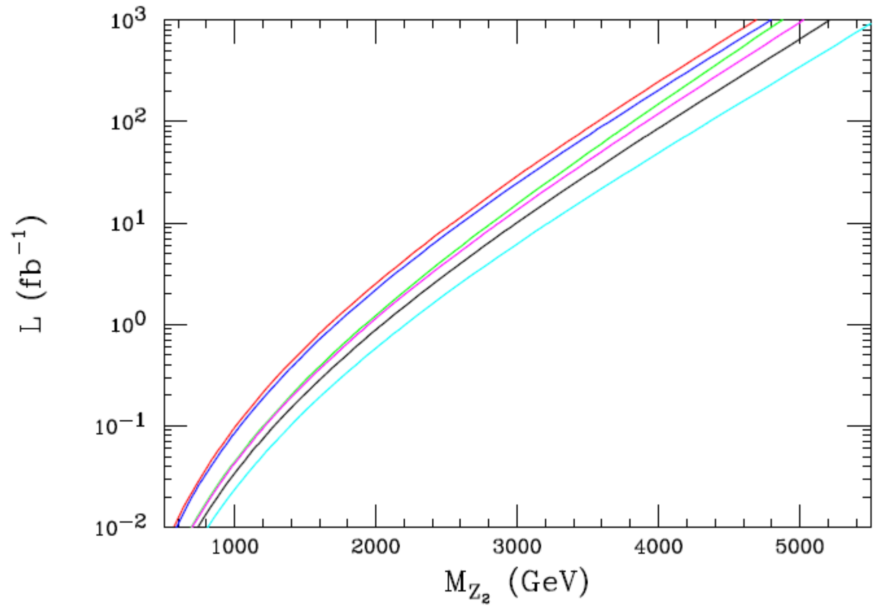
2.5 TeV Z'

to

5 TeV

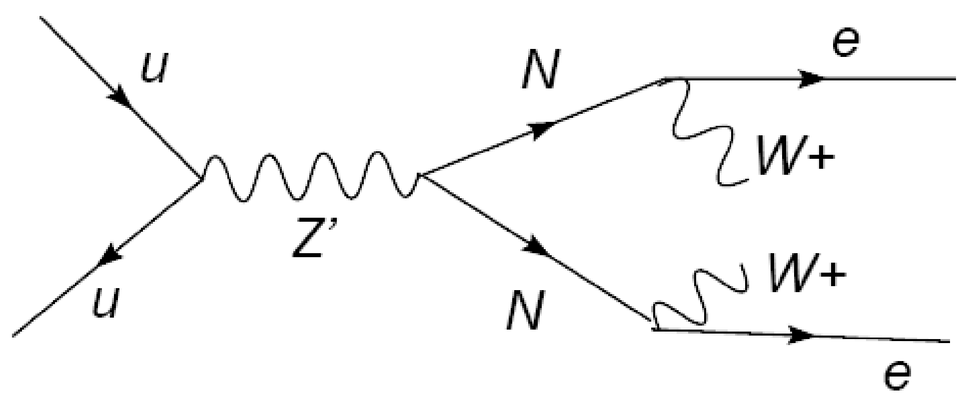
LHC Reach of Z'

- 14 TeV



TeV Seesaw with B-L forces (Z')

- **Seesaw effect observable at LHC even with tiny $\nu - N$ mixings as in generic neutrino models.**
- pp→ Z'+X; Z'→NN followed by N-decay;
- Like sign dileptons is the tell-tale seesaw signal.



LHC Signals for seesaw

- LHC production of WR: $u\bar{d} \rightarrow W_R \rightarrow l^+ N$
  $u\bar{u} \rightarrow Z' \rightarrow NN$
- N-decay gives signals:
- **Type I case:** $N \rightarrow l^\pm jj \;, l^\pm l^\mp \nu$
- Seesaw signals:
  $l^\pm l^\pm + jj; l^\pm l^\mp l \nu + jj$
- **Inverse seesaw:** Only

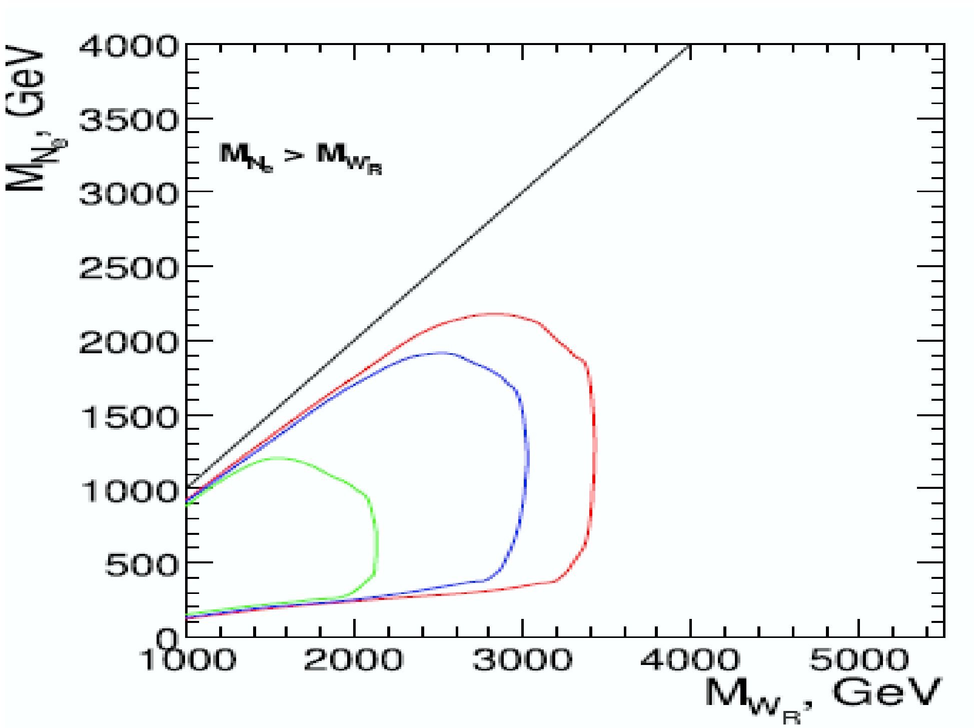
Trileptons; no like sign dileptons

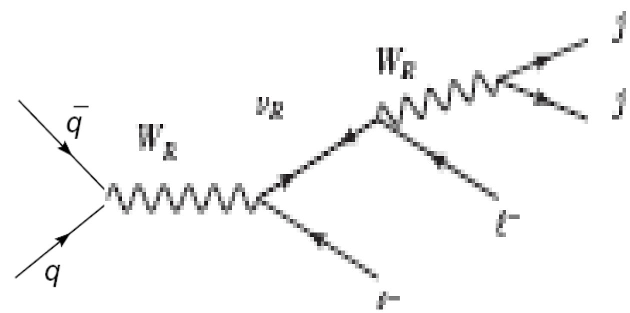
(Aguilar-Saavedra)→

Signals	$\ell^\pm\ell^\pm$	$\ell^\pm\ell^\pm\ell^\mp$	Backgrounds	$\ell^\pm\ell^\pm$	$\ell^\pm\ell^\pm\ell^\mp$
E^+E^- (Σ_M)	1.6	26.3	$t\bar{t}nj$	194	156
$E^\pm N$ (Σ_M)	240.0	192.2	tW	6	6
NN ($Z'_\lambda N_M$)	202.1	252.6	$Wt\bar{t}nj$	12	47
			$Zt\bar{t}nj$	3	20
$E_i^+E_i^-$ (Σ_D)	4.2	80.9	$WWnj$	15	0
$E_i^\pm N$ (Σ_D)	12.3	398.3	$WZnj$	24	38
NN ($Z'_\lambda N_D$)	8.1	481.9	$ZZnj$	4	5
			$WWWnj$	7	12

LHC Reach for WR (Case I)

Azuelos et al; Gnienko et al





Observing this mode via WR decay will rule out simple GUTs.

Testing Left-right type I seesaw with exotic Higgs

- Seesaw requires symmetry breaking by B-L=2 Higgs:

$$\Delta = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^{+} & \Delta^{++} \\ \Delta^{0} & -\frac{1}{\sqrt{2}}\Delta^{+} \end{pmatrix}$$

- Doubly charged Higgs which can have sub-TeV mass.
- Very different from known Higgs in that it couples only to leptons and <u>not</u> to quarks: Coupling not small.
- One coupling to left and another to the right sector:
- Both decay to lepton pairs (from $LL\Delta$ coupling)

$$\Delta^{++} \rightarrow \mu^{+}\mu^{+}, ee, \tau\tau \qquad \Delta^{-} \rightarrow \mu^{-}\nu_{\mu}, e^{-}\nu_{e}, \tau^{-}\nu_{\tau}$$

Present lower bounds on doubly charged Higgs mass:

- Drell-Yan pair production main mechanism at hadron colliders: Signal: pp --> $\tau^-\tau^-\ \mu^+\mu^+$ or all muon
- Collider: CDF, D0: $M_{\Delta^{++}} \geq 136$ GeV
- HERA > 141 GeV
- Low energy: Muonium-anti-muonium osc. (PSI)
- $A_{\mu^+e^-\to\mu^-e^+} \leq 3G_F \times 10^{-3} \approx \frac{f_{ee} f_{\mu\mu}}{8M_\Delta^{\ 2}}\sqrt{2}$ PRISM goal $G_F \times 10^{-4}$
- For $f_{ee} \approx f_{\mu\mu} \approx 0.1$, M++ >250 GeV..

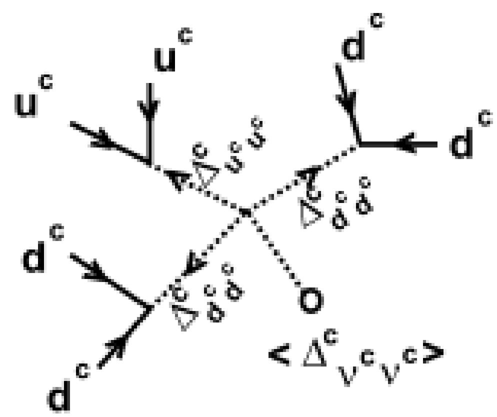
Q-L unify TeV seesaw

- SU(2)$_L$xU(1)$_R$xU(1)$_{B-L}$ $\subset$ SU(2)$_L$xU(1)$_R$xSU(4)$_{PS.}$
- $$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \begin{matrix} u_R \; \nu_R \\ d_R \; e_R \end{matrix} \rightarrow \begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R};$$
- (Pati, Salam)
- **Recall** Origin of RH nu mass for seesaw is from $NN\Delta_{\nu_R\nu_R}$
- **Q-L unif. implies quark partners for** $\Delta_{\nu_R\nu_R}$ **i.e.** $\Delta_{u^c u^c}$ **- <u>color sextet scalars coupling to up quarks</u>** ; similar for dd- only right handed quarks couple. **Come from (1, 3, 10)**
- SU(4)$_{PS}$ **breaks to** U(1)$_{B-L}$ **above 100 TeV**

Baryon violation graph

- $$\mathcal{L}_I = \frac{h_{ij}}{2}\Delta_{d^c d^c} d^c_i d^c_j + \frac{l_{ij}}{2}\Delta_{u^c u^c} u^c_i u^c_j + \lambda\, \Delta_{u^c u^c}\Delta_{d^c d^c}\Delta_{d^c d^c}\Delta_{\nu_R \nu_R}$$
  $$+ \text{ h. c.}$$
- ΔB=2 but no ΔB=1; hence **proton is stable** but neutron can convert to anti-neutron!
- N-N-bar diagram

  (Marshak, RNM'80)



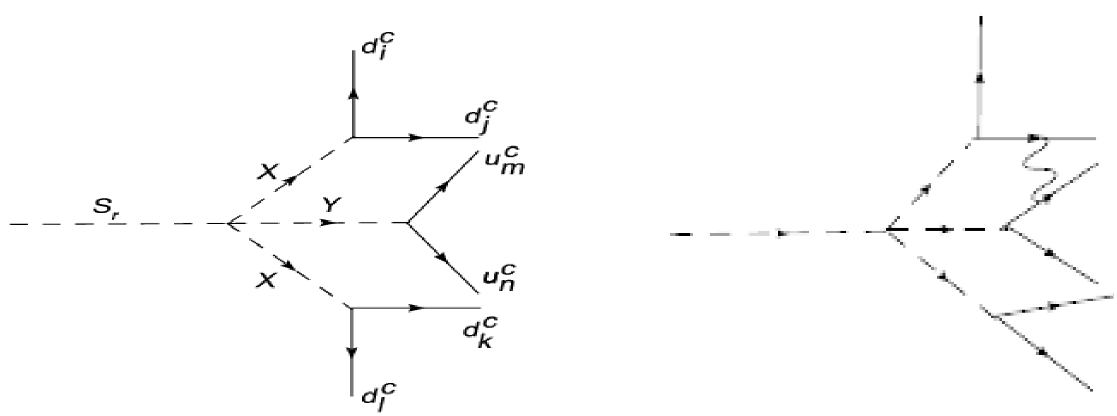
- λ coupling crucial to get baryogenesis (see later)

A new low scale Scenario for Origin of matter

- (Babu, Nasri, RNM, 2006)
- Call Re $\Delta_{\nu_R \nu_R}$ = S**r** ; TeV mass : S-vev generates seesaw

  Baryon number is broken once $\langle S \rangle \neq 0$

  leading to B-violating decays $S_r \to 6q, \quad S_r \to 6\overline{q}$
- **Baryogenesis:** Due to high dimension of operator, B-violating process goes out of eq. below 100 GeV.



Upper limits on S_r and color sextet masses:

- Two key constraints:

$$\frac{\epsilon_B^{\text{vertex}}}{\text{Br}} \simeq -\frac{\alpha_2}{4}\frac{6\ \text{Im}\ [f_{31}^2 m_t V_{tb} m_b f_{33}^* m_t V_{tb} m_b]}{(\text{Tr}[f^\dagger f])^3 M_W^2 M_S^2}$$

→ M_S < 500-700 GeV **to get right amount of baryons.**

- **Decay before QCD phase transition temp:**

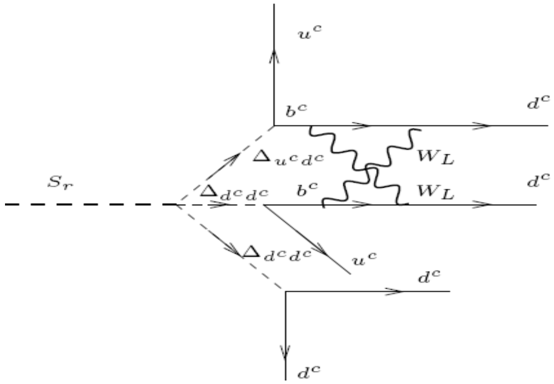
$$\Gamma(S_r \to 6q) \simeq \frac{18P\lambda_2^2 h^2 g^2 M_{S_r}^{13}}{(2\pi)^9 (6M_X)^{12}}$$

- **Implies $M_S < M_X < 2\,M_S$.**

Two experimental implications:

- $n-\bar{n}$ **oscillation:** successful baryogenesis implies that color sextets are light (< TeV) (Babu, RNM, Nasri,06; Babu, Dev, RNM'08);

  $n-\bar{n}$ **arises via the diagram:**



$$\tau_{n\bar{n}} \approx 10^9 - 10^{11}\,\text{sec}.$$

- **Present limit: ILL >10^8 sec. similar bounds from Soudan,S-K etc.**
- **10^11 sec. reachable with available facilities !!**
- **A collaboration for NNbar search with about 40 members exists-Exploration of various reactor sites under way for a second round search.**

Color sextet scalars at LHC

- Low seesaw scale + baryogenesis requires that sextet scalars must be around or below a TeV:
- Two production modes at LHC:

(I) **Single production**: $uu \to \overline{\Delta}_{u^c u^c} \to tt$ or $t + \text{jet}$

xsection calculated in (RNM, Okada, Yu'07;) resonance peaks above SM background- decay to tt or tj depending on RH nu Majorana coupling; directly measures seesaw parameters.

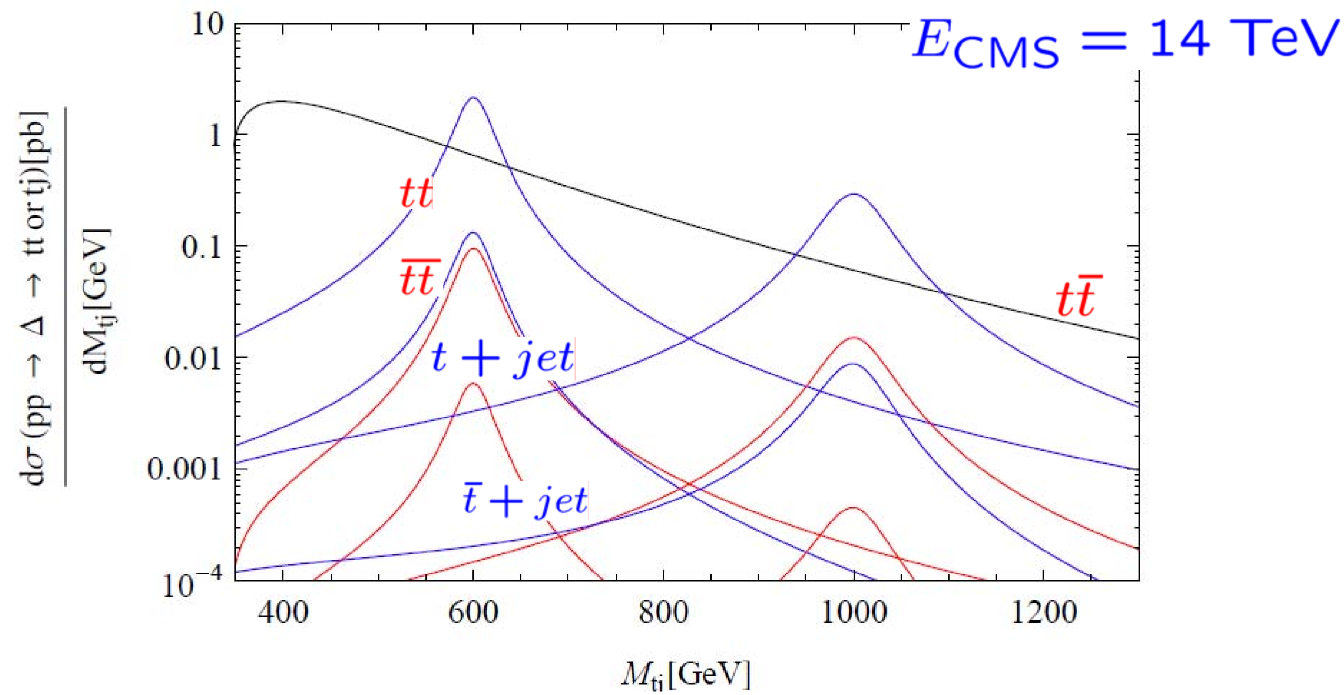
(II) **Drell-Yan pair production**: $q\bar{q} \to G \to \Delta_{u^c u^c} \Delta^*_{u^c u^c}$

(Chen, Klem, Rentala, Wang, 08)

- Leads to $tt\bar{t}\bar{t}$ final states: **LHC reach < TeV**

SINGLE SEXTET PRODUCTION AT LHC:

$$f_{ij} = \begin{bmatrix} 0.3 & 0 & 0.3 \\ 0 & 0 & 0 \\ 0.3 & 0 & 0.3 \end{bmatrix}$$



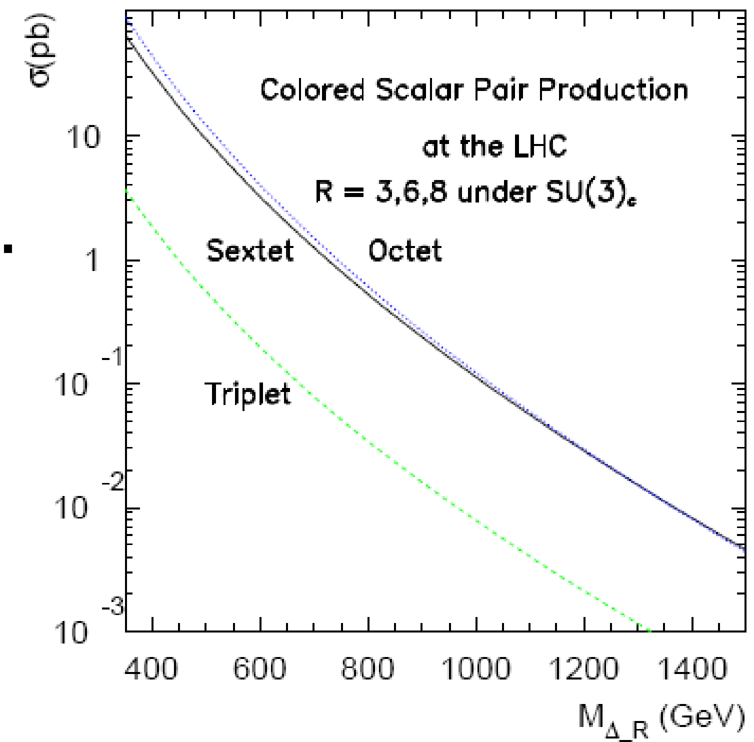
Diquark has a baryon number & LHC is ``pp'' machine

→ $\sigma(tt) \gg \sigma(\overline{tt}), \quad \sigma(t + jet) \gg \sigma(\bar{t} + jet)$

Depends on Yukawa coupling

Pair Production of Deltas

- Due to color sextet nature, Drell-Yan production reasonable- independent of Yukawa coupling
- Leads to $tt\bar{t}\bar{t}$ final states:
- Can be probed upto a TeV using like sign dilepton mode.

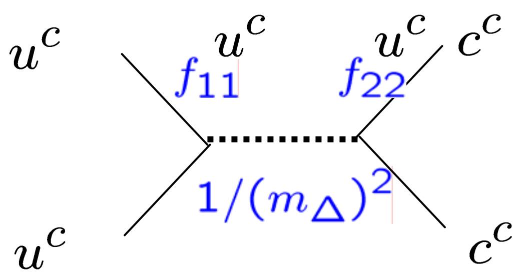


Phenomenological Aspects

$$W_Y \supset f\psi^c\Delta^c\psi \to f_{ij}\Delta_{u^cu^c}\, u_i^c u_j^c + f_{ij}\Delta_{d^cd^c}\, d_i^c d_j^c + \dots$$

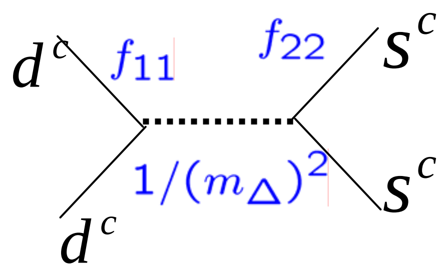
<u>**Constraints by rare processes**</u>

$D^0 - \overline{D^0}$ **mixing**



$\Delta_{u^cu^c}$ *exchange*

$K - \overline{K}$



$\Delta_{d^cd^c}$

Similarly B-B-bar etc. Can generate neutrino masses - satisfying FCNC

Details of FCNC constraints:

- **Hadroni**

$$\frac{f_{uu_{11}} f_{uu_{22}}}{[m_{\Delta^0_{uu}}(\mathrm{TeV})]^2} \leq 1.26 \times 10^{-6}$$

$$\frac{f_{dd_{11}} f_{dd_{22}}}{[m_{\Delta^0_{dd}}(\mathrm{TeV})]^2} \leq 2.2 \times 10^{-6}$$

$$\frac{f_{dd_{22}} f_{dd_{33}}}{[m_{\Delta^0_{dd}}(\mathrm{TeV})]^2} \leq 1.29 \times 10^{-4}$$

$$\frac{f_{11_{dd}} f_{33_{dd}}}{[m_{\Delta^0_{dd}}(\mathrm{TeV})]^2} \leq 5.42 \times 10^{-6}$$

$\mu \rightarrow e + \gamma$

$$\frac{f_{11} f_{12}}{[m_{\Delta^{++}}(\mathrm{TeV})]^2} = G_F \sqrt{BR_1} \leq 1.17 \times 10^{-5}$$

Examples of color sextet couplings that work.

- Down sector

$$f_{dd} = \begin{pmatrix} 0 & 0.95 & 1 \\ 0.95 & 0 & 0.01 \\ 1 & 0.01 & -0.0627357 \end{pmatrix} 10^{-2}$$

$$f_{uu} = \begin{pmatrix} .3 & * & * \\ * & 0 & * \\ * & * & .3 \end{pmatrix}$$

- Fits neutrino mass via type I seesaw.

Is TeV Seesaw compatible with leptogenesis ?

- Leptogenesis details:

- Buchmuller, Di Bari, Plumacher papers:

Basic idea

- **Proposal:** Heavy ν_R decays: (Fukugita and Yanagida ,1986)

$$\nu_R \rightarrow L + H \qquad R = (1+\varepsilon)$$

$$\nu_R \rightarrow \overline{L} + \overline{H} \qquad \overline{R} = (1-\varepsilon)$$

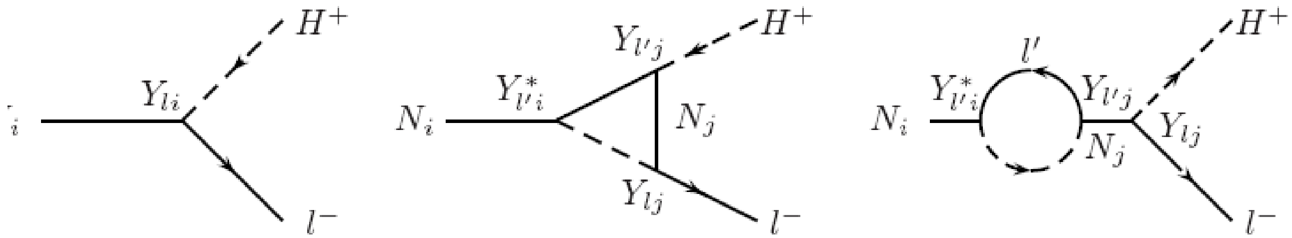
- **Generates lepton asymmetry:**
- **Gets converted to baryons via sphaleron interactions;**

(Kuzmin,Rubakov,Shaposnikov)

- **No new interactions needed other than those already used for generating neutrino masses !!**
- **Seesaw provides a common understanding of both neutrino masses and origin of matter in the Universe.**

Leptogenesis: High vs Low scale

- **Diagrams:**



- Two classes of models depending on RH mass pattern
- **High Scale leptogenesis**: Expected in GUT theories: Adequate asymmetry→ $M \geq 10^9\, GeV$ for lightest RH (for hierarchical masses) (Buchmuller, Plumacher,di Bari; Davidson, Ibarra)
- **Resonant leptogenesis**: **degenerate** N's, self energy diagram dominates: $\sim \frac{1}{M_i^2 - M_j^2 + M\Gamma}$; Resonance when $M_i \cong M_j$;works for **all B-L scales**.

(Liu and Segre'94; Covi et al'95 ; Flanz et al.'95 Pilaftsis'97)

An Issue with High scale SUSY Leptogenesis

- Recall the lower bound on the lightest RH neutrino mass $M_N \geq 10^9 GeV$ for enough baryons in GUTs
- **Problem for supersymmetric models:**

  they have gravitinos with TeV mass that are produced during inflation reheat along with all SM particles-
- **Will overclose the universe if stable for $T_R > 10^9$ GeV.**
- **If unstable, Once produced they live too long -affect BBN.** $T_R \leq 10^{6-7} GeV$. (Kohri et al.)
- **No such conflict for TeV scale resonant leptogenesis !! Goes well with TeV seesaw !**

Does leptogenesis work with TeV Z' and WR ?

- **Conditions:**

(i) RH neutrinos must be degenerate in mass to the level of $M_1 - M_2 \sim 10^{-10} M$ since h~10^-5 ;

(ii) Since there are fast processes at that temperature, the net lepton asymmetry and primordial lepton asym are related by

$$\eta_B \simeq 10^{-2} \sum_{i,\alpha} \varepsilon_{i\alpha}\, \kappa_{i\alpha}$$

where κ <1- depends on Z' mediated $e^+e^- \to NN$ and inverse decay $lH \to N$

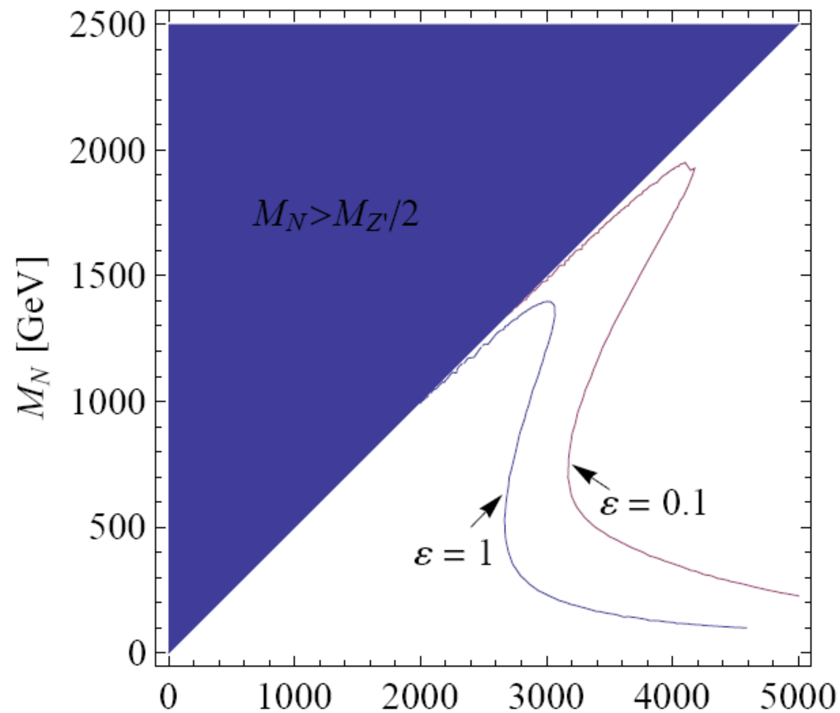
Not clear that a TeV scale Z' is even allowed by baryogenesis due to rapid rates ?

Lower bound on Z' mass from leptogenesis

- Lower the Z' mass, faster the scattering and less the efficiency

implying a lower limit on Z' mass !!

- (Blanchet, Chacko, Granor, RNM: arXiv:0904.2974)



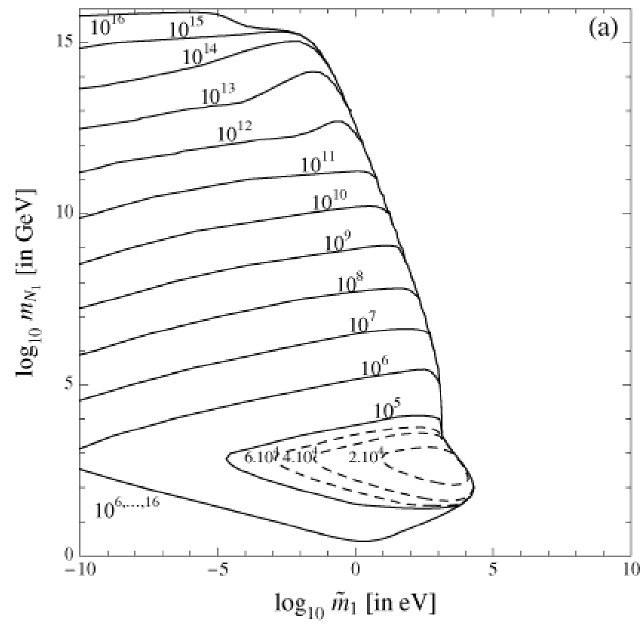
★ $M_{Z'} > 2.5$ -3.2 TeV for $M_{Z'} > 2M_N$ (Accessible at LHC)

WR limit for leptogenesis: case I

- M_{W_R} > 18 TeV, L-violating scatterings e.g.

$$e_R + u_R \rightarrow N + d_R$$ **will** erase lepton asymmetry.

(Frere, Hambye and Vertongen)



Weaker limit for Inverse seesaw, since L=2 Suppressed by mu !!

What if RH neutrinos are TeV scale but non-degenerate ?

- Can one have seesaw scale around a TeV so LHC can see it and still understand the origin of matter related to seesaw physics ?

- Yes- baryogenesis can arise from seesaw related physics below 100 GeV (but not from RH N decay) (post-sphaleron baryogenesis)

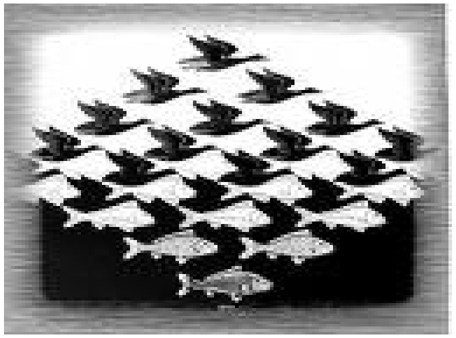
  (Babu, RNM, Nasri'06)
- Predicts light color sextet Higgs (< TeV) that can be observed at LHC via decay to two tops.

SUSY GUTs and Neutrino Mass Physics

Lecture III

Grand unification

Hypothesis: **all forces and all matter become one at high energies no matter how different they are at low energies.** Leptons→



quarks→

become same.

----Aestetically appealing

----Explains charge quantization;

----High scale goes well with ideas in cosmology ;

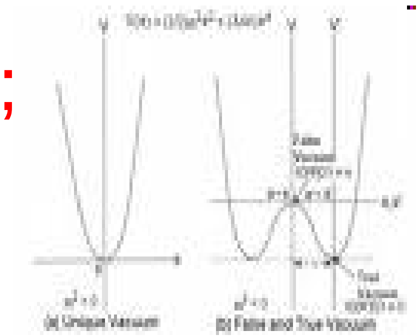
Supersymmetric Route

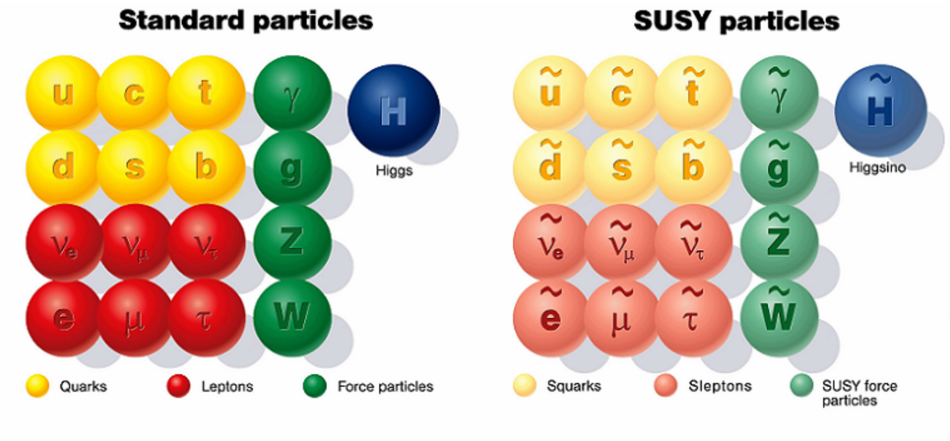
- **We follow the supersymmetry route;**

(i) It stabilizes the Higgs mass ;

(ii) Explains <H> via radiative corrections;

(iii) Provides a dark matter candidate



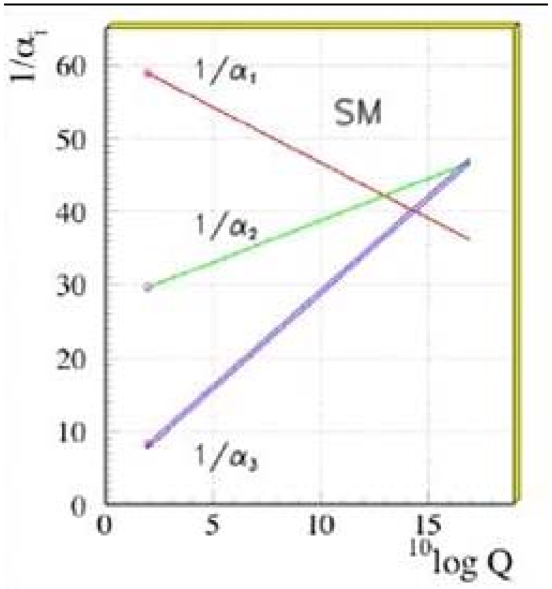


Coupling unification formulae:

- Renormalization determines running: For gauge theories with fermions and scalars,
- $$\frac{d\alpha_i}{dt} = \frac{\beta(\alpha_i)}{2\pi}$$

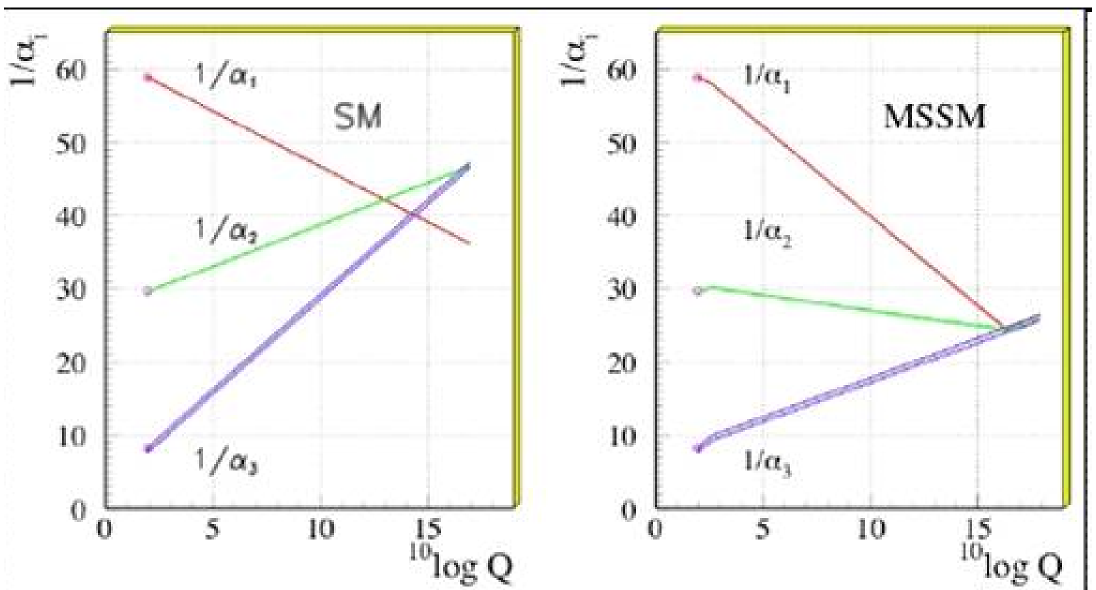
SUSY $\beta(\alpha) = -3N + 2n_g + T_H$

non-susy: $\beta(\alpha) = -\frac{11}{3}N + \frac{4}{3}n_g + \frac{1}{3}T_H$



Supersymmetry hypothesis provided extra boost

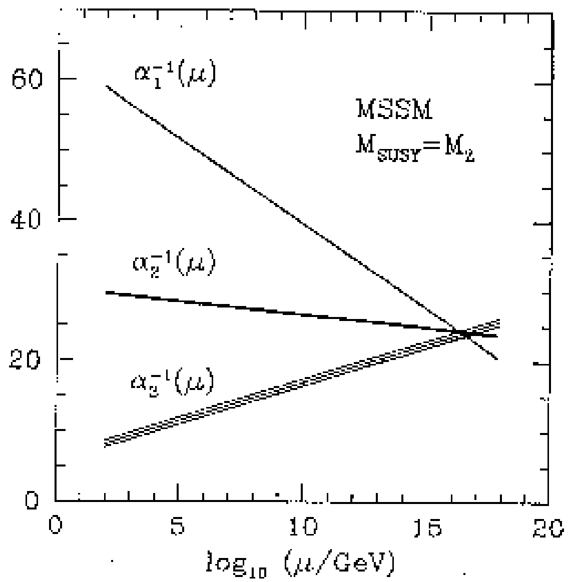
- **Coupling unification in MSSM** (Dimopoulos, Raby, Wilczek)

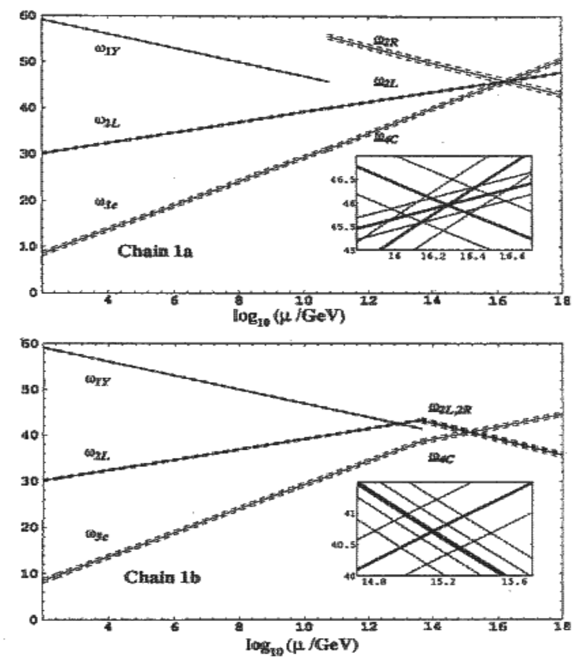


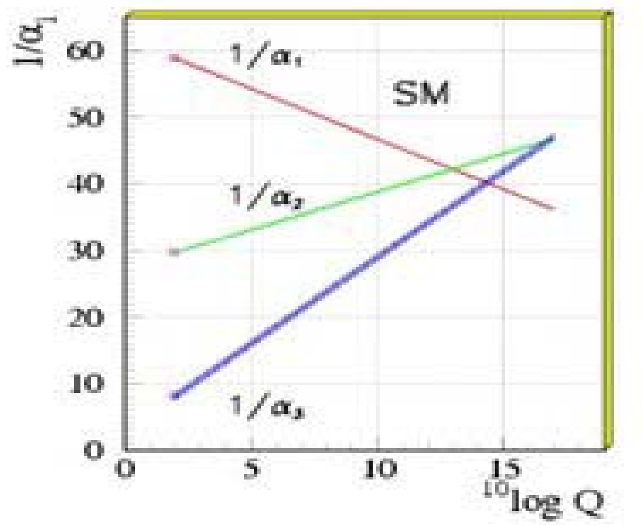
- → There could be one grand unifying group, raising the hope for predicting parameters of SM (e.g. fermion masses)

Some examples:

- SUSY — Non-SUSY SO(10) with seesaw — SM







Simplest SUSY GUT: SU(5)

- The simplest GUT model (circa 1980s)
  - Fermions: $5 = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e^- \end{pmatrix}$ and $10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & u_3 \\ & & 0 & u_3 & d_3 \\ & & & & e^+ \\ & & & & 0 \end{pmatrix}$
  - : Higgs $5 \oplus 5 \oplus 24$.
  - Predicts: at M_U, $m_b = m_\tau$; very good prediction

    Also predicts $m_s = m_\mu$; $m_d = m_e$; VERY BAD PREDICTION!!
  - No explanation of neutrino mass:

How to explain neutrino mass ?

Other pros and cons of SUSY SU(5)

- **Pros:**

  (i) Stabilization of weak scale

  (ii) Radiative EWSB

  (iii) Candidate for Dark matter with R-parity

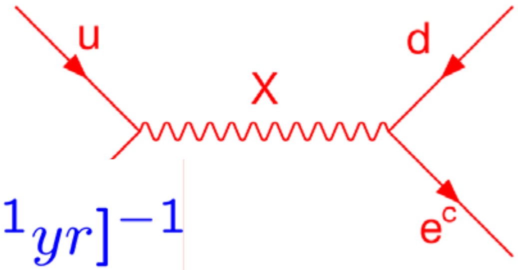
- **Cons**

  (i) No understanding of origin of matter

  (ii) Large edm of neutron (SUSY CP)

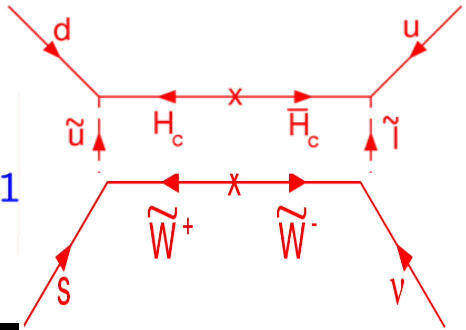
Proton decay Problem for SUSY GUTs

- **Proton decay in SUSY GUTs have two generic sources:**
- **(i) Gauge exchange:**



$$p \to e^+\pi^0,\ \tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [10^{36\pm1} yr]^{-1}$$

- **(ii) Higgsino exchange:**



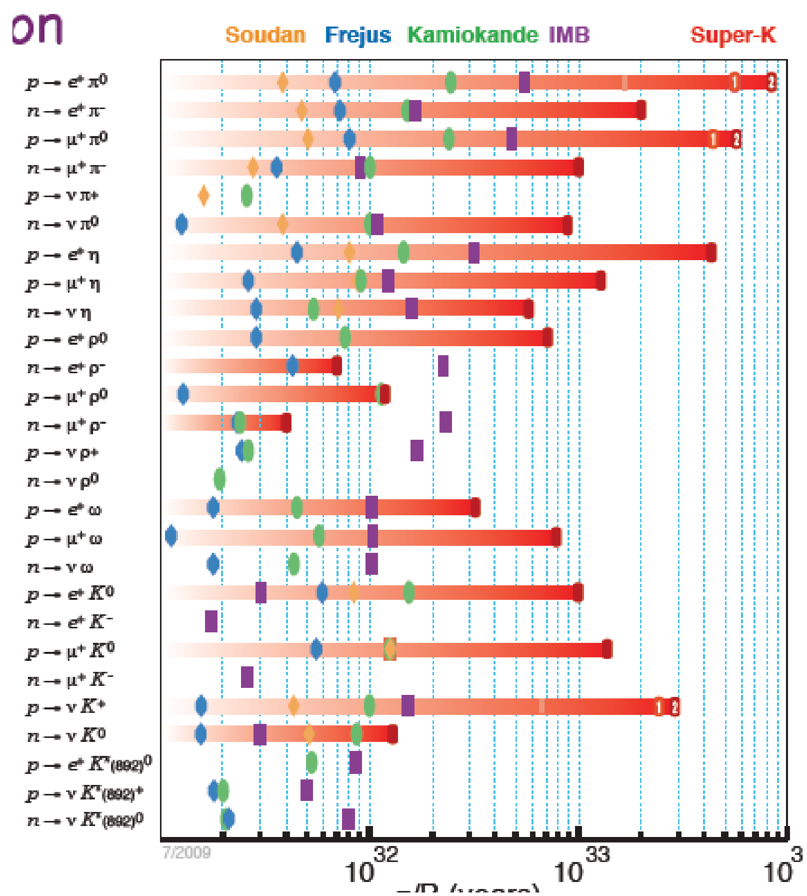
$$p \to \bar{\nu} K^+$$

$$\tau_p^{-1} \approx [\frac{f^2}{M_{H_c} M_{SUSY}}]^2 (\frac{\alpha}{4\pi})^2 m_p^5 \approx [10^{28} - 10^{32} yr]^{-1}$$

- **Second graph too large: SUSY GUT problem; any model must address this issue.**

Experimental status of p-decay

- No evidence for it yet.



Plan of the Talk:

- For **SUSY GUT** program to fulfill its promise, it must be part of a bigger theory that preserves its good features (gauge hierarchy, coupling unification, dark matter) and cure the “bad” ones **e.g.** neutrino mass, susy CP etc.
- To address nu-mass, the GUT group must contain B-L;
- Minimal group SO(10).

B-L Cures proton decay problem of MSSM

- **SM has stable proton- but MSSM takes a step backward !! protons decay in an instant in MSSM.**
- **Culprit: R-parity breaking terms**

$$W' = \underset{\lambda}{LLe^c} + \underset{\lambda'}{QLd^c} + \underset{\lambda''}{u^c d^c d^c}$$

- SUSYLR either version does not allow the last term in renormalizable part. dim-5 term $\frac{1}{M_{Pl}} Q^c Q^c Q^c \chi^c$ - suppressed for TeV inverse seesaw

SO(10)

- Fermion unification:

- {16}-dim. Spinor:
- Includes RH nu:
- (Georgi; Fritzsch, Minkowski)

$$\begin{pmatrix} u \\ u \\ u \\ d \\ d \\ d \\ \nu \\ e \\ u^c \\ u^c \\ u^c \\ d^c \\ d^c \\ d^c \\ \nu^c \\ e^c \end{pmatrix}$$

Symmetry breaking

- GUT sym breaking down to SM: many ways:
- Two popular ones:

  - $SO(10) \xrightarrow{M_G} \mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y(\mathrm{MSSM}) \xrightarrow{M_{\mathrm{SUSY}}} \mathbf{3}_c\mathbf{2}_L\mathbf{1}_Y(\mathrm{SM}) \xrightarrow{M_Z} \mathbf{3}_c\mathbf{1}_Q$

$M_U \geq 10^{16}\,GeV$

  - $SO(10) \xrightarrow{M_G} 3_c2_L2_R1_{B-L} \xrightarrow{M_R} 3_c2_L1_Y(\mathrm{MSSM}) \xrightarrow{M_{\mathrm{SUSY}}} 3_c2_L1_Y(\mathrm{SM}) \xrightarrow{M_Z} 3_c1_Q$

$M_U \geq 10^{16}\,GeV; M_R \approx TeV$

Challenges for TeV scale SUSYLR Grand Unification

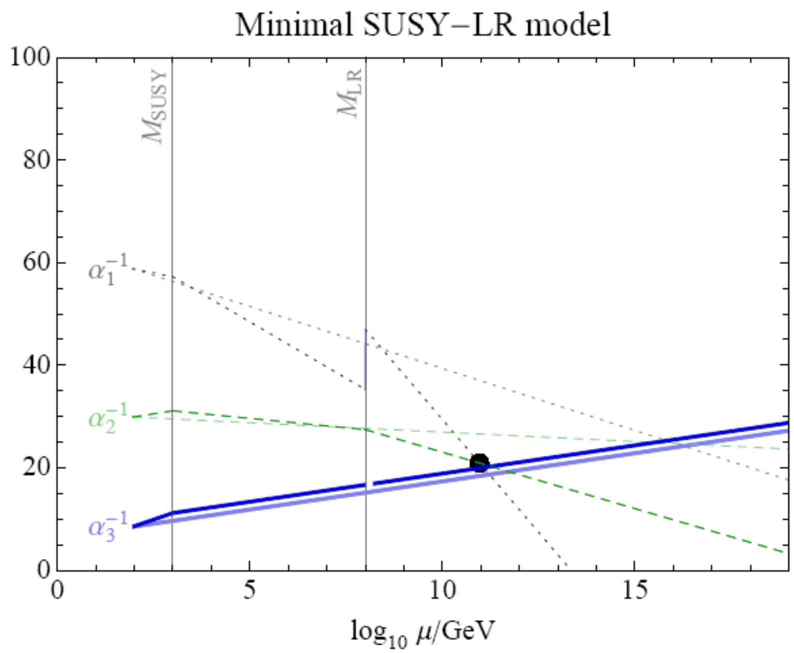
- Running of couplings determined by field content below a scale:
- MSSM at TeV gives right running fir unification; so any new particles (e.g. WR, Z' etc.) will change this and ruin unification.
- To check for unification of any new theory with LR,

Define a GUT indicator: **X**U $\equiv 50b_{2L} - 12.6b_{2R} - 8.4b_{BL} - 3b_{3c}$

If **XU=0 to1, theory unifies.**

- **Type I:** $X_U^{typeI} \approx -90$ **No unification possible.**
- **Inverse seesaw** $X_U^{inverse} \approx -1$ **Unif. OK**

Unification of TeV Type I seesaw does not unify:

- **Does not unify** to SO(10)- too rapid proton decay: **(Kopp, Lindner, Niro,Underwood'09) (Parida, Raichoudhuri, Majee, Sarkar'08)**
- 

Location of Landau pole

M_R (GeV)	μ_0 (GeV)
10^3	7.76×10^{13}
10^5	4.56×10^{14}
10^7	2.56×10^{15}
10^8	6.16×10^{15}
10^9	1.44×10^{16}
10^{10}	3.46×10^{16}
10^{11}	8.31×10^{16}

- Culprit: B-L=2 triplets have high b-coefficient.

New TeV scale SUSYLR theory with gauge unification

- **Requirements:**

(i) B-L and LR breaking at TeV scale;

(ii) Two bidoublets at TeV scale to get realistic fermion masses;

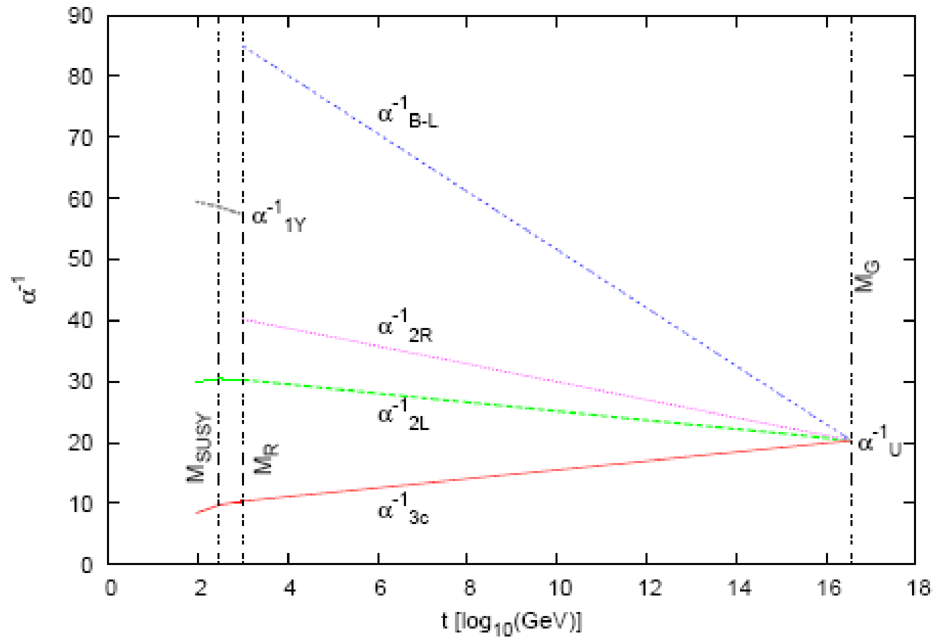
(iii) at least one RH doublet for Inverse seesaw and B-L breaking.

(iv) All multiplets used must be part of an SO(10) multiplet required at GUT scale.

SO(10)Unification with TeV Inverse Seesaw (LR)

- **Inverse seesaw** does unify and give realstic model: with both WR and Z' in TeV range;

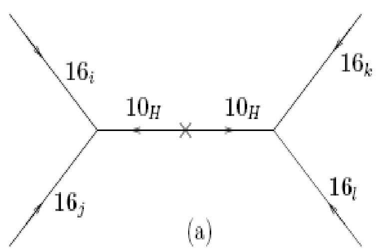
Two bidoublets, two RH doublets + a vector like singlet quark from 45:

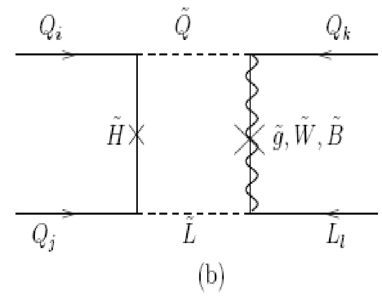


- **New SUSY GUT model for Tev scale nu-physics:**
- **SO(10) Higgs: 16, 10, 54 +45** :(Dev, RNM, 09; PRD);

Proton decay constraints:

- Proton decay arises from Dim 5 operators; need cancellation to fit lower limits along with large squark masses





Decay mode	Experimental lower limit ($\times 10^{33}$ yr)	Predicted upper limit ($\times 10^{33}$ yr)		
		$M_{\tilde{f}} = 1.36$ TeV	$M_{\tilde{f}} = 1.5$ TeV	$M_{\tilde{f}} = 2$ TeV
$p \to K^+ \overline{\nu}$	2.3	2.35	3.5	11
$p \to K^0 \mu^+$	1.3	409.5	606	1.9×10^3
$p \to K^0 e^+$	1.0	2.3×10^5	3.4×10^5	1.1×10^6
$p \to \pi^0 e^+$	8.2	1.4×10^7	2.0×10^7	6.4×10^7
$p \to \pi^0 \mu^+$	6.6	2.4×10^4	3.6×10^4	1.1×10^5
$p \to \pi^+ \overline{\nu}$	0.025	1.5	2.2	7.1

New Dark matter in TeV scale Inverse seesaw:

- If super-partner of RH neutrino is the lightest, it will be stable due to R-parity- become DM.

$$W = W_{MSSM} + h_\nu LHN + f v_R NS + \mu SS$$

- Soft breaking:

$$-L = -L_{soft}^{MSSM} + M^2 \tilde{N}\tilde{N} + M_S^2 \tilde{S}\tilde{S} + A\tilde{L}H\tilde{N} + B\tilde{N}\tilde{S}$$

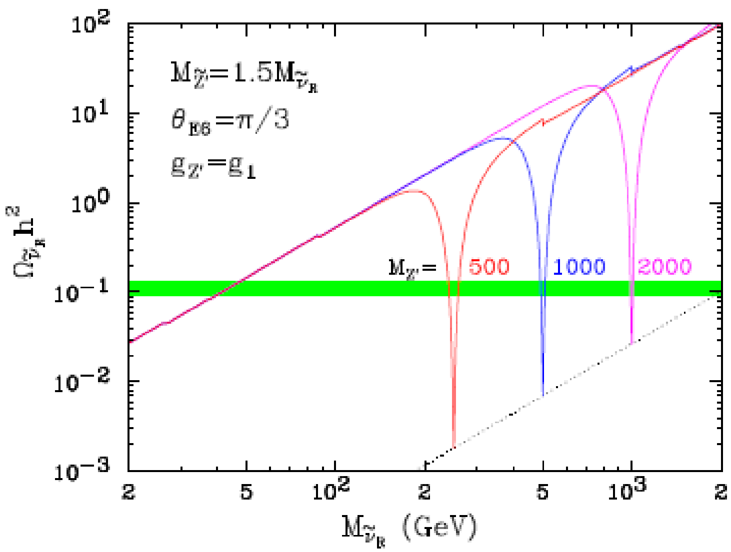
- Lightest linear combination of these is the dark matter:

Relic density

- Inverse seesaw: (Fornengo, Arina, Bazzochi, Romao, Valle; Matchev, Lee, Nasri)
- New DM : $\tilde{\nu}^c$:Two contributions to relic density:

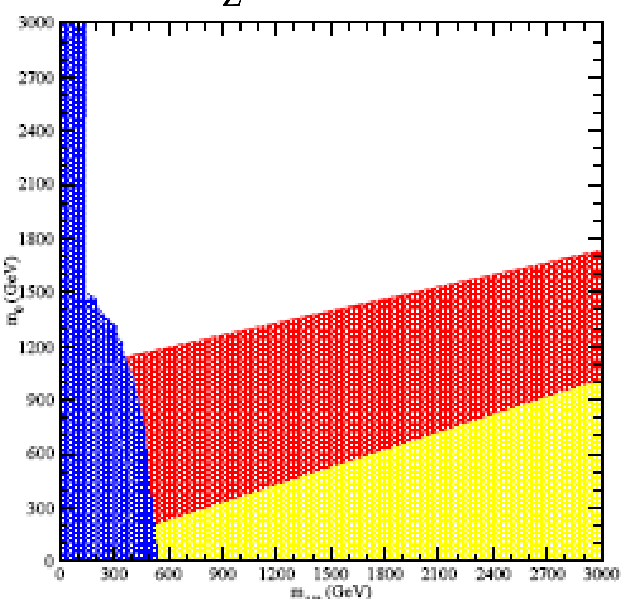
Z' exchange

No or small Z' effect



$M_{\tilde{\nu}^c} < 45GeV$

$M_{Z'} \geq 2.5TeV$



$M_{\nu_1} \approx 100GeV$

Conclusion:

- Neutrino mass theories most likely imply the existence of WR, Z' and N;
- Observing W_R, Z' and N at LHC is not evidence against SUSY GUTs; they can be embeddable in SO(10) GUTs.
- However observing their decay modes e.g. like sign dileptons and doubly charged Higgs Δ^{++} will rule out simple GUTs.

IV: Neutrino Mass and Grand Unification of Flavor

Quark, Lepton flavor: Definitions

- **Key object in Flavor study: Mass Matrix**

Def. $L_{mass} = \overline{Q}_L M_{q=u,d} Q_R + \bar{l}_L M_l l_R + \nu^T m_\nu \nu + h.c.$

- $U_L M_{q,l} U_R^+ = M_{q,l}^{diag}$ **eigenvalues→ masses**
- Mixings:

$V_{CKM} = U_u U_d^+$

$U_{PMNS} = U_l U_\nu^+$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Flavor Puzzle < 1998:

- Quark masses and mixings (at GUT scale)
- Up quarks: $m_u : m_c : m_t = 0.0008 : 0.2 : 82$
- Down quarks: $m_d : m_s : m_b = 0.002 : 0.03 : 1$
- Mixings: $V_{us} \approx 0.22; V_{cb} \approx 0.037; V_{ub} \approx 0.003$
- Leptons: $m_e : m_\mu : m_\tau = 0.0005 : 0.093 : 1.58$
- Note: $m_b \approx m_\tau ; m_\mu \approx 3 m_s$

WHY ?

Attempts to Understand using texture zeros

- **Relation:** $V_{us} \cong \sqrt{\frac{m_d}{m_s}}$ **→d-s Mass matrix** $\begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$
- $a << b \rightarrow m_s = b; m_d = -\frac{a^2}{b}; V_{us} = \frac{a}{b} = \sqrt{\frac{m_d}{m_s}}$

(Weinberg; Wilczek,Zee; Fritzsch)

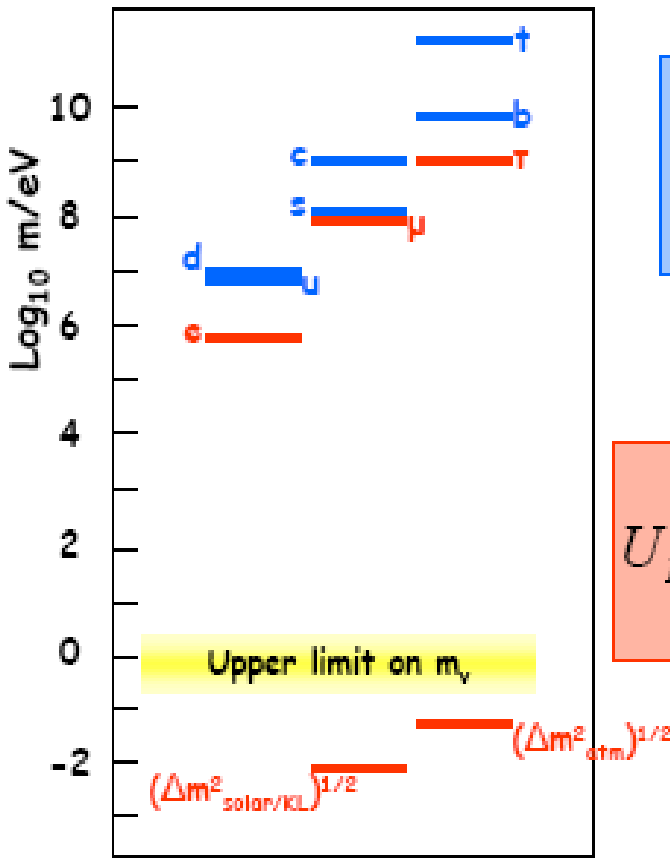
- **Also GUT scale relations:** $m_b \cong m_\tau$

  and $m_e m_\mu \approx m_d m_s \Rightarrow Det[M^l] = Det[M^d]$
- **Finally at GUT scale,** $m_\mu \approx 3m_s$
- **This implies: M$_d$ =** $\begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$ **whereas M$_l$ =** $\begin{pmatrix} 0 & a \\ a & -3b \end{pmatrix}$

(Georgi, Jarlskog)

Neutrino mass discovery has added to this puzzle

- Quark and lepton flavor:



$$V_{CKM} \cong \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 1.00 & 0.04 \\ 0.01 & 0.04 & 1.00 \end{pmatrix}$$

Adapted from PDG 2002

$$U_{PMNS} \cong \begin{pmatrix} 0.84 & 0.54 & 0.1 \\ -0.44 & 0.56 & 0.71 \\ 0.32 & -0.63 & 0.71 \end{pmatrix}$$

An Interesting mixing pattern ?

- **Tri-bi-maximal mixing for neutrinos:**

$$U = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

(Harrison, Perkins, Scott; Xing; Wolfenstein)

- **Is it exact ? If not how big are corrections ?**

New Challenges posed by neutrino masses

- **Flavor issues :**

A.

$$\frac{m_{sol}}{m_{atm}} \approx \theta_C >> \frac{m_\mu}{m_\tau}, \frac{m_s}{m_b}$$

B.

$$V^l_{23} \approx 0.7 >> V_{23}^{CKM} \approx 0.04$$

Quarks and leptons so different-
is a unified description of
Flavor possible ?

Matrices for Masses

- Quark mass matrices very different from lepton mass matrices: up-quark and charged lepton diagonal basis:
- $$M_d = m_b \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} ; \quad M_\nu \cong \begin{pmatrix} \varepsilon_1 & \varepsilon_3 & \varepsilon_3 \\ \varepsilon_3 & 1+\varepsilon_1 & -1+\varepsilon_3 \\ \varepsilon_3 & -1+\varepsilon_3 & 1+\varepsilon_1 \end{pmatrix}$$
- $\varepsilon_i \sim \lambda$ = Cabibbo angle

Strategy for texture

- Key idea: SM has a large sym for zero fermion masses : **[SU(3)]^5**;
- **Choose subgroup**: Discrete subgroup with 3-d. rep.
- **Replace Yukawa's by scalar fields (flavons);**
- **Minima of the flavon theory determines Yukawas:**

Symmetries indicated:

- Charged lepton diagonal basis:

$$\begin{pmatrix} \varepsilon_1 & \varepsilon_3 & \varepsilon_3 \\ \varepsilon_3 & 1+\varepsilon_1 & -1+\varepsilon_3 \\ \varepsilon_3 & -1+\varepsilon_3 & 1+\varepsilon_1 \end{pmatrix}$$

Is invariant under the transformation matrices

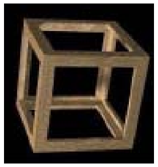
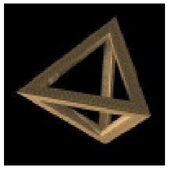
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

Z2 part of A4 group

Many neutrino models using symmetries

- **Successful Family symmetries for TBM**:

$$S_{2(\mu-\tau)} \subset S_3, S_4, A_4, Z_2, \Delta(3n^2), \ldots$$

- **Non-zero θ_{13} will provide important clue about new physics- is it symmetry + corrections or perhaps TBM an accident ?**

(Ma, Rajasekaran; Babu, Ma, Valle, King; Altarelli, Feruglio, Chen, Mahanthappa; Everett, Ramond; Luhn, Nasri, Yu, RNM, Hagedorn, Morissi,.....)

Basic strategy to unify quark-lepton flavor:

- **Assumption (I):** **Suppose a theory gives:**

(Dutta, Mimura, RNM'PRD-09)

- $$M_u = M_0 + \delta_u$$
$$M_d = rM_0 + \delta_d$$
$$M_l = rM_0 + \delta_l$$

$$m_\nu = f v_L$$

$$\delta_{u,d,l} << M_0$$

- **Choose basis so f diagonal.** Then lepton mixings are given by the matrix that diagonalizes**;** M_l .
- **For anarchic M0, quark mixings are small while lepton mixings are large.**

How to see that ?

- **Suppose:** $U_0 M_0 U_0^+ = M^{diag}$
- **Then** $VU_0(rM_0 + \delta_d)U_0^+ V^+ = M_d^{diag}$
- **Since** $\delta_{u,d,l} << M_0$ **off-diagonal elements of** V **are small.**

$$V_{CKM} = U_0 U_0^+ V^+ = V^+$$

- **On the other hand,** $U_{PMNS} = U_0$ **whose matrix elements are large.**
- *Does not however explain mass hierarchies*

Rank One mechanism and mass hierarchy

- **Assumption (II):** M_0 **has rank one** i.e.

$$M_0 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} a & b & c \end{pmatrix}$$

- gives mass to third gen fermions: t, b, tau + $m_b \cong m_\tau$

  others are massless. Turn on $\delta_{u,d,l} << M_0$

  Other fermions c,s,mu pick up mass with

- $m_{c,s,\mu} << m_{t,b,\tau}$ and relates mixings to masses

Illustration for 2-Gen. case

- Suppose $M_0 = \begin{pmatrix} c \\ s \end{pmatrix} (c \quad s)$ and $f = diag(\varepsilon_2, \varepsilon_3) \propto \delta_{u,d}$
- $\theta = Atm.$ angle; chosen large; f <<h.
- **Predictions:** $m_\tau \cong m_b$, $\frac{m_s}{m_b} \approx -V_{cb} \tan\theta$ } **consistent with observations:**

Rest of the talk

- **Show that this strategy can be realized for three generations naturally in a certain class of SUSY GUT theories of neutrinos:**

- **Idea testable in neutrino experiments e.g. those planning to measure θ_{13} .**

What kind of GUT theory ?

- **SM:** $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R d_R $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ e_R

Add → ν_R

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \overset{P}{\leftrightarrow} \begin{pmatrix} u_R \\ d_R \end{pmatrix} \qquad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \overset{P}{\leftrightarrow} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

- Adding RH neutrino suggests left-right sym.unification based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$

- **Minimal GUT group containing this is SO(10):**

Left-Right and SO(10) just right for our ansatz !!

- **Recall ansatz:** $M_u = M_0 + \delta_u$ $M_d = rM_0 + \delta_d$ **as** $\delta_{u,d} \to 0, M_u \propto M_d$

- In SM, u_R d_R singlets- so M**u**, M**d** **unrelated.**
- We need a theory where, $\begin{pmatrix} u_R \\ d_R \end{pmatrix}$ are in a doublet.

- Left-Right symmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ and SO(10) (which contains LR) are precisely such theories.

SUSY SO(10) Features

- Minimal GUT group with complete fermion unification (per family) is SO(10)-its spinor rep contains **all 16** SM fermions (including RH nu) in single rep.

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$$

- **Has B-L needed to understand why $M_R << M\_{Pl}$**
- **Theory below GUT scale is MSSM:**
- **B-L needed for naturally stable dark matter.**

From SO(10) down to the Std Model

SO(10) **Nu mass**

$$\downarrow$$

- **LR Sym.** $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ $\begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix}$

$$\downarrow \quad \Delta(B-L) \neq 0$$

- **Standard Model-** $SU(2)_L \times U(1)_Y \times SU(3)_c$

$$\downarrow$$

-> seesaw $\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$

$$SU(3)_c \times U(1)_{em}$$

SUSY SO(10) and unified understanding of flavor

- **Fermions in {16}:**
  $16_m \times 16_m = \{10\}_H + \{120\}_H + \{126\}_H$
- Only renorm. couplings for fermion masses:
  $$L_Y = h 16 \cdot 16 \cdot 10_H + f 16 \cdot 16 \cdot \overline{126}_H + h' 16 \cdot 16 \cdot [120, 10]_H$$
- **Has SM doublets → contributes to fermion mass**
- **$\{126\}_H$ responsible for both neutrino masses and quark masses:→ helps to connect quark mixings to neutrino mixings: Unifies quark and lepton flavors:** (Babu, Mohapatra, 93)

Fermion mass formulae in renormalizable SO(10)

- **Define** $Y_f = M_f / v_{wk}$
- **The mass formulae:** **Compare with ansatz**

$$
\begin{aligned}
Y_u &= h + r_2 f + r_3 h' \\
Y_d &= r_1(h + f + h') \\
Y_e &= r_1(h - 3f + c_e h') \\
Y_\nu &= h - 3r_2 f + c_\nu h'
\end{aligned}
$$

$$M_u = M_0 + \delta_u$$

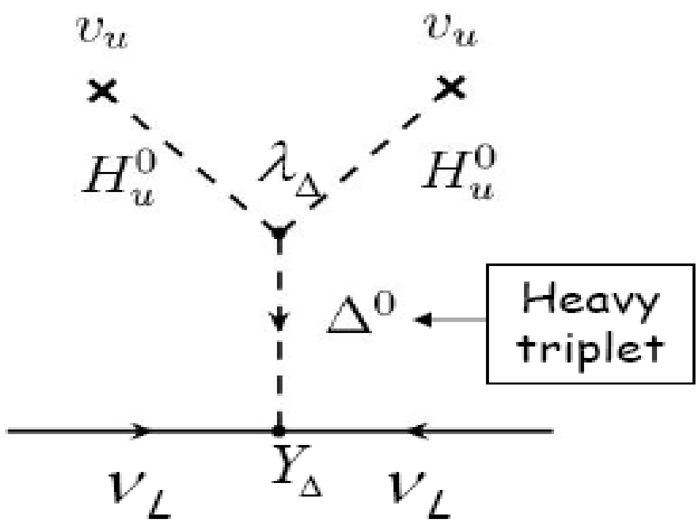
$$M_d = rM_0 + \delta_d$$

$$M_l = rM_0 + \delta_l$$

- **Both sets of formulae identical for** f, h'<< h

Neutrino mass in Renormalizable SO(10):

- {126} has an $SU(2)_L$ triplet with B-L=2:
- New formula for nu-mass:



$$m_\nu = f v_\Delta - M_D \frac{1}{f v_{BL}} M_D^T$$

$$v_\Delta = \lambda_\Delta \mu \frac{v_{wk}^2}{M_\Delta^2}$$

- Type II seesaw: $M_\Delta \approx M_U$ gives naturally small v
- **Two independent parameters:** M_Δ^2, v_R

Type II dominance:

- If $M_\Delta << fv_{BL}$,first term dominates
- **Then the fermion mass formula become:**

$$\begin{aligned} Y_u &= h + r_2 f + r_3 h' \\ Y_d &= r_1(h + f + h') \\ Y_e &= r_1(h - 3f + c_e h') \\ Y_\nu &= h - 3r_2 f + c_\nu h' \end{aligned}$$

:

$$m_\nu \cong fv_\Delta$$

(Bajc, Senjanovic,Vissani'02)

(Babu, Mohapatra'92)

- **Neutrino mass and quark and charged lepton masses connected and all ingredients of our ansatz are realized in SO(10) .**

Rank One mechanism for Flavor

- Generic case does not explain mass hierarchies

$$\begin{aligned} Y_u &= h + r_2 f + r_3 h' \\ Y_d &= r_1(h + f + h') \\ Y_e &= r_1(h - 3f + c_e h') \end{aligned}$$

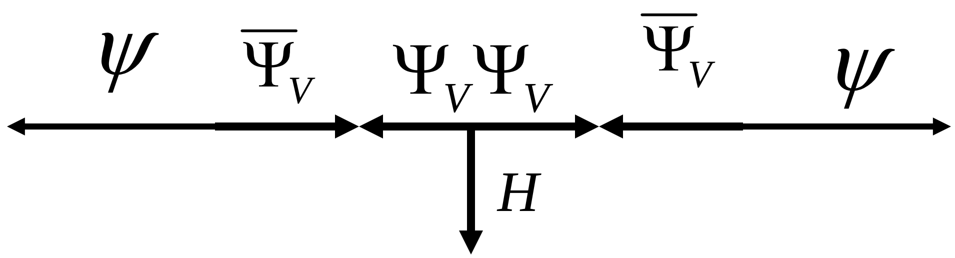
$$m_\nu \cong f v_\Delta$$

- Assume h is rank 1 $h = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} a & b & c \end{pmatrix}$ + $f, h' << h$

- For f, h'=0, only 3rd gen. pick up mass.
- Leads to $m_{s,d} << m_b ; m_{e,\mu} << m_\tau$ with f, h'<<h
- Gives $m_\tau \cong m_b$ and $m_\mu = -3m_s$; $\frac{m_{sol}}{m_{atm}} \sim \theta_C$

Origin of Rank one SO(10)

- Rank one model as an effective theory at GUT scale:
- Add one vector like matter $\Psi_V\{16\} + \overline{\Psi}_V\{\overline{16}\}$
  and singlets: ϕ_i
- **Superpotential:** $W = \phi_i \psi_i \overline{\Psi}_V + \overline{\Psi}_V \overline{\Psi}_V H + M \overline{\Psi}_V \Psi_V$



- **Flavor texture depends on** **< ϕ_i >; with symmetries it can be predicted.**

How to determine the Yukawa alignment ?

- Strategy: Take a discrete group G with 3-dim. Reps:
- Examples: $A_4, S_{4,} \Delta(27), PSL_2(7)$.....
- Take flavons ϕ and matter {16} in 3-d reps of G
- Minimize flavon potential inv under G; this will determine $< \phi >$
- Effective matter Yukawa ~ $\psi\phi\psi\phi H, \psi\phi\psi\phi\overline{\Delta}, ...$

- **Flavon vev's determine the Yukawa texture:**

  since the flavon vevs correspond to minima of theory, Yukawas are determined by dynamics!!

VEV alignment from flat directions and flavor

- Examples: S4 triplet flavon case:

$$W = \frac{1}{2}m\phi^2 - \lambda\phi^3 = \frac{1}{2}m(x^2+y^2+z^2) - \lambda xyz.$$

f F-flat vacua ($\phi \neq 0$) are

$$\phi = \frac{m}{\lambda}\{(1,1,1) \text{ or } (1,-1,-1) \text{ or } (-1,1,-1) \text{ or } (-1,-1,1)\}.$$

- while

$$\begin{aligned} W &= \frac{1}{2}m\phi^2 - \frac{\kappa_1}{M}(\phi^4)_1 - \frac{\kappa_2}{M}(\phi^4)_2 \\ &= \frac{1}{2}(x^2+y^2+z^2) - \frac{\kappa_1}{4M}(x^4+y^4+z^4) - \frac{\kappa_2}{2M}(x^2y^2+y^2z^2+z^2x^2). \end{aligned} \tag{15}$$

The nontrivial F-flat vacua ($\phi \neq 0$) are

$$\phi = \sqrt{\frac{mM}{\kappa_1}}\vec{a}, \quad \sqrt{\frac{mM}{\kappa_1+2\kappa_2}}\vec{b}, \quad \sqrt{\frac{mM}{\kappa_1+\kappa_2}}\vec{c}, \tag{16}$$

where $\vec{a} = (0,0,\pm1),\ (0,\pm1,0),\ (\pm1,0,0)$, $\vec{b} = (\pm1,\pm1,\pm1)$, and $\vec{c} = (0,\pm1,\pm1)$,

Realistic 3-generation model for Flavor:

- Our proposal after diagonalization of h
- $$h \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ with appropriately rotated f and h'.
- Different ansatzes for f and h' lead to different realizations of this idea:

A specific realization with predictive textures:

- Group: SO(10)x**S4** $\supset 3_1 + 3_2 + 2 + 1_1 + 1_2$
- **Consider flavons** $\phi_{1,2,3} \subset 3_{1,2}$ **; matter {16}** $\subset 3_2$
- **Inv effective superpotential at GUT scale:**

$$W = (\phi_1\psi)(\phi_1\psi)H + (\phi_2\psi)(\phi_2\psi)\bar{\Delta} + \phi_3\psi\psi\bar{\Delta} + \phi_2\psi\psi H'$$

- **The flavon vevs align as:** $\phi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$
- **Leading to f=** $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ **and h'=** $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$
- **Gives realistic model for fermion masses and mixings**

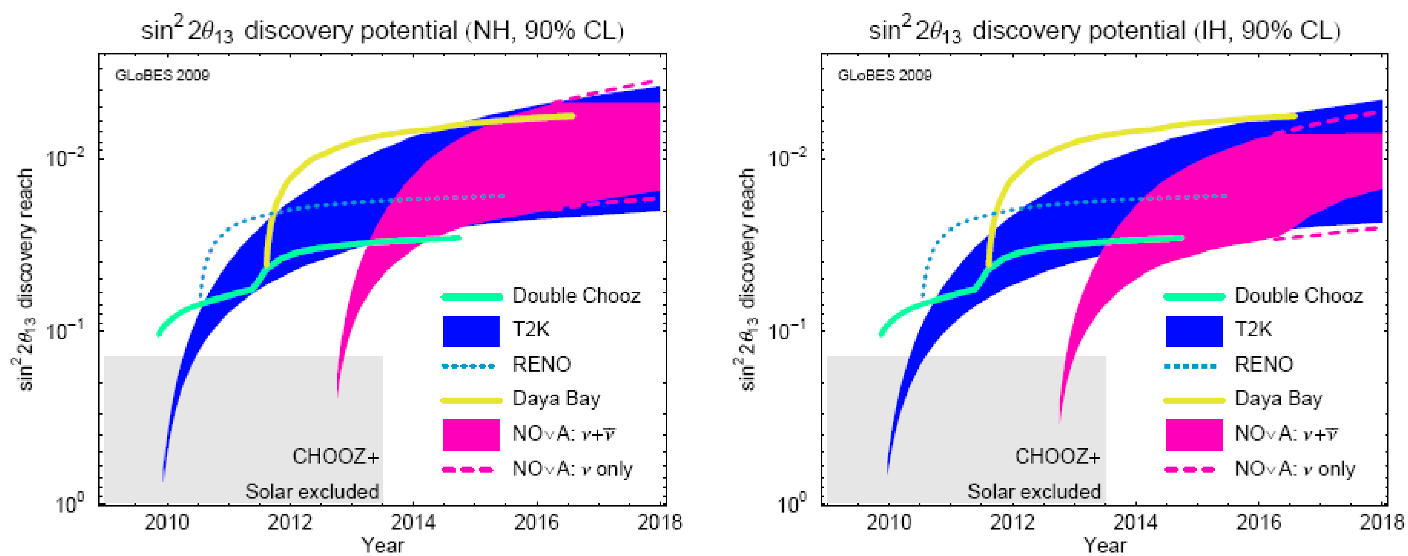
Prediction of SO(10)xS4

- **Solar mass** $\frac{m_{solar}}{m_{atm}} \cong \lambda \cong \theta_c$
- **Bottom-tau:** $m_b \approx m_\tau$ **and** $m_\mu = -3m_s$
- **Leading order PMNS:** $= \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$
- **Testable prediction:** $\theta_{13} = \frac{\theta_c}{3\sqrt{2}} \cong 0.05$
- Double beta mass 3 meV. **Dutta, Mimura, RNM arXiv:0911.2242**

Prospects for measuring θ_{13}

- Reactor, Long base line e.g. T2K, NoVA:

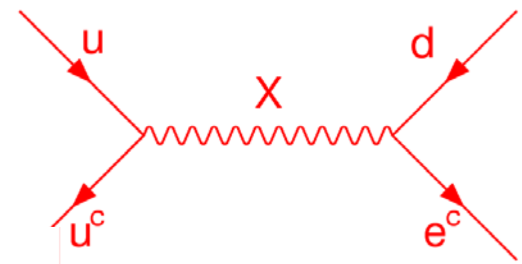
(Lindner, Huber,Schwetz,Winter'09)



Our prediction $\sin^2 2\theta_{13} > 0.01$

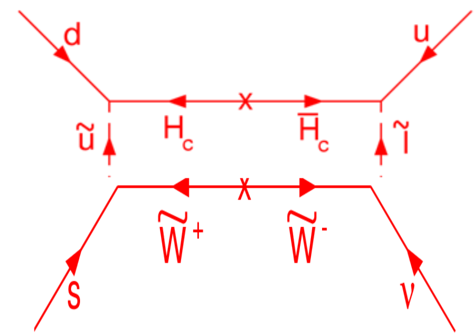
GUTs and Proton decay

- **Proton decay in SUSY GUTs have two generic sources:**



- **(i) Gauge exchange:**

$$p \to e^+\pi^0,\ \tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [10^{36\pm1} yr]^{-1}$$



- **(ii) Higgsino exchange:**

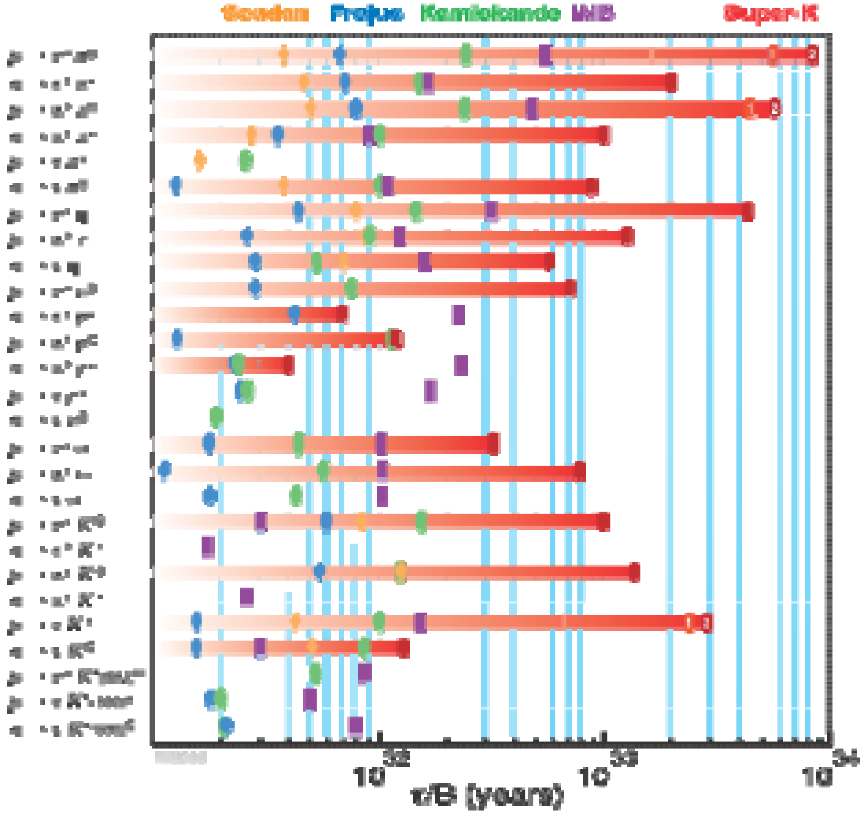
$$p \to \bar{\nu} K^+$$

$$\tau_p^{-1} \approx [\frac{f^2}{M_{H_c} M_{SUSY}}]^2 (\frac{\alpha}{4\pi})^2 m_p^5 \approx [10^{28} - 10^{32} yr]^{-1}$$

- **Present limit:** $\tau_{\bar{\nu}K^+} > 2.3 \times 10^{33}\, yrs$

Present experimental limits

- Super-K, Soudan, IMB, Frejus



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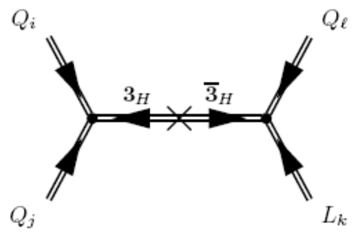
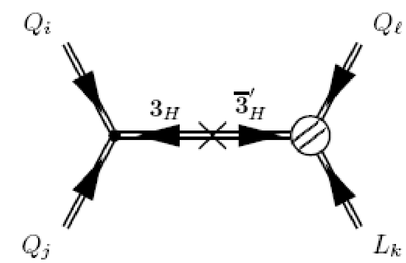
Rank one also solves the proton decay problem

- Proton decay problem in SU(5): one Higgs pair s



$$\rightarrow A_p \propto Y_u Y_d$$

- In SO(10), there are more Higgs fields and if flavor structure is such that triplet Higgs do not connect, no p-decay problem:



- Choice flavor structure that does it (Dutta, Mimura, RNM'05)

$$h_{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \; h_{126} = \begin{pmatrix} 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix};$$

$$h_{120} = \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix};$$

Conclusion:

(i) New ansatz to unify diverse profiles of quark and lepton flavor patterns.

(ii) SO(10) GUT with type II seesaw provides a natural framework for realization of this ansatz.

(iii) Predicts measurable θ_{13} and solves proton decay problem of susy GUTs.