

# *Amplitudes meet Cosmology*

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(Quantum Field Theory Department)

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# QFT & Scattering Amplitudes: The Group



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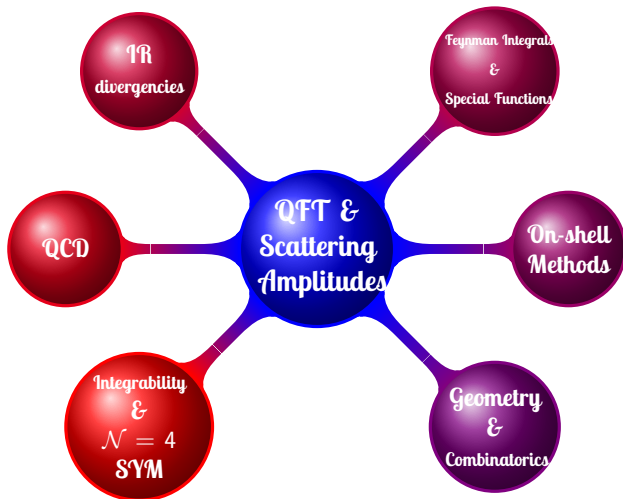


Alex Tumanov

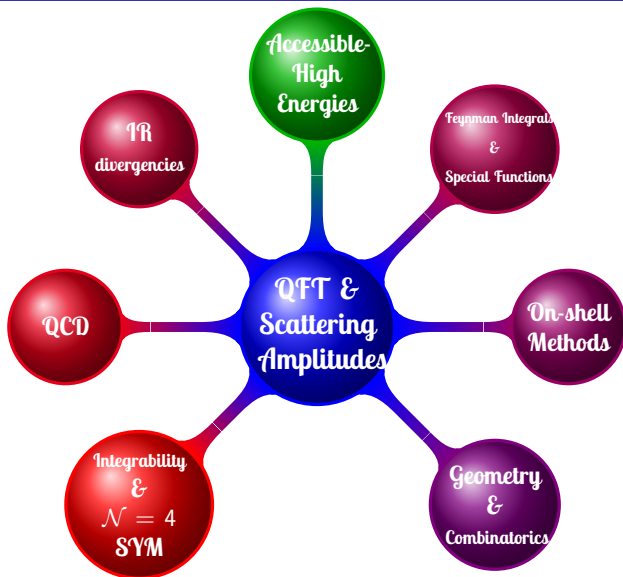


Fabian Wagner

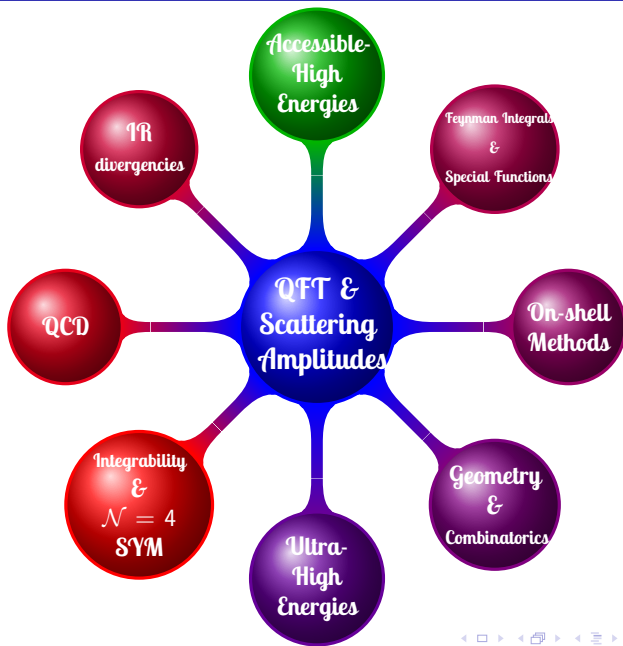
# QFT & Scattering Amplitudes: Our Research



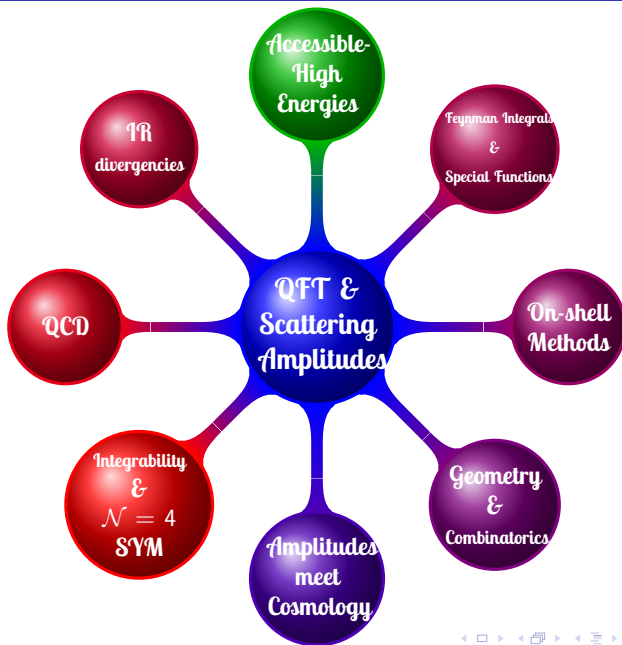
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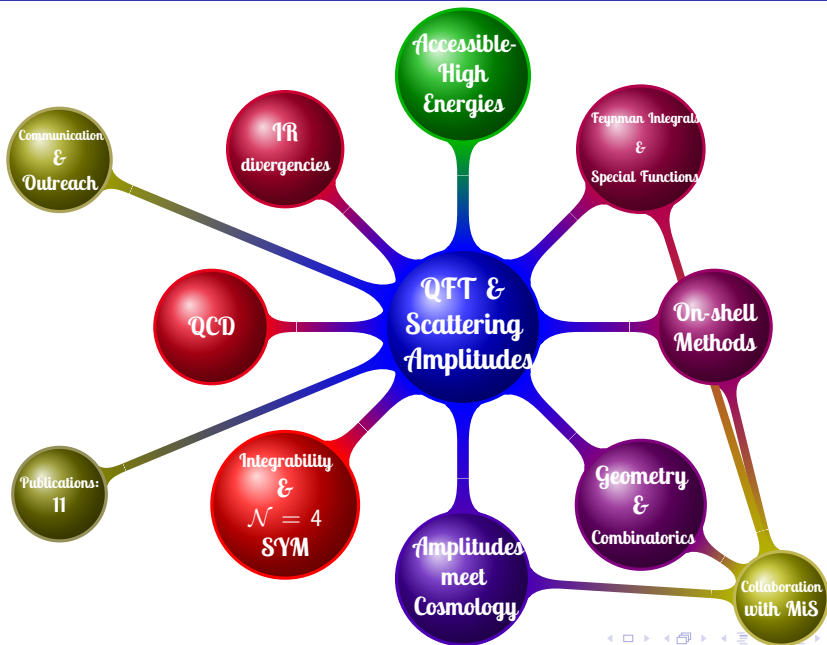
# QFT & Scattering Amplitudes: Our Research



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# Scattering Amplitudes: Physics at Accessibly-High Energies

Unitarity

Locality

Causality

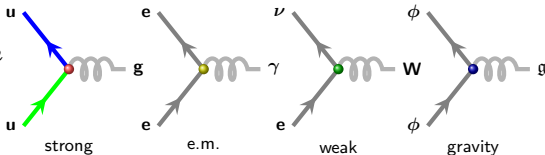




# Scattering Amplitudes: Physics at Accessibly-High Energies

Unitarity

Three-Particle Amplitudes



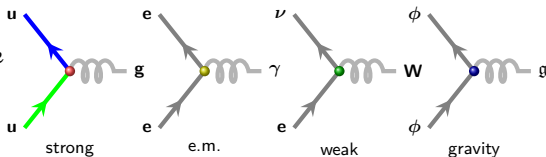
Locality

Causality

# Scattering Amplitudes: Physics at Accessibly-High Energies

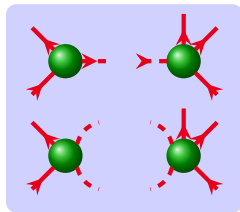
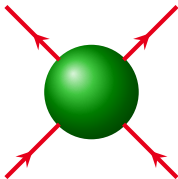
## Unitarity

### Three-Particle Amplitudes



## Locality

### Four-Particle Amplitudes

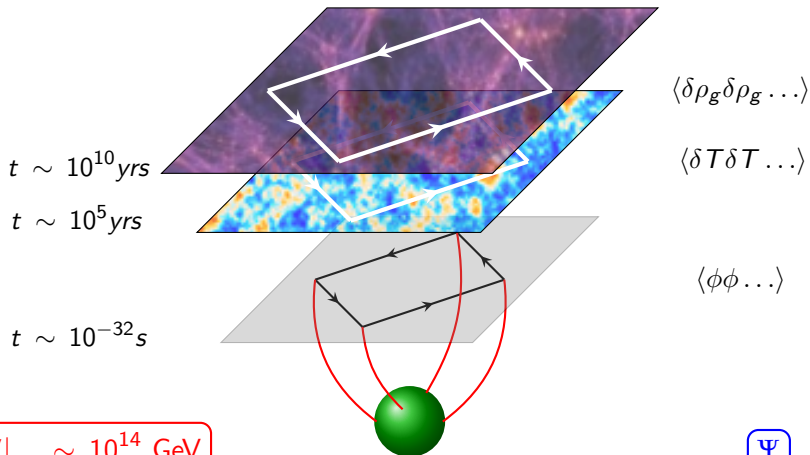


## Causality

- Particles:  $s = 0, 1/2, 1, 3/2, 2$
  - Charge conservation (spin 1)
  - Spin-1 self-interactions for just different species
  - Spin  $> 2$ : No self-interactions; no interactions with  $\leq 2$
  - Yang's (Weinberg-Witten) theorem
  - Equivalence principle (spin 2)
  - Graviton uniqueness theorem
  - $\mathcal{N} = 1$  SUGRA
- No elementary massive particles.

# What about Physics at Higher Energies?

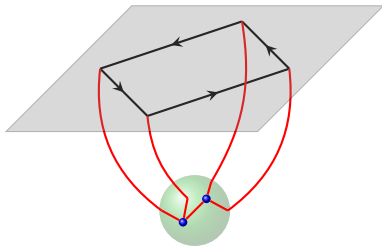
Cosmology as window on the physics at ultra-high energies



# What about Physics at Higher Energies?

Cosmology as window on the physics at ultra-high energies

$$t \sim 10^{-32} \text{ s}$$



$$\langle \phi \phi \dots \rangle$$

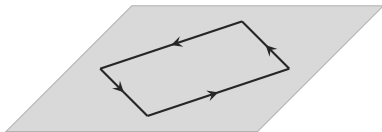
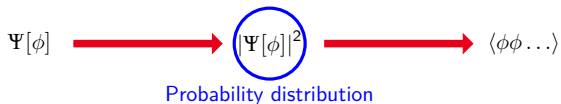
$$\mathcal{H}|_{\text{infl.}} \sim 10^{14} \text{ GeV}$$



# What about Physics at Higher Energies?

Cosmology as window on the physics at ultra-high energies

What are the rules governing physical processes at these energies?



$\langle \phi \phi \dots \rangle$

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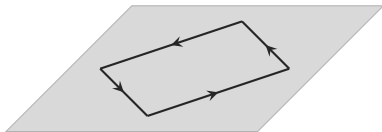
Cosmology as window on the physics at ultra-high energies

What are the rules governing physical processes at these energies?

~~Lorentz  
Invariance~~

Causality

Unitarity



$\langle \phi \phi \dots \rangle$

$\mathcal{H}|_{\text{infl.}} \sim 10^{14} \text{ GeV}$

$\Psi$



# Questions

- Can we reach a similar understanding of *ultra-high* energy processes as the *accessibly-high* energy ones?
- What is the imprint of causality and unitarity on the quantum mechanical observables in cosmology?  $\Psi[\phi], \langle \phi\phi \dots \rangle$
- What are their consequences?

*What are the invariant properties that  $\Psi$  ought to satisfy in order to come from a consistent causal evolution in cosmological space-times?*

- What is the most suitable language to describe the physics at this regime?



*A Twofold Way for  
Ultra-High Energy Processes*





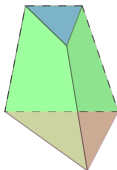
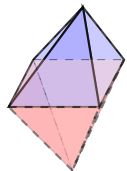
# A Twofold Way for Ultra-High Energy Processes

- 1 The *responsible* way: Understanding  $\Psi$  as a function of boundary data

$$\left( \sum_{v \in \mathcal{V}} x_v \right) \psi_n = \sum_{e \in \mathcal{E}} \psi_L \text{ --- } \psi_R + \sum_{e \in \mathcal{E}} \psi_{\text{boundary}}$$

[Arkani-Hamed, **P.B.**, Postnikov, 17]  
[**P.B.**, 19]

- 2 The *irresponsible* way: *Guessing* the mathematical structure underlying  $\Psi$



[Arkani-Hamed, **P.B.**, Postnikov, 17]  
[Arkani-Hamed, **P.B.**, 18]  
[**P.B.**, 18-19]  
[**P.B.**, Parisi, 20]  
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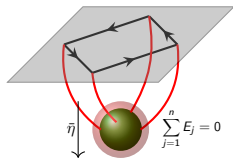
Their intrinsic definition  
determines all their features



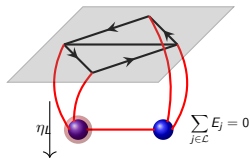
# The Responsible Way: $\Psi$ and Boundary Data

What have we learnt?

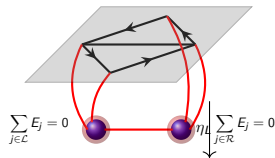
## 1 Factorisation properties



Scattering Amplitude  $\mathcal{A}$



$\mathcal{A}_L \times \tilde{\Psi}_R$



$\mathcal{A}_L \times \mathcal{A}_R$

[Maldacena, Pimentel, 12], [Arkani-Hamed, P.B., Postnikov, 17]

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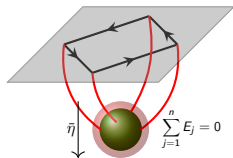
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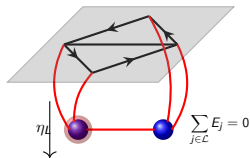
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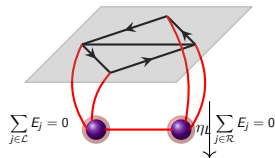
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$\mathcal{A}_{\mathcal{L}} \times \mathcal{A}_{\mathcal{R}}$

### 2 Feynman-like tree theorem:

[**P.B.** 18]

$$\begin{array}{c}
 \text{Diagram: A red circle with two blue dots on the left and right, labeled } x \text{ and } x'. \\
 \end{array}
 = \int_{-i\infty}^{+i\infty} d\varepsilon \begin{array}{c}
 \text{Diagram: A red line segment with three blue dots. The left dot is labeled } x_1 + \varepsilon, \text{ the middle dot is } x', \text{ and the right dot is } x_2 - \varepsilon. \text{ The segment is labeled } y_1 \text{ above the left part and } y_2 \text{ above the right part.} \\
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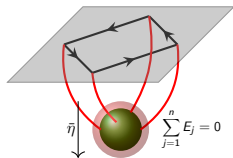
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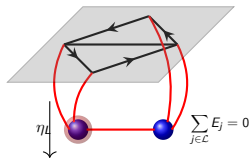
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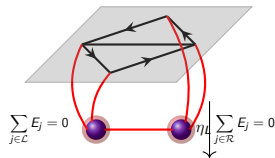
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[Goodhew, Jazayeri, Pajer, 20], [Melville, Pajer, 21]

### 3 Cosmological optical theorem:

[Goodhew, Jazayeri, Gordon-Lee, Pajer, 21]

$$\psi_n(\{E_j\}) + \psi_n^\dagger(\{-E_j\}) = - \sum_{\text{cuts}} \psi_n$$

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[Meltzer, 21]

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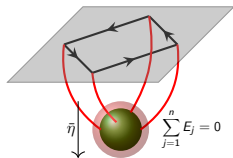
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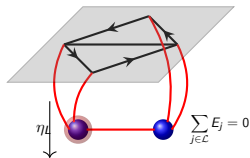
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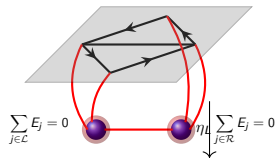
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Incorrect flat-space limit

Not unique

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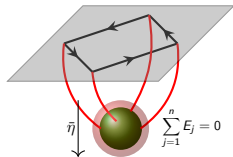
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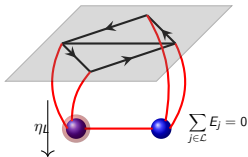
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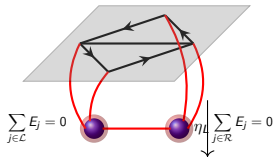
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Existence of distributional terms  $\iff$  Flat-space limit

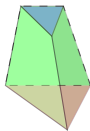
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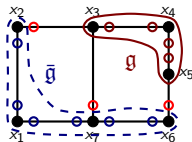
Max-Planck-Institut für Physik  
(Heisenberg Institute)

# The Irresponsible Way: Guessing a New Language

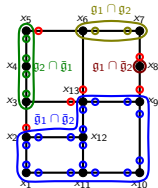
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Questions about  $\Psi$  in combinatoric-geometrical terms



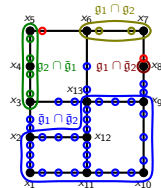
Emergent Flat-Space  
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Emergent Flat-Space  
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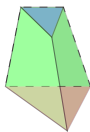


Causality-like relations  
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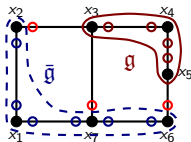


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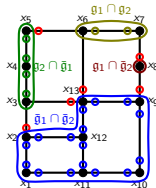
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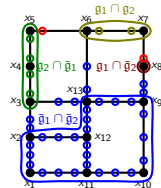


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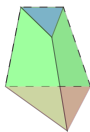
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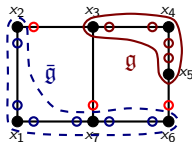


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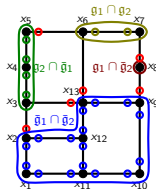
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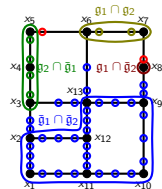


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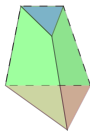
3 Systematic procedure for classifying and writing down representations for  $\psi_n$  and  $\mathcal{A}_n$

[P.B., Torres Bobadilla, to appear]

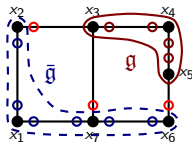


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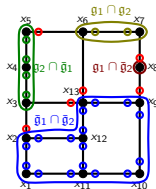
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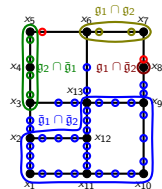
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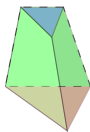
4 Invariant definition of unitarity, cutting rules as triangulations

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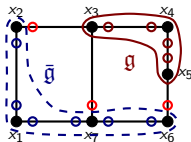


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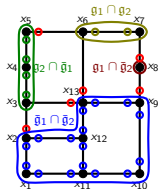
What have we learnt?



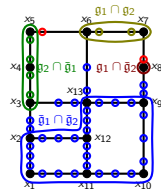
Questions about  $\Psi$  in combinatoric-geometrical terms



Emergent Flat-Space  
Unitarity



Emergent Flat-Space  
Causality



Causality-like relations  
for  $\Psi$

[Arkani-Hamed, P.B., Postnikov, 17], [Arkani-Hamed, P.B., 18]

[P.B., 18], [P.B., 19], [P.B., McLeod, Vergu, 20]

2 Reconstructing the wavefunction  $\psi_n^{\text{tree}}$  from the flat-space amplitude  $\mathcal{A}_n^{\text{tree}}$

[P.B., 18]

3 Systematic procedure for classifying and writing down representations for  $\psi_n$  and  $\mathcal{A}_n$

[P.B., Torres Bobadilla, to appear]

4 Invariant definition of unitarity, cutting rules as triangulations

[P.B., Duaso Pueyo, w.i.p.]

5 Math: direct and graph theoretic way of finding triangulations of polytopes

[P.B., Torres Bobadilla, to appear]



# Looking at the Future

- Reconstructing  $\psi_n$  from first principles.
- Constraints on the interactions from consistency conditions on  $\psi_n$ .
- Combinatoric-geometrical description for the full  $\psi_n$ .
- Further generalisations
- Are the causality-like relations an avatar of the causality of time evolution?
- Systematic exploration of the symmetries and how symmetries emerge in flat-space.
- From  $\Psi$  to  $|\Psi|^2$  and to more general observables.
- What are the right observables?
- Formulating the most suitable language to describe processes at ultra-high energies



# Looking at the Future: We Are Still at the Beginning!

