

Exact verification of the strong Birch–Swinnerton-Dyer conjecture for some absolutely simple modular abelian surfaces

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The strong Birch–Swinnerton-Dyer conjecture and in particular the exact order of the Shafarevich–Tate group for abelian varieties over the rationals has only been known for elliptic curves (dimension 1) or in higher dimension where the conjecture could be reduced to dimension 1. We give the first absolutely simple examples of dimension 2 where the conjecture can be verified:

Let X be (1) a quotient of the modular curve $X_0(N)$ by a subgroup generated by Atkin-Lehner involutions such that its Jacobian J is an absolutely simple modular abelian surface, or, more generally, (2) an absolutely simple factor of $J_0(N)$ isomorphic to the Jacobian J of a genus-2 curve X . We prove that for all such J from (1), the Shafarevich–Tate group of J is trivial and satisfies the strong Birch–Swinnerton-Dyer conjecture. We further indicate how to verify strong BSD in the cases (2) in principle and in many cases in practice.

To achieve this, we compute the image and the cohomology of the mod- p Galois representations of J , show effectively that almost all of them are irreducible and have maximal image, make Kolyvagin–Logachev effective, compute the Heegner points and Heegner indices, compute the p -adic L -function, and perform p -descents. Because many ingredients are involved in the proof, we will give an overview of the methods involved and give more details regarding the computation of the Galois representations.

This is joint work with Michael Stoll.

Presenter: KELLER, Timo (Universität Bayreuth)

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