

Recursive Towers of Function Fields over Finite Fields

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An infinite sequence $\mathcal{F} = (F_0, F_1, \dots)$ of function fields F_n/\mathbb{F}_q of transcendence degree one with full constant field \mathbb{F}_q is called a tower of function fields if all extensions F_{n+1}/F_n are finite separable and the genus $g(F_n)$ tends to infinity as $n \rightarrow \infty$.

A tower is called asymptotically good if its limit $\lambda(\mathcal{F}) := \lim_{n \rightarrow \infty} \frac{N(F_n)}{g(F_n)}$ is positive where $N(F_n)$ denotes the number of \mathbb{F}_q -rational places in F_n .

Good towers can then be used to construct Goppa codes with good parameters. Unfortunately, many of the known good towers are constructed with methods which involve class field theory or modular curves and these constructions usually do not provide explicit presentations of the function fields F_n .

However, a special type of towers are the recursive towers which are recursively defined by bivariate polynomials $f(x, y)$ over \mathbb{F}_q .

Here, we have a sequence (x_0, x_1, \dots) such that F_n is of the explicit form $\mathbb{F}_q(x_0, \dots, x_n)$ and $f(x_n, x_{n+1}) = 0$ holds for all $n \in \mathbb{N}_0$.

In 2005, Beelen-Garcia-Stichtenoth conjectured that a good recursive tower has to have rational splitting, i.e. there is a place in F_0 which splits completely in all extensions F_n/F_0 . In this talk, we will discuss this conjecture.

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