## Recursive Towers of Function Fields over Finite Fields

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An infinite sequence $\mathcal{F}=\left(F_{0}, F_{1}, \ldots\right)$ of function fields $F_{n} / \mathbb{F}_{q}$ of transcendence degree one with full constant field $\mathbb{F}_{q}$ is called a tower of function fields if all extensions $F_{n+1} / F_{n}$ are finite separable and the genus $g\left(F_{n}\right)$ tends to infinity as $n \rightarrow \infty$.
A tower is called asymptotically good if its limit $\lambda(\mathcal{F}):=\lim _{n \rightarrow \infty} \frac{N\left(F_{n}\right)}{g\left(F_{n}\right)}$ is positive where $N\left(F_{n}\right)$ denotes the number of $\mathbb{F}_{q}$-rational places in $F_{n}$.
Good towers can then be used to construct Goppa codes with good parameters. Unfortunately, many of the known good towers are constructed with methods which involve class field theory or modular curves and these constructions usually do not provide explicit presentations of the function fields $F_{n}$.
However, a special type of towers are the recursive towers which are recursively defined by bivariate polynomials $f(x, y)$ over $\mathbb{F}_{q}$.
Here, we have a sequence $\left(x_{0}, x_{1}, \ldots\right)$ such that $F_{n}$ is of the $\operatorname{explicit~form~} \mathbb{F}_{q}\left(x_{0}, \ldots, x_{n}\right)$ and $f\left(x_{n}, x_{n+1}\right)=$ 0 holds for all $n \in \mathbb{N}_{0}$.

In 2005, Beelen-Garcia-Stichtenoth conjectured that a good recursive tower has to have rational splitting, i.e. there is a place in $F_{0}$ which splits completely in all extensions $F_{n} / F_{0}$. In this talk, we will discuss this conjecture.

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