

Tagung der Fachgruppe Computeralgebra

Report of Contributions

Contribution ID: 1

Type: **not specified**

The laws of Gaussian conditional independence

Thursday, 10 March 2022 16:30 (30 minutes)

Conditional independence is a ternary relation from probability theory between subcollections of jointly distributed random variables. Among normally distributed (i.e., “Gaussian”) variables, these relations are characterized by the vanishing of specific subdeterminants of the distribution’s positive definite covariance matrix.

The combinatorial structures realizable as conditional independence relations of positive definite matrices can be studied algebraically and this theory is in many ways similar to that of (oriented) matroids in synthetic geometry. However, points in a vector space are replaced by random variables and linear independence by stochastic independence.

This talk gives an overview of the Gaussian conditional independence inference problem with an emphasis on the parallels to matroid theory, including the method of final polynomials and universality theorems. Example computations underline the significance of the positive definiteness restriction for the combinatorial structure.

Presenter: BOEGE, Tobias (Max-Planck-Institut für Mathematik in den Naturwissenschaften Leipzig)

Session Classification: Vorträge 1

Contribution ID: 2

Type: **not specified**

FirstOrderSolve - A MAPLE Package

Wednesday, 9 March 2022 17:10 (30 minutes)

Given a first order autonomous algebraic ordinary differential equation, i.e. an equation of the form

$$F(y, y')=0 \text{ with } F \in K[y, y'],$$

where K is a computable field such as the rational numbers, we present a method to compute all formal Puiseux series solutions. In fact, all of the solutions are convergent Puiseux series.

By considering y and y' as independent variables, F implicitly defines an affine plane curve where (rational) local parametrizations can be computed via Puiseux expansions. We show a sufficient and necessary condition on such a local parametrization to obtain a formal Puiseux series solution of the original differential equation by substitution. This leads to a complete characterization of initial values with respect to the number of distinct solutions extending them. Moreover, by choosing a particular initial value, we give an algorithm computing the coefficients of all solutions starting with this initial value up to an arbitrary order.

These results have been extended to systems of autonomous ordinary differential equations by using the Thomas decomposition.

The theoretic results have been implemented in Maple in the Software package FirstOrderSolve. In the talk we give an overview on the theoretic results and illustrate them by using this software package.

Presenter: FALKENSTEINER, Sebastian (RISC Hagenberg, Johannes Kepler University Linz)

Session Classification: Vorträge 1

Contribution ID: 3

Type: **not specified**

Moment Varieties of Non-Gaussian Graphical Models

Thursday, 10 March 2022 17:10 (30 minutes)

In this talk we shed light on varieties of moments arising from graphical models. A directed graph corresponds to a statistical model where the nodes represent random variables and arrows encode relations between them. The polynomial relations between entries of the covariance matrices have been previously studied from an algebraic and combinatorial point of view. Dropping Gaussianity makes moment tensors of higher order meaningful. We explain how to extend the combinatorial description from covariance matrices to higher-order tensors and use computational algebra to obtain a description for the third-order moment variety for trees. We also describe the polytopes coming from the moment parametrisation and explain the situation for graphs with hidden variables.

Presenter: GROSDOS, Alexandros (TU München)

Session Classification: Vorträge 1

Contribution ID: 4

Type: **not specified**

MaRDI

Wednesday, 9 March 2022 16:30 (30 minutes)

It is important that results published in papers are understandable and verifiable by other mathematicians. Even if they read them a hundred years from now. If a paper is highly dependent on a software component this may often be difficult. The software could be out of date, very difficult to install or contain bugs that were never checked for. Computed results might not be available anymore or stored in a datatype which no one other than the authors are able to comprehend. The MaRDI aims to combat these issues by developing standards, confirmable workflows and a robust research data infrastructure.

Presenter: HANSELMAN, Jeroen (TU Kaiserslautern)

Session Classification: Vorträge 1

Contribution ID: 5

Type: **not specified**

The OSCAR Computer Algebra System

Wednesday, 9 March 2022 15:10 (30 minutes)

We give an introduction to the OSCAR Computer Algebra System (see <https://oscar.computeralgebra.de>) for computations in algebra, geometry, and number theory.

Presenter: HORN, Max (TU Kaiserslautern)

Session Classification: Vorträge 1

Contribution ID: 6

Type: **not specified**

Exact verification of the strong Birch–Swinnerton-Dyer conjecture for some absolutely simple modular abelian surfaces

Wednesday, 9 March 2022 15:10 (30 minutes)

The strong Birch–Swinnerton-Dyer conjecture and in particular the exact order of the Shafarevich–Tate group for abelian varieties over the rationals has only been known for elliptic curves (dimension 1) or in higher dimension where the conjecture could be reduced to dimension 1. We give the first absolutely simple examples of dimension 2 where the conjecture can be verified:

Let X be (1) a quotient of the modular curve $X_0(N)$ by a subgroup generated by Atkin-Lehner involutions such that its Jacobian J is an absolutely simple modular abelian surface, or, more generally, (2) an absolutely simple factor of $J_0(N)$ isomorphic to the Jacobian J of a genus-2 curve X . We prove that for all such J from (1), the Shafarevich–Tate group of J is trivial and satisfies the strong Birch–Swinnerton-Dyer conjecture. We further indicate how to verify strong BSD in the cases (2) in principle and in many cases in practice.

To achieve this, we compute the image and the cohomology of the mod- p Galois representations of J , show effectively that almost all of them are irreducible and have maximal image, make Kolyvagin–Logachev effective, compute the Heegner points and Heegner indices, compute the p -adic L -function, and perform p -descents. Because many ingredients are involved in the proof, we will give an overview of the methods involved and give more details regarding the computation of the Galois representations.

This is joint work with Michael Stoll.

Presenter: KELLER, Timo (Universität Bayreuth)

Session Classification: Vorträge 2

Contribution ID: 7

Type: **not specified**

Recursive Towers of Function Fields over Finite Fields

Thursday, 10 March 2022 17:10 (30 minutes)

An infinite sequence $\mathcal{F} = (F_0, F_1, \dots)$ of function fields F_n/\mathbb{F}_q of transcendence degree one with full constant field \mathbb{F}_q is called a tower of function fields if all extensions F_{n+1}/F_n are finite separable and the genus $g(F_n)$ tends to infinity as $n \rightarrow \infty$.

A tower is called asymptotically good if its limit $\lambda(\mathcal{F}) := \lim_{n \rightarrow \infty} \frac{N(F_n)}{g(F_n)}$ is positive where $N(F_n)$ denotes the number of \mathbb{F}_q -rational places in F_n .

Good towers can then be used to construct Goppa codes with good parameters. Unfortunately, many of the known good towers are constructed with methods which involve class field theory or modular curves and these constructions usually do not provide explicit presentations of the function fields F_n .

However, a special type of towers are the recursive towers which are recursively defined by bivariate polynomials $f(x, y)$ over \mathbb{F}_q .

Here, we have a sequence (x_0, x_1, \dots) such that F_n is of the explicit form $\mathbb{F}_q(x_0, \dots, x_n)$ and $f(x_n, x_{n+1}) = 0$ holds for all $n \in \mathbb{N}_0$.

In 2005, Beelen-Garcia-Stichtenoth conjectured that a good recursive tower has to have rational splitting, i.e. there is a place in F_0 which splits completely in all extensions F_n/F_0 . In this talk, we will discuss this conjecture.

Presenter: KUHN, Dietrich (CvO Universität Oldenburg)

Session Classification: Vorträge 2

Contribution ID: 8

Type: **not specified**

Checking the CB-property and related problems

Thursday, 10 March 2022 16:30 (30 minutes)

Given a 0-dimensional projective scheme X in the projective n -space over a field K , we are interested in studying the Cayley-Bacharach property (CB-property) of X . Classically, a finite set of points X has the CB-property if every hypersurface of certain degree which contains all points of X but one automatically contains the last point. In this talk we show how to check the CB-property of a 0-dimensional projective scheme X and present some related problems.

Presenter: LONG LE, Ngoc (Universität Passau)

Session Classification: Vorträge 2

Contribution ID: 9

Type: **not specified**

Algorithmic Computation of Arrangements of Lines

Thursday, 10 March 2022 15:10 (30 minutes)

We develop an algorithm based on a greedy algorithm of Cuntz from his 2021 paper “A Greedy Algorithm to Compute Arrangements of Lines in the Projective Plane,” to find arrangements of lines for which a “Mod 2-net coloring” is possible. In such a coloring we color the lines of the arrangement with one of two colors so, that for all lines going through a given point, we have an even number of lines of each color. The question of Mod 2-nets arised in a talk of Yoshinaga based on his 2020 paper “Double coverings of arrangement complements and 2-torsion in Milnor fiber homology,” where it was used to show 2-torsion in the homology of the Milnor fiber of an arrangement. Our talk is based on our Masters Thesis.

Presenter: MUNKACSI, David (Leibniz Universität Hannover)

Session Classification: Vorträge 2

Contribution ID: **10**

Type: **not specified**

Calculating the Frattini subgroup of a polycyclic group

Thursday, 10 March 2022 11:30 (30 minutes)

We present a novel, practical method to determine the Frattini subgroup of a polycyclic group. This method is based on new theoretical investigations about complements and module structure of elementary abelian sections in polycyclic groups. We have implemented our method in GAP and include a discussion of this implementation.

Presenter: NEUMANN-BROSIG, Matthias (TU Braunschweig)

Session Classification: Vorträge 1

Contribution ID: 11

Type: **not specified**

C²-finite Sequences: A Computational Approach

Thursday, 10 March 2022 11:30 (30 minutes)

We define a class of sequences which satisfy a linear recurrence with coefficients that, in turn, satisfy a linear recurrence with constant coefficients themselves, i.e., are C-finite. These C²-finite sequences are a natural generalization of P-finite sequences. We show that C²-finite sequences form a ring and study additional computational properties. It turns out that, compared to P-finite sequences, the algorithmic aspects are much more involved and are related to difficult problems in number theory. Furthermore, we present and discuss an implementation of these methods in the computer algebra system SageMath.

Presenter: NUSPL, Philipp (JKU Linz)

Session Classification: Vorträge 2

Contribution ID: 12

Type: **not specified**

Neues in Maple 2022

Wednesday, 9 March 2022 17:50 (30 minutes)

Wir zeigen die wichtigsten Neuerungen in Maple 2022 aus dem Bereich symbolische Mathematik, darüber hinaus einiges aus den Themen Numerik, Programmierung und Performance, Benutzeroberfläche und Grafik sowie Connectivity.

Presenter: RICHARD, Thomas (Maplesoft GmbH)

Session Classification: Vorträge 1

Contribution ID: 13

Type: **not specified**

Constructive Category Theory and Tilting equivalences via Strong Exceptional Sequences (1/2)

Wednesday, 9 March 2022 16:30 (30 minutes)

In this talk, we'll demonstrate how to implement categories on a computer using our software project CAP - Categories, Algorithms, and Programming. We will discuss how to construct the bounded homotopy category of an additive or Abelian category, as well as the obstacles that arise and their practical categorical solutions.

We use these homotopy categories to demonstrate some applications to homological algebra: We can verify whether a given set of objects in a bounded homotopy category forms a full strong exceptional sequence and implement the associated tilting equivalences.

Presenter: SALEH, Kamal (Universität Siegen)

Session Classification: Vorträge 2

Contribution ID: 14

Type: **not specified**

Compiling towers of categories to unlock the full potential of constructive category theory (2/2)

Wednesday, 9 March 2022 17:10 (30 minutes)

As can be seen in the preceding talk ("Constructive Category Theory and Tilting equivalences via Strong Exceptional Sequences" by Kamal Saleh), building constructive towers of categories allows us to reach advanced and complex applications while retaining (relative) simplicity of both the mathematical descriptions and the organization of the software. Our software project CAP (Categories, Algorithms, and Programming) is explicitly designed to facilitate the creation and organization of such towers. This approach allows for modular, reusable, high-level algorithms which are easy to comprehend and check. The only drawback is the computational overhead introduced by the categorical abstraction.

In this talk, we will see various examples of sources of such overhead and how we can again make use of our highly structured setting to compile the high-level code into low-level code by using a compiler explicitly designed for CAP. The compilation process is fully automated and systematic, but can also be easily extended by giving hints for possible optimizations to the compiler. Due to this, the compiler produces better results than either a generic compiler or a manual compilation process could.

This process lifts all the performance restrictions introduced by the categorical abstraction and thus unlocks the full potential of constructive category theory.

Presenter: ZICKGRAF, Fabian (Universität Siegen)

Session Classification: Vorträge 2

Contribution ID: 15

Type: **not specified**

Varieties over Module Homomorphisms and their Correspondence to Free Algebras

Thursday, 10 March 2022 15:10 (30 minutes)

Homomorphisms of modules are fundamental objects in algebra and applied mathematics. Recent developments in symbolic computation allow a formal verification and exploration of identities between homomorphisms. Systems of those identities define varieties similar as in classical algebraic geometry. For constant-free systems one has a Galois connection to free algebras, and morphisms of varieties can be associated to homomorphisms of factor algebras via a faithful functor. This yields a purely algebraic characterization of isomorphic varieties which can be addressed with Gröbner bases from computer algebra. The suggested formalism finds application in theory on generalized inverses and in recent developments concerning equivalent parametrizations of stabilizing controllers.

Presenter: SCHMITZ, Leonard (U Greifswald)

Session Classification: Vorträge 1

Contribution ID: 16

Type: **not specified**

Power series formulas with m-fold hypergeometric term coefficients

Thursday, 10 March 2022 10:50 (30 minutes)

Power series remain the best tool for effective evaluations of combinations of elementary functions. We overview a recent result concerning the symbolic computation of univariate formal power series.

Given a field K , of characteristic zero, a term $a(n)$ is called m -fold hypergeometric for a positive integer m (mostly $m > 1$), if the ratio $a(n+m)/a(n)$ is rational in $K(n)$. When the value of m is not specified, the phrase “ m -fold hypergeometric terms” denotes all such terms. Several power series formulas are simplified when rewritten with m -fold hypergeometric term coefficients. Moreover, some other power series cannot be computed automatically without using m -fold hypergeometric terms. We present an algorithm that converts holonomic functions into power series with m -fold hypergeometric term coefficients, whenever such coefficients exist.

We demonstrate this result using our implementation in the computer algebra system Maple.

This is joint work with Wolfram Koepf.

Presenter: TEGUIA TABUGUIA, Bertrand (Universität Kassel / Max Planck Institute for Mathematics in the Sciences)

Session Classification: Vorträge 2

Contribution ID: 17

Type: **not specified**

On programming with algebraic groups

Thursday, 10 March 2022 10:50 (30 minutes)

In group theory, we have a certain type of groups, i.e. algebraic groups. Thanks to an additional underlying structure, we have certain rules for the action of the groups on themselves and their associated Lie algebras.

We are interested in solving several problems concerning algebraic groups (for example linked to representation theory) in small characteristic. These complex problems often require computation. This talk will look into a possible solution for a programme concerned with orbit calculations over an algebraic closure of a field with small characteristic.

Presenter: VOGGESBERGER, Laura (TU Kaiserslautern)

Session Classification: Vorträge 1

Contribution ID: 18

Type: **not specified**

Invariant Varieties and Collision-Freeness for Polynomial Systems

Wednesday, 9 March 2022 14:00 (1 hour)

The concept of invariant sets is an important tool to investigate the behaviour of solutions of ODE systems. A set M is called invariant for a specified system if every trajectory of the system that starts in M stays in M for all times. Considering varieties and polynomial ODE systems, we show how Groebner bases provide an algorithmic test for invariance. As an application, we investigate collision-freeness for polynomial ODE systems, which is a structural property in particle dynamics. An ODE system is called collision-free if every trajectory of the system that has distinct components at some initial time has distinct components for all times. We show that this property is closely related to the concept of invariant varieties, which again provides an algorithmic test for collision-freeness.

Presenter: HARMS, Melanie (RWTH Aachen)

Session Classification: Hauptvorträge

Contribution ID: 19

Type: **not specified**

Investigating Terao's freeness conjecture with computer algebra

Thursday, 10 March 2022 09:15 (1 hour)

Motivated by singularity theory, Hiroaki Terao introduced a module of logarithmic derivations associated with a hyperplane arrangement. This talk is concerned with Terao's freeness conjecture which asserts that the freeness of this derivation module is determined by the underlying combinatorics of the arrangement.

To investigate this conjecture, we have enumerated all matroids of rank 3 with up to 14 hyperplanes whose characteristic polynomial splits over the integers and saved it in a public database. Subsequently we have computed the moduli space and the free locus of the derivation module of each of these matroids as a quasi-affine set. As the main result, this yields a computational proof of Terao's freeness conjecture for rank 3 arrangements with up to 14 hyperplanes over arbitrary characteristic.

In this talk, I will explain the background of this conjecture without assuming prior knowledge and demonstrate the database and highlights of the computations.

This talk is based on joint work with Mohamed Barakat, Reimer Behrends, Christopher Jefferson, and Martin Lerner.

Presenter: KÜHNE, Lukas (MPI Leipzig)

Session Classification: Hauptvorträge

Contribution ID: 20

Type: **not specified**

Lattice isomorphism and the integral matrix similarity problem

Thursday, 10 March 2022 14:00 (1 hour)

We introduce the integral matrix similarity problem as a special instance of the conjugacy problem from group theory. While this problem has been known to be decidable for over 40 years due to work of Grunewald and Sarkisyan, no upper bound on the complexity has been known and practical algorithms have only been available for special cases. We will present recent work with Werner Bley and Henri Johnston, where we exploit the connection to the lattice isomorphism problem for orders to provide a practical algorithm with proven complexity.

Presenter: HOFMANN, Tommy (Universität Siegen)

Session Classification: Hauptvorträge

Contribution ID: 21

Type: **not specified**

The SONC Cone: Primal and Dual Perspectives

Friday, 11 March 2022 09:15 (1 hour)

Countless applied problems in various sciences can be expressed as polynomial optimization problems. Solving these nonlinear problems essentially requires to certify nonnegativity of multivariate, real polynomials, a classical problem from real algebraic geometry.

A classical way to certify nonnegativity are sums of squares (SOS). An alternative way are sums of nonnegative circuit polynomials (SONC), which I introduced joint with Iliman in 2014. The latter forms a convex cone with algebraic boundary.

Moreover, motivated from a dualization process, one can obtain a particular (strict but full-dimensional) subcone of the SONC cone leading to certificates which have, despite being weaker than SONC, the great benefit to be obtainable via linear programming.

In this talk I will speak about the SONC cone and its subcone. It is based on joint work with Jens Forsgaard and upcoming work with Janin Heuer.

Presenter: DE WOLFF, Timo (TU Braunschweig)

Session Classification: Hauptvorträge

Contribution ID: 22

Type: **not specified**

Inequalities and computer algebra

Friday, 11 March 2022 11:00 (1 hour)

It is very common nowadays to use tools from symbolic computation for applications in many different areas of mathematics (or physics, or computer science, etc.). One backbone of symbolic computation is Cylindrical Algebraic Decomposition (CAD). In the past years, we have applied CAD to problems arising in applied mathematics. Among the prominent methods in computer algebra are algorithms to prove and derive identities of expressions satisfying difference or differential equations of certain type. Thanks to work of Gerhold and Kauers, CAD can be used to (semi-)automatically prove inequalities including this type of input. In this talk, I give an overview on some of these results.

Presenter: PILLWEIN, Veronika (RISC Linz)

Session Classification: Hauptvorträge

Contribution ID: 23

Type: **not specified**

Neues aus der Fachgruppe

Thursday, 10 March 2022 12:10 (20 minutes)

Presenter: FRÜHBIS-KRÜGER, Anne (Uni Oldenburg)

Session Classification: Vorträge 1