On integrability and the relation to defects and generalised T-duality

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(Classical) integrability: $\boxed{\text{solvable by quadrature}}$

i) Liouville integrability: Hamiltonian system on $\dim M = 2n$ mfd.

↳ n conserved quantities F_i

↳ $\{F_i, F_j\} = 0$ (in involution)

ii) Lax approach: (for 2d field theory)

$\mathcal{L}_\alpha$ connection: $\mathcal{L} = \begin{cases} \mathcal{L}_t = V(t,x,\lambda) \\ \mathcal{L}_x = U(t,x,\lambda) \end{cases}$ $\boxed{F_{\alpha\beta}(\mathcal{L}) = 0 \iff \text{e.o.m}}$

↳ $F_{\alpha\beta}(\mathcal{L}) = 0 \implies \underbrace{\partial_t T(t,\lambda) = [V(t,a,\lambda), T(t,\lambda)]}$ .... $T(t,\lambda) = \mathcal{P} e^{\int_a^b dx\, U(t,x,\lambda)}$

$\Rightarrow F_i = \mathrm{tr}\,(T^i(\lambda, t))$ conserved

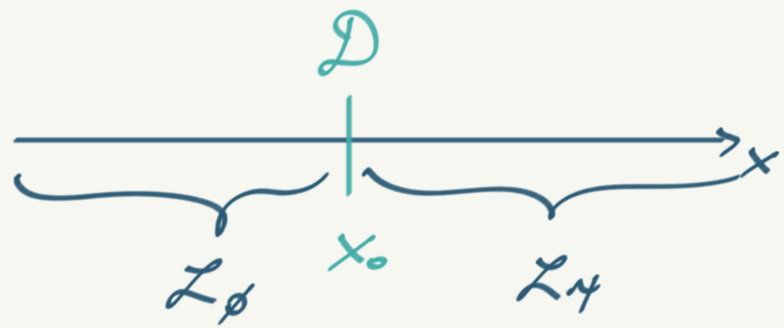
↳ $\{F_i(\lambda), F_j(\mu)\} = 0$? $\rightsquigarrow$ classical r-matrix, Yang-Baxter equation, ...

Defects: ... on the real line

(Corrigan, Zambon 2018)

- $\mathcal{L} = \theta(x_0 - x)\mathcal{L}_\phi + \theta(x - x_0)\mathcal{L}_\psi + \delta(x - x_0)\mathcal{D}(\phi,\psi)$

  (e.g. Affine Toda field theory)



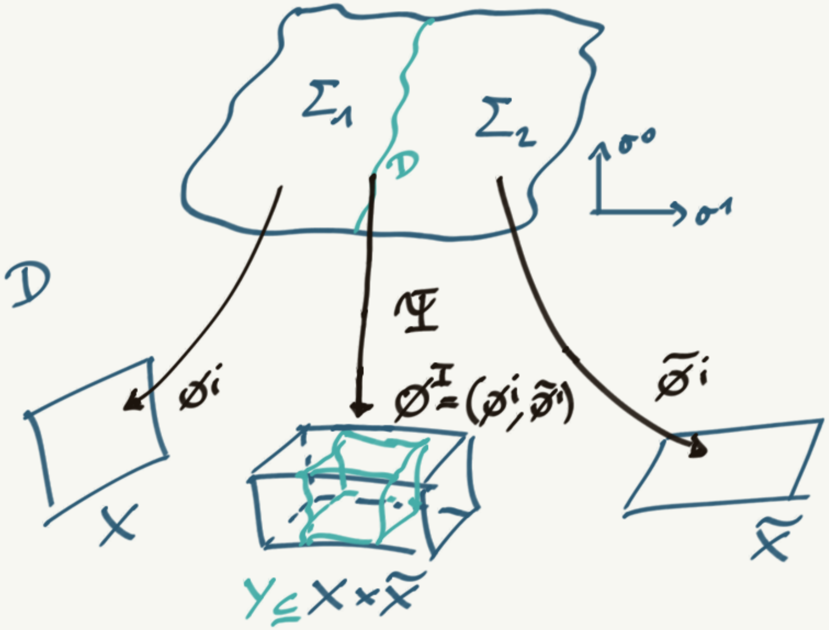
↳ goal: preserve
- integrability,
- energy-momentum conservation
- ⋮

$\Longrightarrow$ defect/gluing conditions at x_0 $\Rightarrow$ $\boxed{\text{find admissible } \mathcal{D}}$

- Preserve Integrability: $\boxed{\partial_t K = K U_\psi - U_\phi K}$

$\Longrightarrow$ $\underline{Q = \mathrm{tr}\left(T(-\infty, x_0, \lambda) K T(x_0, \infty, \lambda)\right)}$ conserved

Defects: ... on two-dimensional worldsheet Σ



- $S_\Sigma = S_{\Sigma_1} + S_{\Sigma_2} + S_D$, $D \subset \Sigma$ one-dim submanifold

↳ topological defects: $T^{(1)} = T^{(2)}, \ \bar{T}^{(1)} = \bar{T}^{(2)} \ \forall p \in D$

- σ-model action on Σ_1, Σ_2:

$$S = \int_{\Sigma_1} d^2\sigma \left(-\frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - \frac{1}{2} b_{ij} \varepsilon^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j \right) + \int_{\Sigma_2} d^2\sigma \, (\tilde{g}, \tilde{b})$$

$$+ \int_D \underbrace{d\sigma^0 A_I \partial_0 \phi^I}_{\Psi^* A}$$

↳ $A = A_I \, d\phi^I$ connection one-form with values in $Y \subseteq X \times \tilde{X}$

& associated curvature $F = dA$

$\Longrightarrow$ demand of having topological defect restricts admissible pairs of (Y, F)

(Kapustin, Setter 2010)

(Generalised) T-duality:

$$S \propto \int_\Sigma d^2\sigma \left(\sqrt{-h}\, h^{\alpha\beta} G_{\mu\nu}\, \partial_\alpha X^\mu \partial_\beta X^\nu + \varepsilon^{\alpha\beta} B_{\mu\nu}\, \partial_\alpha X^\mu \partial_\beta X^\nu + \alpha' \sqrt{-h}\, R\, \Phi\right)$$

- abelian T-duality: isometry group G abelian

$$S = S(G, B, \Phi) \Longleftrightarrow \text{dual } \tilde{S} = \tilde{S}(\tilde{G}, \tilde{B}, \tilde{\Phi})$$

(G, B, Φ) and $(\tilde{G}, \tilde{B}, \tilde{\Phi})$ related by Buscher rules

- non-abelian T-duality: G non-abelian $\rightsquigarrow$ no Buscher rules

choosing $Y, \mathcal{A}$ s.t. $(Y, \mathcal{F})$ defines top. defect

$\Longrightarrow$ T-dual backgrounds (abelian / non-abelian)

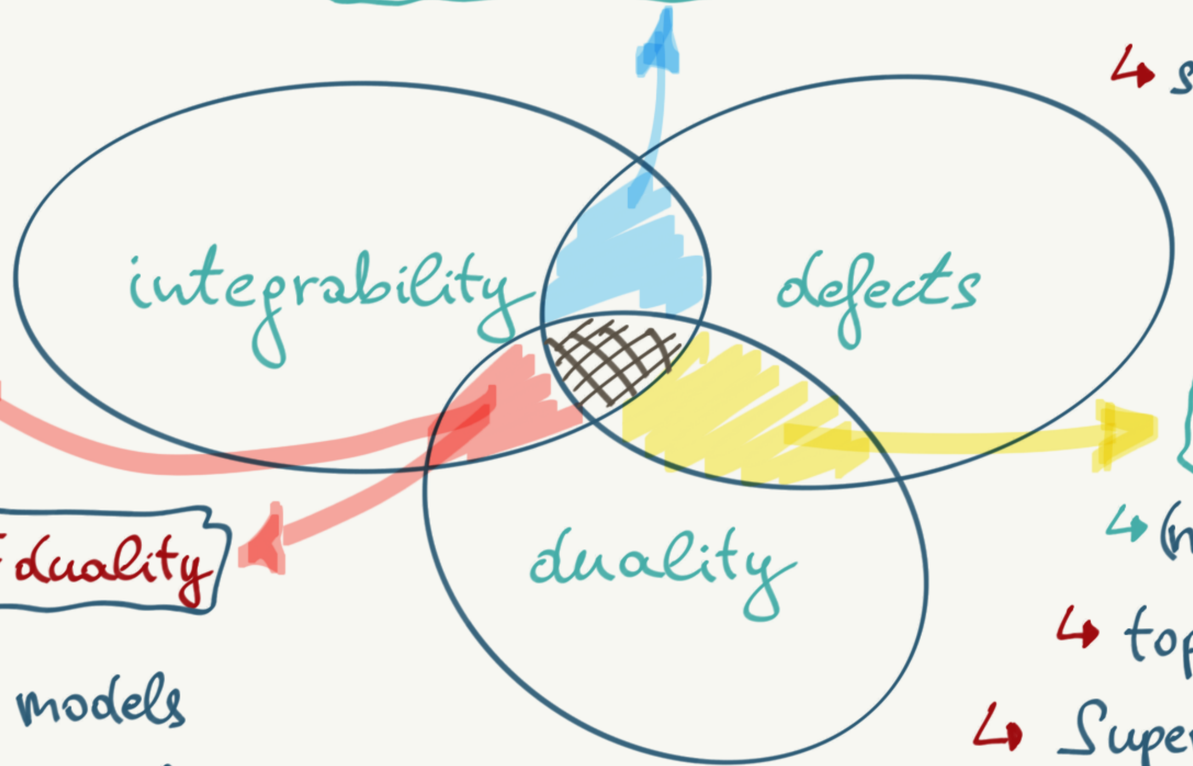
(Gevorgyan, Sarkissian 2013)

Conclusions & Outlook:

integrable defects for theories on real line

↳ Which theories can support defects,
e.g Affine Toda F.T.

↳ solitons & defects



AdS/CFT

Poisson-Lie T-duality

↳ integrable models
(η-deformation)

topological defects

↳ (non-)abelian T-duality

↳ top. defects & SUSY

↳ Superstring theory:

- T-duality for D-branes
  „Fourier-Mukai transform"
- heterotic string th.