IR-resummed bispectrum for Effective Field Theory of Large Scale Structure

Jungwon Lim
ETH Zürich

Contents

- Motivations
- Details on IR-resummation of bispectrum
- Result and discussion

Motivations cut into three parts

③ IR-resummed bispectrum
for Effective Field Theory of
② Large Scale Structure
①

IR-resummed bispectrum for Effective Field Theory of Large Scale Structure

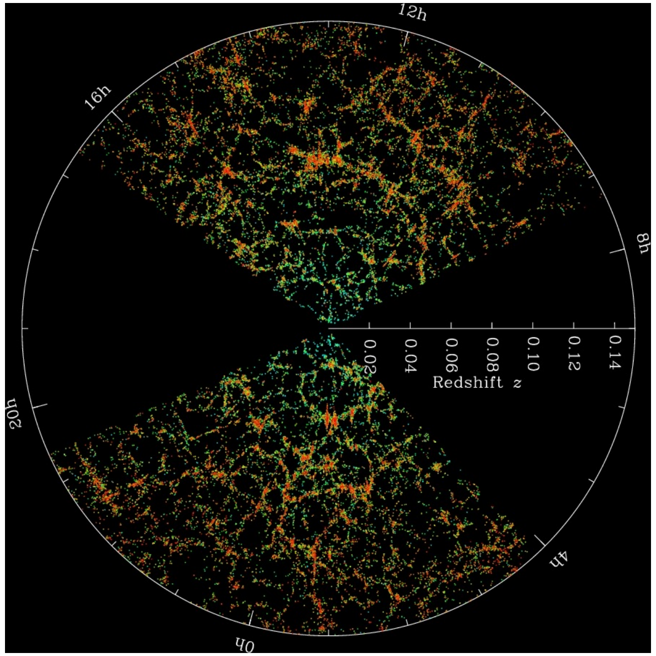
①

Perturbation Theory of Large Scale Structure (LSS)

- The Large Scale Structure of the universe refers to the pattern of galaxies and matter on scales much larger than individual galaxies or groupings of galaxies.

- Small primordial fluctuations were amplified due to the gravitational interaction of collisionless cold dark matter particles.

- We obtain about 10^9modes while the Planck satellite has about 10^6modes (CMB has upper limit due to Silk damping)



Galaxies discovered by the Sloan Digital Sky Survey (SDSS)

Eulerian Perturbation Theory

From Dark Matter particles to Cosmic fluids

- Density

$\rho(\mathbf{x},\tau) \equiv \bar{\rho}(\tau)[1+\delta(\mathbf{x},\tau)]$

with density contrast $\delta(\mathbf{x})$

- Velocity

$\mathbf{v}(\mathbf{x},\tau) \equiv \mathcal{H}\mathbf{x} + \mathbf{u}(\mathbf{x},\tau)$

with peculiar velocity $\mathbf{u}$

- Potential

$\phi(\mathbf{x},\tau) \equiv -\frac{1}{2}\frac{\partial\mathcal{H}}{\partial\tau}x^2 + \Phi(\mathbf{x},\tau)$

with gravitation potential Φ

- Equation of motion

$$\frac{d\mathbf{p}}{d\tau} = -am\nabla\boldsymbol{\Phi}(\mathbf{x}) \qquad \text{With } \mathbf{p} = am\mathbf{u}$$

Lagrangian Perturbation Theory

- Lagrangian dynamics follows the trajectories of particles or fluid elements.

- In Lagrangian dynamics object of interest is the displacement field $\boldsymbol{\Psi}(\mathbf{q})$, mapping the initial particle positions $\mathbf{q}$ into the final Eulerian particle positions $\mathbf{x}$.

$$\mathbf{x}(\tau) = \mathbf{q} + \Psi(\mathbf{q}, \tau)$$

- The equation of motion for particle trajectories $\mathbf{x}(\tau)$ is

$$\frac{d^2\mathbf{x}}{d\tau^2} + \mathcal{H}(\tau)\frac{d\mathbf{x}}{d\tau} = -\nabla\boldsymbol{\Phi}$$

IR-resumed bispectrum for Effective Field Theory of Large Scale Structure

②

①

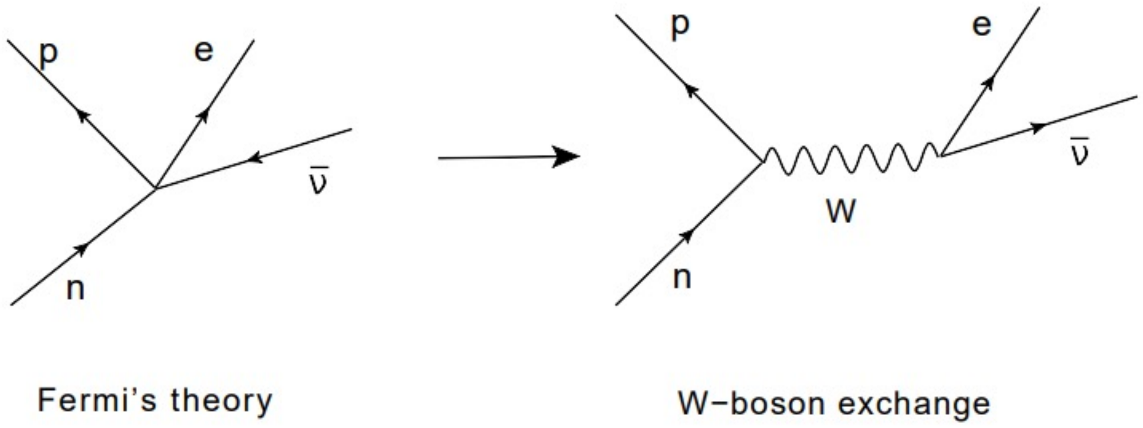
Effective Field Theory

- At very high energy, we don't know the exact physics
- Effective field theory is based on simple ideas that physical phenomena at low-energies are not sensitive to the details of the high-energy structure of particles or field.
- We can use effective field theory to describe physics at a given energy scale to a given accuracy in terms of quantum field theory with a finite set of parameters.

For example, Fermi theory on weak interaction

$$G_F = 1.166\times10^{-5}\text{GeV}^{-2}$$

$$G_F = \frac{\sqrt{2}g^2}{8m_w^2}$$



Rajasekaran, G. (2014). Fermi and the theory of weak interactions. *Resonance*, *19*(1), 18-44.

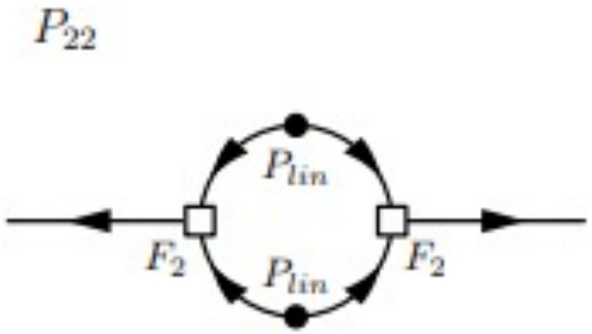
Motivation of Effective Field Theory of Large Scale Structure

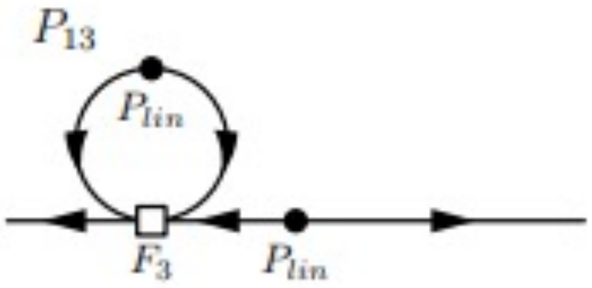
First order power spectrum

$$P_{1-\text{loop}}(k,\tau) = D^4(\tau)[P_{22}(k) + P_{13}(k)]$$

$$P_{22}(k) = 2\int_q F_2^2(q, k-q) P_{\text{lin}}(q) P_{\text{lin}}(|k-q|)$$

$$P_{31}(k) = 6P_{\text{lin}}(k)\int_q F_3(q, -q, k) P_{\text{lin}}(q)$$





Simonović, M., Baldauf, T., Zaldarriaga, M., Carrasco, J. J., & Kollmeier, J. A. (2018). Cosmological perturbation theory using the FFTLog: formalism and connection to QFT loop integrals. *Journal of Cosmology and Astroparticle Physics*, *2018*(04), 030.

Motivation of Effective Field Theory of Large Scale Structure

- Standard perturbation theory (SPT) of LSS cannot be used to describe the small scales.

  → The series do not converge so no resummation of diagrams will fix the problem.

- In loop calculations, those small scales affect large scale observables as the loop integral cover all momenta and the errors in the small scales pollute large scale results.

⇨ This has led to the development of the Effective Field Theory of Large Scale Structure

IR-resummed bispectrum for Effective Field Theory of Large Scale Structure

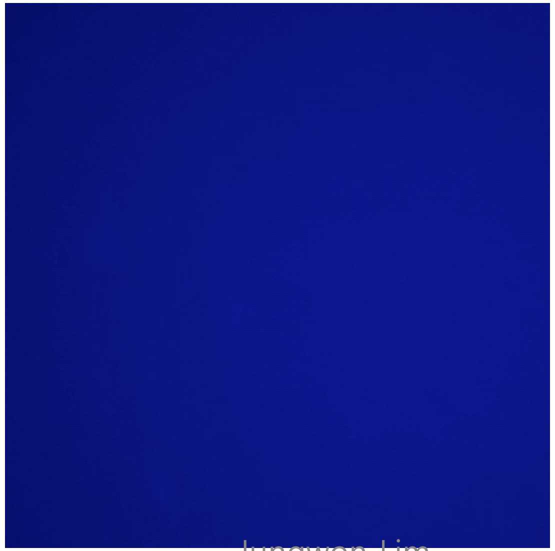
③ ② ①

Motivation for Largangian Dynamics on Effective Field Theory

- The Effective Field Theory was originally developed in Eulerian dynamics and used to compute one and two-loop corrections to the matter power spectrum.

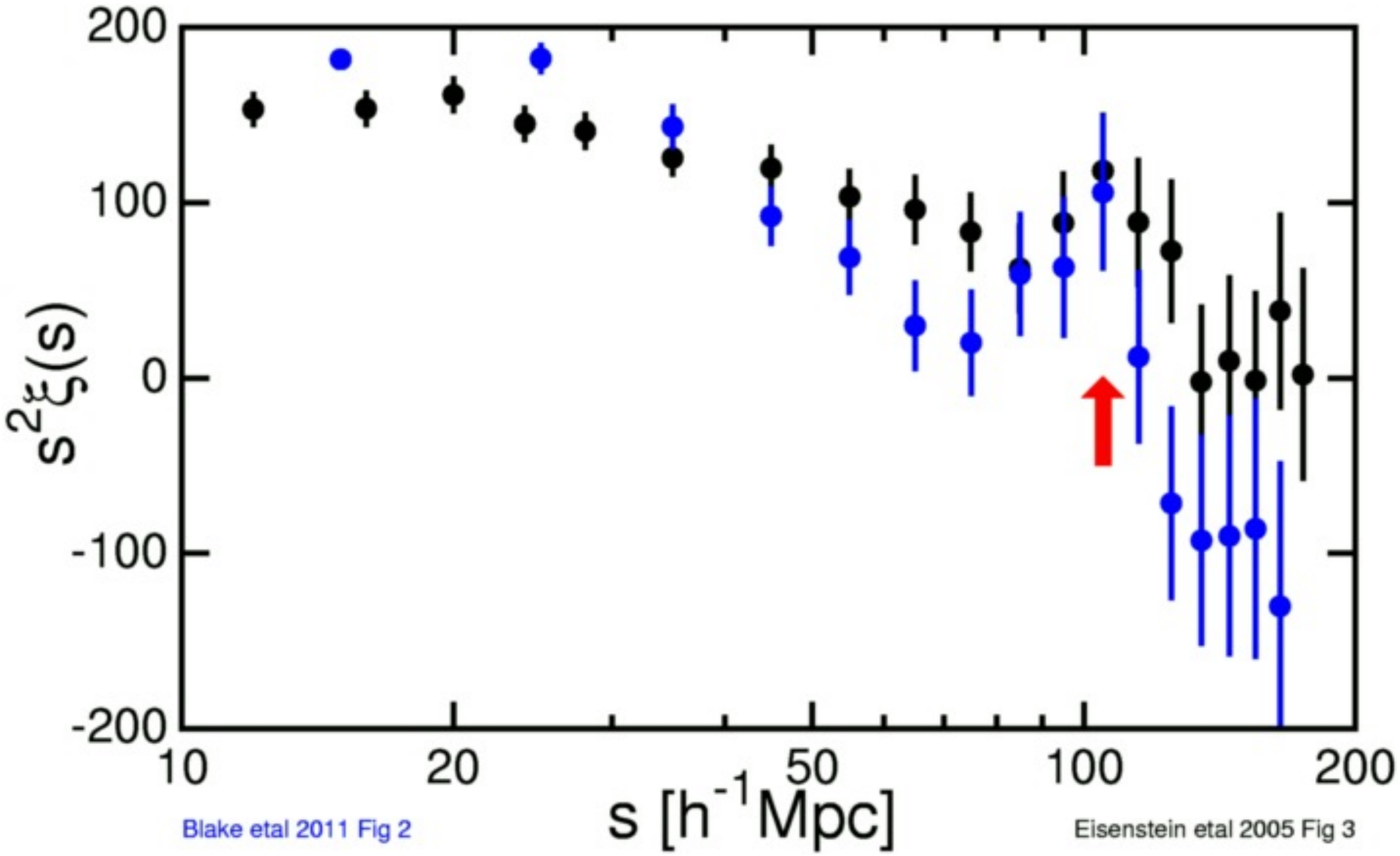
- However, Eulerian effective field theory of large scale structure shows 2% residual whi ch appeared related to Baryon Acoustic oscillation (BAO).

- For Baryon Acoustic oscillation (BAO), perturbation theory in Lagrangian space is better than Eulerian perturbation.



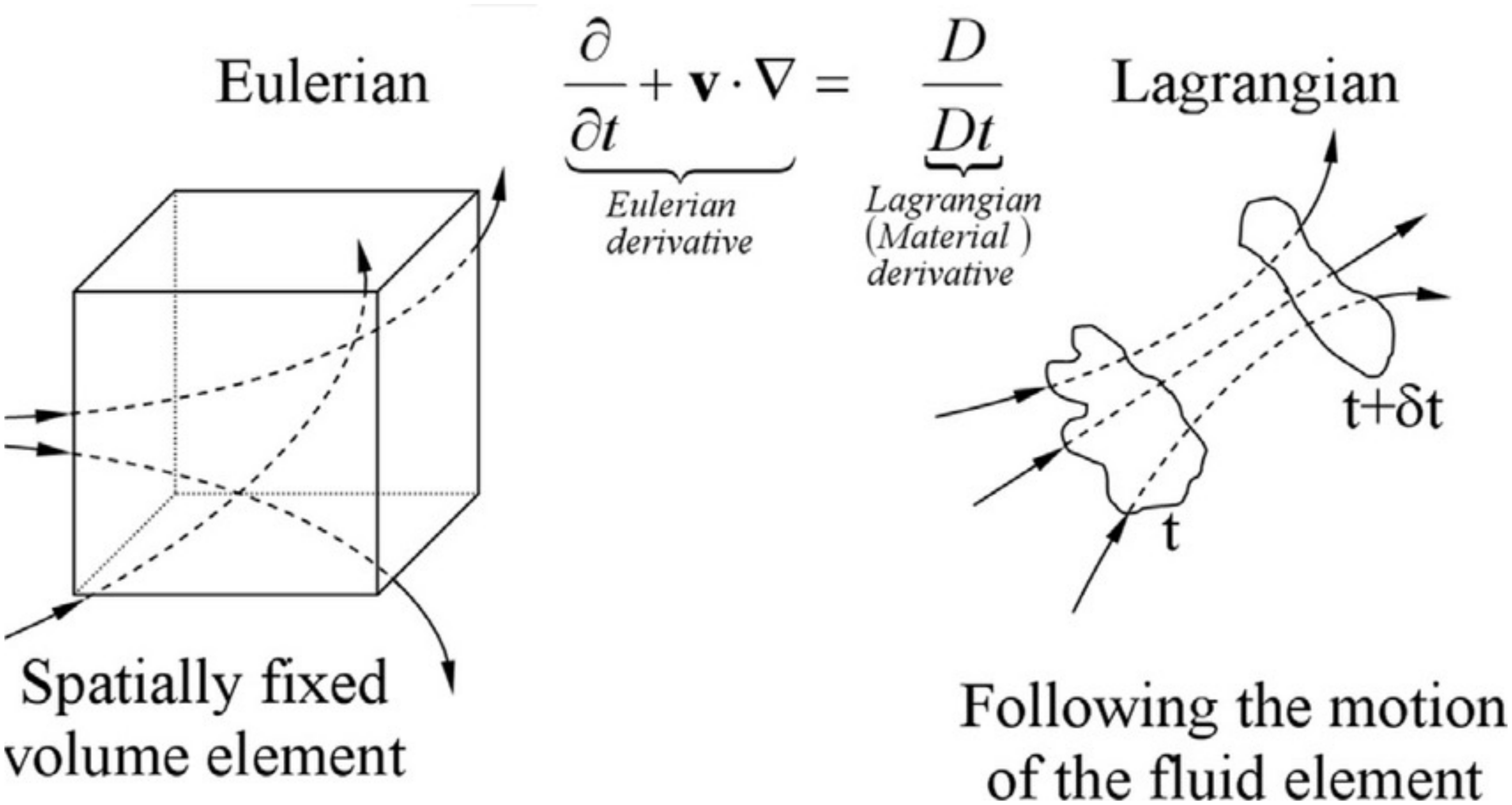
https://svs.gsfc.nasa.gov/13768

Baryon Acoustic Oscillation (BAO)



http://www.astro.ucla.edu/~wright/BAO-cosmology.html

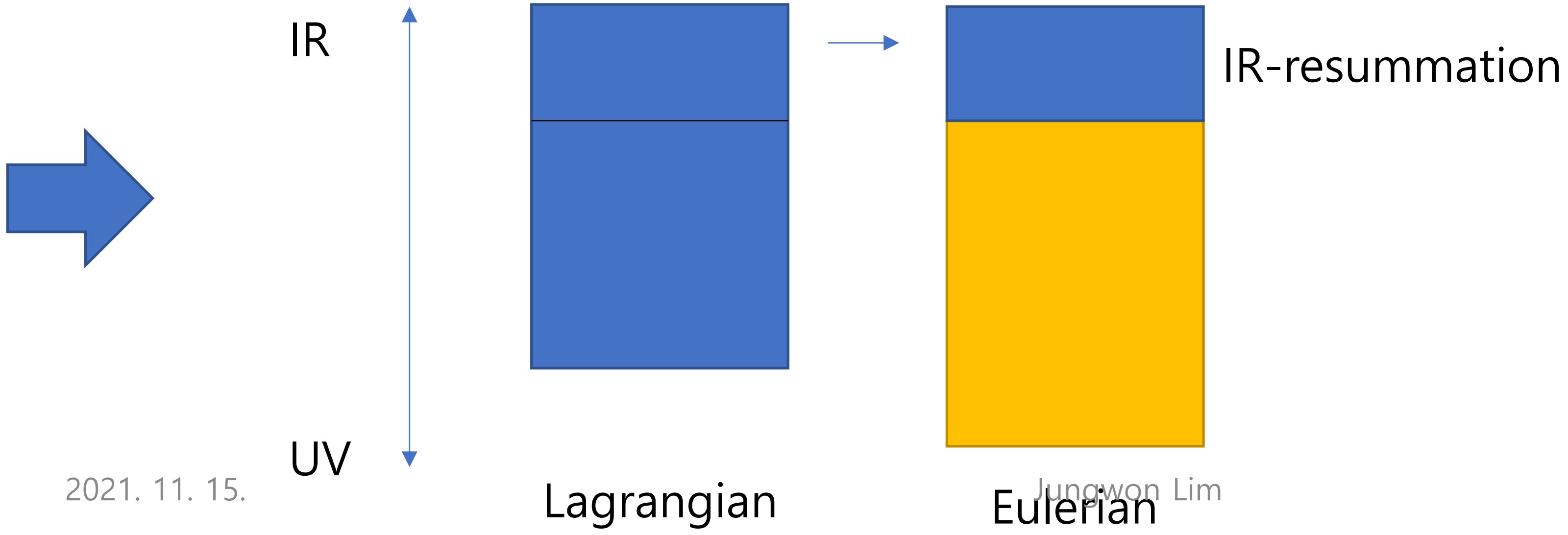
Eulerian picture vs Lagrangian picture



Shadloo, M. S. & Le Touzé, David & Oger, Guillaume. (2016). Smoothed particle hydrodynamics method for fluid flows, towards industrial applications-Motivations, current state, and challenges.

Motivation for IR-resummation

- Fundamental quantities of Eulerian scheme are local fields like the over-density and velocity in the fluid picture.
- The Eulerian picture is not good at describing bulk (IR) displacement effects.
- By resumming the effects coming from long wavelength to make Lagrangian and Eulerian one-to-one correspondent.



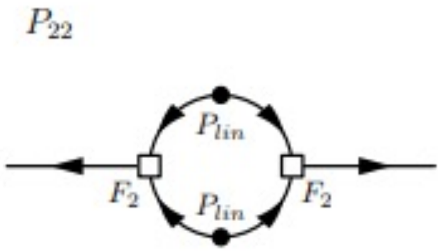
Details for IR-resummed bispectrum

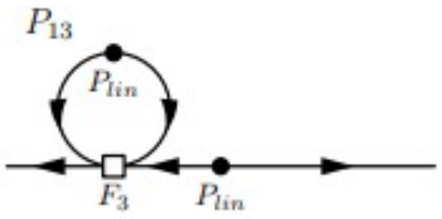
Coefficients of non-linearities

Three types of coefficients coming from one-loop power spectrum

$$P_{22}(k) = 2\int_q F_2^2(q, k-q)P_{\text{lin}}(q)P_{\text{lin}}(|k-q|)$$

$$P_{31}(k) = 6P_{\text{lin}}(k)\int_q F_3(q, -q, k)P_{\text{lin}}(q)$$





$$\epsilon_{\delta_<} = \int_0^k \frac{d^3k'}{(2\pi)^3} P_{\text{lin}}(k')$$

(related to IR density fluctuation)

$$\epsilon_{s_>} = k^2 \int_k^\infty \frac{d^3k'}{(2\pi)^3} \frac{P_{\text{lin}}(k')}{k'^2}$$

(related to UV displacement)

$$\epsilon_{s_<} = k^2 \int_0^k \frac{d^3k'}{(2\pi)^3} \frac{P_{\text{lin}}(k')}{k'^2}$$

(related to IR displacement)

This seems to be cancelled in equal-time correlator

Need to be resummed

IR-resummed correlation function

The three-point correlation function at all orders in $\epsilon_{s_<}$ and expanded to order N in $\epsilon_{\delta_<}$ and $\epsilon_{s_>}$ is given by

$$\xi_{\delta\delta\delta}(\vec{r},\vec{r'};t_1,t_2,t_3)\Big|_N = \sum_{j=0}^{N}\int d^3q\, d^3q' R_{N-j}(\vec{r}-\vec{q},\vec{r'}-\vec{q'};\vec{r},\vec{r'})\xi_j^E(\vec{q},\vec{q'})$$

Where $\xi_j^E(\vec{q},\vec{q'})$ is the j-th order contribution to the Eulerian three-point correlation function, and the kernels R_{N-j} contain the information about the long wavelength displacements (the form of the kernels R_{N-j} is derived from the Lagrangian picture)

Correlation Function

Fourier Transformation

IR-resummed bispectrum for Effective Field Theory of Large Scale Structure

Motivation on Wiggle/no-Wiggle Method

$$\xi_{\delta\delta\delta}(\vec{r},\vec{r'};t_1,t_2,t_3)\Big|_N = \sum_{j=0}^{N}\int d^3q\, d^3q' R_{N-j}(\vec{q},\vec{q'};\vec{r},\vec{r'})\xi_j^E(\vec{r}-\vec{q},\vec{r'}-\vec{q'})$$

$$B(\vec{k_1},\vec{k_2};t_1,t_2,t_3)\Big|_N = \sum_{j=0}^{N}\int d^3r d^3r' e^{-i\vec{k_1}\cdot\vec{r}}e^{-i\vec{k_2}\cdot\vec{r'}}\int d^3q\, d^3q' R_{N-j}(\vec{q},\vec{q'};\vec{r},\vec{r'})\xi_j^E(\vec{r}-\vec{q},\vec{r'}-\vec{q'})$$

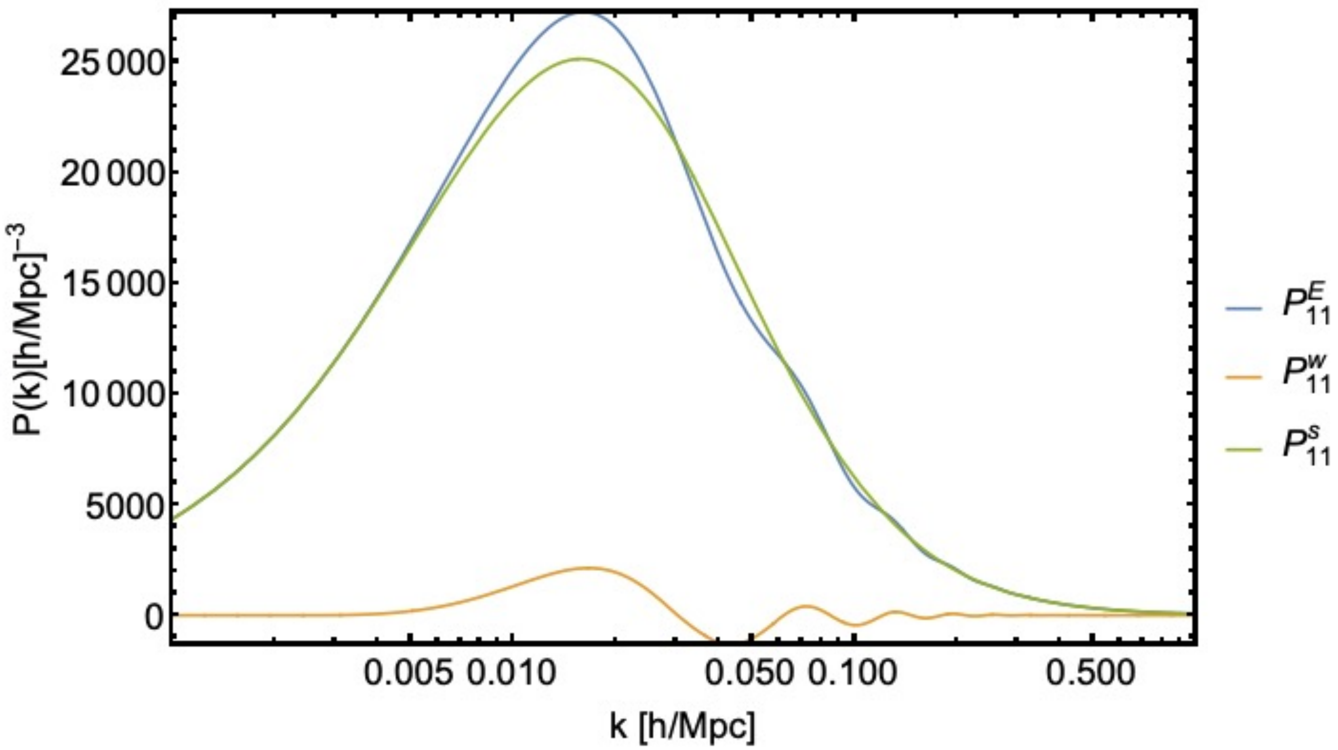
- To get IR-resummed bispectrum, we need to take Fourier transformation of the IR-resummed correlation function

- R_{N-j} has the dependence on q, q', $q\cdot q'$, $q\cdot r$, $q'\cdot r'$ and $r\cdot r'$

→ Very challenging to take integration numerically

Wiggle/No-Wiggle method for power spectrum

By splitting smooth part and oscillating part of power spectrum, one can get simpler expression for IR-rsummed bispectrum.



$$P_{11} = P_{11}^s + P_{11}^w$$

Wiggle/No-Wiggle method for power spectrum

$$\xi_{\delta\delta}(\vec{r};t_1,t_2)\Big|_N = \sum_{j=0}^{N}\int d^3q\, R_{N-j}(\vec{r})\xi_j^E(|\vec{r}-\vec{q}|)$$

$$P(\vec{k};t_1,t_2)\Big|_N = \sum_{j=0}^{N}\int d^3r e^{-i\vec{k}\cdot\vec{r}}\int d^3q\, R_{N-j}(\vec{r})\xi_j^E(|\vec{r}-\vec{q}|)$$

$$P_{11} = P_{11}^s + P_{11}^w$$

- wiggle power spectrum ⟷ peaked correlation function

$$P^w(k)\Big|_1 \rightarrow \exp\left[-\frac{1}{2}k^2\overline{A_0}\right]\left(\left(1+\frac{1}{2}k^2\overline{A_0}\right)P_{\rm lin}^w(k) + P_1^w(k)\right)$$

$\xi^p(|\vec{r}-\vec{q}|)$ can be treated as Dirac-delta function for R_{N-j}

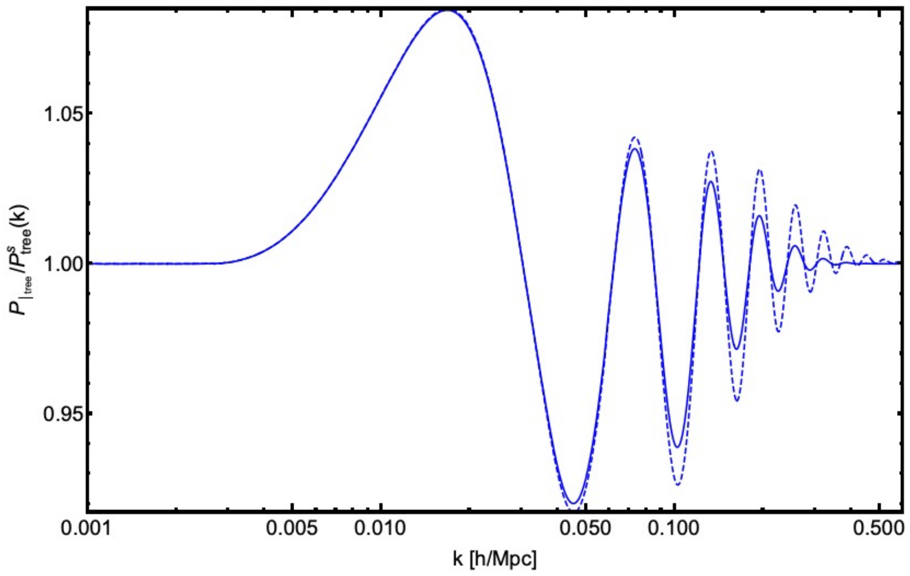
- smooth power spectrum ⟷ smooth correlation function

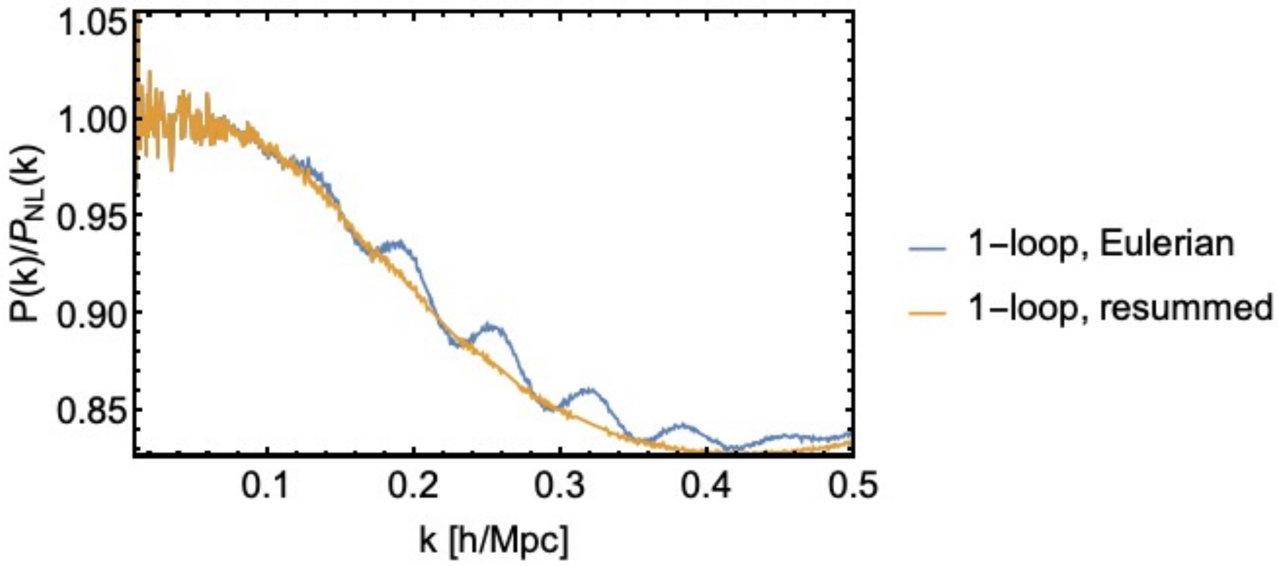
$$P^s(k)\Big|_1 \rightarrow P_{\rm lin}^s(k) + P_1^s(k)$$

$\xi^s(|\vec{r}-\vec{q}|) \rightarrow \xi^s(r)$ for R_{N-j}

Wiggle/No-Wiggle method for power spectrum

$$P(\vec{k};t_1,t_2)\Big|_1 = \exp\left[-\frac{1}{2}k^2\overline{A_0}\right]\left(\left(1+\frac{1}{2}k^2\overline{A_0}\right)P^w_{\mathrm{lin}}(k)+P^w_1(k)\right)+\mathrm{P}^s_{\mathrm{lin}}(k)+P^s_1(k)$$





Solid line : IR-resummed, dashed line : Eulerian

Wiggle/No-Wiggle method for bispectrum

As we divided the power spectrum by two (wiggle and no-wiggle part), we can also divide the bispectrum as

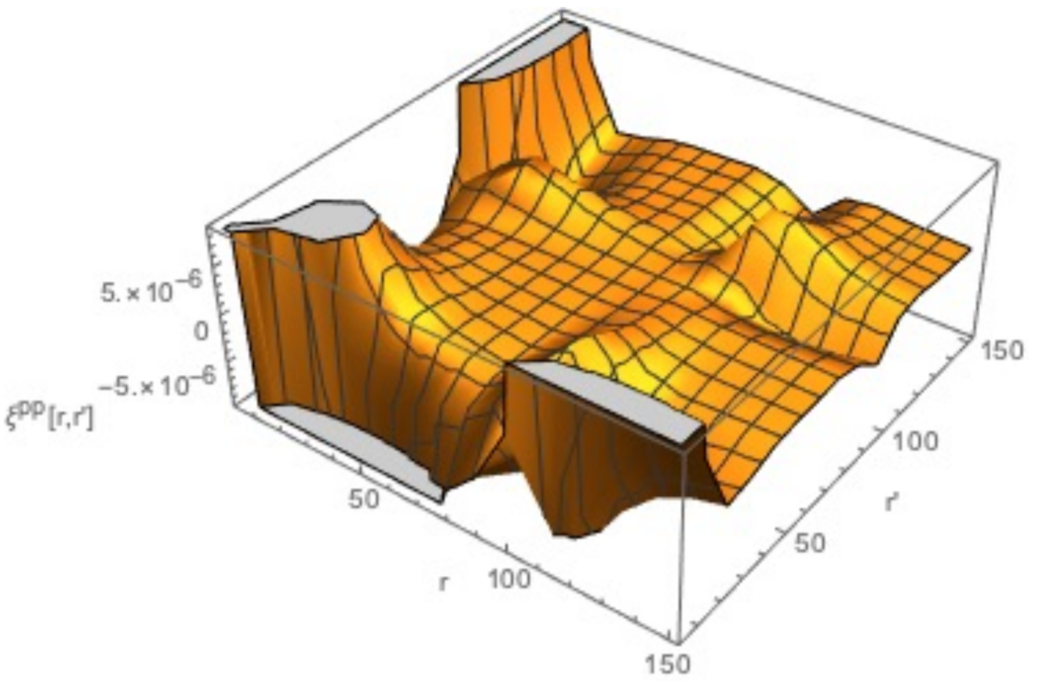
$$B_{tree}(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3}) = 2\left(P_{11}^{s}(\boldsymbol{k_1}) + P_{11}^{w}(\boldsymbol{k_1})\right)\left(P_{11}^{s}(\boldsymbol{k_2}) + P_{11}^{w}(\boldsymbol{k_2})\right)F_2^{(s)}(\boldsymbol{k_1},\boldsymbol{k_2}) + \text{permutations}$$

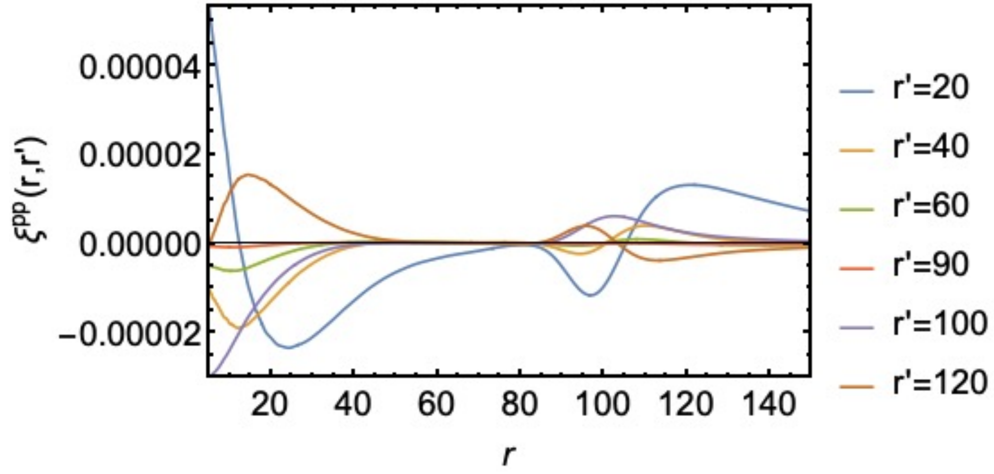
$$= 2\left(P_{11}^{s}(\boldsymbol{k_1})P_{11}^{s}(\boldsymbol{k_2}) + P_{11}^{s}(\boldsymbol{k_1})P_{11}^{w}(\boldsymbol{k_2}) + P_{11}^{w}(\boldsymbol{k_1})P_{11}^{s}(\boldsymbol{k_2}) + P_{11}^{w}(\boldsymbol{k_1})P_{11}^{w}(\boldsymbol{k_2})\right)F_2^{(s)}(\boldsymbol{k_1},\boldsymbol{k_2}) + \text{permutations}$$

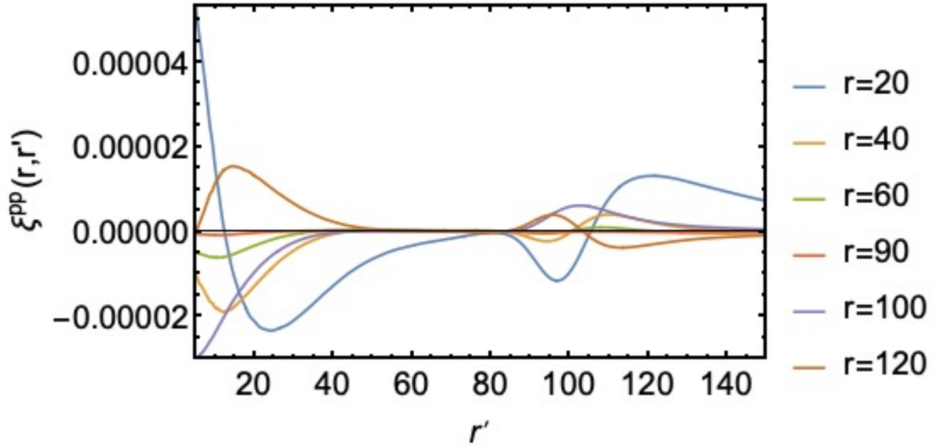
$$= 2\left(\underbrace{B^{ss}(\boldsymbol{k_1},\boldsymbol{k_2})}_{\text{smooth-smooth}} + \underbrace{B^{sw}(\boldsymbol{k_1},\boldsymbol{k_2})}_{\text{smooth-wiggle}} + \underbrace{B^{ws}(\boldsymbol{k_1},\boldsymbol{k_2})}_{\text{wiggle-smooth}} + \underbrace{B^{ww}(\boldsymbol{k_1},\boldsymbol{k_2})}_{\text{wiggle-wiggle}}\right)F_2^{(s)}(\boldsymbol{k_1},\boldsymbol{k_2}) + \text{permutations}$$

Wiggle/No-Wiggle method for bispectrum

- Wiggle-wiggle bispectrum B^{ww} ⟷ Peaked-peaked correlation function ξ^{pp}







Wiggle/No-Wiggle method

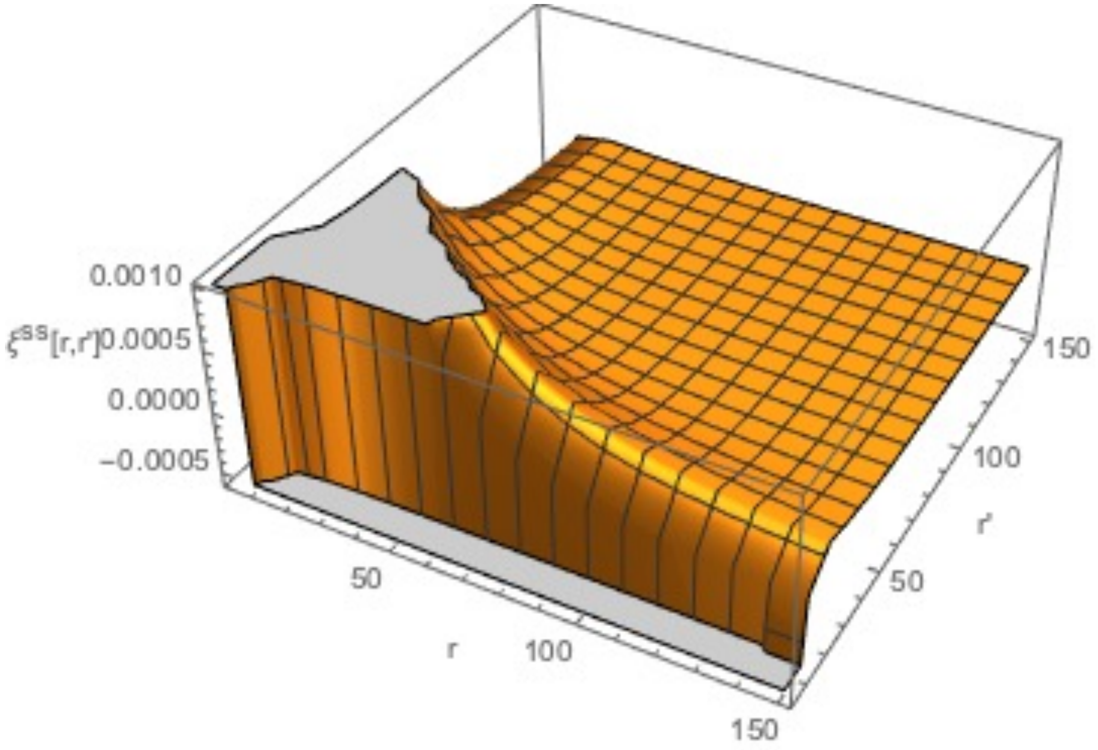
$$B^{ww}(\overrightarrow{k_1},\overrightarrow{k_2})\Big|_1 = \frac{1}{(2\pi)^3}\int d^3r d^3r' \frac{1}{|A_0|^{1/2}}\frac{1}{|C_0|^{1/2}}\int d^3q d^3q' e^{i\vec{q}\cdot\overrightarrow{k_1}} e^{i\overrightarrow{q'}\cdot\overrightarrow{k_2}} \exp\left[-\frac{1}{2}\bar{A}_0^{-1}q'^2 - \frac{1}{2}\bar{C}_0^{-1}q^2\right]$$
$$\times\left(\left(4 - \frac{1}{2}\bar{A}_0^{-1}q'^2 - \frac{1}{2}\bar{C}_0^{-1}q^2\right)\xi^{pp}_{tree}(\vec{r}-\vec{q},\overrightarrow{r'}-\overrightarrow{q'}) + \xi^{pp}_1(\vec{r}-\vec{q},\overrightarrow{r'}-\overrightarrow{q'})\right)$$

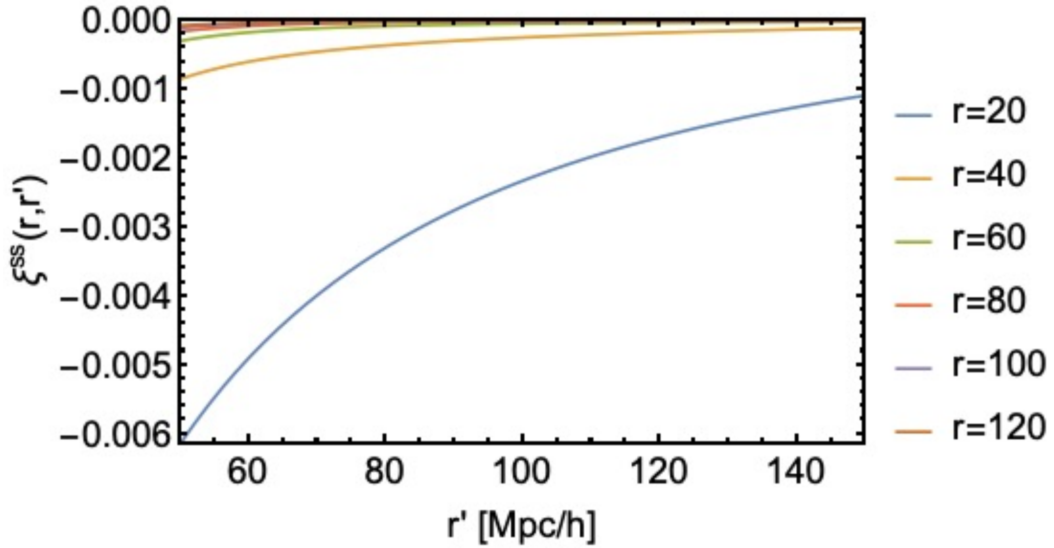
↓ Treat the peaked function as Dirac-delta function

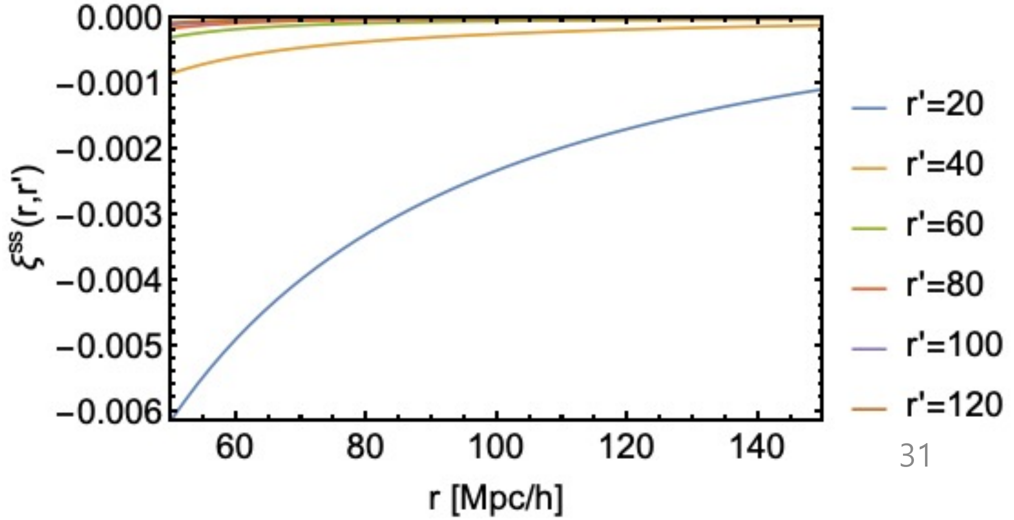
$$= \exp\left[-\frac{1}{2}\bar{A}_0 k_2^2 - \frac{1}{2}\bar{C}_0 k_1^2\right]\left(\left(1+\frac{1}{2}\bar{A}_0 k_2^2 + \frac{1}{2}\bar{C}_0 k_1^2\right) B^{ww}_{tree}(\overrightarrow{k_1},\overrightarrow{k_2}) + B^{ww}_1(\overrightarrow{k_1},\overrightarrow{k_2})\right)$$

Wiggle/No-Wiggle method

- smooth-smooth bispectrum B^{ss} ⟷ smooth-smooth correlation function ξ^{ss}







Wiggle/No-Wiggle method

$$B^{ss}\left(\overrightarrow{k_1},\overrightarrow{k_2}\right)\Big|_1$$
$$= \frac{1}{(2\pi)^3}\int d^3r d^3r' \frac{1}{|A_0|^{1/2}}\frac{1}{|C_0|^{1/2}}\int d^3q d^3q' e^{i\vec{q}\cdot\overrightarrow{k_1}} e^{i\overrightarrow{q'}\cdot\overrightarrow{k_2}} \exp\left[-\frac{1}{2}\bar{A}_0^{-1}q'^2\right.$$
$$\left.-\frac{1}{2}\bar{C}_0^{-1}q^2\right]\left(\left(4-\frac{1}{2}\bar{A}_0^{-1}q'^2-\frac{1}{2}\bar{C}_0^{-1}q^2\right)\xi^{ss}_{tree}\left(\vec{r}-\vec{q},\overrightarrow{r'}-\overrightarrow{q'}\right)+\xi^{ss}_1\left(\vec{r}-\vec{q},\overrightarrow{r'}-\overrightarrow{q'}\right)\right)$$

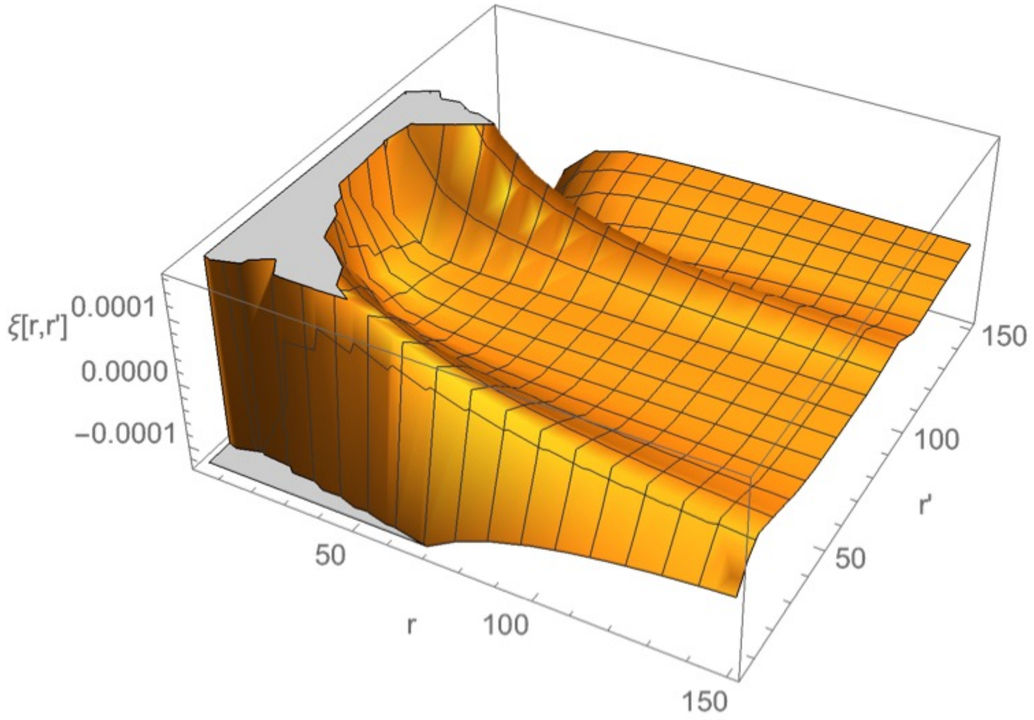
$$\xi^{ss}\left(\vec{r}-\vec{q},\overrightarrow{r'}-\overrightarrow{q'}\right)\to\xi^{ss}\left(\vec{r},\overrightarrow{r'}\right)$$

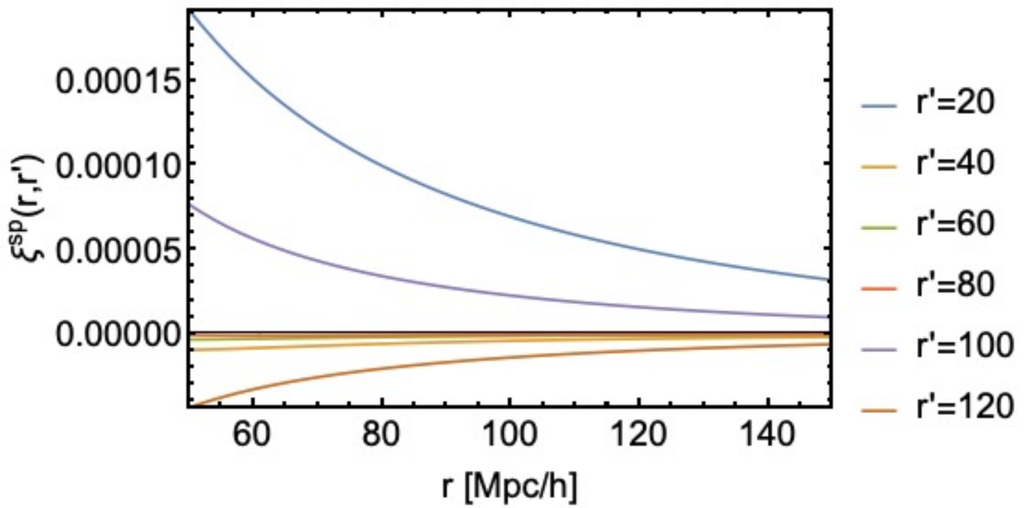
$$= B^{ss}_{tree}\left(\overrightarrow{k_1},\overrightarrow{k_2}\right)+B^{ss}_1\left(\overrightarrow{k_1},\overrightarrow{k_2}\right)$$

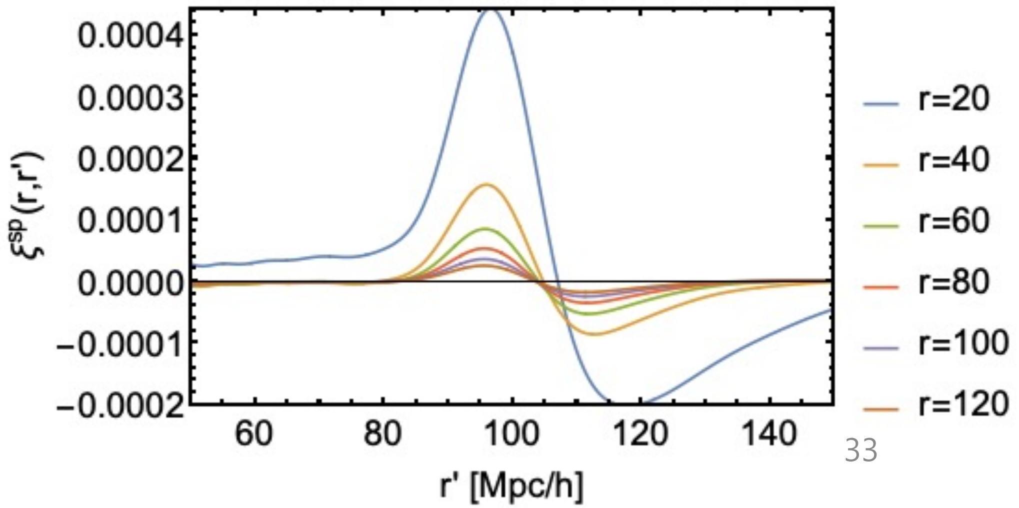
One can check IR-resummation does not change truly smooth function if it is featureless

Wiggle/No-Wiggle method

- Wiggle-smooth bispectrum B^{ws} ⟷ Peaked-smooth correlation function ξ^{ps}







Wiggle/No-Wiggle method

$$B^{sw}(\overrightarrow{k_1},\overrightarrow{k_2})\Big|_1$$
$$= \frac{1}{(2\pi)^3}\int d^3r d^3r' \frac{1}{|A_0|^{\frac{1}{2}}}\int d^3q' e^{i\overrightarrow{q'}\cdot\overrightarrow{k_2}}\exp\left[-\frac{1}{2}\bar{A}_0^{-1}q'^2\right]\left(\left(\frac{5}{2}-\frac{1}{2}\bar{A}_0^{-1}q'^2\right)\xi^{sp}_{tree}(\vec{r},\overrightarrow{r'}-\overrightarrow{q'})+\xi^{sp}_1(\vec{r},\overrightarrow{r'}-\overrightarrow{q'})\right)$$

Treat the peaked part as Dirac-delta function
smooth part as featureless function

$$= \exp\left[-\frac{1}{2}\bar{A}_0k_2^2\right]\left(\left(1+\frac{1}{2}\bar{A}_0k_2^2\right)B^{sw}_{tree}(\overrightarrow{k_1},\overrightarrow{k_2})+B^{sw}_1(\overrightarrow{k_1},\overrightarrow{k_2})\right)$$

IR-resummed bispectrum

$$B(\boldsymbol{k_1},\boldsymbol{k_2})|_1 = B^{ww}(\boldsymbol{k_1},\boldsymbol{k_2})|_1 + B^{sw}(\boldsymbol{k_1},\boldsymbol{k_2})|_1 + B^{ws}(\boldsymbol{k_1},\boldsymbol{k_2})|_1 + B^{ss}(\boldsymbol{k_1},\boldsymbol{k_2})|_1$$

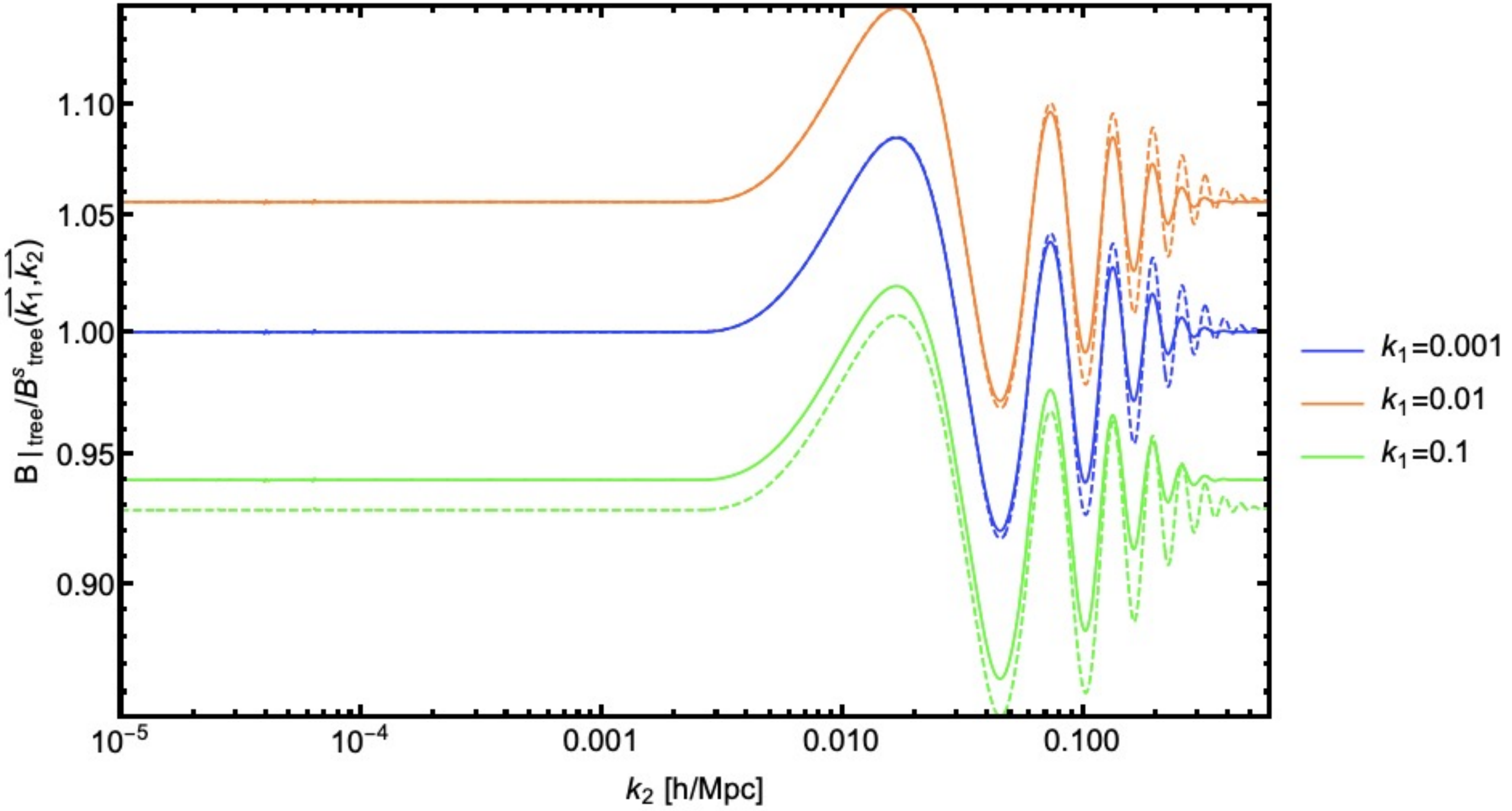
$$= \exp\left[-\frac{1}{2}\overline{A_0}k_2^2 - \frac{1}{2}\overline{C_0}k_1^2\right]\left(\left(1+\frac{1}{2}\overline{A_0}k_2^2+\frac{1}{2}\overline{C_0}k_1^2\right)B^{ww}_{\mathrm{tree}}(\boldsymbol{k_1},\boldsymbol{k_2}) + B^{ww}_1(\boldsymbol{k_1},\boldsymbol{k_2})\right)$$

$$+ \exp\left[-\frac{1}{2}\overline{A_0}k_2^2\right]\left(\left(1+\frac{1}{2}\overline{A_0}k_2^2\right)B^{sw}_{\mathrm{tree}}(\boldsymbol{k_1},\boldsymbol{k_2}) + B^{sw}_1(\boldsymbol{k_1},\boldsymbol{k_2})\right)$$

$$+\exp\left[-\frac{1}{2}\overline{C_0}k_1^2\right]\left(\left(1+\frac{1}{2}\overline{C_0}k_1^2\right)B^{ws}_{\mathrm{tree}}(\boldsymbol{k_1},\boldsymbol{k_2}) + B^{ws}_1(\boldsymbol{k_1},\boldsymbol{k_2})\right)$$

$$+ B^{ss}_{\mathrm{tree}}(\boldsymbol{k_1},\boldsymbol{k_2}) + B^{ss}_1(\boldsymbol{k_1},\boldsymbol{k_2})$$

IR-resummed bispectrum



Solid line : IR-resummed bispectrum, dashed line : Eulerian bispectrum

Summary

- IR-resummed bispectrum can be built through hybrid of Lagrangian and Eulerian perturbation theory.
- For simplifying computation, approximation through wiggle/no-wiggle decomposition can be done.
- We can recover damped BAO by resumming IR effect.

Discussion

- Angular effect
- Explicit expression
- Loop integral of bispectrum with wiggle/no-wiggle methods?

THANK YOU