Amplituhedron-Geometrical Approach to Scattering Amplitudes

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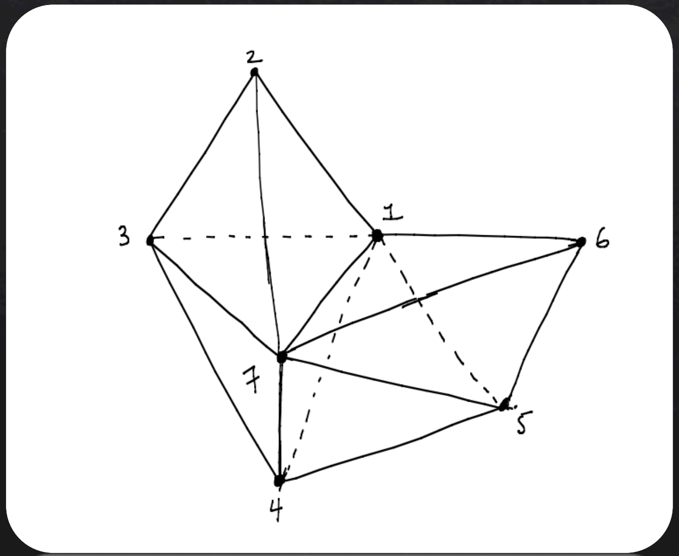
What is the Amplituhedron and what do we need it for?

- Conventional approach to scattering amplitudes inefficient due to large amounts of gauge redundancy
- complexity of calculations is a defect of the Feynman diagram approach which make locality and unitarity manifest
- We look for a „dual“ approach which unveils the hidden symmetries and exploits the simplicity
- Initiator was an observation for NMHV tree amplitudes led by Hodges:
- Amplitude can be thought of as the „volume“ of polytope in momentum twistor space
- Try to GENERALIZE this idea for all (planar) N=4 SYM
- AMPLITUHEDRON → jewel-shaped geometrical object

How can we deal with the Amplituhedron ?

- The important question: How can we parametrize a jewel- shaped Amplituhedron ?
  - Mimick ideas of simple geometries like the triangle and generalize them

$$Y = C_1 Z_1 + C_2 Z_2 + C_3 Z_3$$



$Y = ?$

Amplituhedron

- Amplituhedron: mathematically defined via

$$\text{map } \Phi_Z : G_+(k,n) \rightarrow G(k,m+k),$$

where $C \in G_+(k,n)$ and the Amplituhedron $A_{(n,m,k)} \subseteq G(k,m+k)$

Calculate the map by parametrizing the Amplituhedron as linear combination of kinematic data Z :

$$Y = C \cdot Z$$

with C & Z having positive minors.

Think of parametrization of the interior of a triangle.

- Extract scattering amplitude from canonical form associated to the Amplituhedron
- Canonical form obtained from minor positivity
- Amplitude = $\int\uparrow$▒$\Omega\downarrow nkL$

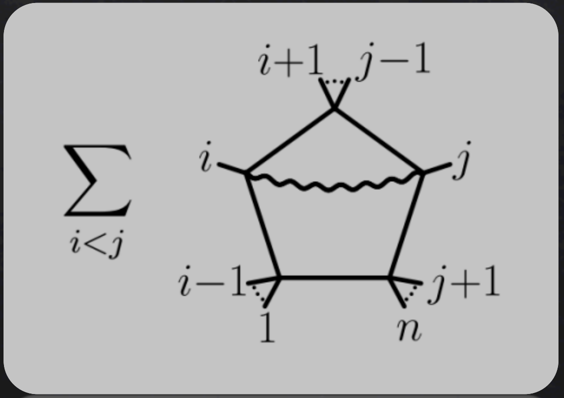
Central questions for the project

1) How is the Amplituhedron described using topological/ combinatoric methods?

- Amplituhedron characterized by winding number → natural
- **Amplituhedron characterized by k sign flips**
- Where does the sign flip definition come from?
- homeomorphic B-amplituhedron

Central questions for the project

- Application of Amplituhedron to n= 4 and n= 5 scattering for $l = 1, 2$
- Deriving the canonical forms
- $\Omega_{l=1} = \sum_{i<j} \frac{\langle AB(1\, i\, i{+}1) \cap (1\, j\, j{+}1)\rangle^2}{\langle AB1\, i\rangle \langle AB1\, i{+}1\rangle \langle AB i\, i{+}1\rangle \langle AB1\, j\rangle \langle AB1\, j{+}1\rangle \langle AB j\, j{+}1\rangle}$
- Comparing to classical approaches:
- Local expansion in chiral pentagons
- Prooving equivalence between Amplituhedron approach and pentagon expansion

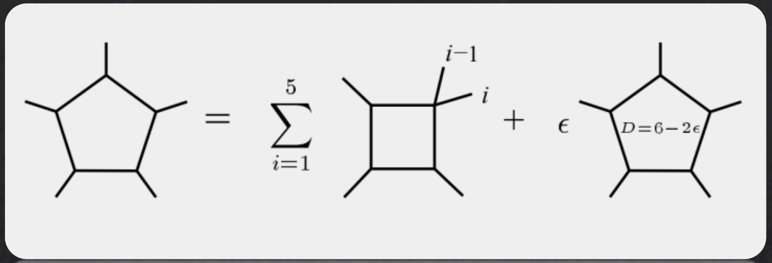


Central questions for the project

- Analysing the gluon scattering via box expansion
- Result not exact → What is the missing term ?
- Discussion on integrand and integral level
- Integral level → Deriving the

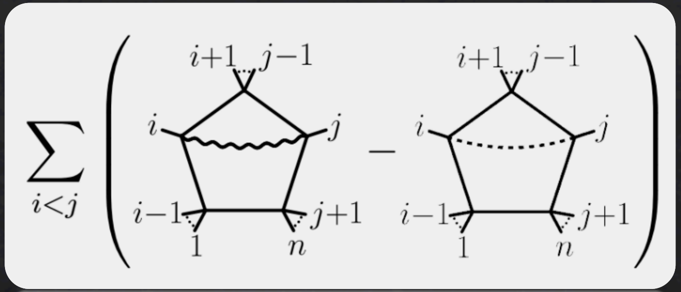
  partial integration technique

  $I_n = \frac{1}{2N_n}\left(\sum \gamma I_{n-1} + (n-5+2\varepsilon)\Delta_n I_n^{6-2\varepsilon}\right)$



- Integrand level:

  Emergence of parity odd term:

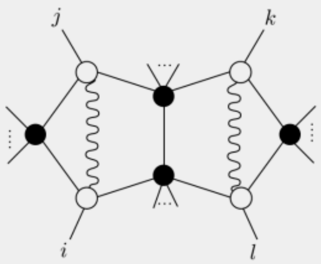


- What do we have to do to get exact result via box expansion?

Central questions for the project

- At $l = 2$, prove that chiral double pentagon expansion is equivalent to Amplituhedron

$$\mathcal{A}_{\mathrm{MHV}}^{2-\mathrm{loop}} = \sum_{i<j<k<l<i}$$



$$\mathcal{A}_{MHV}^{2-loop} = \sum_{i<j<k<l<i} \frac{\langle AB(i-1ii+1)\cap(j-1jj+1)\rangle\langle ijkl\rangle}{\langle ABi-1i\rangle\langle ABii+1\rangle\langle ABj-1j\rangle\langle ABjj+1\rangle\langle ABCD\rangle} * \frac{\langle CD(k-1kk+1)\cap(l-1ll+1)\rangle}{\langle CDk-1k\rangle\langle ABkk+1\rangle\langle CDl-1l\rangle\langle CDll+1\rangle}$$

What comes next? What are related topics?

- Idea that positive geometries (X, $X_{\geq 0}$) encodes scattering amplitudes is applicable to a variety of theories

1) Correlahedron $C_{n,k}$: geometric encodingof n-point stress tensor correlator function. Extract the correlation function from canonical form

   Correlahedron geometry can be projected down to amplituhedron space by taking light-like limits

2) Associahedron A_n :"Amplituhedron" for the scalar bi-adjoint φ^3 theory at tree level

Define $A_n = \Delta_n \cap H_n$, where Δ_n is a positive region and H_n an affine space

$$\Omega(\mathcal{A}_n) = \sum_{p=1}^{C_{n-2}} \mathrm{sign}(v_p) \bigwedge_{a=1}^{n-3} \mathrm{dlog} X_{i_a,j_a}$$

4) Momentum Amplituhedron

Amplituhedron expressed via spinor helicity variables instead of momentum twistor variables

5) Cosmological polytope

Positive geometries in Cosmology relate wavefunction of universe to polyhedral geometry

Open problems

- Understanding known geometries more generally
- At the moment knowledge based on case-by-case study
- Finding new geometries
- How can we define the „Loop Momentum Amplituhedron"?
- Using Momentum Amplituhedron as starting point to define non-planar theories
- Finding geometries for QCD
- Finding a single geometry underlying $\Psi\downarrow\Omega$
- What role does the Associahedron play for (open) strings?

Sources

- See Master thesis

Questions