Probing lepton dipole moments at a high-energy Muon Collider

(A model-independent high-energy test of new physics for leptonic g-2)

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Plan of the presentation

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- The muon magnetic moment
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  2. New Physics explanation for the muon g-2 anomaly
  3. Testing the muon g-2 anomaly at a high-energy Muon Collider
- The tau magnetic moment
  1. Current status of the tau g-2
  2. New Physics contributions to the tau g-2
  3. Testing the tau g-2 at a high-energy Muon Collider
- Conclusions and future prospects

The lepton magnetic moments: an introduction

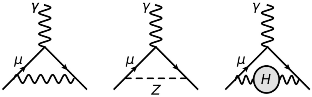
Lepton magnetic moments g_ℓ and anomalous magnetic moments a_ℓ ($\ell = e, \mu, \tau$)

$$\vec{\mu}_\ell = \frac{g_\ell \, e}{2m_\ell}\vec{S}_\ell, \qquad a_\ell \equiv \frac{g_\ell - 2}{2}$$

where $g_\ell \sim$ strength of interaction between the magnetic field and the spin of the lepton. At tree-level: $g_\ell = 2$. The Standard Model (SM) prediction for a_ℓ is:

$$a_\ell^{SM} = a_\ell^{QED} + a_\ell^{EW} + a_\ell^{Had}$$

- a_ℓ^{QED} accounts for all the diagrams containing only leptons and photons;
- a_ℓ^{EW} accounts for those diagrams containing massive bosons m_W, m_Z, m_h;
- a_ℓ^{Had} accounts for QED diagrams involving hadrons.



a_ℓ offers a unique opportunity to test the different sectors of the SM Lagrangian!

The anomalous magnetic moment of the muon

- The experimental value of a_μ comes from the average of the experiments E821 at BNL and E989 at FNAL (B. Abi et al., Phys.Rev.Lett. 126):

$$a_\mu^{Exp} = 116592061(41) \times 10^{-11}$$

- The SM prediction for a_μ reads (T. Aoyama et al., Phys.Rept. 887 (2020)):

$$a_\mu^{SM} = 116591810(43) \times 10^{-11}$$

$$\Delta a_\mu = a_\mu^{Exp} - a_\mu^{SM} = 251(59) \times 10^{-11} \sim 4.2\ \sigma \ \ \textit{discrepancy!!!}$$

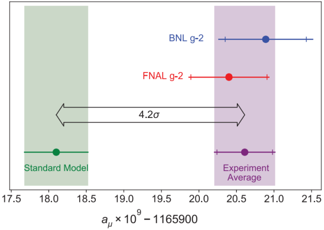


Figure: From top to bottom: experimental values of a_μ from BNL E821, from FNAL E989, and the combined average (B. Abi et al., Phys.Rev.Lett. 126).

New Physics for the muon $g-2$: at which scale?

- Δa_μ discrepancy at $\sim 4.2\,\sigma$ level:

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{g_{weak}^2}{16\pi^2}\frac{m_\mu^2}{v^2} \approx 2 \times 10^{-9}$$

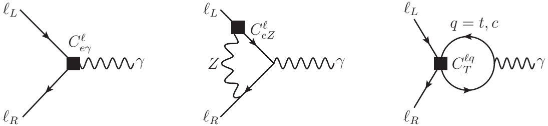
  - NP is at the weak scale ($\Lambda \approx v$) and weakly coupled to SM particles.*
  - NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.
  - NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.

*Favoured by the *hierarchy problem* and by a WIMP DM candidate but disfavoured by the LEP and LHC bounds (supersymmetry being the most prominent example).

$\Lambda \gg v$: the muon g-2 in the Standard Model EFT (SMEFT)

- **SMEFT Lagrangian relevant for Δa_ℓ**

$$\mathcal{L} = \sum_{V=B,W} \frac{C^\ell_{eV}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C^{\ell q}_T}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$



$$\Delta a_\ell \simeq \frac{4 m_\ell^2}{e \Lambda^2} \frac{v}{m_\ell} \left(C^\ell_{e\gamma} - \frac{3\alpha}{2\pi} \frac{c_W^2 - s_W^2}{s_W c_W} C^\ell_{eZ} \log \frac{\Lambda}{m_Z} \right) - \sum_{q=c,t} \frac{4 m_\ell^2}{\pi^2} \frac{m_q}{m_\ell} \frac{C^{\ell q}_T}{\Lambda^2} \log \frac{\Lambda}{m_q}$$

$$\frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left(\frac{250\,\text{TeV}}{\Lambda} \right)^2 |C^\mu_{e\gamma}| \qquad \frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left(\frac{50\,\text{TeV}}{\Lambda} \right)^2 |C^\mu_{eZ}|$$

$$\frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left(\frac{100\,\text{TeV}}{\Lambda} \right)^2 |C^{\mu t}_T| \qquad \frac{|\Delta a_\mu|}{3 \times 10^{-9}} \approx \left(\frac{10\,\text{TeV}}{\Lambda} \right)^2 |C^{\mu c}_T|$$

- **Strongly coupled NP:** $C^\mu_{e\gamma}, C^{\mu t}_T \sim g^2_{\rm NP}/16\pi^2 \lesssim 1$ implying $\Lambda \lesssim$ *few* x 100 TeV, beyond the direct production reach of any foreseen collider.
- **Weakly coupled NP:** $C^\mu_{e\gamma}, C^{\mu t}_T \lesssim 1/16\pi^2$ implying $\Lambda \lesssim 20$ TeV maybe within the direct production reach of a very high-energy Muon Collider

Connecting $(g-2)_\mu$ with high-energy processes

- **SMEFT Lagrangian relevant for Δa_ℓ**

$$\mathcal{L} = \sum_{V=B,W} \frac{C^\ell_{eV}}{\Lambda^2} (\bar\ell_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C^{\ell q}_T}{\Lambda^2} (\bar\ell_L \sigma_{\mu\nu} e_R)(\bar Q_L \sigma^{\mu\nu} q_R) + h.c.$$

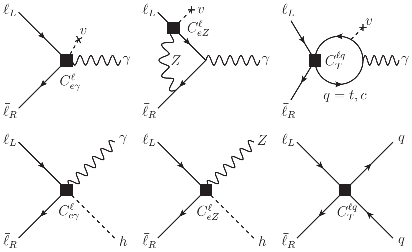


Figure: Connection between the Feynman diagrams for leptonic g-2 (upper row) and high-energy scattering processes (lower row) in the SMEFT: $H = v + h/\sqrt{2}$

$$\Delta a_\mu \sim \frac{m_\mu v}{\Lambda^2} C_{eV,T} \quad \Longleftrightarrow \quad \sigma_{\mu\mu\to f} \sim \frac{s}{\Lambda^4} |C_{eV,T}|^2 \quad (f = e\gamma, eZ, q\bar q)$$

- **Connecting $\mu^+\mu^- \to h\gamma$ with Δa_μ**

$$\sigma_{\mu\mu\to h\gamma} = \frac{s}{48\pi}\frac{|C^\mu_{e\gamma}|^2}{\Lambda^4} \approx 0.7\,\mathrm{ab}\left(\frac{\sqrt{s}}{30\,\mathrm{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2$$

- **SM irreducible background:**
  - $\sigma^{\rm SM}_{\mu\mu\to h\gamma} \approx (\alpha y_\mu^2/4s)\times \ln(s/m_\mu^2)|_{\sqrt{s}=30\,\mathrm{TeV}} \sim 4\times 10^{-3}$ ab: negligible!

- **SM reducible background:**

$$\frac{d\sigma_{\mu\mu\to Z\gamma}}{d\cos\theta} \sim \frac{\pi\alpha^2}{4s}\frac{1+\cos^2\theta}{\sin^2\theta} \qquad\qquad \frac{d\sigma_{\mu\mu\to h\gamma}}{d\cos\theta} = \frac{|C^\mu_{e\gamma}|^2}{\Lambda^4}\frac{s}{64\pi}(1-\cos^2\theta)$$

  - The significance of the signal $S = N_S/\sqrt{N_B+N_S}$ maximal for $|\cos\theta| \lesssim 0.6$.

$$\sigma^{\rm cut}_{\mu\mu\to h\gamma} \approx 0.53\,\mathrm{ab}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2, \qquad \sigma^{\rm cut}_{\mu\mu\to Z\gamma} \approx 82\,\mathrm{ab} \qquad (\sqrt{s}=30\,\mathrm{TeV})$$

  - S/B isolation: i) angular distributions and ii) h/Z invariant mass reconstruction.
  - Cut-and-count exp. with $b\bar{b}$ final state, $\mathcal{B}(h/Z\to b\bar{b}) = 0.58/0.15$ and $\epsilon_b = 80\%$.
  - For a Z/h misident. prob. of 10%, $N_{S(B)} = 22(88)$ and $S=2$ at $\sqrt{s}=30$ TeV.

Testing the muon g-2 anomaly at a high-energy Muon Collider

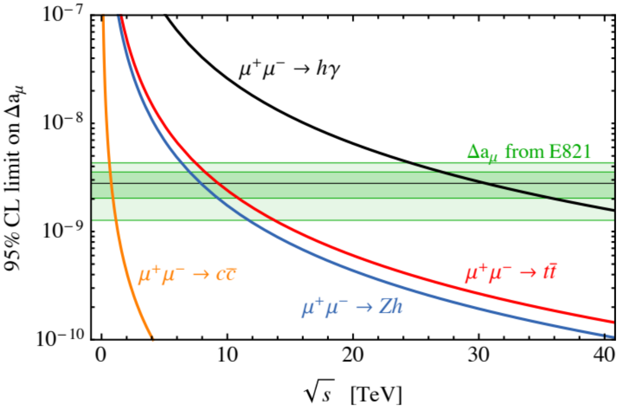


Figure: 95% C.L. reach on the muon $g-2$ as a function of the c.o.m energy $\sqrt{s}$ of the Muon Collider assuming an integrated luminosity $\mathcal{L} = \left(\frac{\sqrt{s}}{10\,\text{TeV}}\right)^2 \times 10\,\text{ab}^{-1}$.

The anomalous magnetic moment of the tau

The most stringent measurement of a_τ comes from the DELPHI Collaboration (J. Abdallah, et al, Eur.Phys.J.C35):

$$-0.052 < a_\tau < 0.013.$$

The SM prediction for a_τ reads (S. Eidelman, M. Passera, Mod.Phys.Lett.A22):

$$a_\tau^{SM} = 117721(5) \times 10^{-8}.$$

The sensitivity of the best existing measurements is still more than one order of magnitude worse than the leading QED effect $a_\tau = \frac{\alpha}{2\pi} \sim 10^{-3}$.

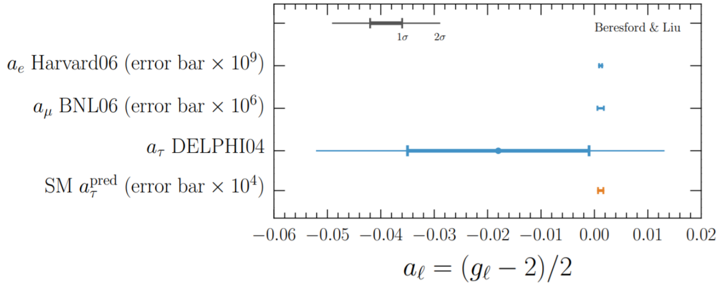


Figure: From top to bottom: experimental values for a_e, a_μ, a_τ and SM prediction for a_τ.

Testing the tau g-2 at a high-energy Muon Collider

- **SMEFT Lagrangian relevant for Δa_ℓ**

$$\mathcal{L} = \sum_{V=B,W} \frac{C^\ell_{eV}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C^{\ell q}_T}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$

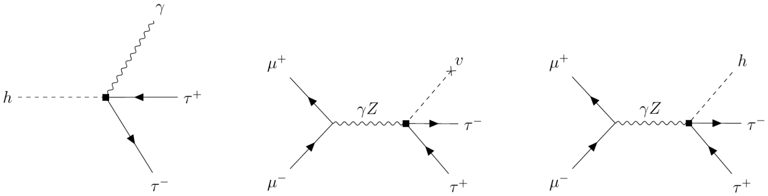


Figure: Processes at a MC sensitive to the same new physics effects of the tau $g-2$

- $h \to \tau^+\tau^-\gamma$: a huge-number of Higgs bosons should be produced at a high-energy MC $\Rightarrow$ possibility to look for rare Higgs decays
- $\mu^+\mu^- \to \tau^+\tau^-$: we expect $\sigma^{\rm NP}_{\tau\tau} \sim v^2/\Lambda^4$ while $\sigma^{\rm SM}_{\tau\tau} \sim 1/s$
- $\mu^+\mu^- \to \tau^+\tau^- h$: we expect $\sigma^{\rm NP}_{\tau\tau h} \sim s/\Lambda^4$ while $\sigma^{\rm SM}_{\tau\tau h} \sim y^2_\tau/s$

At high-energy $\sqrt{s} \gg v$ (multi TeV regime) the SM background is kept under control while the NP signal can be simultaneously enhanced!

$h \to \tau^+\tau^-\gamma$

$h \to \tau^+\tau^-\gamma$: a huge-number of Higgs bosons should be produced at a high-energy MC $\Rightarrow$ possibility to look for rare Higgs decays

$$\frac{\mathcal{B}_{h\ell\ell\gamma}}{\mathcal{B}_{h\ell\ell}} \simeq \frac{\mathcal{B}^{\rm SM}_{h\ell\ell\gamma}}{\mathcal{B}^{\rm SM}_{h\ell\ell}} - \frac{\alpha}{4\pi}\frac{m_h^2}{m_\ell^2}\Delta a_\ell + \frac{\alpha}{192\pi}\frac{m_h^4}{m_\ell^4}\left(\Delta a_\ell\right)^2$$

$$\mathcal{B}(h \to \tau^+\tau^-\gamma) \approx 10^{-3}, \qquad \mathcal{B}(h \to \tau^+\tau^-) \approx 6 \times 10^{-2}$$

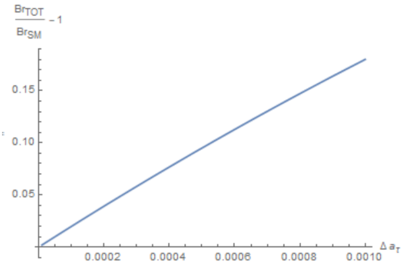


Figure: $\mathcal{B}^{\rm NP}_{h\tau\tau\gamma}/\mathcal{B}^{\rm SM}_{h\tau\tau\gamma}$ as a function of Δa_τ.

A multi TeV Muon Collider can realistically probe $|\Delta a_\tau| \simeq 10^{-4}$!

$\mu^+\mu^- \to \tau^+\tau^-$

- $\mu^+\mu^- \to \tau^+\tau^-$: we expect $\sigma_{\tau\tau}^{\text{NP}} \sim v^2/\Lambda^4$ while $\sigma_{\tau\tau}^{\text{SM}} \sim 1/s$
- We define the significance of the signal $S = \dfrac{N_S}{\sqrt{N_B + N_S}}$, with N_S and N_B the *signal* and the *background events*. To test Δa_τ at 95% C.L we impose $S = 2$.

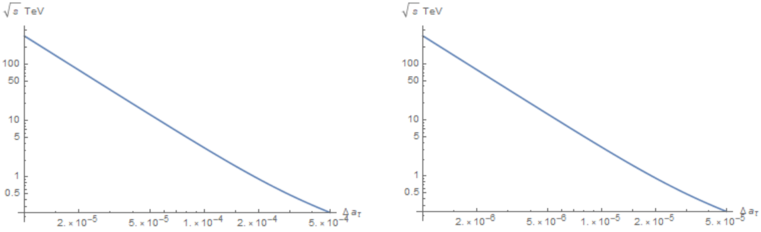


Figure: 95% C.L. reach on the Δa_τ in the case where $C_{e\gamma}^\tau \neq 0$ and $C_{eZ}^\tau = 0$ (left) and $C_{e\gamma}^\tau = 0$ and $C_{eZ}^\tau \neq 0$ (right) as a function c.o.m. energy of the MC.

A multi TeV Muon Collider can realistically probe $|\Delta a_\tau| \lesssim 10^{-4}$!

Conclusions and future prospects

- A muon collider running at center-of-mass energies of several TeV provides a unique, model-independent test of new physics in the muon g-2 through the study of the high-energy processes $\mu^+\mu^- \to h\gamma, hZ, q\bar{q}$ (with $q = c, t$).
  - A 30 TeV collider can probe the electromagnetic dipole operator at the level of $\Delta a_\mu \times 10^{-9}$, comparable to the present anomaly.
  - If the g-2 anomaly arises at loop-level from quark-lepton interactions, this could already be tested at a few TeV collider.
- These results rely on measurements with $\mathcal{O}(1)$ accuracy, and thus do not require a precise control of systematic or theoretical uncertainties.
- A muon collider running at $\sqrt{s} \sim \textit{few TeV}$ can probe the tau g-2 at the level of $10^{-5} \lesssim |\Delta a_\tau| \lesssim 10^{-4}$ by the processes $h \to \tau^+\tau^-\gamma$ and $\mu^+\mu^- \to \tau^+\tau^-(h)$
- At a high-energy lepton collider Δa_τ can also be efficiently probed through the processes $\mu^+\mu^- \to \mu^+\mu^-\tau^+\tau^-(\bar{\nu}\nu\,\tau^+\tau^-)$ which enjoys a very large cross-section driven by vector-boson-fusion (left to a future work!).