Some Theoretical Aspects of Multi-Higgs Doublet Models

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Overview

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Introduction

A Multi-Higgs doublet model consists of an extension of the Standard Model where one considers additional scalar doublets. There is no known restriction to the number of scalars one can add.

These models intend to address problems such as Baryogenesis and Dark Matter. Here, we discuss two model building issues one may encounter and explore these models in the Dark matter context.

NHDM and Stability Conditions

The scalar potential for a Higgs Model with N scalar doublets (NHDM) takes the form:

$$V_{\text{NHDM}} = Y_{ij} \left(\Phi_i^\dagger \Phi_j\right) + Z_{ij,kl} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_k^\dagger \Phi_l\right)$$

The first step to perform when studying a NHDM is to find the necessary and sufficient conditions for the potential to be bounded from below (BFB).

However, with increasing number of scalars this task becomes very challenging.

A_4-symmetric potential

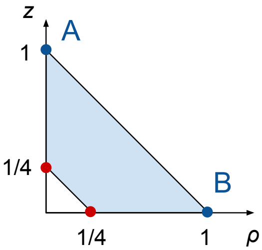
$$
\begin{aligned}
V_{A_4} \quad = \quad & -\frac{M_0}{\sqrt{3}}\left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right) + \frac{\Lambda_0}{3}\left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right)^2 \\
& + \frac{\Lambda_3}{3}\left[(\phi_1^\dagger\phi_1)^2 + (\phi_2^\dagger\phi_2)^2 + (\phi_3^\dagger\phi_3)^2 - (\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) - (\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1)\right] \\
& + \Lambda_1\left[(\Re\phi_1^\dagger\phi_2)^2 + (\Re\phi_2^\dagger\phi_3)^2 + (\Re\phi_3^\dagger\phi_1)^2\right] + \Lambda_2\left[(\Im\phi_1^\dagger\phi_2)^2 + (\Im\phi_2^\dagger\phi_3)^2 + (\Im\phi_3^\dagger\phi_1)^2\right] \\
& + \Lambda_4\left[(\Re\phi_1^\dagger\phi_2)(\Im\phi_1^\dagger\phi_2) + (\Re\phi_2^\dagger\phi_3)(\Im\phi_2^\dagger\phi_3) + (\Re\phi_3^\dagger\phi_1)(\Im\phi_3^\dagger\phi_1)\right]
\end{aligned}
$$

Defining $r_0 = (\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3)/\sqrt{3}$ and expressing the potential as:

$$
V = -M_0 r_0 + r_0^2 v_4\,, \quad \text{where} \quad v_4 = \Lambda_0 + \Lambda_1 x + \Lambda_2 y + \Lambda_3 z + \Lambda_4 t
$$

The variables (x, y, z, t) define the orbit space (Γ) of the theory.

Geometric Minimization: Rephasing invariant potential



For the rephasing invariant potential ($\Lambda_4 = 0$ and $\Lambda_1 = \Lambda_2$):

$$
\begin{aligned}
v_4 &= \Lambda_0 + \Lambda_1 \rho + \Lambda_3 z \,, \qquad \rho \equiv x + y \\
v_4 &= \Lambda_0 - \vec{n} \cdot \vec{x} \,, \qquad\qquad \vec{n} \equiv -(\Lambda_1, \Lambda_3)
\end{aligned}
$$

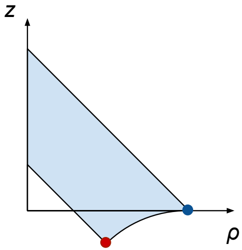
$$V_{min} = -\frac{M_0^2}{4 v_{4,min}}$$

The BFB conditions are obtained by making $v_4 \geq 0$ in Γ:

$$\Lambda_0 + \Lambda_3 \geq 0 \,, \quad \Lambda_0 + \Lambda_1 \geq 0 \,, \quad \Lambda_0 + \frac{\Lambda_3}{4} \geq 0 \,, \quad \Lambda_0 + \frac{\Lambda_1}{4} \geq 0$$

All the literature we found on A_4-symmetry listed incomplete or incorrect BFB conditions.

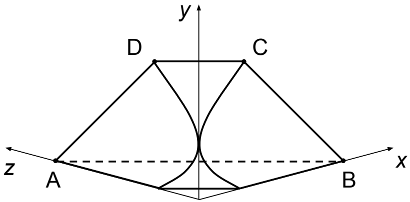
Soft Symmetry Breaking



For the case of exact symmetry and working in a neutral minimum, if the orbit space is convex, and all the Higgs masses squared positive, then the neutral minimum is automatically a global minimum.

When adding soft symmetry breaking terms, the above assertion is not necessarily true. Then, we must assure the potential is bounded in the charge breaking directions as well.

S_4 BFB conditions



Neutral directions:

$$\Lambda_0 + \Lambda_3 \geq 0$$
$$\Lambda_0 + \Lambda_1 \geq 0$$
$$\Lambda_0 + \frac{\Lambda_1 + 3\Lambda_2}{4} \geq 0$$
$$\Lambda_0 + \frac{\Lambda_3 + 3\Lambda_2}{4} \geq 0$$

Charge breaking directions:

$$\Lambda_0 + \frac{\Lambda_1}{4} \geq 0\,, \qquad \Lambda_0 + \frac{\Lambda_1 + 3\Lambda_2}{16} \geq 0\,, \qquad (\text{and } \Lambda_1 \to \Lambda_3)$$

$$\Lambda_0 + \frac{\Lambda_1\Lambda_2}{\Lambda_2 + 3\Lambda_1} \geq 0 \quad \text{if} \quad \Lambda_1 > 0 \text{ and } \Lambda_1 > |\Lambda_2| \quad (\text{and } \Lambda_1 \to \Lambda_3)$$

Conjecture: exact A_4 BFB conditions

Assuming the orbit space of the A_4 potential to be convex we conjecture that the BFB conditions for the exact symmetry and in a neutral global minimum are:

$$\Lambda_0 + \Lambda_3 \geq 0\,, \quad \Lambda_0 + \Lambda_1 \geq 0\,, \quad \Lambda_0 + \frac{\Lambda_1 + 3\Lambda_2 - \sqrt{3}|\Lambda_4|}{4} \geq 0\,,$$

$$\Lambda_0 + \frac{\Lambda_3}{4} + \frac{3}{8}\left(\Lambda_1 + \Lambda_2 - \sqrt{(\Lambda_1 - \Lambda_2)^2 + \Lambda_4^2}\right) \geq 0$$

For further details of this work check Igor P. Ivanov, FV: JHEP 11 (2020), 104.

This conjecture was later proved true in arXiv:2104.11428.

Decoupling

A decoupling limit happens when the additional scalars are sufficiently massive that the interactions with the remaining light scalars are negligible.

The couplings of the h_{125} to gauge bosons and the heaviest charged fermions are known to coincide with couplings expected in the SM, with errors of order 20% or better. This feature is easy to explain in models which have a so-called decoupling limit.

The 2HDM with $\mathbb{Z}_2$ symmetry

The symmetry group is represented by:

$$\phi_1 \to \phi_1, \quad \phi_2 \to -\phi_2$$

$$\begin{aligned} V_{\mathbb{Z}_2} \quad = \quad & -m_{11}^2 \left(\phi_1^\dagger \phi_1\right) - m_{22}^2 \left(\phi_2^\dagger \phi_2\right) + \frac{\lambda_1}{2} \left(\phi_1^\dagger \phi_1\right)^2 + \frac{\lambda_2}{2} \left(\phi_2^\dagger \phi_2\right)^2 \\ & + \lambda_3 \left(\phi_1^\dagger \phi_1\right) \left(\phi_2^\dagger \phi_2\right) + \lambda_4 \left(\phi_1^\dagger \phi_2\right) \left(\phi_2^\dagger \phi_1\right) + \left[\frac{\lambda_5}{2} \left(\phi_2^\dagger \phi_1\right)^2 + H.c.\right] \end{aligned}$$

$$\Phi_i = \begin{bmatrix} \varphi_i^+ \\ v_i + (H_i + i\chi_i)/\sqrt{2} \end{bmatrix}$$

$$(M_\pm^2)_{ij} = \left.\frac{\partial^2 V_H}{\partial \varphi_i^+ \partial \varphi_j^-}\right|_{\{t_a\},(\varphi_b^+ \varphi_b^-, H_b, \chi_b) \to 0}$$

$$(M_{\text{neutral}}^2)_{ij} = \left.\frac{\partial^2 V_H}{\partial (H,\chi)_i \partial (H,\chi)_j}\right|_{\{t_a\},(\varphi_b^+ \varphi_b^-, H_b, \chi_b) \to 0}$$

The mass matrix for the charged scalars when the VEV is $(v, 0)$, is:

$$M_{\pm}^2 = \begin{pmatrix} 0 & 0 \\ 0 & m_{22}^2 + \frac{\lambda_3 v^2}{2} \end{pmatrix}$$

The mass matrix for the charged scalars when the VEV is (v_1, v_2), is:

$$M_{\pm}^2 = \begin{pmatrix} -\frac{1}{2}\left(\lambda_4 + \lambda_5\right) v_2^2 & \frac{1}{2}\left(\lambda_4 + \lambda_5\right) v_1 v_2 \\ \frac{1}{2}\left(\lambda_4 + \lambda_5\right) v_1 v_2 & -\frac{1}{2}\left(\lambda_4 + \lambda_5\right) v_1^2 \end{pmatrix}$$

Theorem

The vev $(v, 0)$ conserves the symmetry of the potential, and has decoupling. The vev (v_1, v_2) breaks the symmetry of the potential and has no decoupling limit.

- Faro, Romão and Silva proved that this property holds for all possible 2HDM's.
- Using the same approach as before we proved it also holds for all 3HDM's.

Theorem: A NHDM has a decoupling limit if and only if the vacuum conserves the symmetry of the potential.

The proof of this theorem and its interesting consequences can be found in Sergio Carrolo, Jorge C. Romão, João P. Silva, FV: Phys.Rev.D 103 (2021) 7, 075026.

Inert Doublet Model

$$\phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG^0\right) \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}$$

In this model $m_H < m_A, m_{H^\pm}$. Then, H is the dark matter candidate.

The Inert Doublet Model is a 2HDM with a $\mathbb{Z}_2$-symmetry. This symmetry forbids Yukawa interactions between the second doublet and the fermions, rendering it a good dark matter candidate.

Scan and Results

We performed a scan of the variable space (m_H^2, m_A^2, $m_{H^\pm}^2$, λ_2 and $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$) and then tested each point with the existing theoretical and experimental constraints.

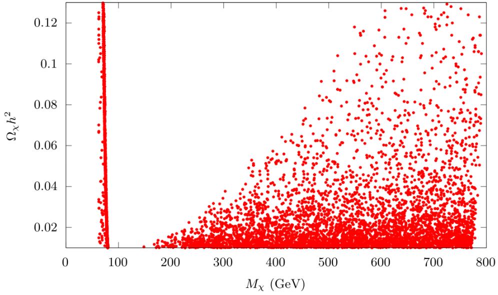


Figure: Relic density as a function of the dark matter mass.

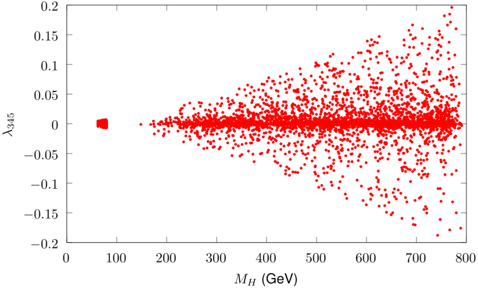


Figure: Coupling parameter as a function of the dark matter mass.

Questions?