GENEVA Monte Carlo:
Status and recent developments

Simone Alioli

MILANO-BICOCCA UNIVERSITY & INFN

MPI WS RINGBERG CASTLE 1/11/2022

The Geneva method

- The goal is to have fully-differential event generation at higher-orders (NNLO)
- Resummation plays a key role in the defining the events in a physically sensible way
- Results at partonic level are further evolved by the shower matching and hadronization

Resolution parameters for N extra emissions

- The key idea is the introduction of a resolution variable r_N that measure the hardness of the $N+1$-th emission in the Φ_N phase space.

- For color singlet production one can have $r_0 = q_T, p_T^j, k_T$-ness,....

- N-jettiness is a valid resolution variable: given an M-particle phase space point with $M \geq N$

$$\mathcal{T}_N(\Phi_M) = \sum_k \min\{\hat{q}_a \cdot p_k, \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k, \ldots, \hat{q}_N \cdot p_k\}$$

- The limit $\tau_N \to 0$ describes a N-jet event where the unresolved emissions are collinear to the final state jets/initial state beams or soft

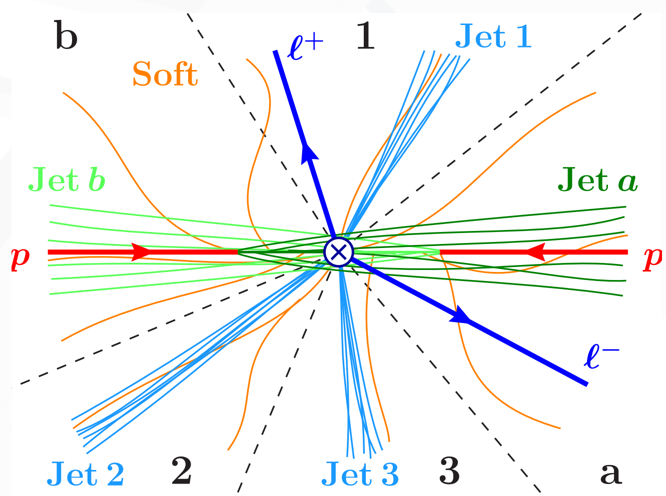
- For color-singlet final states, it reduces to 0-jettiness

$$\mathcal{T}_0 = \sum_k |\vec{p}_{kT}|\, e^{-|\eta_k - Y|}$$

[Stewart, Tackmann,Waalewijn `09,`10]

- When an extra jet is present 1-jettiness used for r_1

$$\mathcal{T}_1 = \sum_k \min\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\}$$

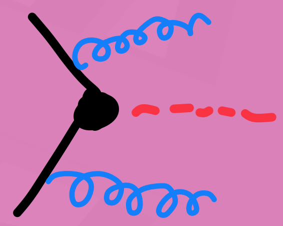


UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

Partitioning phase space with resolution cuts

NNLO example : start with two widely separated emission.
Can be described well with LO_2 matrix elements.
What happens when emissions start growing closer and closer ?

$$\frac{d\sigma}{d\Phi_2}(r_0 > r_0^{cut}, r_1 > r_1^{cut}) =$$



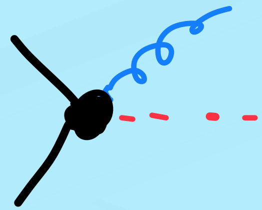
$r_0 > r_0^{cut}$
$r_1 > r_1^{cut}$

The logarithms of the resolution parameters grow larger and larger. They need to be resummed to give a physically sensible description. This takes care of their IR divergencies.
Generated events must have integrated cross section LO_2 accurate and the full N+2-body kinematics must be retained.

Partitioning phase space with resolution cuts

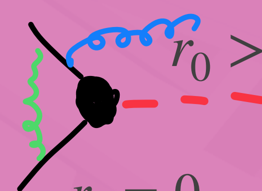
Next: one hard and one unresolved

$$\frac{d\sigma}{d\Phi_1}(r_1^{cut}) = \begin{cases} \\ \end{cases}$$



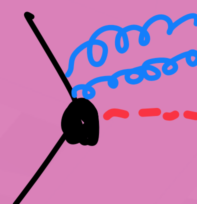
$r_0 > r_0^{cut}$

$r_1 = 0$



$r_0 > r_0^{cut}$

$r_1 = 0$



$r_1 < r_1^{cut}$

$r_0 > r_0^{cut}$

$$d\Phi_1 = d\Phi_0 dr_0 dz d\varphi$$

When one emission becomes unresolved r_1^{cut} must be resummed.

Integrated quantities require NLO$_1$ accuracy via local subtraction $\frac{d\Phi_2}{d\Phi_1}\theta(r_1 < r_1^{cut})$.

Φ_2 differential information below r_1^{cut} is lost during projection to Φ_1.

No difference for preserved quantities, in general can be made a power correction in r_1^{cut}.

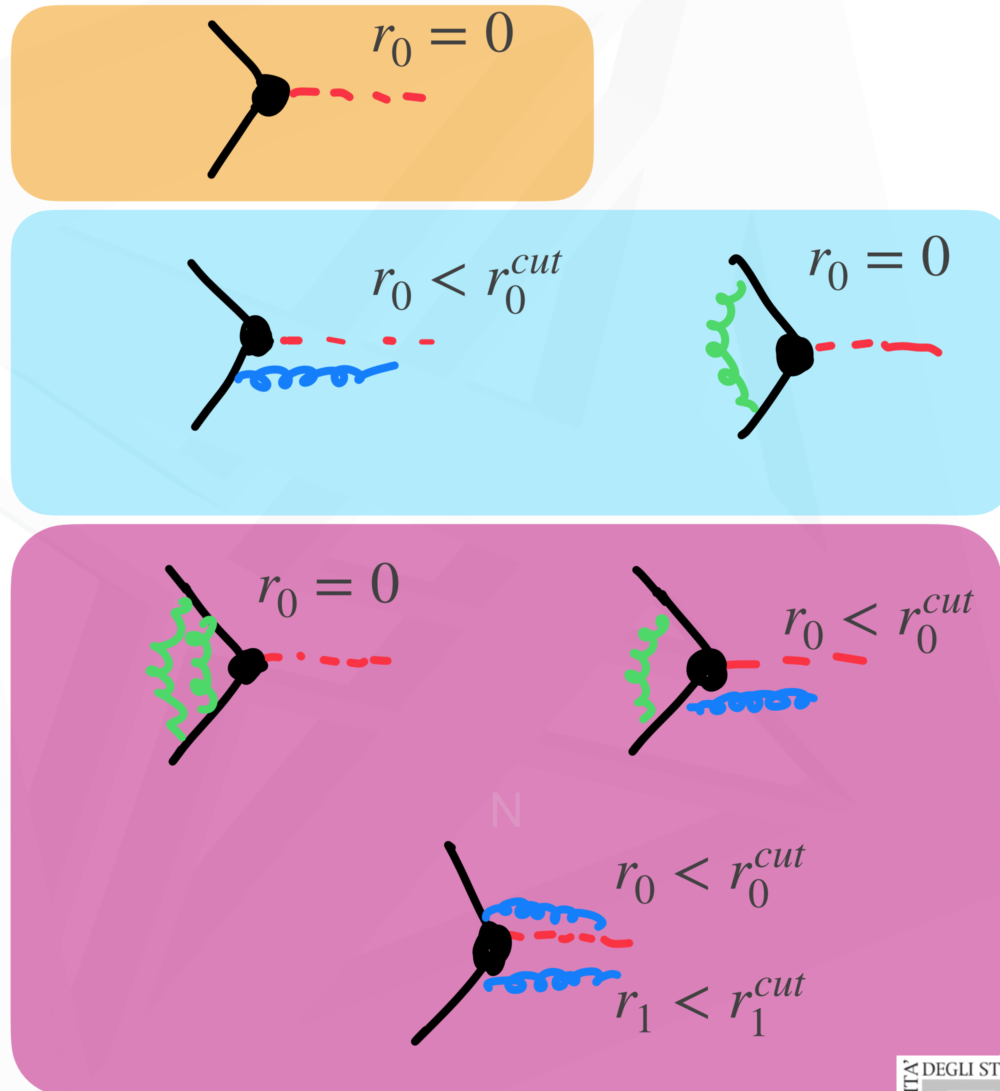
Mapping that preserves r_0 singular behavior is required for correct event definition.

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

Partitioning phase space with resolution cuts

Last: two unresolved

$$\frac{d\sigma}{d\Phi_0}(r_0^{cut}) = \Bigg\{$$



Zero jet bin must have NNLO_0 integrated accuracy. N-jettiness subtraction used.

The resummation of both r_0^{cut} and r_1^{cut} ensures physically sensible xsec and IR-finite events.

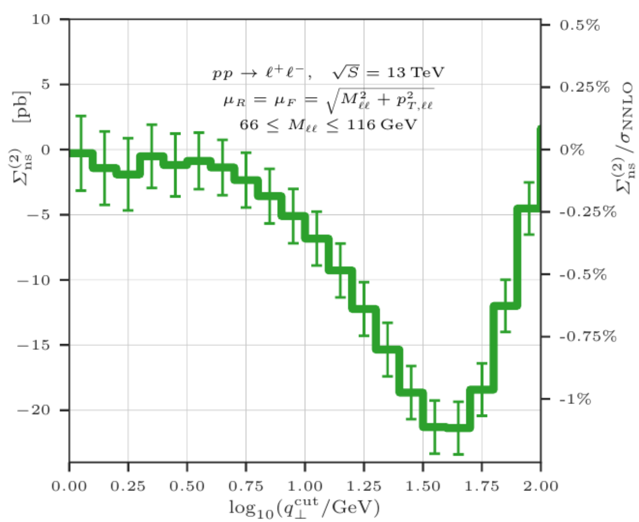
Resummation of resolution parameters

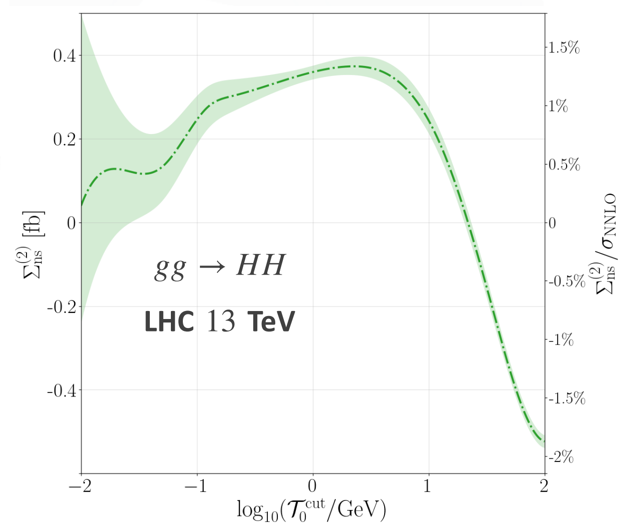
Resumming resolutions parameters not really a new idea, SMCs do it since the '80s with Sudakov factors

The key difference is that using the proper resummation at the proper order has several benefits: systematically improvable (NLL, NNLL,N3LL,…), lowering theoretical uncertainty at each step. Including primed accuracy captures the exact singular behaviour at $\delta(r_N)$.

The higher the accuracy the lower the cuts can be pushed without risking missing higher logarithms being numerically relevant. The lower the cuts the smaller the nonsingular power corrections due to phase-space projections will affect the results differentially.

For NNLO event generation one needs at least NNLL'$_{r_0}$ + NNLO accuracy to control the full α_s^2 singular contributions.





From resummation to event generation

Ideally one would want also NLL'$_{r_1}$ to capture the 1/2-jet separation.

When $r_0 \sim r_1 \ll Q$ the joint resummation should be performed (not yet there)

Unitarity can be exploited to perform the double resummation of r_0 and r_1 , at the price of losing ability to systematically improve particular regions of the phase space

Final GENEVA partonic formulae combine resummation and matching to fixed-order

$$\frac{d\sigma^{\mathrm{MC}_0}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) + \frac{d\sigma_0^{\mathrm{nons}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{d\sigma_0^{\mathrm{nons}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{d\sigma_0^{\mathrm{NNLO}_0}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\right]_{\mathrm{NNLO}_0}$$

$$\frac{d\sigma^{\mathrm{MC}_1}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) = \frac{d\sigma_{\geq 1}^{C}}{d\Phi_1} U_1(\Phi_1, \mathcal{T}_1^{\mathrm{cut}})\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{d\sigma_1^{\mathrm{match}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}})$$

$$\frac{d\sigma^{\mathrm{MC}_{\geq 2}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) = \frac{d\sigma_{\geq 1}^{C}}{d\Phi_1} U_1'(\Phi_1, \mathcal{T}_1)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\Big|_{\Phi_1 = \Phi_1^{\mathcal{T}}(\Phi_2)} \times \mathcal{P}(\Phi_2)\, \theta(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) + \frac{d\sigma_{\geq 2}^{\mathrm{match}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})$$

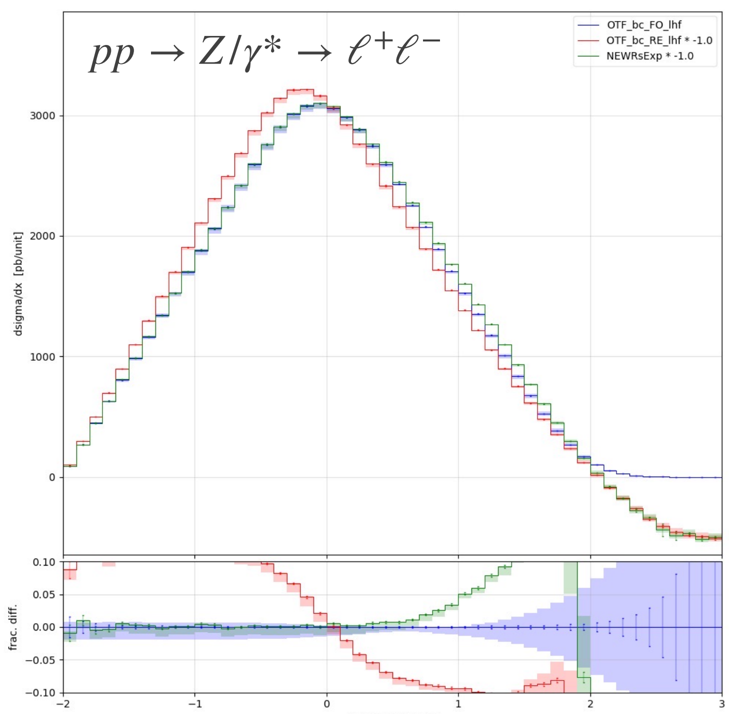
Spreading out the resummation

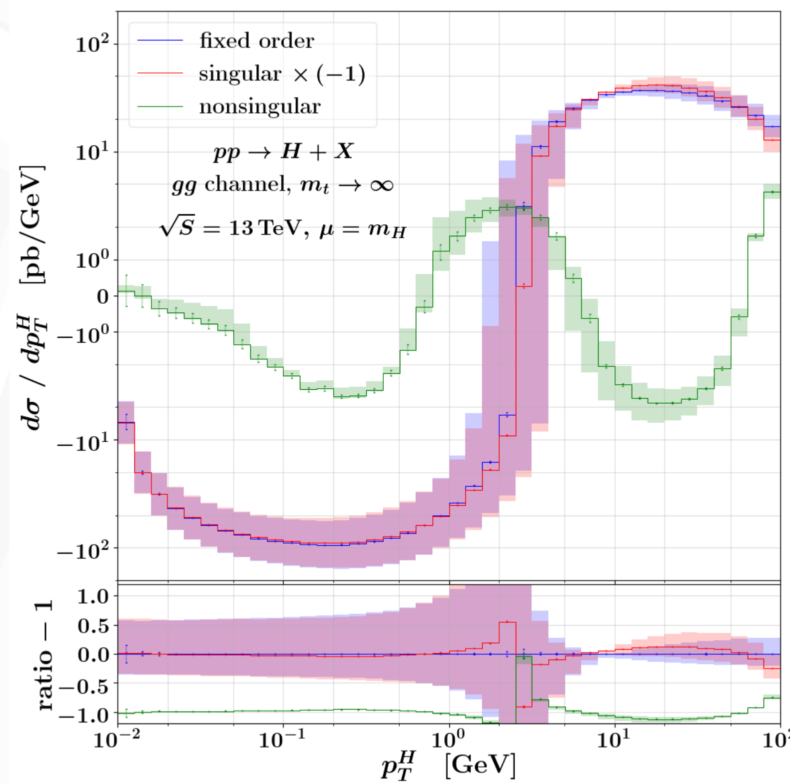
Splitting functions are required to make resummed spectrum fully-differential.

$$\frac{d\sigma}{d\Phi_N dr_N} P_{N\to N+1} \to \frac{d\sigma}{d\Phi_{N+1}}$$

New on-the-fly evaluation and better functional forms captures better the singular behavior of matrix elements also for different resolution variables.

$$P_{N\to N+1}(\Phi_{N+1}) = \frac{f_{kj}(\Phi_N, \mathcal{T}_N, z)}{\sum_{k'=1}^{N+2} \int_{z_{\min}^{k'}(\Phi_N,\mathcal{T}_N)}^{z_{\max}^{k'}(\Phi_N,\mathcal{T}_N)} dz'\, J_{k'}(\Phi_N, \mathcal{T}_N, z')\, I_{\phi}^{k'}(\Phi_N, \mathcal{T}_N, z') \sum_{j'=1}^{n_{\text{split}}^{k'}} f_{k'j'}(\Phi_N, \mathcal{T}_N, z')}$$





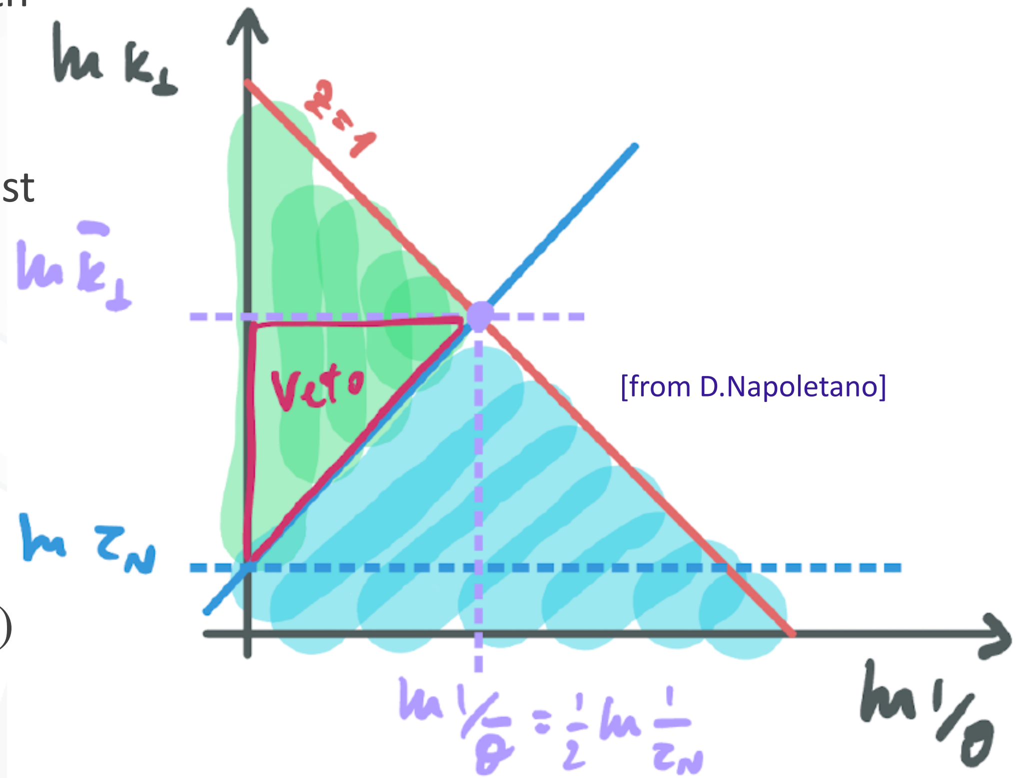
Interface with the parton shower

$\mathscr{T}_N(\Phi_{N+1})$ measures hardness of N+1-th emission

If shower ordered in k_T, start from largest value allowed by N-jettiness

Let the shower evolve unconstrained.

Veto an event if after $M \geq 1$ shower emissions $\mathscr{T}_N(\Phi_{N+M}) > \mathscr{T}_N(\Phi_N + 1)$ and retry the whole shower.



[from D.Napoletano]

All the relevant phase space is correctly covered to ensure double log accuracy

0-jet and 1-jet bins are treated differently: starting scale is resolution cutoff.

Method independent from shower used: done with PYTHIA8, DIRE & SHERPA.

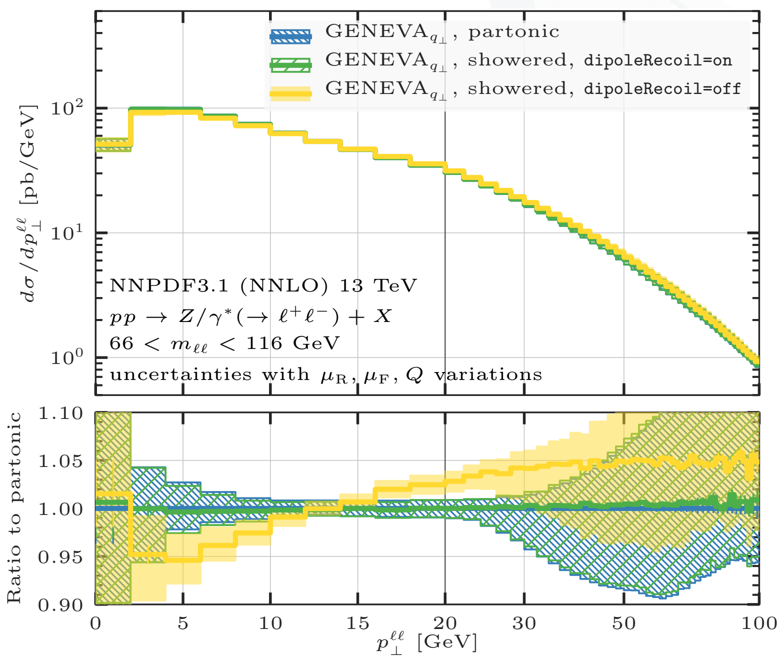
Interface with the parton shower

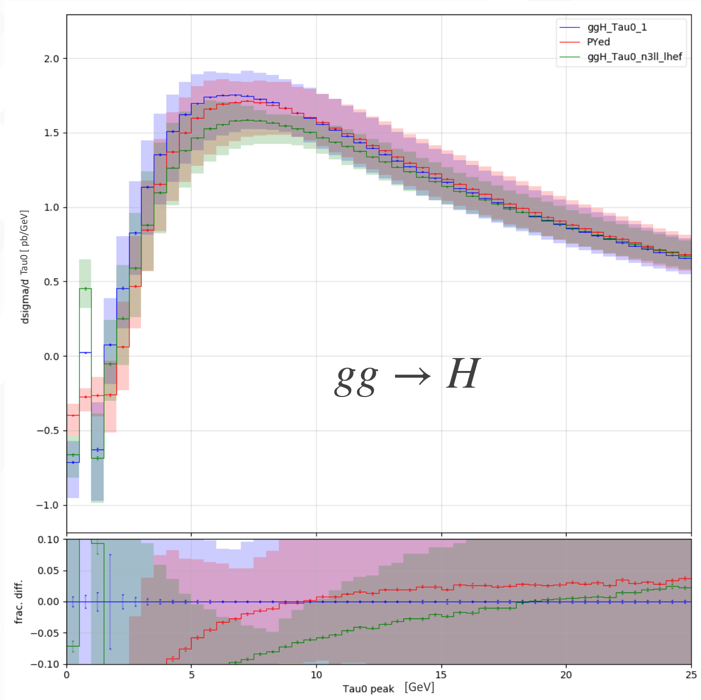
Effect of shower on resolution parameter directly resummed is not known analytically, but numerically is very small.

For 0-jettiness where $< \mathcal{T}_2 > \sim \alpha_s^2 \mathcal{T}_0$ one can estimate that the average shift due to the first shower emission is of similar size of the missing higher-order N3LL contributions $\alpha_s^3 / \mathcal{T}_0$

[SA et al. 1508.01475]

For q_T one can take advantage off the shower recoil maps to preserve the quantity being resummed.





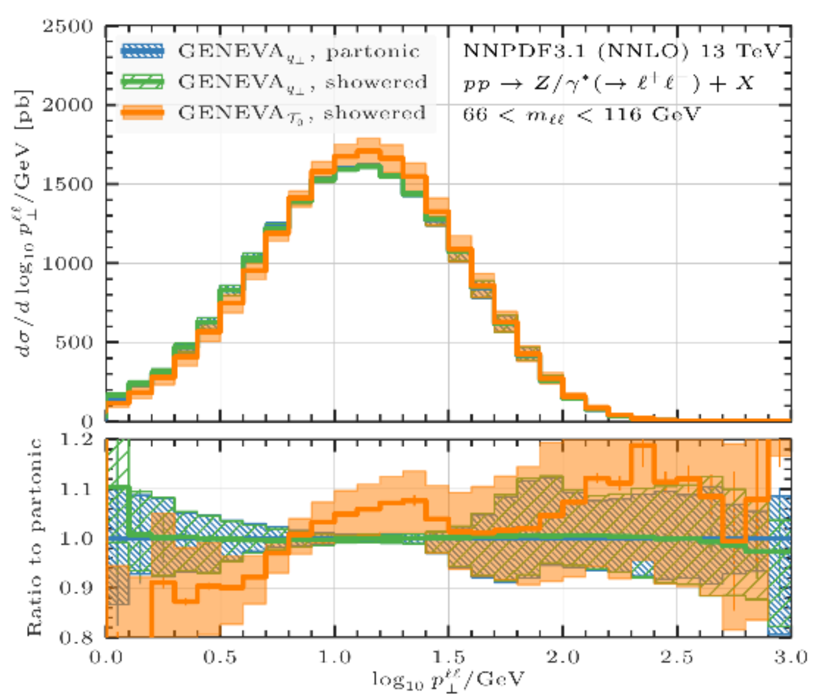
UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

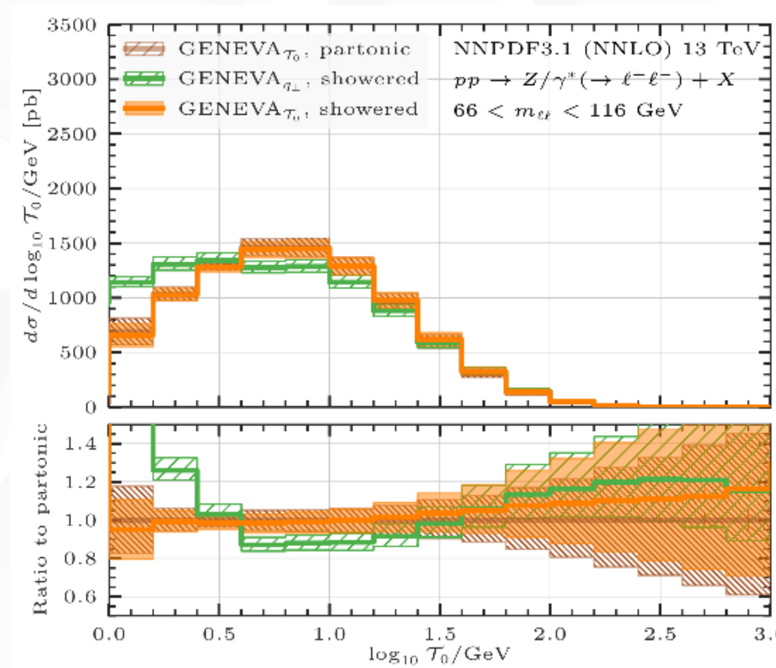
Interface with the parton shower

Effect of shower on resolution variables different from what is resummed more marked.

GENEVA framework allows this comparison for DY when resumming q_T or $\mathscr{T}_0$

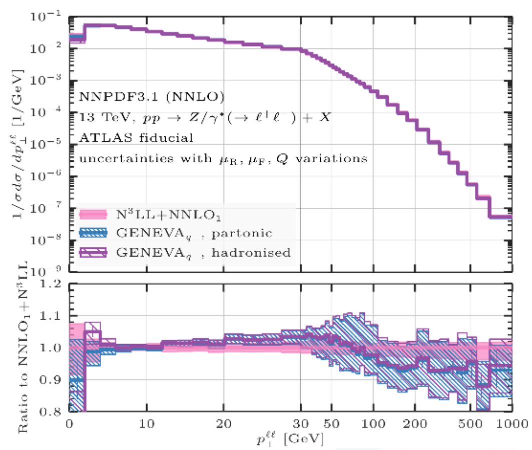
Correct approach here would be joint $(\mathscr{T}_0, \vec{q}_T)$ resummation, avoids need of splitting func.

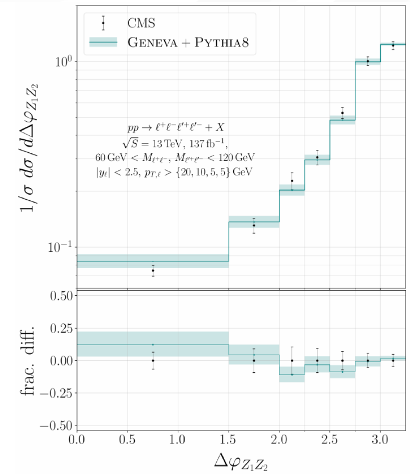


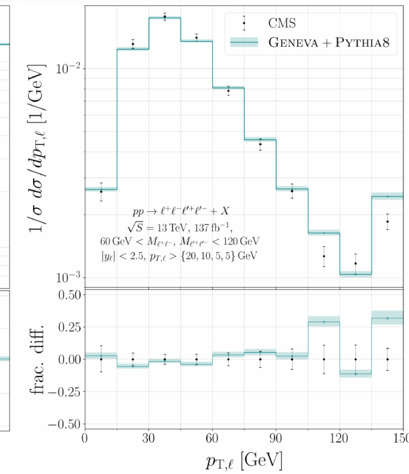


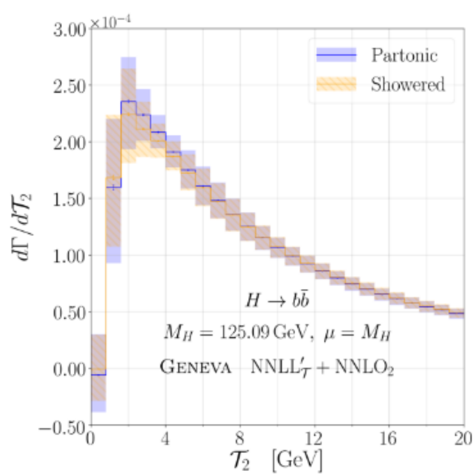
Implemented processes

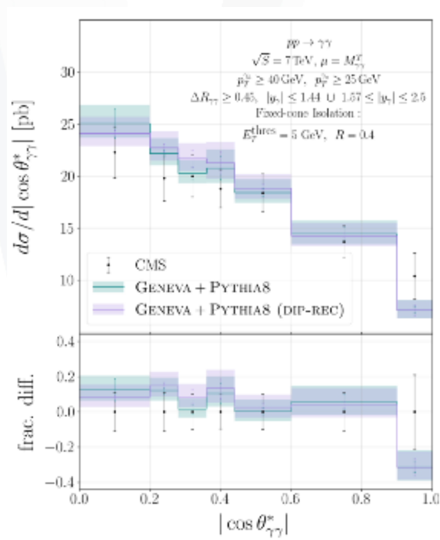
Method has been tested and validated with several color singlet production processes:
DY, ZZ, $W\gamma$, VH, $\gamma\gamma$, Higgs decays

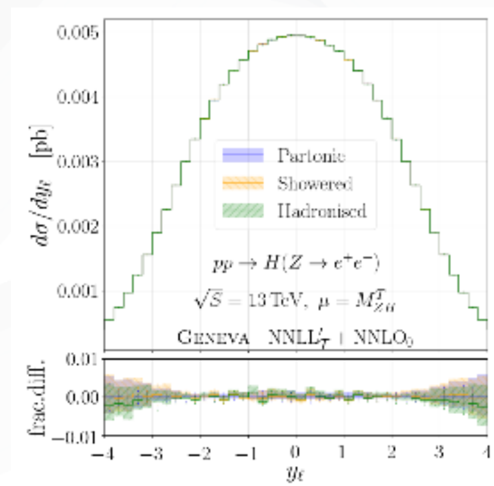












SIMONE ALIOLI - MPI WS

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

Higgs production via gluon fusion

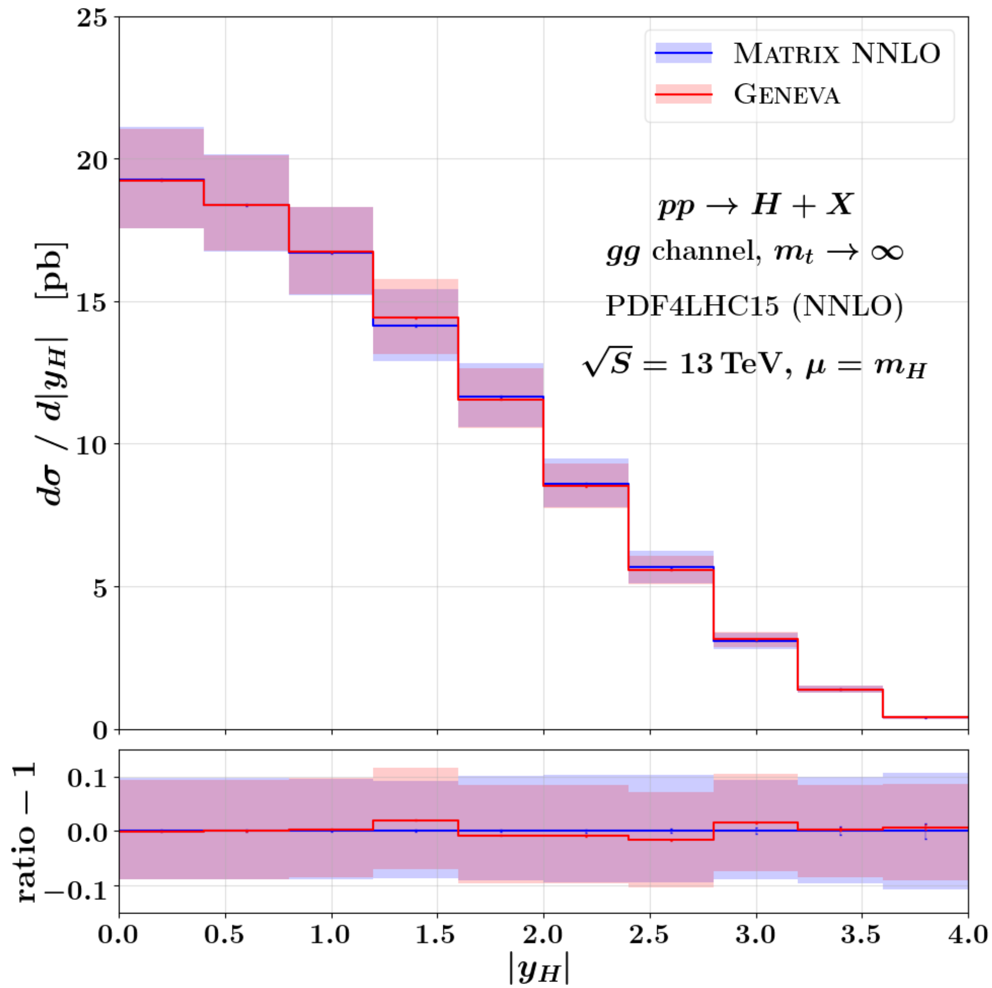
Heavy top-quark limit rescaled with exact LO m_t dependence (rEFT).

NNLL' $\mathcal{T}_0$ resummation requires gluon beam functions

Include 7-point scale variations, introducing explicit μ_F dependence in beam function and their pert. matching coefficients I_{ij}.

Additional variations must be considered also in the resummation region to probe independent μ_B and μ_F changes.

Variations combine in inclusive quantities and give perfect agreement in both central values and FO variations with MATRIX.



Resummation of timelike logarithms

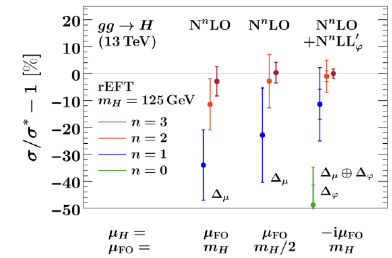
Timelike processes: $q^2 = Q^2 > 0$

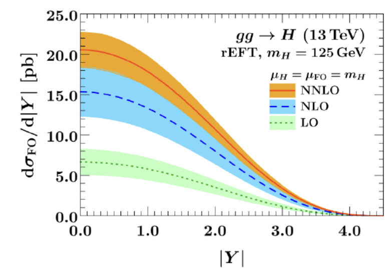
$$H(q^2, \mu) \propto L \equiv \ln\left(\frac{-q^2 - i0}{\mu^2}\right) = 2\ln\left(\frac{-iQ}{\mu}\right) = -i\pi + 2\ln\left(\frac{Q}{\mu}\right)$$

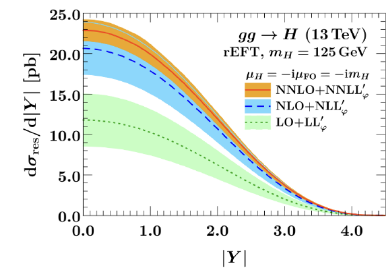
$\mu = Q$ classical choice

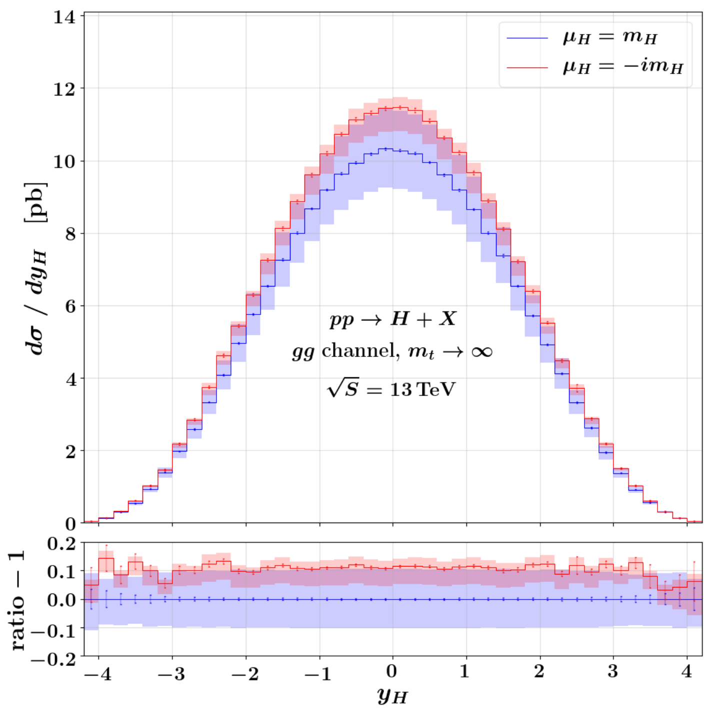
$\mu = -iQ = Qe^{-i\varphi}$ timelike resummation

- $\varphi = 0$ central value of FO prediction (no resummation);
- $\varphi = \frac{\pi}{2}$ central value of resummed prediciton;
- $\varphi \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ phase variation of $\pm\frac{\pi}{4}$.









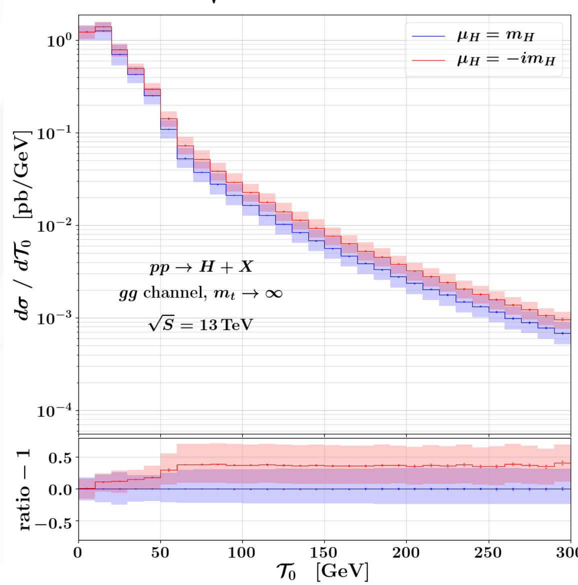
Δ_μ fixed-order uncertainty;

Δ_φ resummation uncertainty;

$$\Delta_\mu \oplus \Delta_\varphi = \sqrt{\Delta_\mu^2 + \Delta_\varphi^2}.$$

[arXiv:1702.00794]

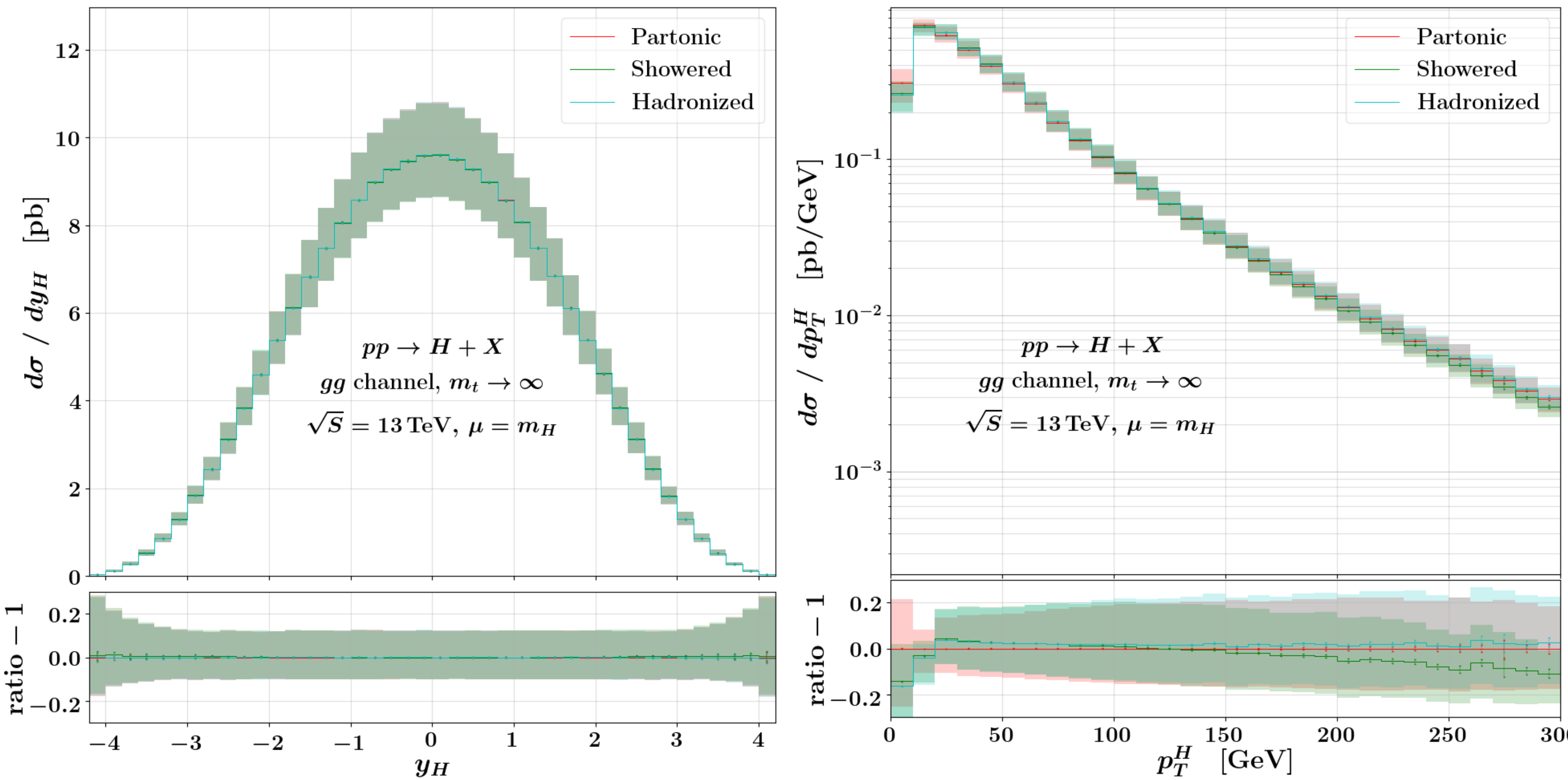
Ebert, Michel and Tackmann



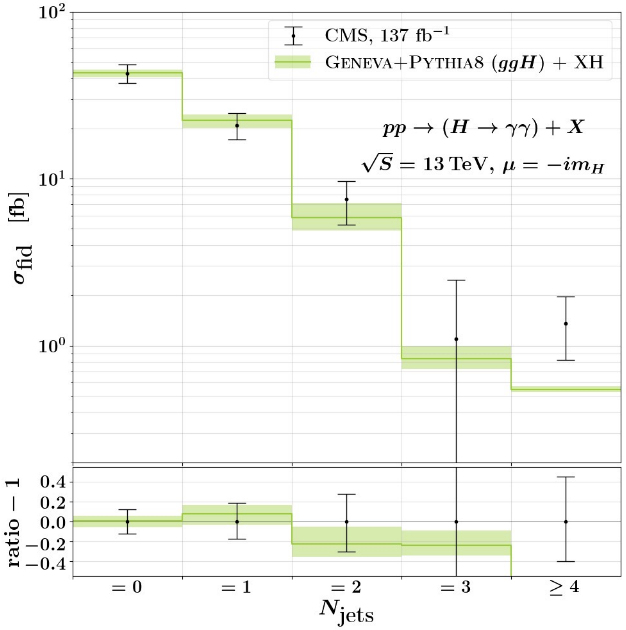
UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

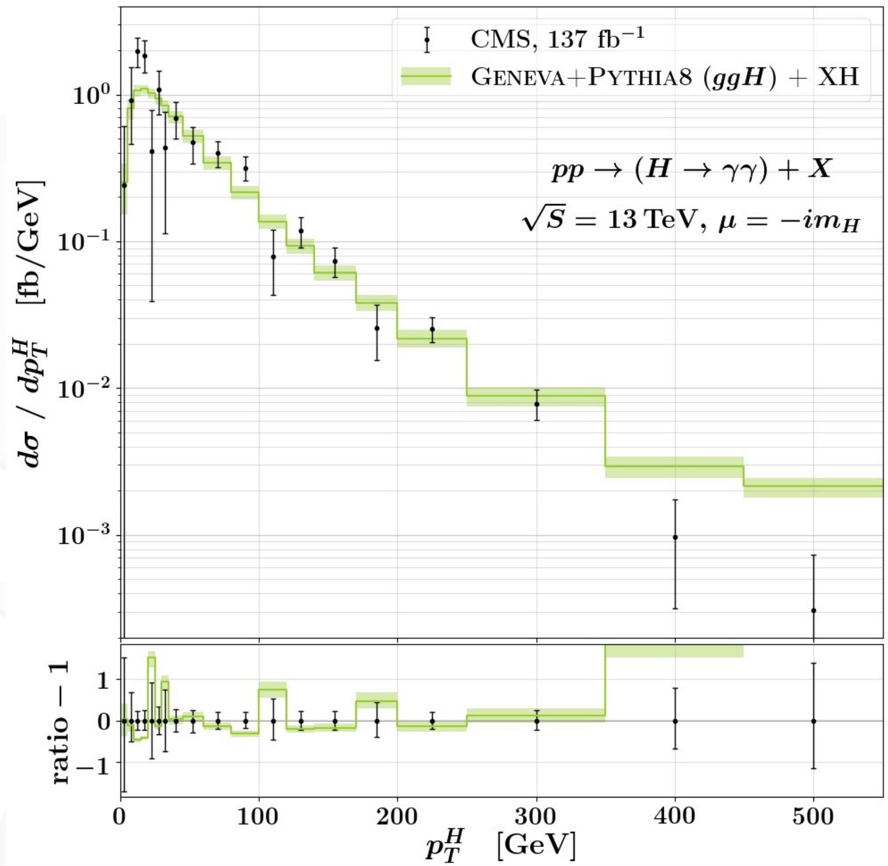
Showered results

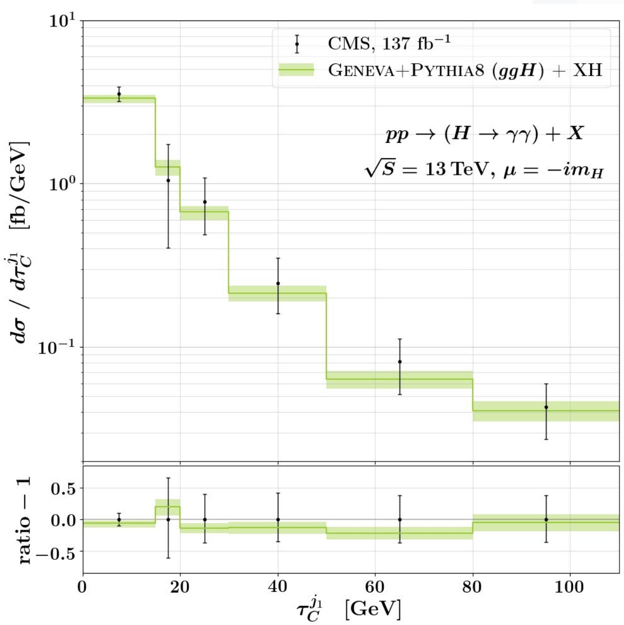
PYTHIA8 showering gives expected results for inclusive quantities.
Reasonable differences for more exclusive ones.



Comparison with LHC data at fiducial level







Fiducial cross section affected by resummation, parton shower effects, hadron decays, mainly due to photon isolation requirements, etc…

Comparison with ATLAS shows similar agreement. High p_T tail sensitive to m_t effects.

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

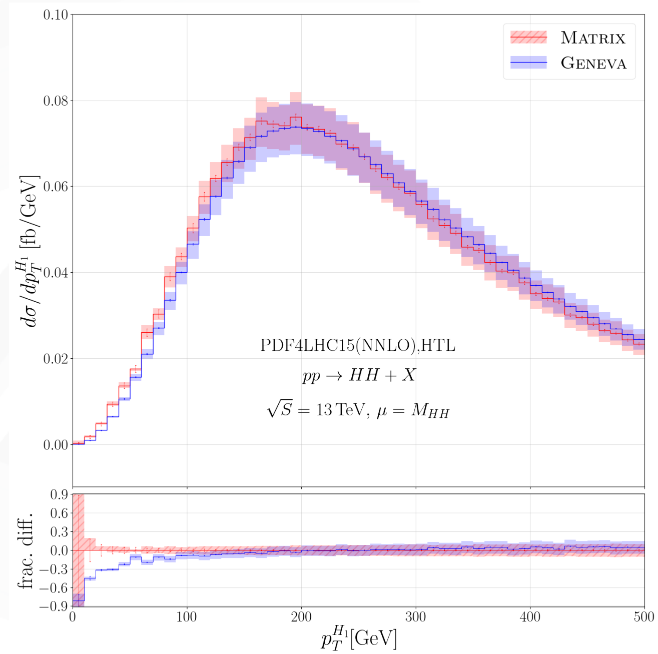
Double Higgs production

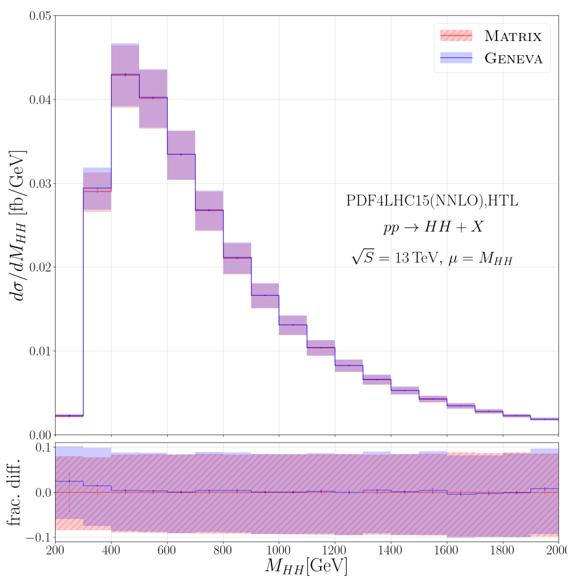
Calculation in infinite top-quark mass limit

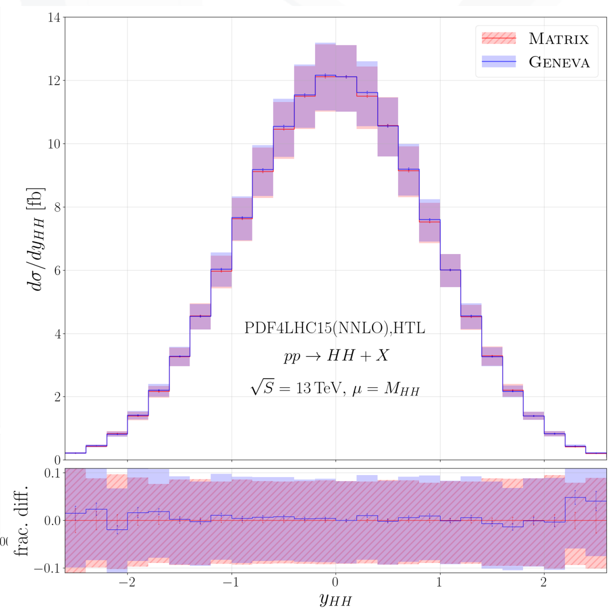
Very good agreement for invariant mass and rapidities

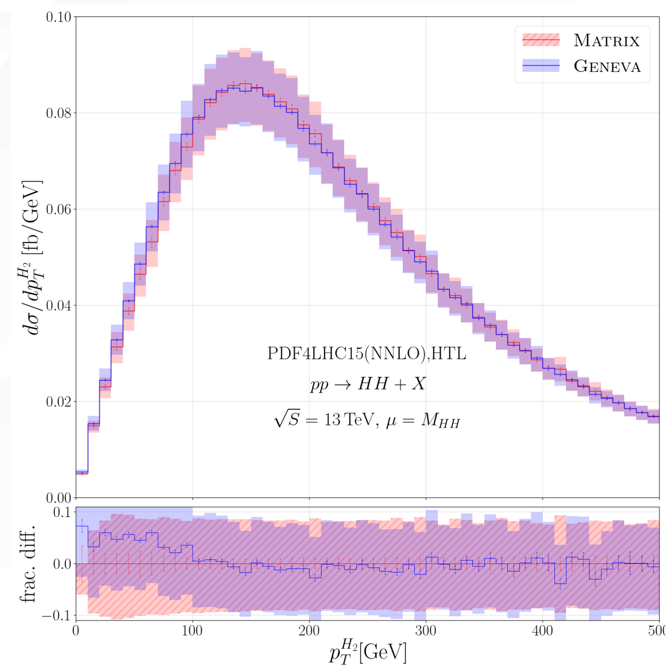
Interesting discrepancy when p_T of the hardest Higgs boson H_1 goes to zero.

Signals inadequacy of fixed-order calculations when $p_{T,HH} \to 0$



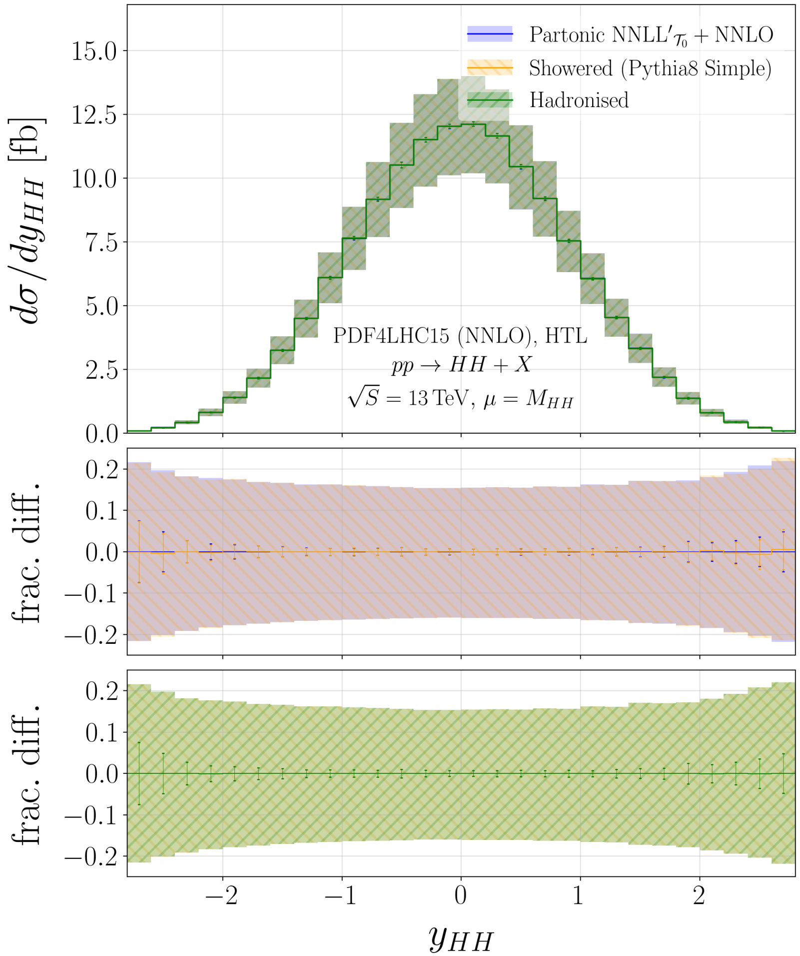


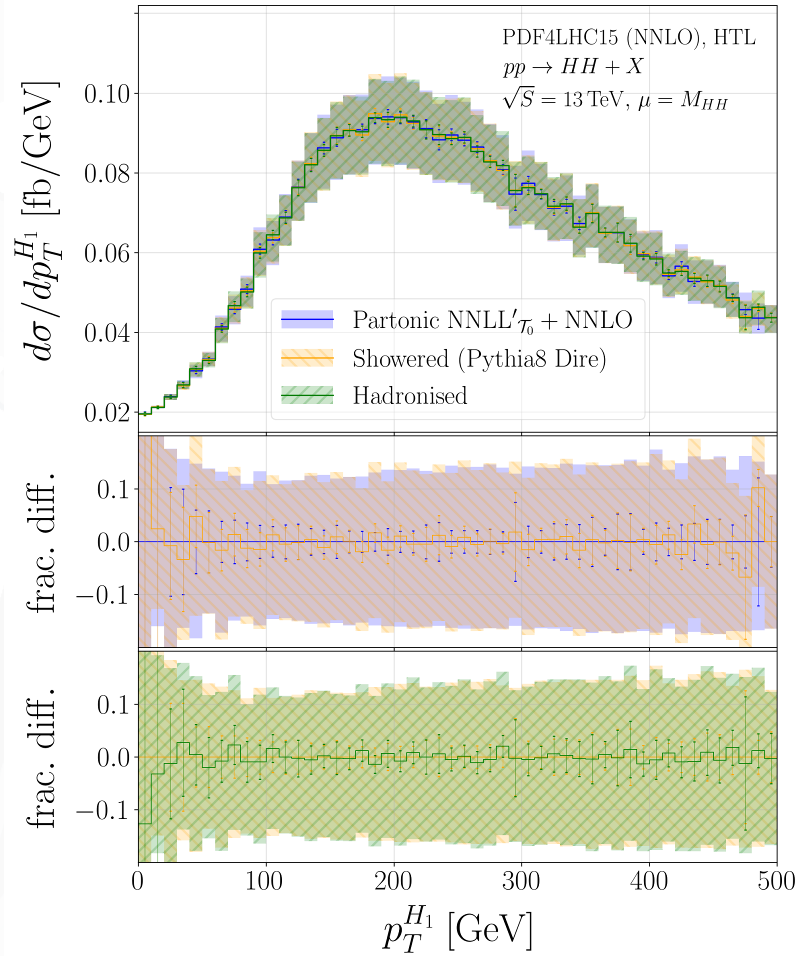




Double Higgs production: showered results

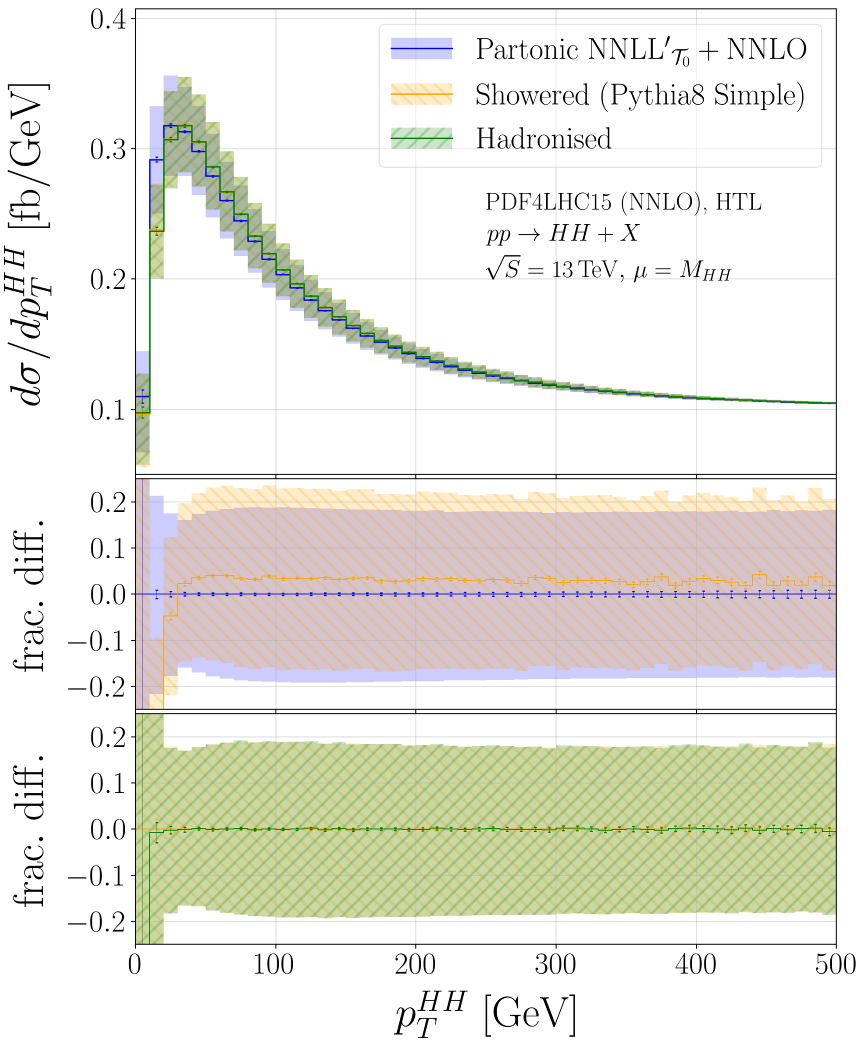
Exploring different shower models: PYTHIA8 standard (simple) and DIRE showers

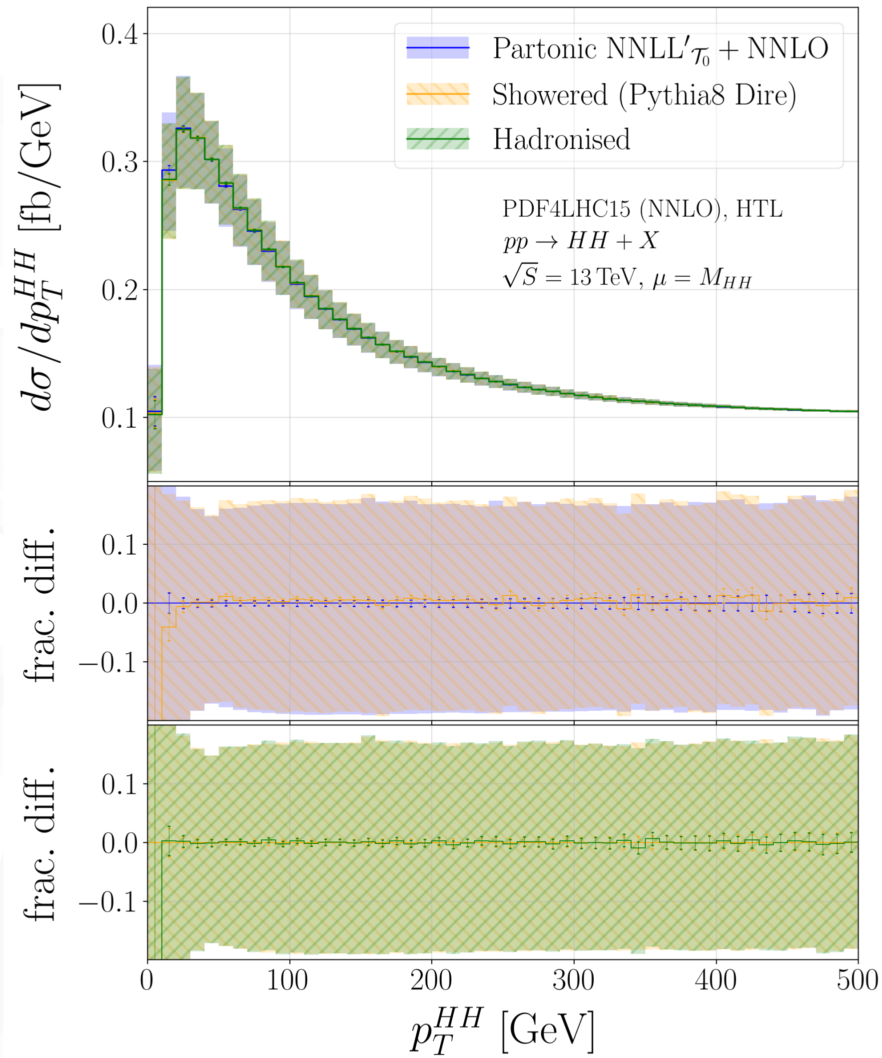




Inclusive quantities correctly described by both showers.

Double Higgs production: showered results





DIRE shower has a better treatment of recoil (less global) which reduces the shower effects in exclusive quantities.

Zero-jettiness factorization for top-quark pairs

Factorization formula derived using SCET+HQET in the region where $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales. [SA et al. 2111.03632]

In case of boosted regime $M_{t\bar{t}} \gg m_t$ one would instead need a modified two-jettiness [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21]

$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b\, B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)\, \mathrm{Tr}\left[\mathbf{H}_{ij}(\Phi_0, \mu)\, \mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N3LO

Hard functions (color matrices)

Soft functions (color matrices)

It is convenient to transform the soft and beam functions in Laplace space to solve the RG equations, the factorization formula is turn into a product of (matrix) functions

$$\mathscr{L}\left[\frac{d\sigma}{d\Phi_0 d\tau_B}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_i\left(\ln\frac{M\kappa}{\mu^2}, z_a\right) \tilde{B}_j\left(\ln\frac{M\kappa}{\mu^2}, z_b\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^2}{\mu^2}, \Phi_0\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^2}{\kappa^2}, \Phi_0\right)\right]$$

Zero-jettiness resummation for top pairs

Resummed formula valid at any logarithmic order

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B} = U(\mu_h, \mu_B, \mu_s, L_h, L_s) \times \mathrm{Tr}\Big\{ \mathbf{u}(\beta_t, \theta, \mu_h, \mu_s)\, \mathbf{H}(M, \beta_t, \theta, \mu_h)\, \mathbf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s) \Big\} \times \tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B) \tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B) \frac{1}{\tau_B^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})} \, .$$

where

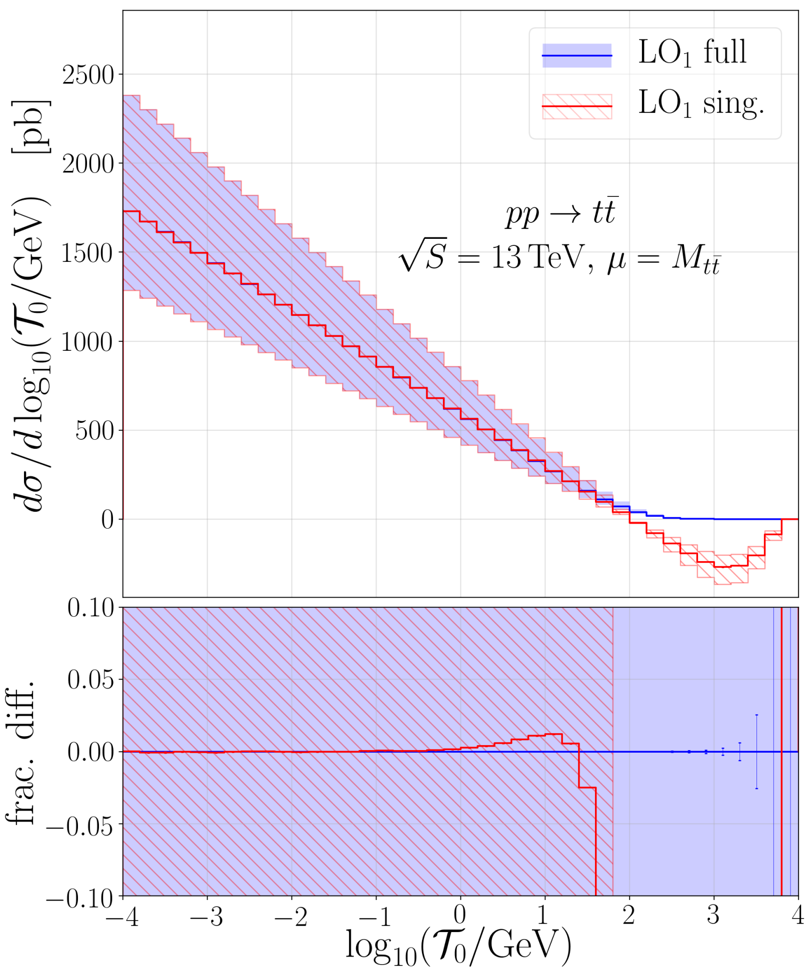
$$U(\mu_h, \mu_B, \mu_s, L_h, L_s) = \exp\Big[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_\Gamma(\mu_h, \mu_B)\, L_h - 2a_\Gamma(\mu_s, \mu_B)\, L_s \Big]$$

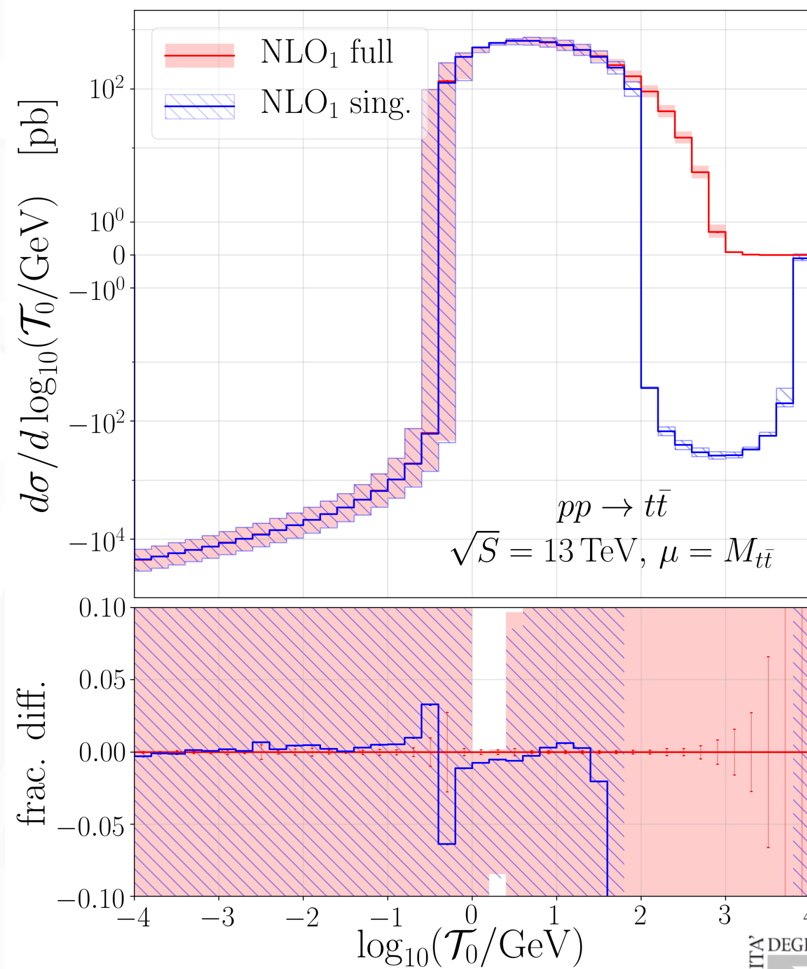
and $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$ and $\eta_{\mathrm{tot}} = 2\eta_S + \eta_B + \eta_{B'}$

The final accuracy depends on the availability of the perturbative ingredients

Singular cross section

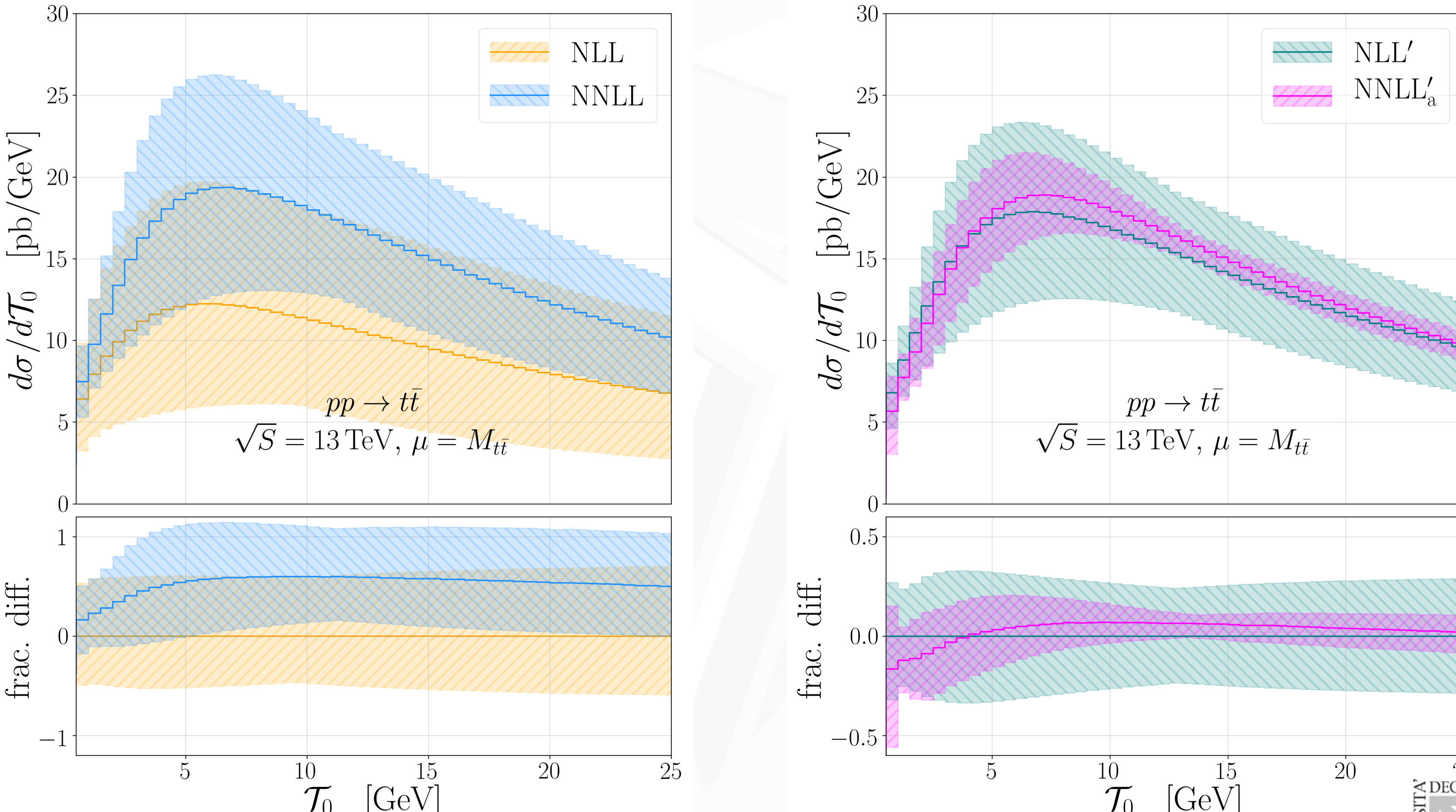
Fixed-order comparisons, approximate NLO and approximate NNLO vs LO_1 and NLO_1





Resummed results

NNLL'_{a} is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales

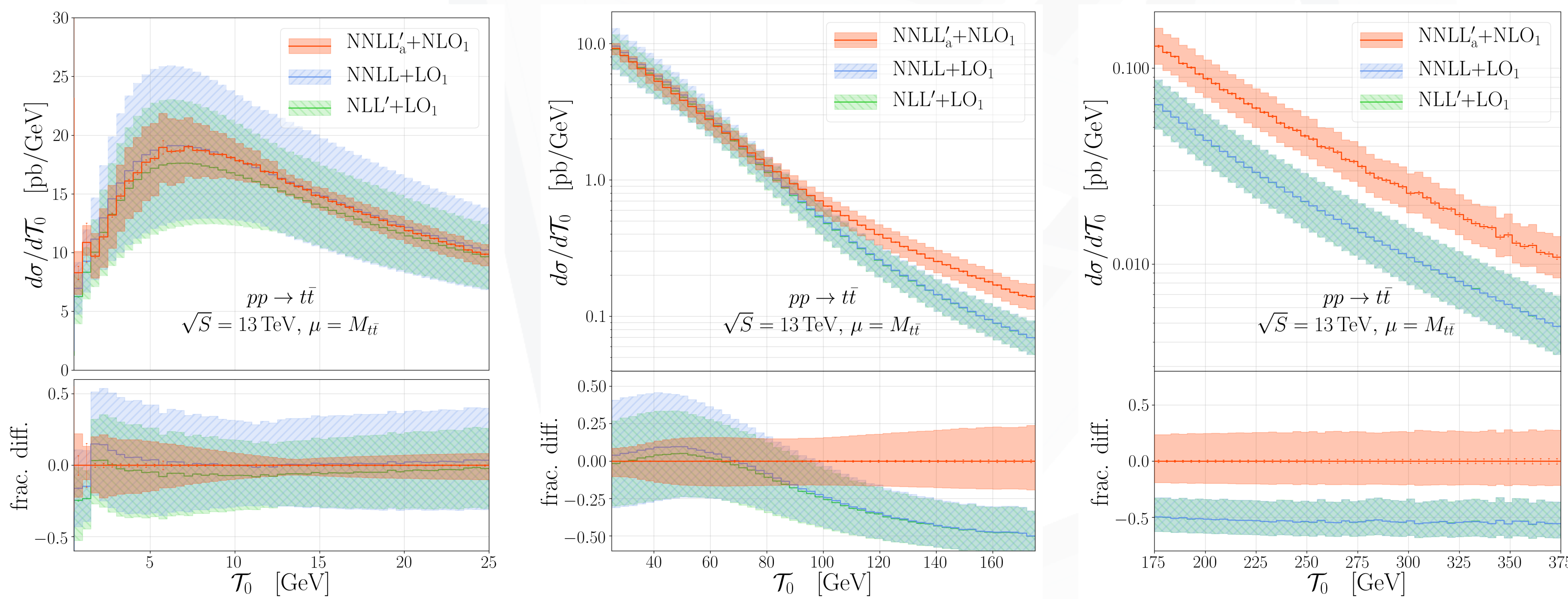


UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

Matched results

Matching to $t\bar{t}+j$ @NLO improves the perturbative accuracy across the whole spectrum

$$\frac{d\sigma^{\rm match}}{d\mathcal{T}_0} = \frac{d\sigma^{\rm resum}}{d\mathcal{T}_0} + \frac{d\sigma^{\rm FO}}{d\mathcal{T}_0} - \left[\frac{d\sigma^{\rm resum}}{d\mathcal{T}_0}\right]_{\rm FO}$$



Extension to full NNLL' and to event generation is ongoing …

Conclusion and outlook

- GENEVA method allows for fully exploiting higher-order resummation of resolution variables in event generation
- Tested and implemented for several color singlet production processes
- Plan to continue implementation for heavy-quarks and extension to other processes with colored particles at the Born level.
- Joint resummation of different resolution parameters is the next challenge, to extend precision to every corner of the available phase space.

Thank you for your attention.