University of Zurich[UZH]

q_T subtraction for massive final states: Wbb@NNLO in 4FS

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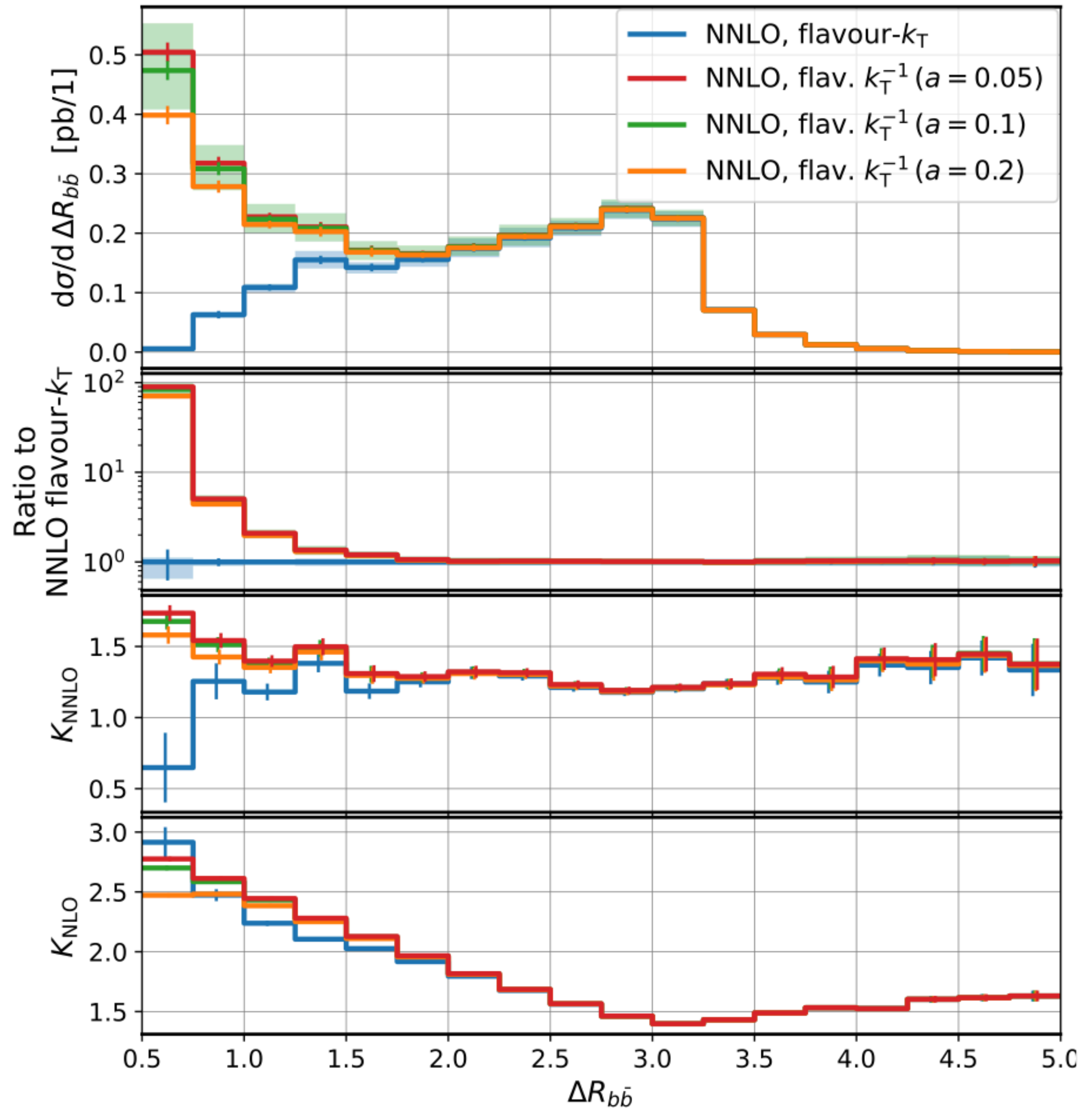
Workshop on Tools for High Precision LHC Simulations
Castle Ringberg - 1st November 2022

Intro

W+1bj and W+2bj interesting signatures

- tests of QCD at LHC   *see talk by A. Huss*
- background to $WH(H \to b\bar{b})$ and single top $\bar{b}t(t \to Wb)$
- **bottom quarks modelling:** massive effects, bottom in the PDF, flavour tagging

HPPZ



NLO corrections (massless bottom quarks)

[Ellis, Veseli, 1999]

NLO corrections (massive bottom quarks)

[Febres Cordero, Reina, Wackeroth, 2006, 2009]

NLO corrections (4FS+5FS)

[Campbell, Ellis, Febres Cordero, Maltoni, Reina, Wackeroth, Willenrock, 2009] [Campbell, Caola, Febres Cordero, Reina, Wackeroth,2011]

NLO+PS

[Oleari, Reina,2011] [Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, 2011]

POWHEG+MiNLO

[Luisoni, Oleari, Tramontano, 2015]

Wbb + up to 3 jets

[Anger, Febres Cordero, Ita, Sotnikov, 2018]

Analytical Two-loop W+4partons amplitude in Leading Colour (LC)

[Badger, Hartanto, Zoia, 2021] [Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov, 2021]

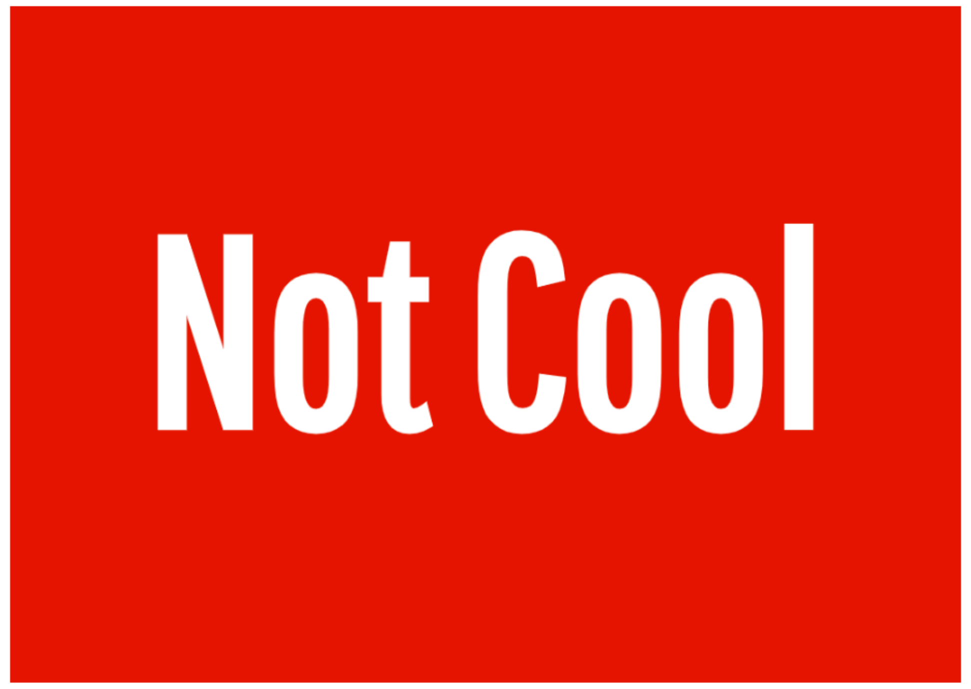
see talk by L. Tancredi

NNLO corrections (massless bottom quarks)

[Hartanto, Poncelet, Popescu, Zoia, 2022]

Intro: slicing methods

Slicing $\int |\mathcal{M}|^2 \, F_J \, \mathrm{d}\phi_d = \int_0^\delta \left[|\mathcal{M}|^2 \, F_J \, \mathrm{d}\phi_d \right]_{\mathrm{simp}} + \int_\delta^1 |\mathcal{M}|^2 \, F_J \, \mathrm{d}\phi_4 + \mathcal{O}(\delta)$



Slicing $\int |\mathcal{M}|^2 \, F_J \, \mathrm{d}\phi_d = \int_0^\delta \left[|\mathcal{M}|^2 \, F_J \, \mathrm{d}\phi_d \right]_{\mathrm{simp}} + \int_\delta^1 |\mathcal{M}|^2 \, F_J \, \mathrm{d}\phi_4 + \mathcal{O}(\delta)$

One of the main reasons for the slicing comeback is the increase in computing power available for such computations.

CONs

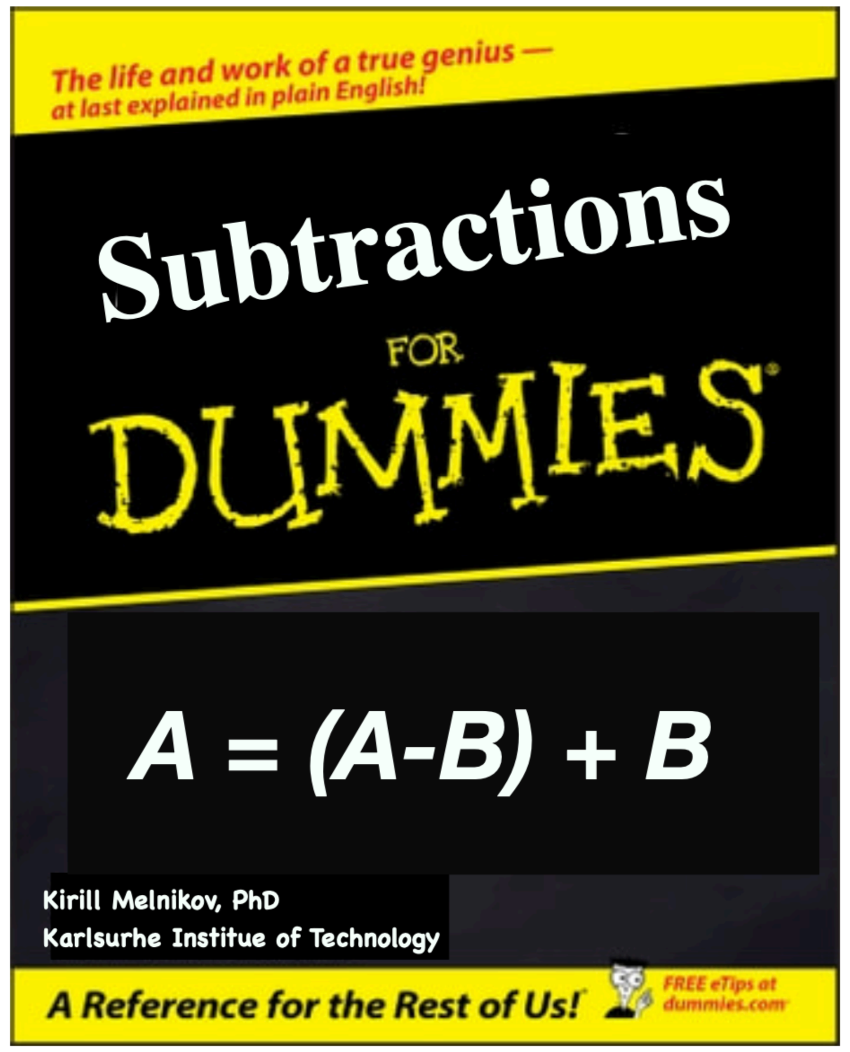
- large global cancellation
- residual power corrections

PROs

see talk by A. Huss

- usually simpler (allowed to reach N^{3}LO for color singlet production)
- connection with **factorisation** theorems and **resummation**
- implications for higher-order matching (MiNNLO/GENEVA)

see MiNNLO sessions and S. Alioli



Intro: slicing methods: q_T subtraction formalism for massive final states

q_T subtraction **initially formulated for color singlet processes** [Catani, Grazzini, 2007] and successfully applied for the calculation of NNLO QCD corrections [Grazzini, Kallweit, Wiesemann 2018]

q_T subtraction formalism extended to the case of **heavy quarks** production [Catani, Grazzini, Torre, 2014]

Successful employed for computation of NNLO QCD corrections to the production of

- a **top pair** [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan 2019]
- a **bottom pair** production [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, 2021]

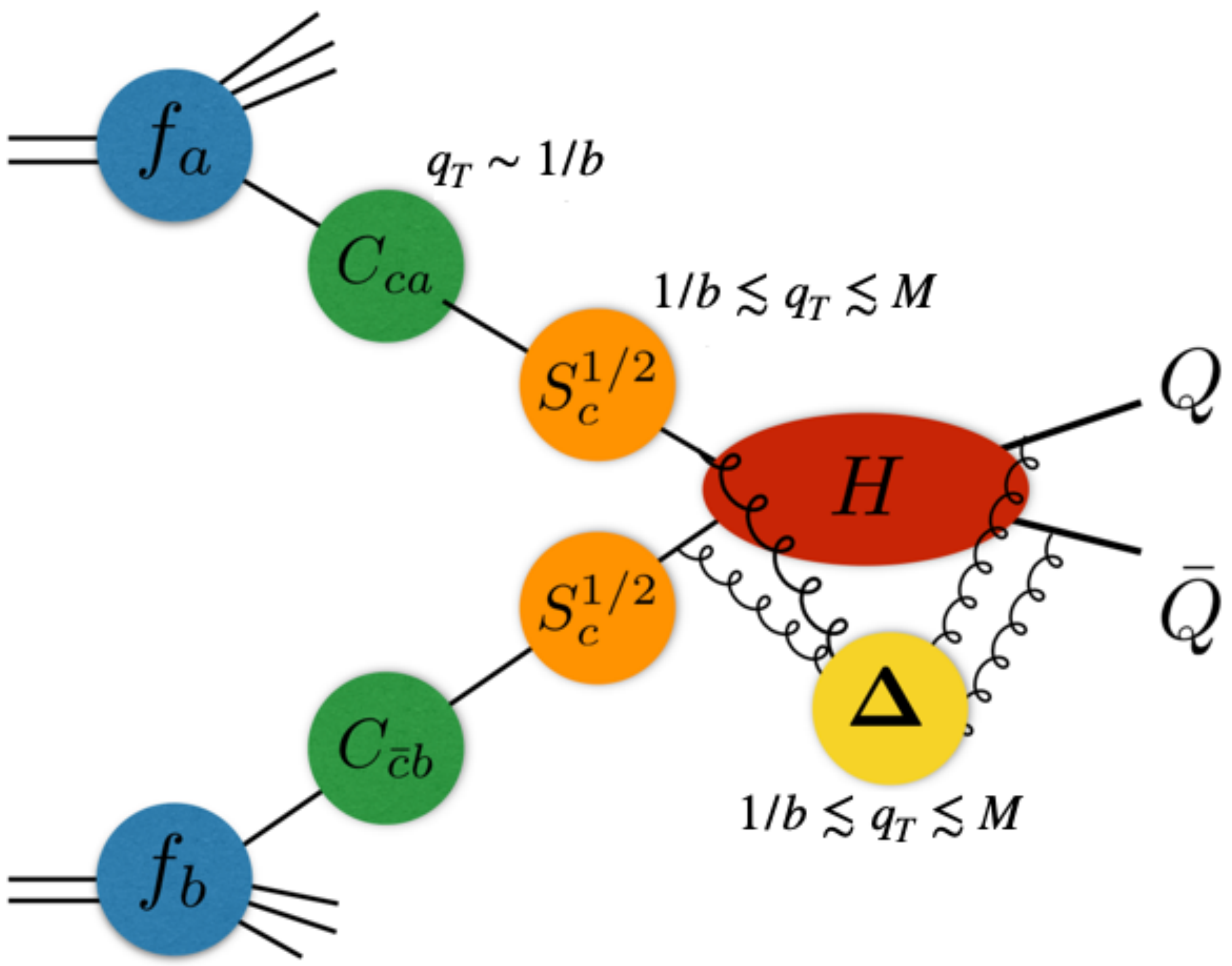
and the computation of Mixed QCD-EW corrections to

- a **charged current Drell-Yan** [LB, Grazzini, Kallweit, Savoini, Tramontano (2021)]
- a **neutral current Drell-Yan** [Bonciani, LB, Grazzini, Kallweit, Rana, Tramontano, Vicini (2021)]

$$d\sigma_{N^kLO} = \mathcal{H} \otimes d\sigma_{LO} + \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T/Q > r_{\text{cut}}} + \mathcal{O}(r^{\ell}_{\text{cut}})$$

Intro: slicing methods: q_T subtraction formalism for massive final states

The resummation formula shows a **richer structure** because of additional soft singularities (four coloured partons at LO)



- Soft logarithms controlled by the **transverse momentum anomalous dimension Γ_t** known up to NNLO [Mitov, Sterman, Sung, 2009], [Neubert, et al 2009]
- Hard coefficient gets a **non-trivial** colour structure (matrix in colour-space)
- Non trivial azimuthal correlations

Non trivial ingredient

- **Two-loop soft function** [Catani, Devoto, Grazzini, Mazzitelli, in preparation]

Intro: slicing methods: q_T subtraction formalism for massive final states

Resolution variable (for example in Drell-Yan)

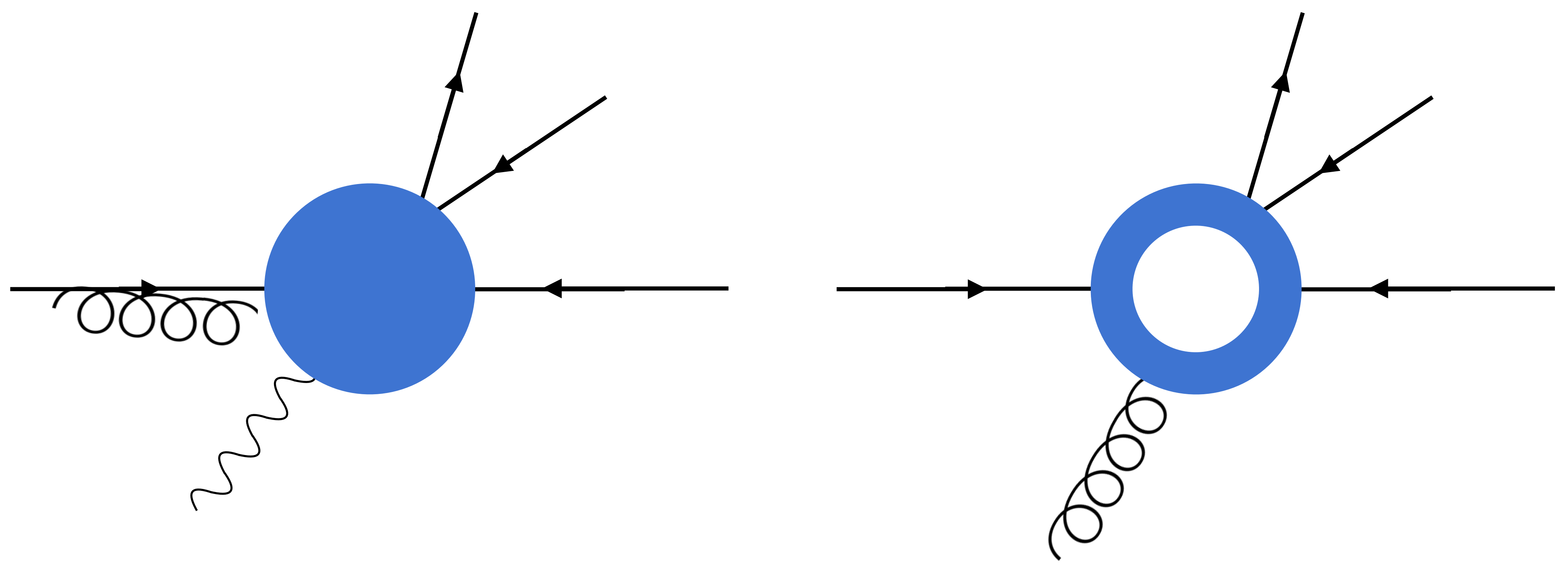
$q_T :=$ **transverse momentum** of the dilepton final state
$Q :=$ invariant mass of the dilepton final state

Final state must be massive!

for jets in the final state see talk by L. Rottoli

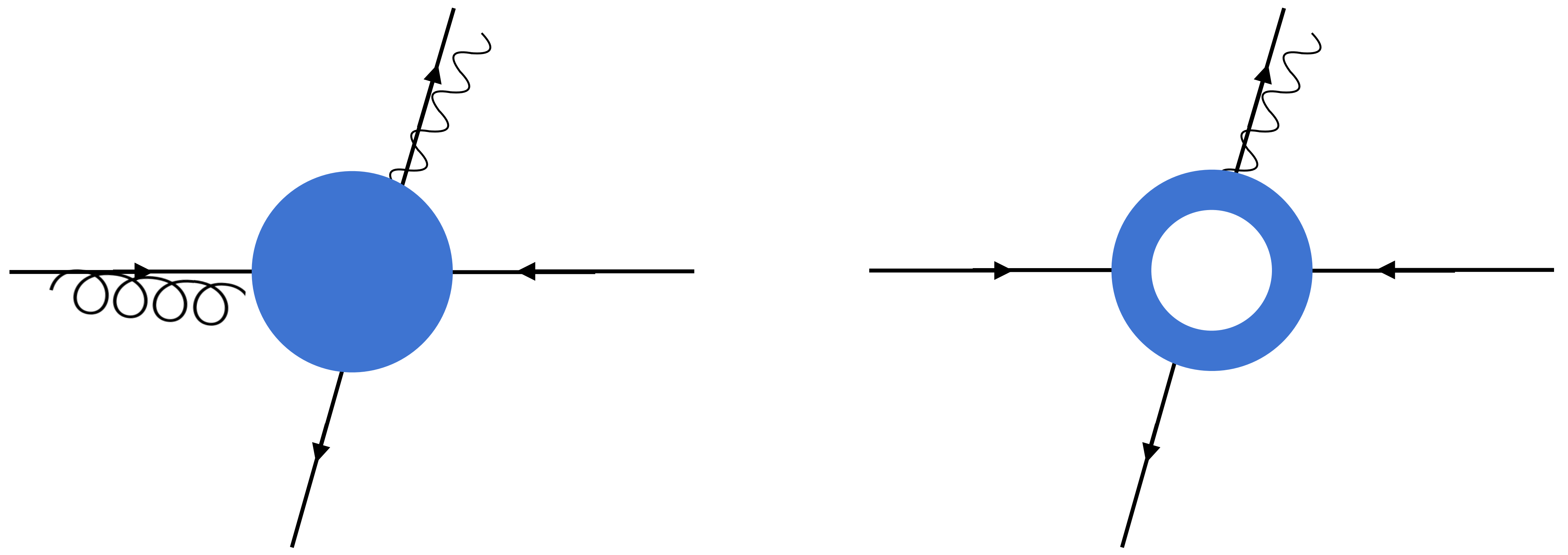
Initial-state radiation

For $q_T/Q > 0$ **one emission is always resolved**



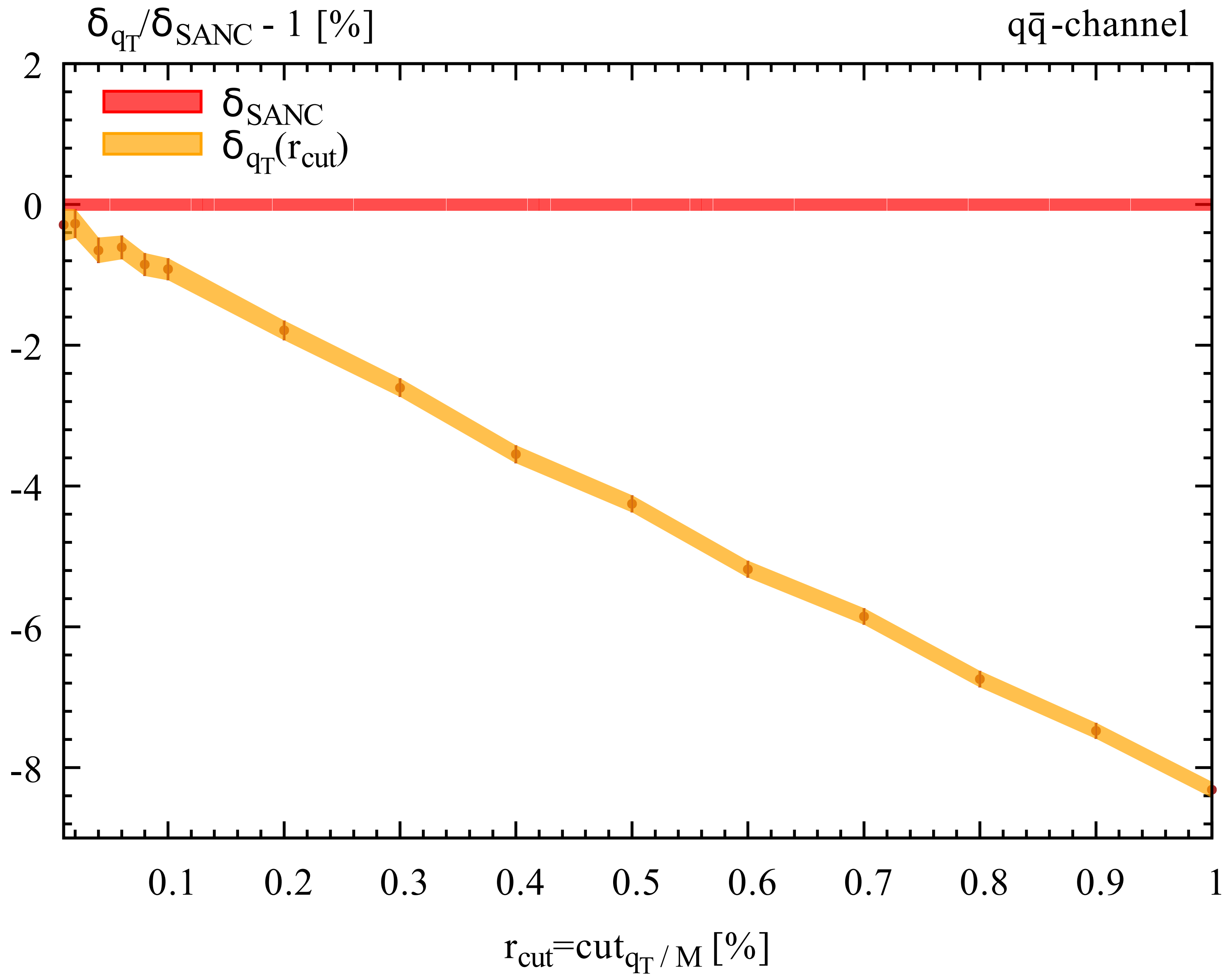
Final-state (collinear) radiation

There are configurations with $q_T/Q > 0$ and **two unresolved emission if leptons are massless**



Intro: slicing methods: q_T subtraction formalism for massive final states

- Massive final state **linear** ($m = 1$) power corrections due to final-state emission



analytical insight for **inclusive cross section** in pure QED

$$\sigma^{NLP}(s; r_{cut}) = -\frac{3\pi}{8}\frac{\alpha}{2\pi} r_{cut} \left[\frac{6(5-\beta^2)}{3-\beta^2} + \frac{-47+8\beta^2+3\beta^4}{\beta(3-\beta^2)} \log\frac{1+\beta}{1-\beta} \right] \sigma_B(s)$$

$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

[LB, Grazzini, Tramontano, 2019]

- At NNLO: linear (m=1) + log enhancement

in general we have to rely on an extrapolation procedure!

Intro: slicing methods: q_T subtraction formalism for massive final states

MATRIX

[Grazzini, Kallweit, Wiesemann 2018]

MUNICH (by S. Kallweit)

- **efficient multichannel** phase space generation
- **bookkeeping** all subprocesses
- automatic implementation of dipole subtraction

AMPLITUDES

- 1-loop amplitudes: **OpenLoops, Recola** (Collier, CutTOols,...)
- 2-loop: dedicated 2-loop codes (VVamp, GiNac, TDHPL,...)

SUBTRACTION SCHEME

- @NLO: dipole and q_T subtraction
- @NNLO: q_T subtraction

Progress

- **SHARK**: of **soft function for arbitrary kinematics** see talk by J. Mazzitelli
- first $2 \rightarrow 3$ calculation: **associated production of a top-anti-top pair and a Higgs** see talk by M. Grazzini

Wbb with massive bottom quarks (4FS) desirable. I will report on on-going work

Two-loop amplitude is the main bottleneck: our strategy is to rely on the **massification procedure**

Outline

- Two-loop Amplitude
- Preliminary results: comparison with HPPZ
- Conclusions

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Massification procedure

[Mitov, Moch, 2007]

The massification procedure is based on the **factorisation properties** of QCD amplitudes

Basic idea: in the small mass limit, the massive amplitude $\mathcal{M}^{[p],(m)}$ and the massless one $\mathcal{M}^{[p],(m=0)}$ are connected as the mass screens some collinear divergences "trading" poles in the dimensional regulator ϵ into logarithms of the mass

This can be viewed as a **change in the renormalisation scheme** which leads to a universal **"multiplicative renormalization"** relation between (*ultraviolet renormalised*) massive and massless amplitudes

$$\mathcal{M}^{[p],(m)} = \prod_{i\in\{\text{all legs}\}} \left(Z_{[i]}^{(m|0)}\right)^{\frac{1}{2}} \mathcal{M}^{[p],(m=0)} + \mathcal{O}(m^k)$$

- The function $Z_{[i]}^{(m|0)}$ are universal, depends only on the external parton (quark or gluon) and admit perturbative expansion in α_s:

$$Z_{[i]} = 1 + \sum_k \left(\frac{\alpha_s}{2\pi}\right)^k Z_{[i]}^k$$

$$\mathcal{M}^{[p],(m)} = \sum_{k=0} \left(\frac{\alpha_s}{2\pi}\right)^k \mathcal{M}_{(k)}^{[p],(m)}$$

$\rightarrow$

$$\mathcal{M}_0^{Wbb,(m)} = \mathcal{M}_0^{Wbb,(m=0)}$$

$$\mathcal{M}_{(1)}^{Wbb,(m)} = \mathcal{M}_{(1)}^{Wbb,(m=0)} + Z_{[q]}^{(1)} \mathcal{M}_{(0)}^{Wbb,(m=0)}$$

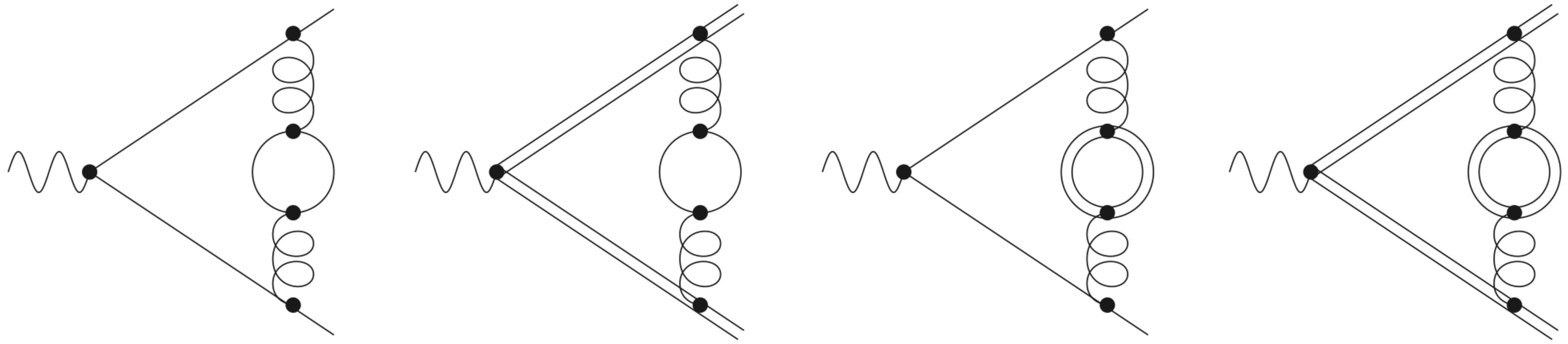
$$\mathcal{M}_{(2)}^{Wbb,(m)} = \mathcal{M}_{(2)}^{Wbb,(m=0)} + Z_{[q]}^{(1)} \mathcal{M}_{(1)}^{Wbb,(m=0)} + Z_{[q]}^{(2)} \mathcal{M}_{(0)}^{Wbb,(m=0)}$$

Massification procedure

[Mitov, Moch, 2007]

$$\mathcal{M}^{[p],(m)} = \prod_{i\in\{\text{all legs}\}} \left(Z_{[i]}^{(m|0)}\right)^{\frac{1}{2}} \mathcal{M}^{[p],(m=0)} + \mathcal{O}(m^k)$$

- The $Z_{[i]}^{(m|0)}$ are given by the **ratio of massive and massless form factors** ($\gamma^* qq$ for the quark case)



ll *hl* *lh* *hh*

- Starting at two loops, contributions from heavy quarks loops (*lh* and *hh*) arise. Their description requires additional process dependent terms and **have been excluded** from the definition of the $Z_{[i]}^{(m|0)}$

The massification procedure predicts **poles, logarithms of mass and mass independent terms (constants)** of $\mathcal{M}^{[p],(m)}$ while **power corrections** in the mass and the contribution of **heavy loops** cannot be retrieved using this approach

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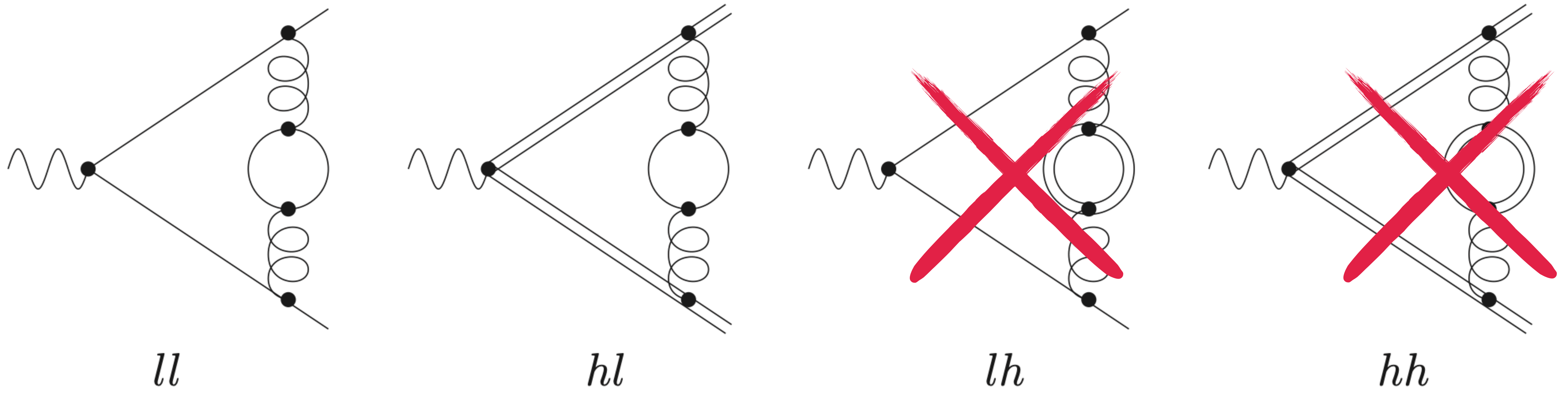

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Massification procedure

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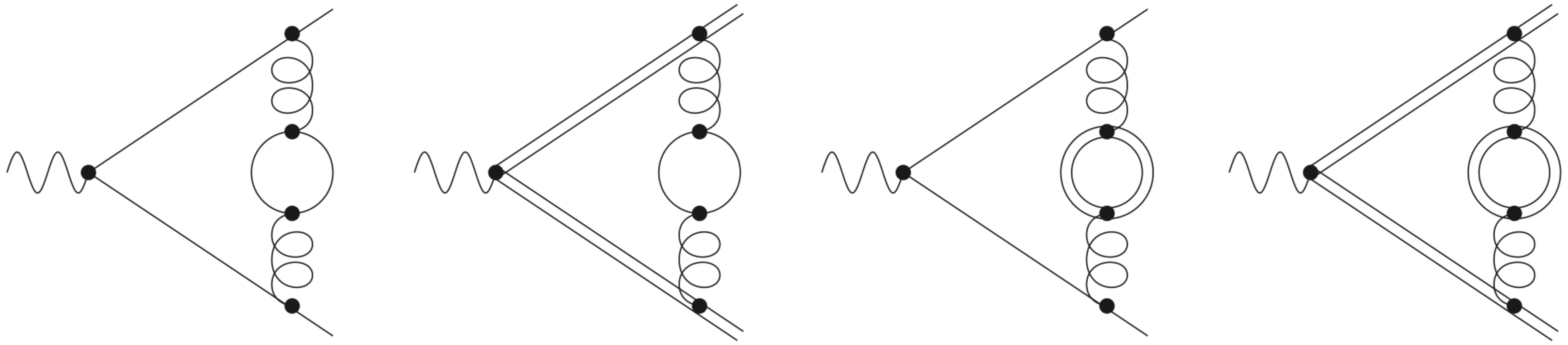


Remarks

- The functions $Z_{[i]}^{(m|0)}$ are **trivial objects in colour space** and are expressed in terms of colour Casimir
- At each perturbative order, $Z_{[i]}^{(k)}$ is given by a Laurent series in ϵ

$$Z_{[q]}^{(1)} = C_F \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\ln \frac{\mu^2}{m^2} + \frac{1}{2} \right) + \ldots \right]$$

→ requires knowledge of the massless one-loop amplitude $\mathcal{M}_{(1)}^{Wbb,(m=0)}$ up to $\mathcal{O}(\epsilon^2)$

Two-loop massless amplitudes

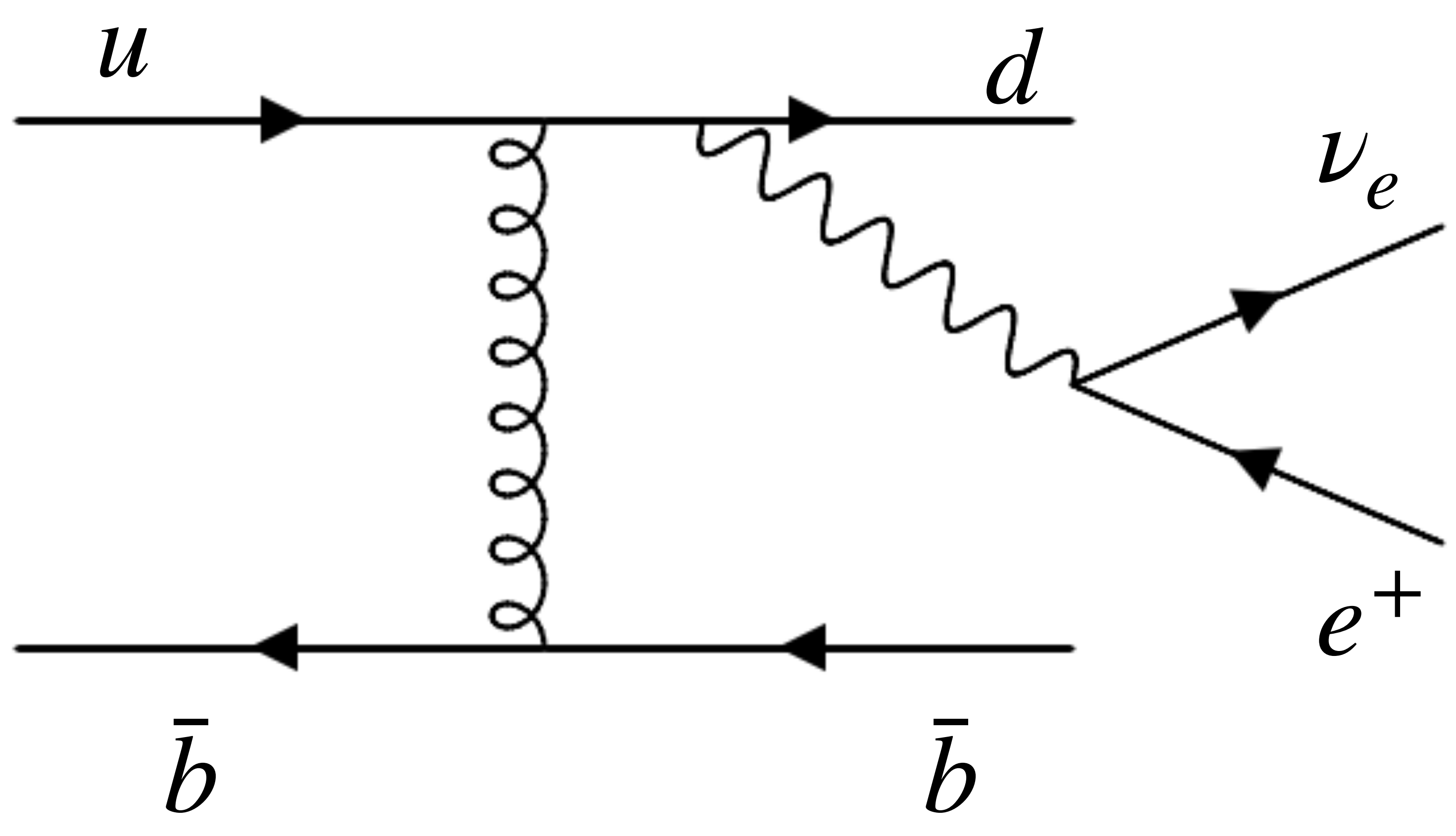
[Abreu, Febres-Cordero, Ita, Klinkert, Page, Sotnikov, 2022]

Leading-colour two-loop helicity virtual amplitudes for the scattering of a W boson and four partons

- analytical expressions obtained within the framework of numerical unitary (using numerical samples)
- the results are expressed in terms of a basis of **one-mass pentagon functions** [Chicherin, Sotnikov, Zoia 2021]
- **off-shell W boson** including its leptonic decay
- publicly available http://www.hep.fsu.edu/~ffebres/W4partons
- **analytical expressions of the one-loop amplitudes up to** $\mathcal{O}(\epsilon^2)$ in the leading colour approximation

Some complications

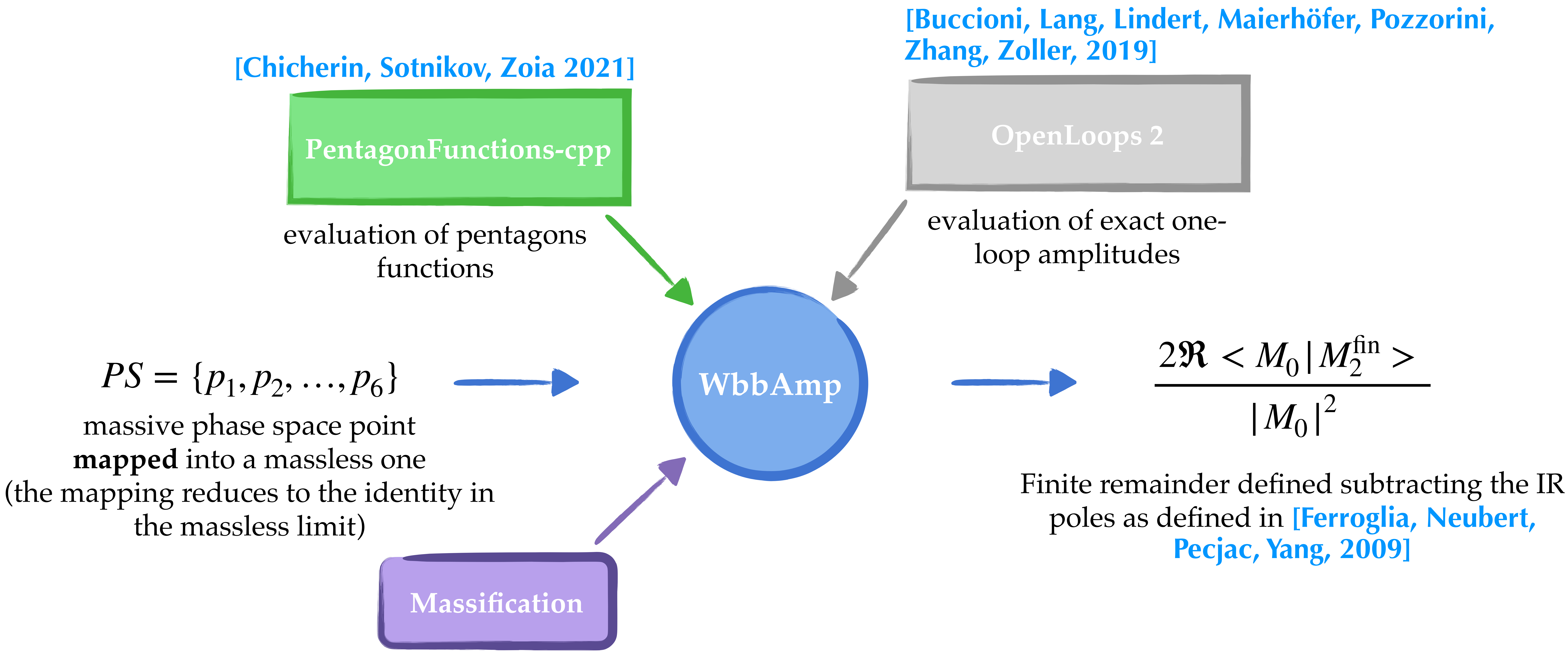
- Amplitudes provided as analytical expressions that can be processed in Mathematica; **this is not suitable for on-the-fly numerical evaluation** for Monte Carlo integration
- Rather long algebraic expressions
- Reference process is $u\bar{b} \to \bar{b}de^+\nu_e$. Initial-final state crossing involves in general **analytic continuation**



WbbAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini WIP]

We have implemented the one-loop and two-loop amplitudes of [Abreu et al, 2022] in a **C++ library** for the efficient numerical evaluation of the **massive** amplitudes



WbbAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini WIP]

Dealing with the complications

One-Loop amplitudes: $\mathcal{O}(1000)$ source files of small-moderate size (< 100 Kb)

- algebraic expressions (rational function of the invariants) simplified using MultiVariate Apart [Heller, von Manteuffel, 2021] at the level of Mathematica before exporting them
- automatised generation of C++ source files from the Mathematica expressions; very simple code optimisation introducing abbreviations (https://github.com/lecopivo/OptimizeExpressionToC)

Two-Loop amplitudes: $\mathcal{O}(3000)$ source files of moderate size (< 250 Kb)

- algebraic expressions **too long and complex**; no pre-simplification step
- breakdown of each expression in small blocks
- automatised generation of C++ source files for each block
- handling of **numerical instabilities**

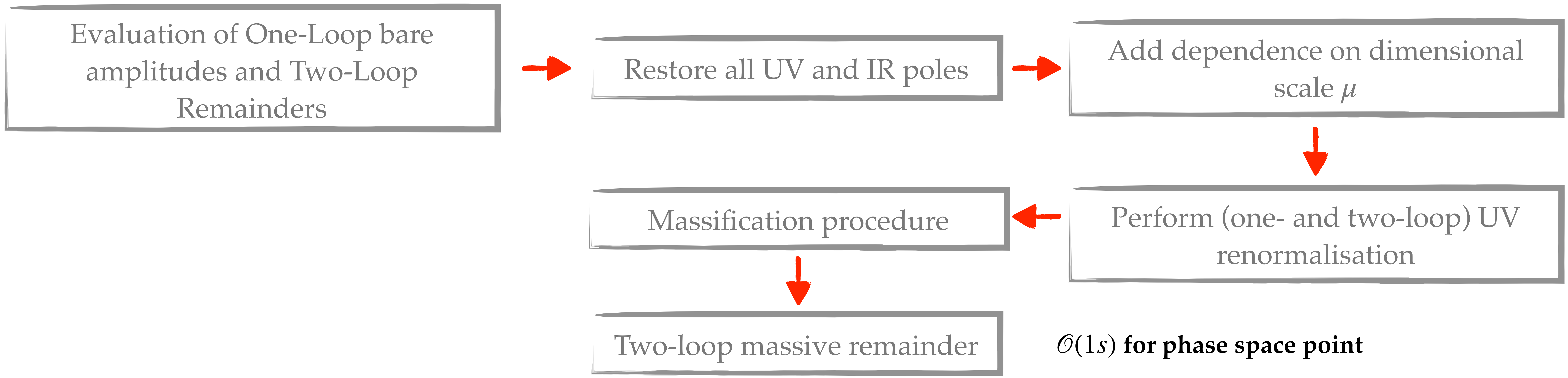
Crossing

- simple permutation of the momenta in the algebraic coefficients
- the action of the permutation transforms the **pentagon functions** into each other, no need for analytic continuation. All permutations available in a Mathematica script [Chicherin, Sotnikov, Zoia 2021]

WbbAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini WIP]

WORKFLOW in a NUTSHELL



Validation and checks

- the C++ code reproduces the massless results obtained with Mathematica for different phase space points and crossing of the amplitudes within the single precision (7-9 digits)
- for the one-loop amplitudes, we have tested both the **massless and massive** amplitudes against the **independent implementation available in MCFM**, which allows to extract the leading colour contribution
- the poles of the massive amplitude cancels against the ones predicted in [Ferroglia, Neubert, Pecjac, Yang, 2009] (at leading colour)

WbbAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini WIP]

Leading Colour and Massification

- we have carried out the massification procedure in the leading colour approximation to explicitly check the cancellation of the poles
- however, in this way we are artificially introducing **spurious miscancellation** between real and virtual contributions
- moreover, the terms introduced with the massification, being enhanced by large logarithms of μ^2/m^2, are generally the dominant contributions and the difference between Full Colour and Leading Colour can be sizeable $C_F/(N_C/2) \sim 0.89$ and $(C_F/(N_C/2))^2 \sim 0.8$

Retain massification contributions at full colour whenever possible!

$$\mathcal{M}_{(2)}^{Wbb,(m)} = \mathcal{M}_{(2)}^{Wbb,(m=0)} + Z_{[q]}^{(1)} \mathcal{M}_{(1)}^{Wbb,(m=0)} + Z_{[q]}^{(2)} \mathcal{M}_{(0)}^{Wbb,(m=0)}$$

$$Z_{[q]}^{(1),2} M_{(1)}^{Wbb,(m=0),-2} + Z_{[q]}^{(1),1} M_{(1)}^{Wbb,(m=0),-1} + Z_{[q]}^{(1),0} M_{(1)}^{Wbb,(m=0),0} + Z_{[q]}^{(1),-1} M_{(1)}^{Wbb,(m=0),1} + Z_{[q]}^{(1),-2} M_{(1)}^{Wbb,(m=0),2}$$

with **OpenLoops2**

cannot be retrieved but it is less problematic, at most a single log

Outline

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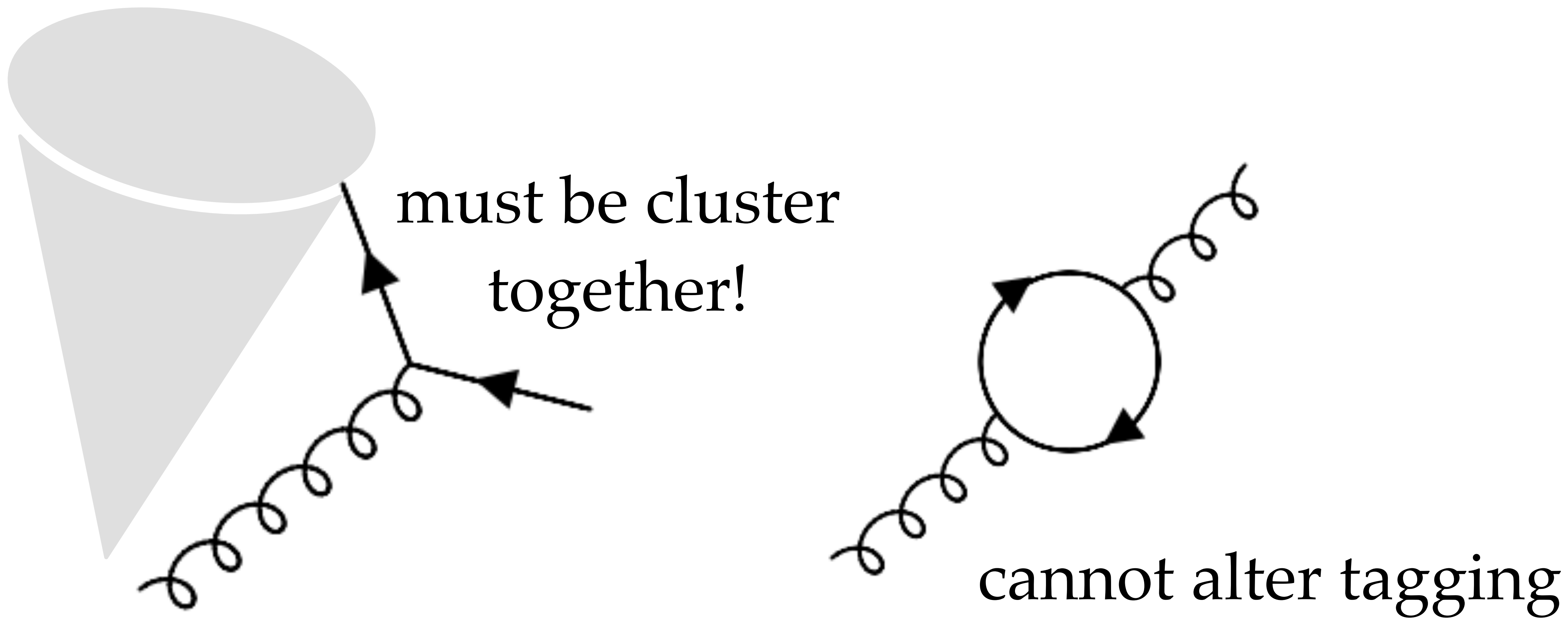
Infrared safety and flavour tagging

Jet clustering algorithms consist in a sequence of two-to-one recombination steps. They are then completely defined once the binary distance d_{ij} and the beam distance d_{iB} are given

For parton level calculation (fixed order), **infrared safety** is a crucial requirement

For observable sensitive to the flavour assignment, infrared safety can be an issue, usually associated **to gluon splitting in the double soft limit** (the problem starts at NNLO)



this may lead to a flavour configuration different from the corresponding virtual one, spoiling KLN cancellation

Two necessary conditions for a wide-angle double-soft limit of two opposite flavoured parton i and j **[Czakon, Mitov, Poncelet, 2022]**

1. d_{ij} vanishes for every R_{ij}
2. d_{ij} vanishes faster than the distance of either i or j to the remaining pseudo-jets

Flavour jet algorithms: flavour anti-k_T

[Czakon, Mitov, Poncelet, 2022]

Standard anti-k_T algorithm

$$d_{ij} = \min\left(k_{T,i}^{-2}, k_{T,j}^{-2}\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^{-2}$$

Flavour anti-k_T algorithm

$$d_{ij}^{(F)} = d_{ij} \times \begin{cases} \mathcal{S}_{ij}, & \text{if both } i \text{ and } j \text{ have non-zero flavour of opposite sign} \\ 1, & \text{otherwise} \end{cases}$$

$$\mathcal{S}_{ij} = 1 - \theta(1-\kappa)\cos\left(\frac{\pi}{2}\kappa\right), \qquad \kappa = \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,max}^2}$$

$$\longrightarrow \quad \mathcal{S}_{ij} \sim E^4 \implies d_{ij}^{(F)} \sim E^2$$

No change in the beam distance required

Parameter a control the turning on of the suppression factor. In the limit $a \to 0$, the standard anti-k_T algorithm is recovered.

Best choice of the parameter a from comparison at NLO (+PS) (minimise unfolding)

Flavour jet algorithms: flavour k_T

[Banfi, Salam, Zanderighi, 2006]

Standard k_T algorithm

$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^2$$

Flavour k_T algorithm (common choice $\alpha = 2$)

$$d_{ij}^{(F)} = R_{ij}^2 \times \begin{cases} \left[\max\left(k_{T,i}^2, k_{T,j}^2\right)\right]^{\alpha} \left[\min\left(k_{T,i}^2, k_{T,j}^2\right)\right]^{2-\alpha}, & \text{if softer of } i,j \text{ is flavoured} \\ \min\left(k_{T,i}^2, k_{T,j}^2\right), & \text{if softer of } i,j \text{ is flavourless} \end{cases}$$

Also beam distance problematic (a soft flavoured parton can be identified as a protojet and removed from the list)

$$d_{iB(\bar{B})}^{(F)} = R_{ij}^2 \times \begin{cases} \left[\max\left(k_{T,i}^2, k_{T,B(\bar{B}}^2\right)\right]^{\alpha} \left[\min\left(k_{T,i}^2, k_{T,B(\bar{B}}^2\right)\right]^{2-\alpha}, & \text{if } i \text{ is flavoured} \\ \min\left(k_{T,i}^2, k_{T,B(\bar{B}}^2\right), & \text{if } i \text{ is flavourless} \end{cases}$$

$$k_{T,B}(y) = \sum_i k_{T,i}\left(\Theta(y_i - y) + \Theta(y - y_i)e^{y_i - y}\right) \qquad k_{T,\bar{B}}(y) = \sum_i k_{T,i}\left(\Theta(y - y_i) + \Theta(y_i - y)e^{y - y_i}\right)$$

Comparison with HPPZ

$$W + 2\ b_{jet} + X\ \textbf{(inclusive)}\ @\ \sqrt{s} = 8\,\text{TeV}$$

[CMS:arXiv:1608.07561]

Selection cuts

$$p_{T,\ell} > 30\ \text{GeV} \qquad |\eta_\ell| < 2.1$$

$$n_b = 2 : p_{T,b} > 25\ \text{GeV} \qquad |\eta_\ell| < 2.4$$

$$p_{T,j} > 25\ \text{GeV} \qquad |\eta_\ell| < 2.4$$

Reference scale

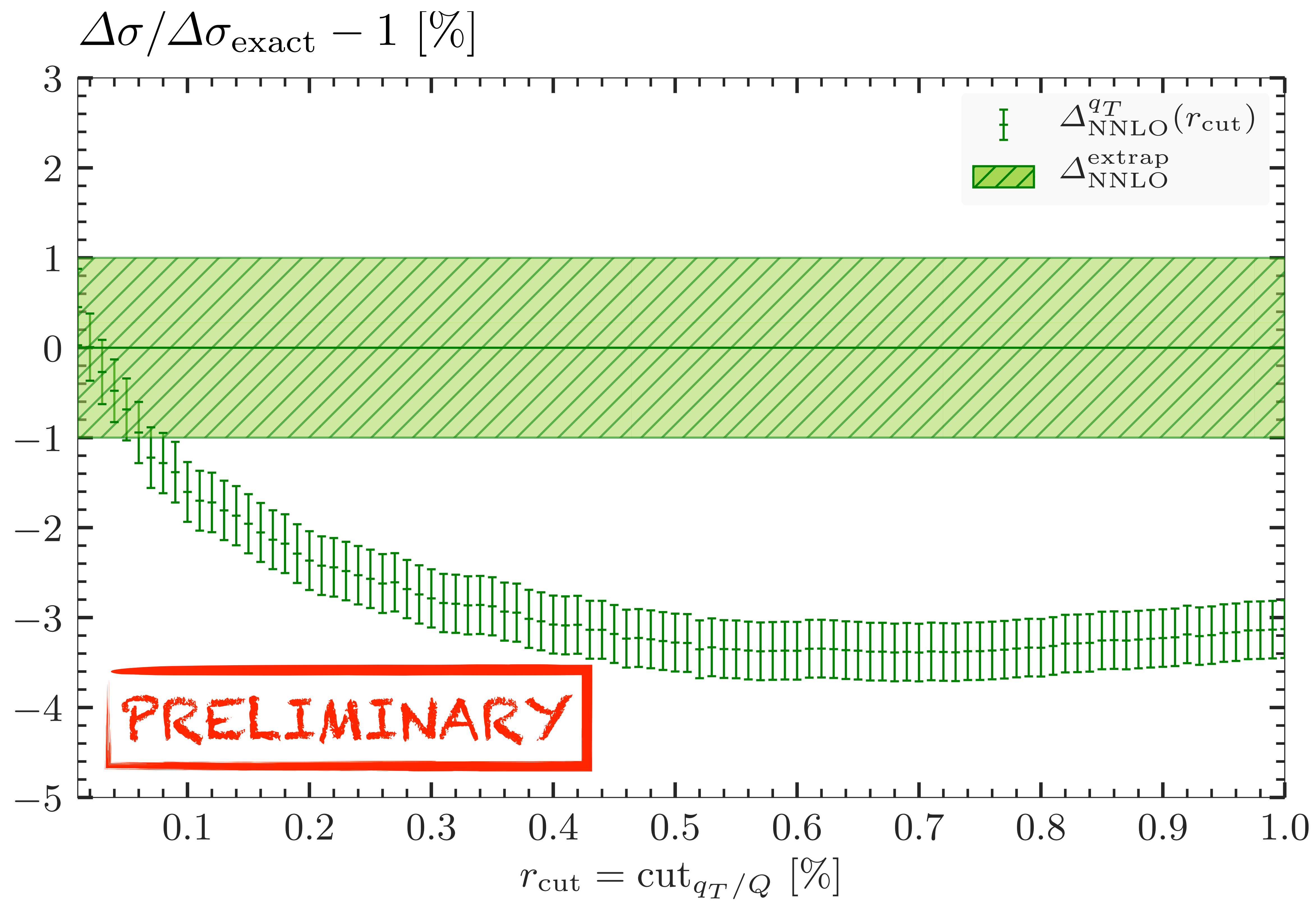
$$H_T = E_T(\ell\nu) + p_T(b_1) + p_T(b_2)$$

$$E_T(\ell\nu) = \sqrt{M^2(\ell\nu) + p_T^2(\ell\nu)}$$

	HPPZ	**This work**
α_s and PDF scheme	5FS	4FS
Jet clustering algortihm	flavour k_T and flavour anti-k_T algorithm (R=0.5)	k_T and anti-k_T algorithm (R=0.5)
pdf sets	`NNPDF31_as_0118` (LO, NLO, NNLO)	`NNPDF30_as_0118_nf_4` (LO) `NNPDF31_as_0118_nf_4` (NLO, NNLO)

Comparison with HPPZ: r_{cut} dependence

$$d\sigma_{N^kLO} = \mathcal{H} \otimes d\sigma_{LO} + \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T/Q > r_{\text{cut}}} + \mathcal{O}(r^{\ell}_{\text{cut}})$$



Behaviour of the power corrections compatible with a **linear scaling** as expected from processes with massive final state

Overall mild power corrections

Control of the NNLO correction at $\mathcal{O}(1\%)$
→ $\mathcal{O}(0.2\%)$ at the level of the total cross section

Comparison with HPPZ: fiducial cross sections

HPPZ

Inclusive $W^+(\to \ell^+\nu)b\bar{b}$ cross sections

Jet algorithm	$\sigma_{\rm LO}$ [fb]	$\sigma_{\rm NLO}$ [fb]	$K_{\rm NLO}$	$\sigma_{\rm NNLO}$ [fb]	$K_{\rm NNLO}$
flavour-$k_{\rm T}$	$213.24(8)^{+21.4\%}_{-16.1\%}$	$362.0(6)^{+13.7\%}_{-11.4\%}$	1.70	$445(5)^{+6.7\%}_{-7.0\%}$	1.23
flavour anti-$k_{\rm T}$ ($a = 0.05$)	$262.52(10)^{+21.4\%}_{-16.1\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	1.91	$690(7)^{+10.9\%}_{-9.7\%}$	1.38
flavour anti-$k_{\rm T}$ ($a = 0.1$)	$262.47(10)^{+21.4\%}_{-16.1\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	1.90	$677(7)^{+10.4\%}_{-9.4\%}$	1.36
flavour anti-$k_{\rm T}$ ($a = 0.2$)	$261.71(10)^{+21.4\%}_{-16.1\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$	1.86	$647(7)^{+9.5\%}_{-8.9\%}$	1.33

PRELIMINARY	LO [fb]	NLO [fb]		NNLO [fb]		
$m_b = 4.92$ GeV	$210.46(2)^{+21.4\%}_{-16.2\%}$	$468.01(5)^{+17.8\%}_{-13.8\%}$	2.22	$627.5(1.5)^{+10.9\%}_{-10.0\%}$	1.34	33K CPU hours
$m_b = 4.2$ GeV	$212.28(2)^{+21.4\%}_{-16.2\%}$	$473.10(5)^{+17.9\%}_{-13.9\%}$	2.23	$641.9(1.5)^{+11.3\%}_{-10.3\%}$	1.36	24K CPU hours

Comparison with HPPZ: Comments

General agreement within scale variations, but 4FS systematically below

Change of scheme @NLO [Cacciari, Nason, Greco, 1998]

1. Use same running coupling and PDF set of the 5FS calculation
2. Add the extra factor (due to the conversion between $\overline{MS}$ and decoupling schemes) : $-\alpha_s \frac{2T_R}{3\pi} \ln \frac{\mu_R^2}{m^2} \sigma_{q\bar{q}}^{\mathrm{LO}}$

   No corrective term for pdfs at this order
3. Take the massless limit $m_b \to 0$

NLO 4FS: 468 fb $\xrightarrow{1,2}$ 481 fb $\xrightarrow{3}$ 493 fb

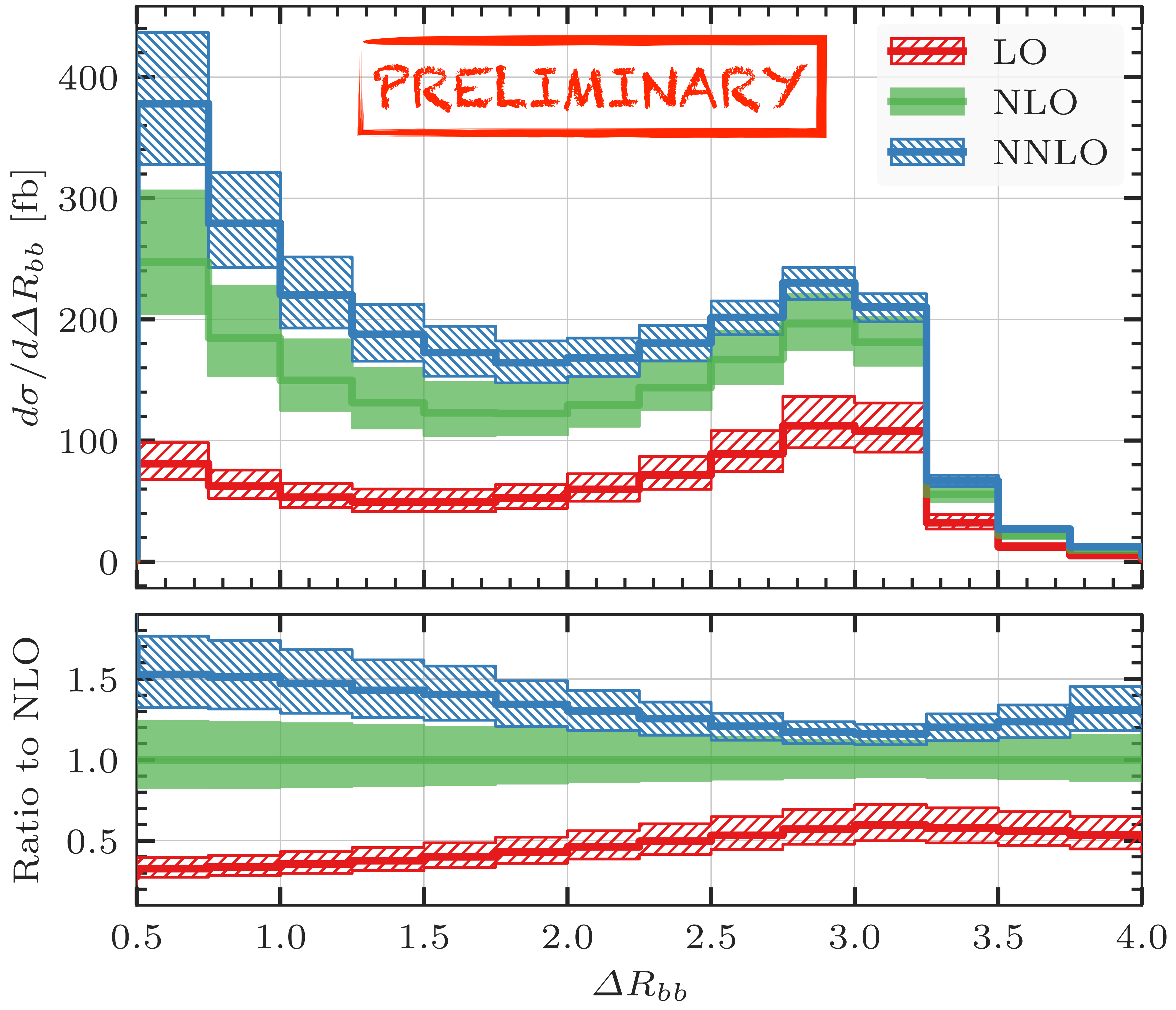
Observations

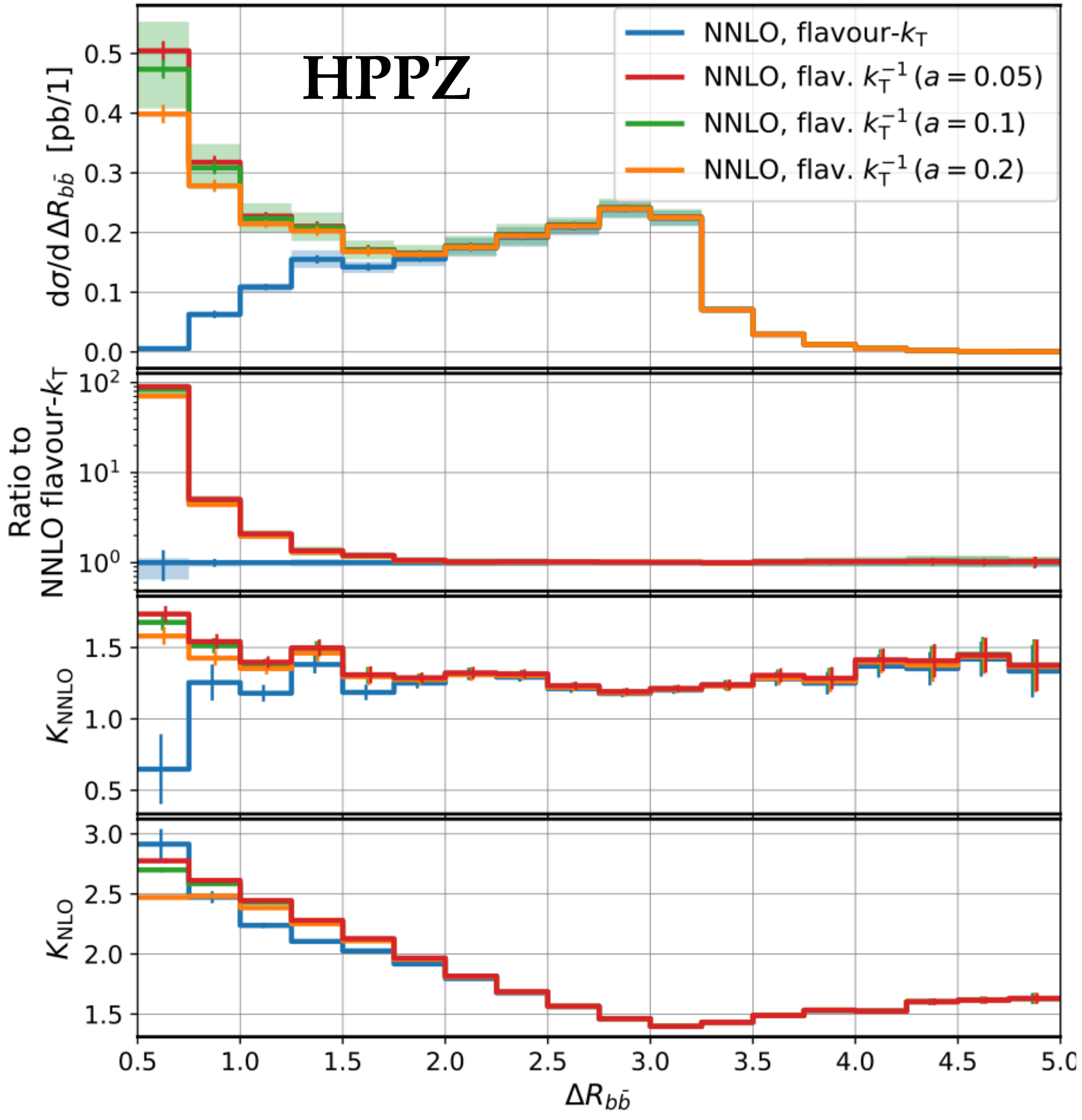
- At NNLO the situation is more delicate: **the massless limit is more problematic and, in principle, ill-defined**
- We have verified that the results obtained with **the standard k_T and anti-k_T algorithms are practically indistinguishable** among each other up to NNLO
- The double real with **4 massive bottom quarks is negligible** (~1 fb)

Comparison with HPPZ: jet clustering algorithms

Flavour k_T favours the clustering of the two bottom quarks in the same leading to the suppression at small ΔR_{bb}

Flavour anti-k_T and standard anti-k_T display a similar behaviour

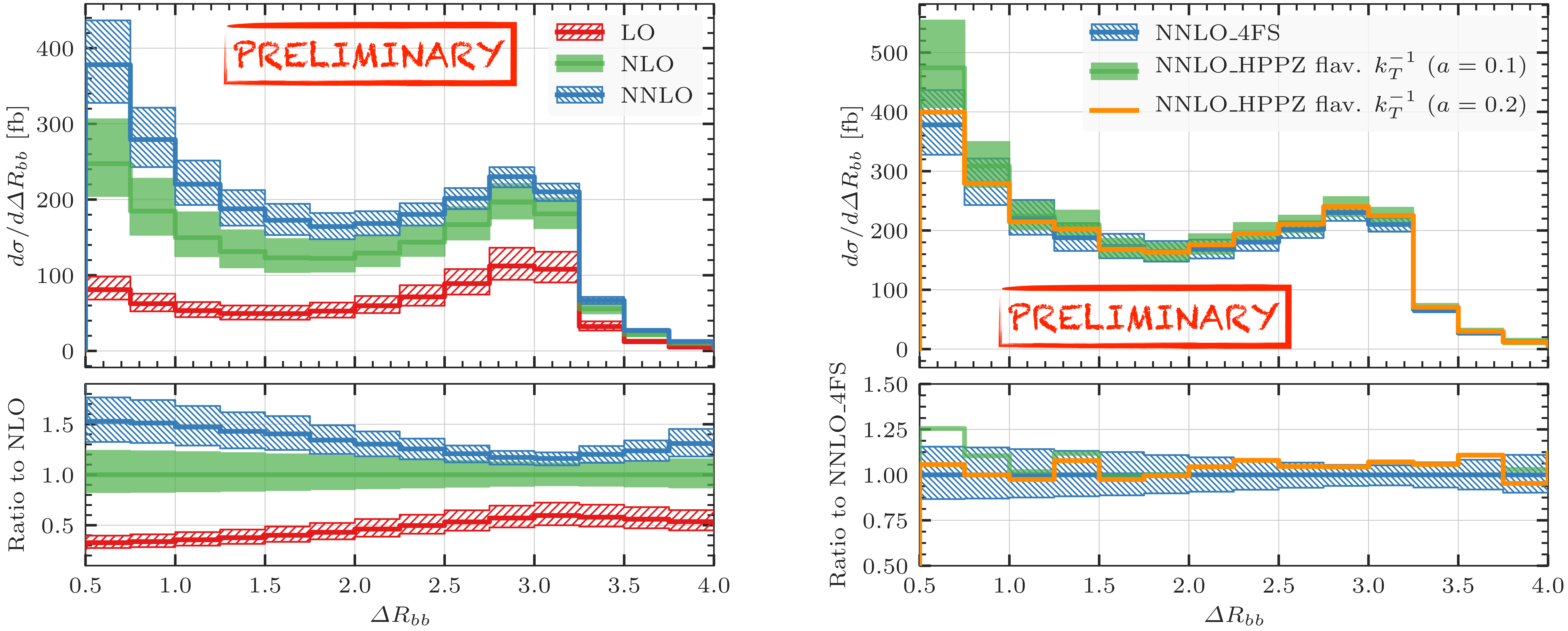




Comparison with HPPZ: jet clustering algorithms

Sizeable NNLO corrections which lead to a steeper slope at small ΔR_{bb} (where the scale uncertainty are larger)

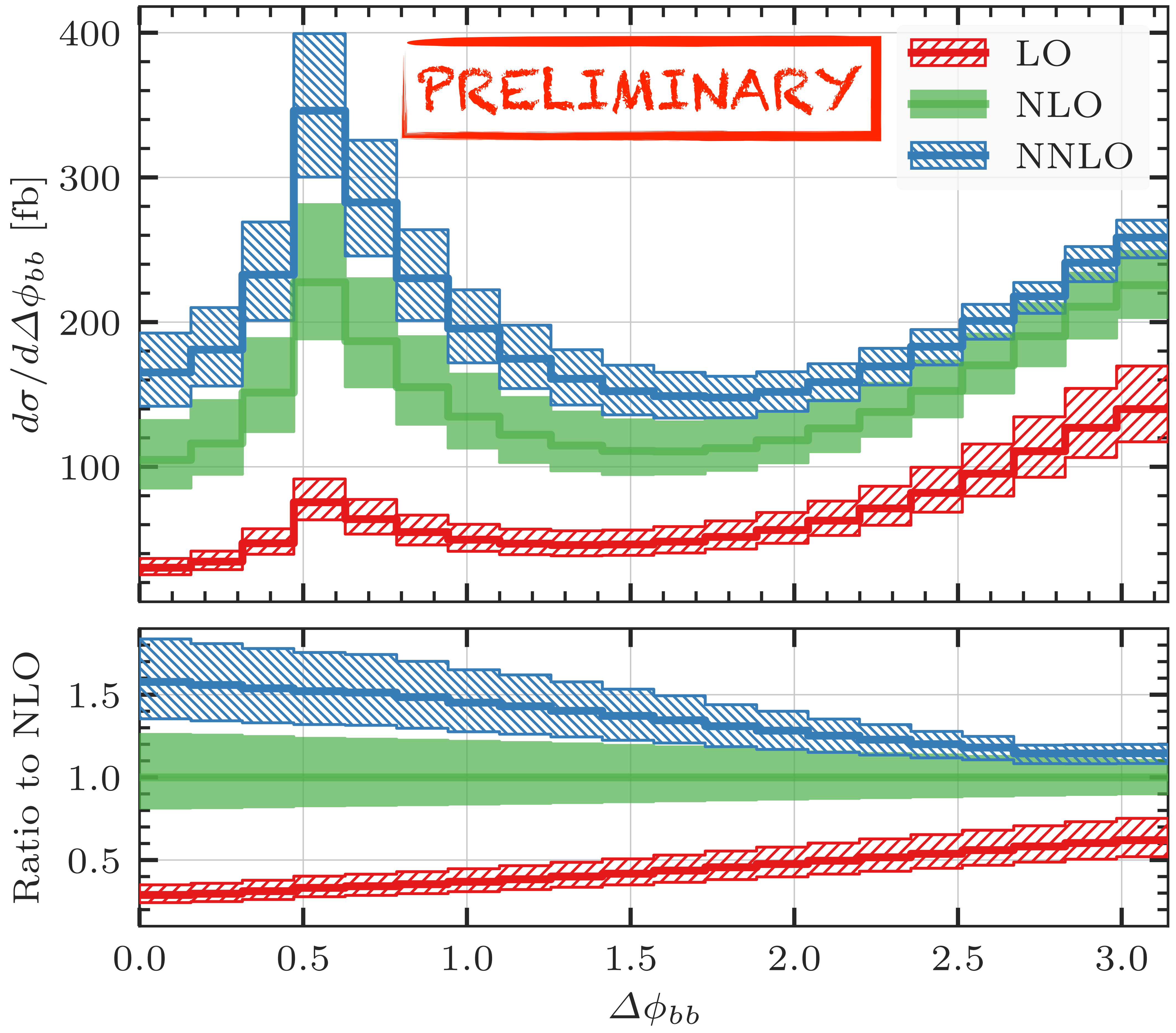
Good agreement between flavour and standard anti-k_T for the largest value $a = 2$

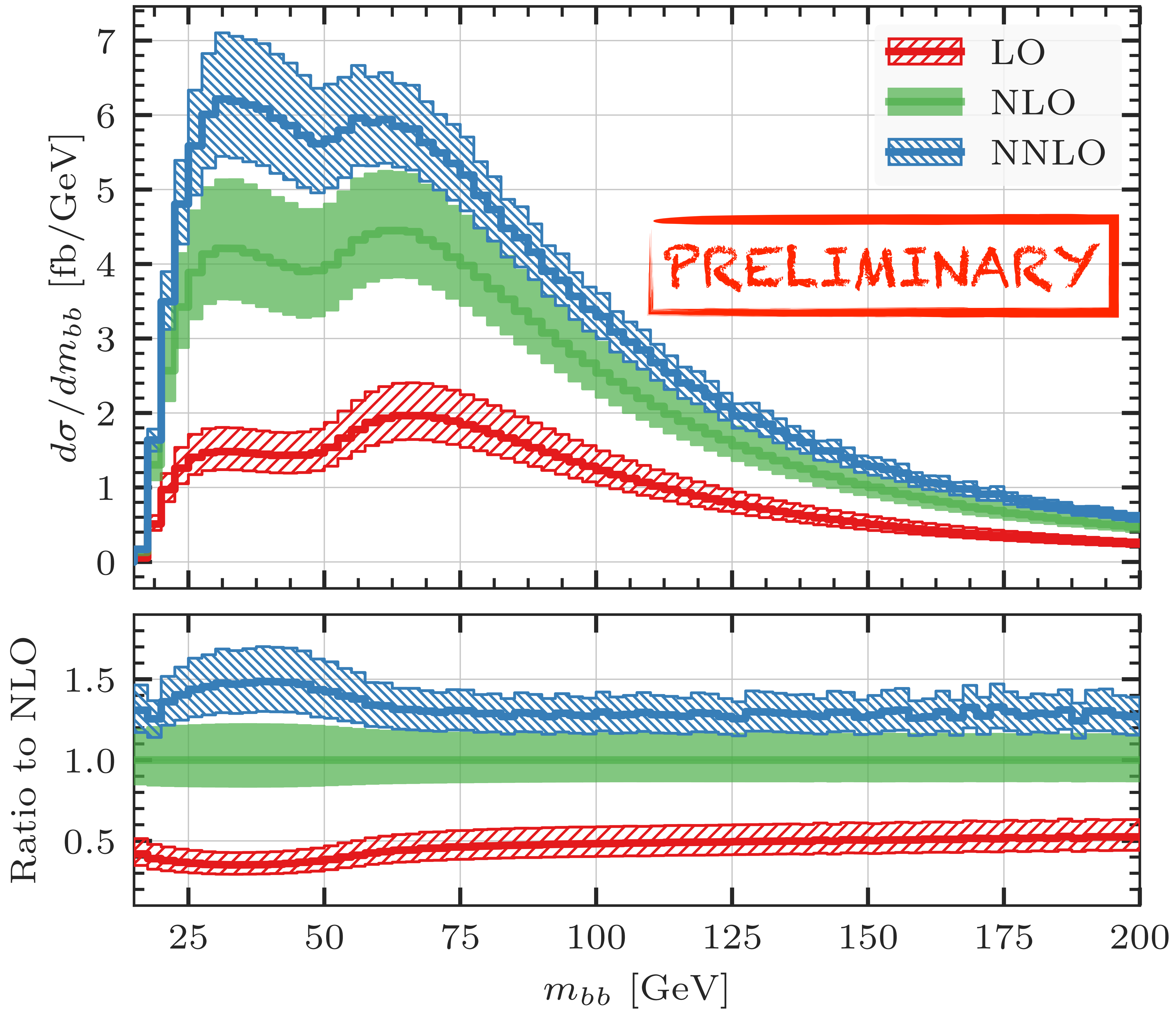


Comparison with HPPZ: additional distributions

Other distributions display similar pattern of the higher-order corrections

The process features two dominant configurations: **gluon splitting** and **t-channel** enhancement (back-to-back bottom quarks and back-to-back leptons)





Outline

- Two-loop Amplitude
- Preliminary results: comparison with HPPZ
- Conclusions

Conclusions

- The calculation of the **soft function for arbitrary kinematics** allows to use the q_T subtraction formalism for the production of a coloured massive final state + a colour singlet system, as ttH and Wbb
- Crucial progress in the calculation of two-loop virtual amplitudes: analytical amplitudes one massive + 4 partons available in the leading colour (non planar topology possibly within the reach in the near future)
- We have report on a **new calculation of Wbb** production in NNLO QCD in the **4FS**
- We rely on the **massification procedure** starting from the corresponding amplitude
- Preliminary results show a qualitative agreement with the massless calculation
- The calculation in the 4FS displays smaller systematic uncertainties associated to the variation of the mass of the bottom quark and it seems to prefer a smaller value of the parameter a of the flavour anti-k_T

Outlooks

- Study of W production in association to a single b (comparison with the combined 4FS+5FS @NLO)
- Matching to parton shower in a NNLO+PS implementation