ttH production at NNLO

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Outline

- Introduction
- Heavy quark production at NNLO
- The extension to $Q\bar{Q}F$ $(F = H, W, Z\ldots)$
- NEW: First results for $t\bar{t}H$
- Summary

Introduction

The current and expected precision of LHC data requires NNLO accurate QCD predictions for the most relevant processes

NNLO results now available for essentially all the relevant $2 \to 1$ and $2 \to 2$ processes and lead to an improved description of the data for many benchmark processes

The q_T subtraction formalism has been widely used to obtain NNLO predictions for the production of many colourless systems and heavy-quark production

These computations have been implemented in the parton level event generator MATRIX

Kallweit, Wiesemann, MG (2017)

The eventual goal is to make NNLO predictions to these processes completely available to the community

The method

Catani, MG (2007)

Consider the hard-scattering process $pp \to F + X$

F colourless system (vector, Higgs boson(s)...) of invariant mass M and transverse momentum q_T

Use a dimensionless resolution variable $r > r_{\rm cut}$ (e.g. $r = q_T/M$)

Real contribution with one additional resolved jet, divergent as $r_{\rm cut} \to 0$

Subtraction counterterm that cancels the $r_{\rm cut} \to 0$ singularity

$$d\sigma^{F+X}_{NNLO} = \mathscr{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\rm jets}_{NLO} - d\sigma^{CT,F}_{NNLO} \right] + \mathcal{O}(r^p_{\rm cut})$$

Virtual after subtraction of IR singularities + collinear contributions

Power suppressed contribution whose size determines the efficiency of the computation

Structure of $\mathscr{H}^F$ and $d\sigma^{CT,F}$ can be obtained from all-order resummation: now available at N^{3}LO

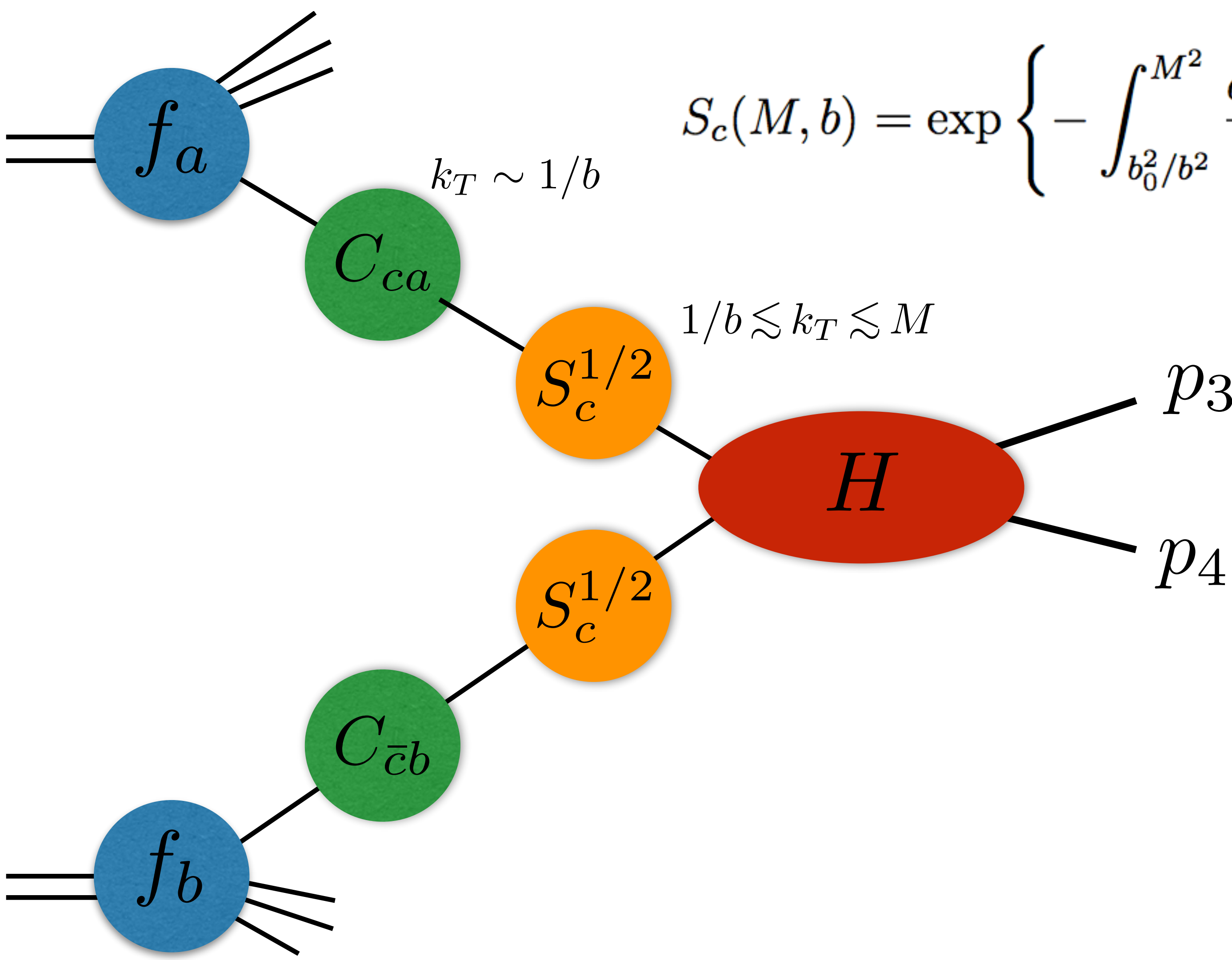
The resummation formula

J.Collins, D.Soper, G.Sterman (1984)
S.Catani, D. de Florian, MG (2000); S.Catani, MG (2010)

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\;dM^2\;dy\;d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma_{c\bar{c},F}^{(0)}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\;\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

$$S_c(M,b)=\exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\;\ln\frac{M^2}{q^2}+B_c(\alpha_S(q^2))\right]\right\}$$



C coefficients embody collinear radiation at scale 1/b

S_c embodies soft and flavour conserving collinear radiation in the region $1/b < k_T < M$

H^F includes hard radiation at scales $k_T \sim M$

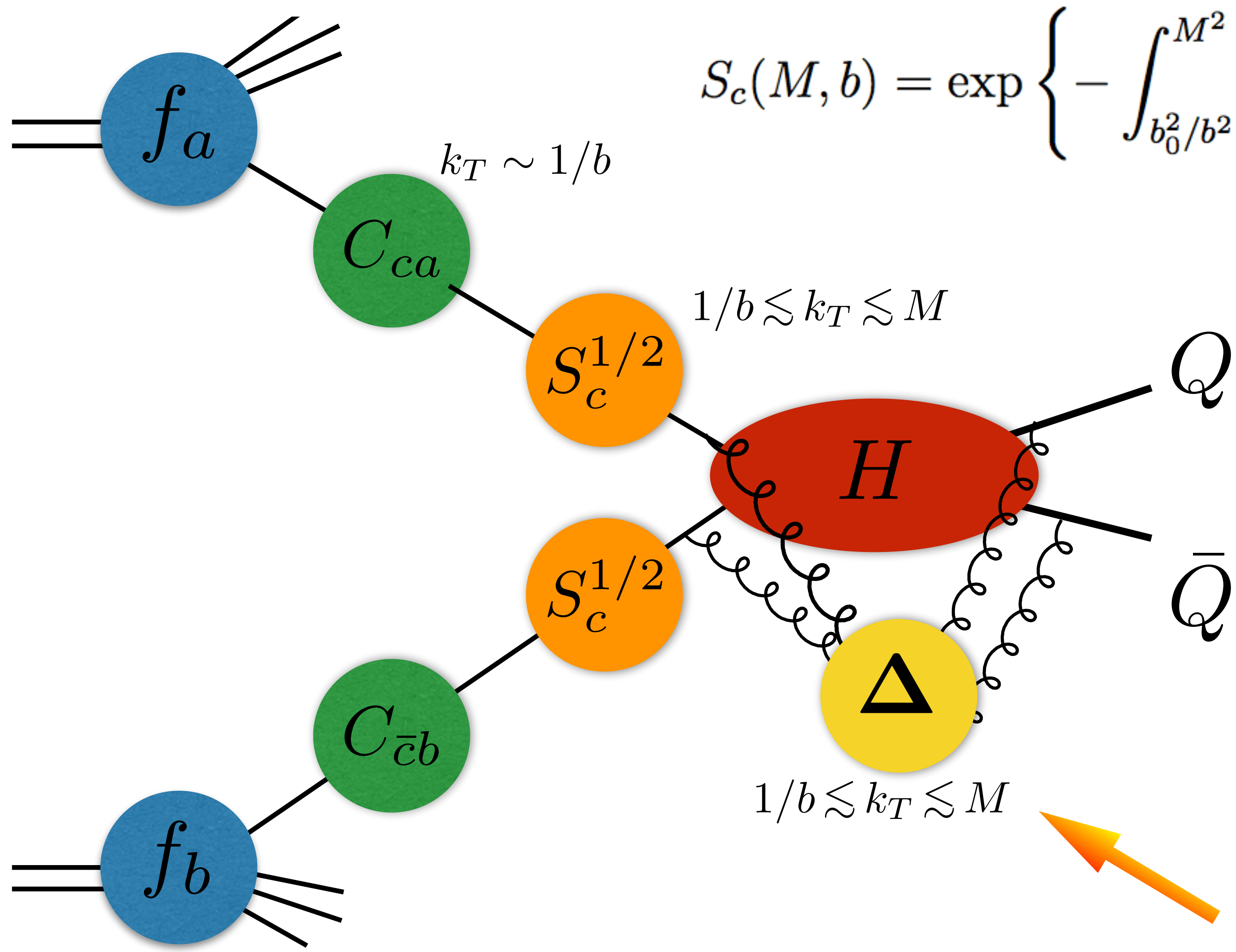
Extension to heavy-quark production

Catani, Torre, MG (2014)

$$\frac{d\sigma^{(\mathrm{sing})}(P_1,P_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\;dM^2\;dy\;d\mathbf{\Omega}}=\frac{M^2}{2P_1\cdot P_2}\sum_{c=q,\bar q,g}\left[d\sigma^{(0)}_{c\bar c}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\;\left[(\mathbf{H}\,\mathbf{\Delta})\,C_1C_2\right]_{c\bar c;a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

$$S_c(M,b)=\exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\;\ln\frac{M^2}{q^2}+B_c(\alpha_S(q^2))\right]\right\}$$



C coefficients embody collinear radiation at scale 1/b

S_c embodies soft and flavour conserving collinear radiation in the region $1/b < k_T < M$

H^F includes hard radiation at scales $k_T \sim M$

Additional radiative factor of purely soft origin (starts to contribute at NLL)

Extension to heavy-quark production

➡ We obtain an analogous structure for the subtraction formula (qq and gg channels contribute at the same order) with some differences

$$d\sigma^{Q\bar{Q}}_{(N)NLO} = \mathscr{H}^{Q\bar{Q}}_{(N)NLO} \otimes d\sigma^{Q\bar{Q}}_{LO} + \left[d\sigma^{Q\bar{Q}+\text{jet}}_{(N)LO} - d\sigma^{CT}_{(N)NLO} \right]$$

✅ Modified subtraction counterterm fully known

Additional perturbative ingredient: soft anomalous dimension Γ_t (known to NNLO) and related to IR singular structure of virtual amplitudes

Mitov, Sterman, Sung (2009)
Neubert et al (2009)

➡ This is enough to compute NNLO corrections in all the off-diagonal channels

Bonciani, Catani, Torre, Sargsyan, MG (2015)

Extension to heavy-quark production

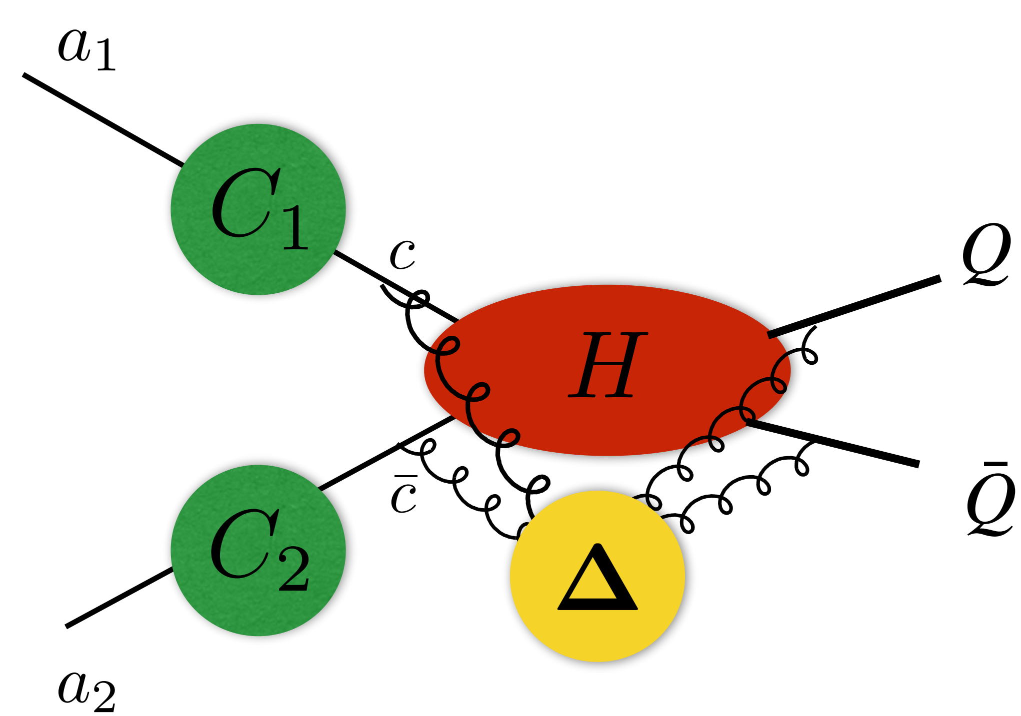
✅ Structure of hard collinear function is analogous

$$\mathscr{H}^{Q\bar{Q}} = \left[H^{Q\bar{Q}} C_1 C_2 \right]_{c\bar{c};a_1 a_2}$$

but now $H^{Q\bar{Q}} \sim \langle \tilde{\mathcal{M}} | \mathbf{\Delta} | \tilde{\mathcal{M}} \rangle$

Additional soft contributions

IR-subtracted virtual amplitude



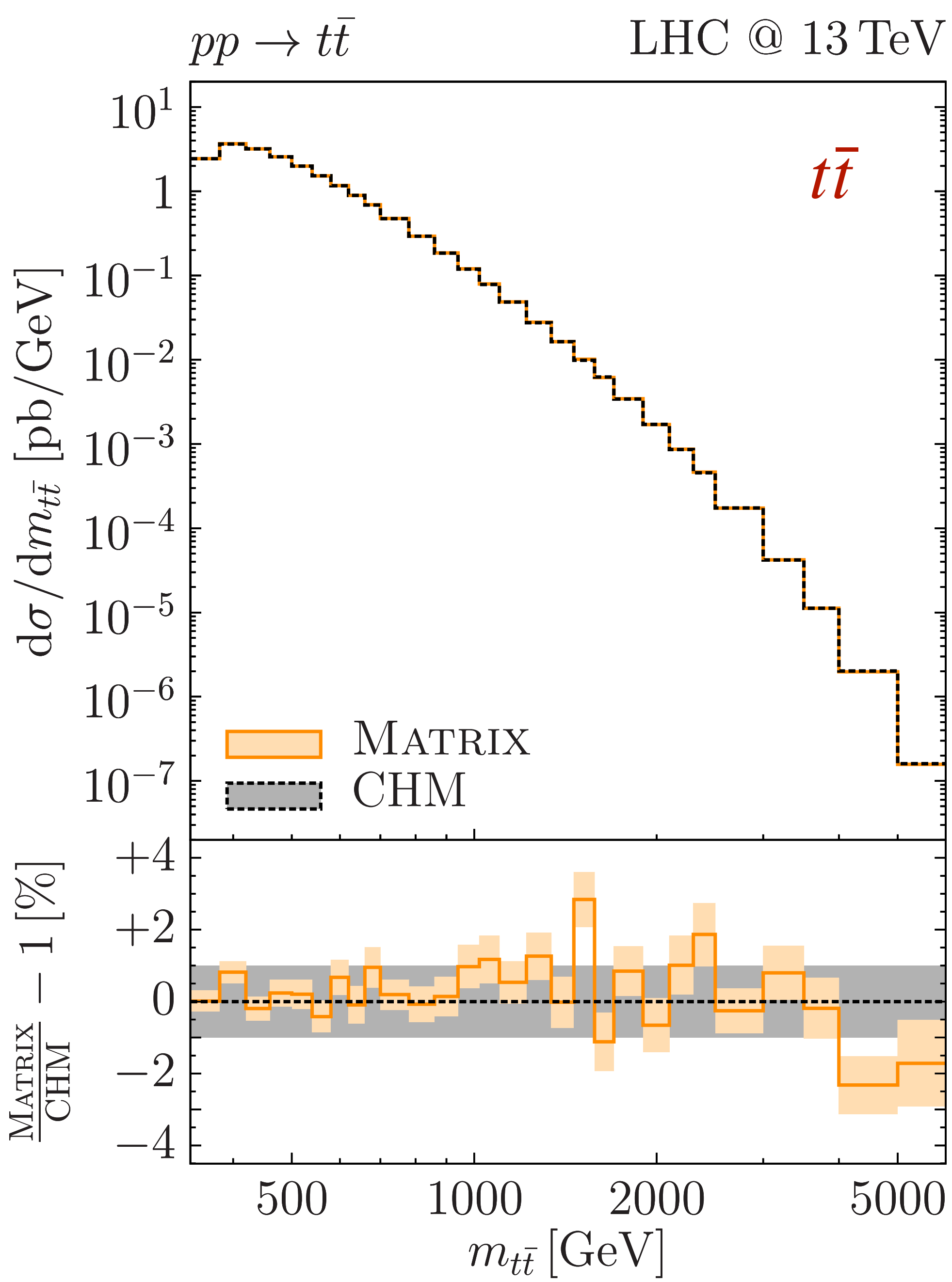
✅ The soft contributions have been computed by integrating a suitably subtracted soft current

Catani, Devoto, Mazzitelli, MG (to appear)

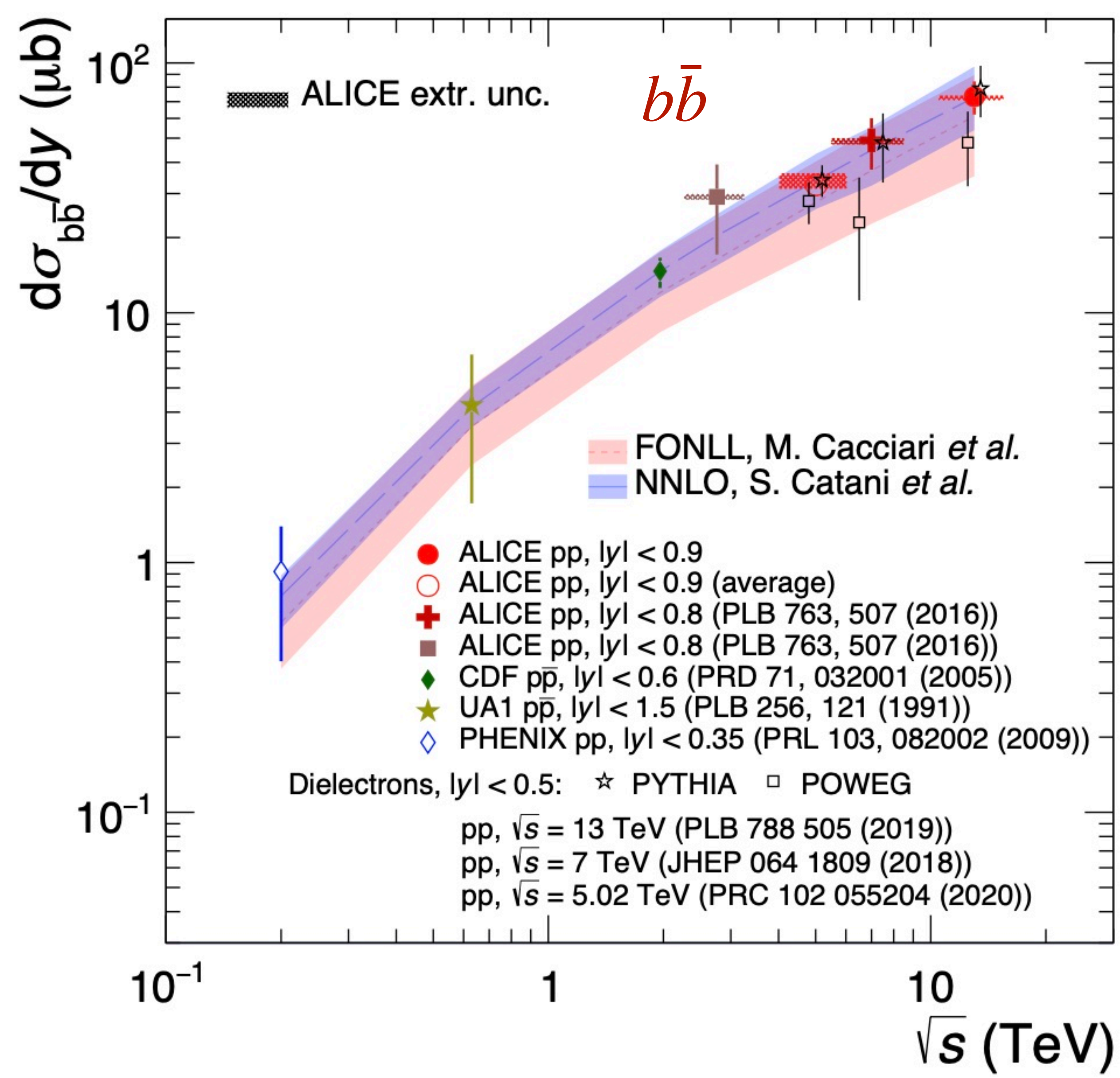
$t\bar{t}$ and $b\bar{b}$ production at NNLO

Catani, Devoto, Kallweit,Mazzitelli, MG (2019, 2020)

Tree and loop amplitudes from Openloops (cross checked with Recola)



Two-loop amplitudes from Czakon et al (2013)



Now publicly available in MATRIX v2.1beta

What else ?

We are working in two main directions:

- **Extension to heavy-quark + colourless (QQF)**

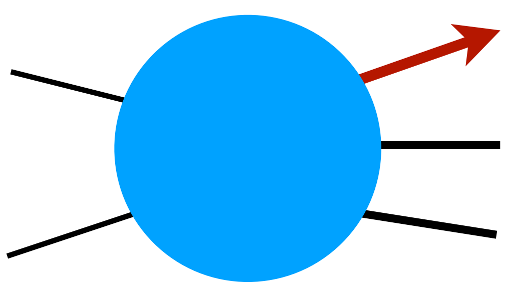
  → ttH (this talk)

  → Wbb (talk by Luca Buonocore)

- **Extension of our method to include jet final states**

  → talk by Luca Rottoli

QQF



When the heavy quark pair is accompanied by a colourless system the resummation and subtraction formalisms can be applied in an analogous way with just two additional complications

Catani, Fabre, Kallweit, MG (2020)

- The colourless system takes away momentum and the computation of the additional soft contributions has to be extended accordingly

Devoto, Mazzitelli (in preparation)

- For some important processes ($t\bar{t}Z$, $WWb\bar{b}$....) three-parton correlators are non vanishing and also contribute to the soft integrals

This is not the case for $t\bar{t}$ and $t\bar{t}H$

See eg Forshaw, Seymour and Siodmok (2012)
Czakon and Fiedler (2014)

➡ **First NNLO results for $t\bar{t}H$**

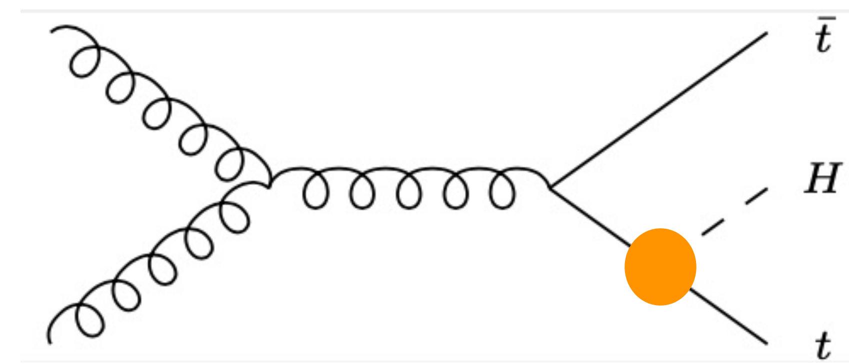
Catani, Devoto, Kallweit,Mazzitelli, Savoini, MG (2022)

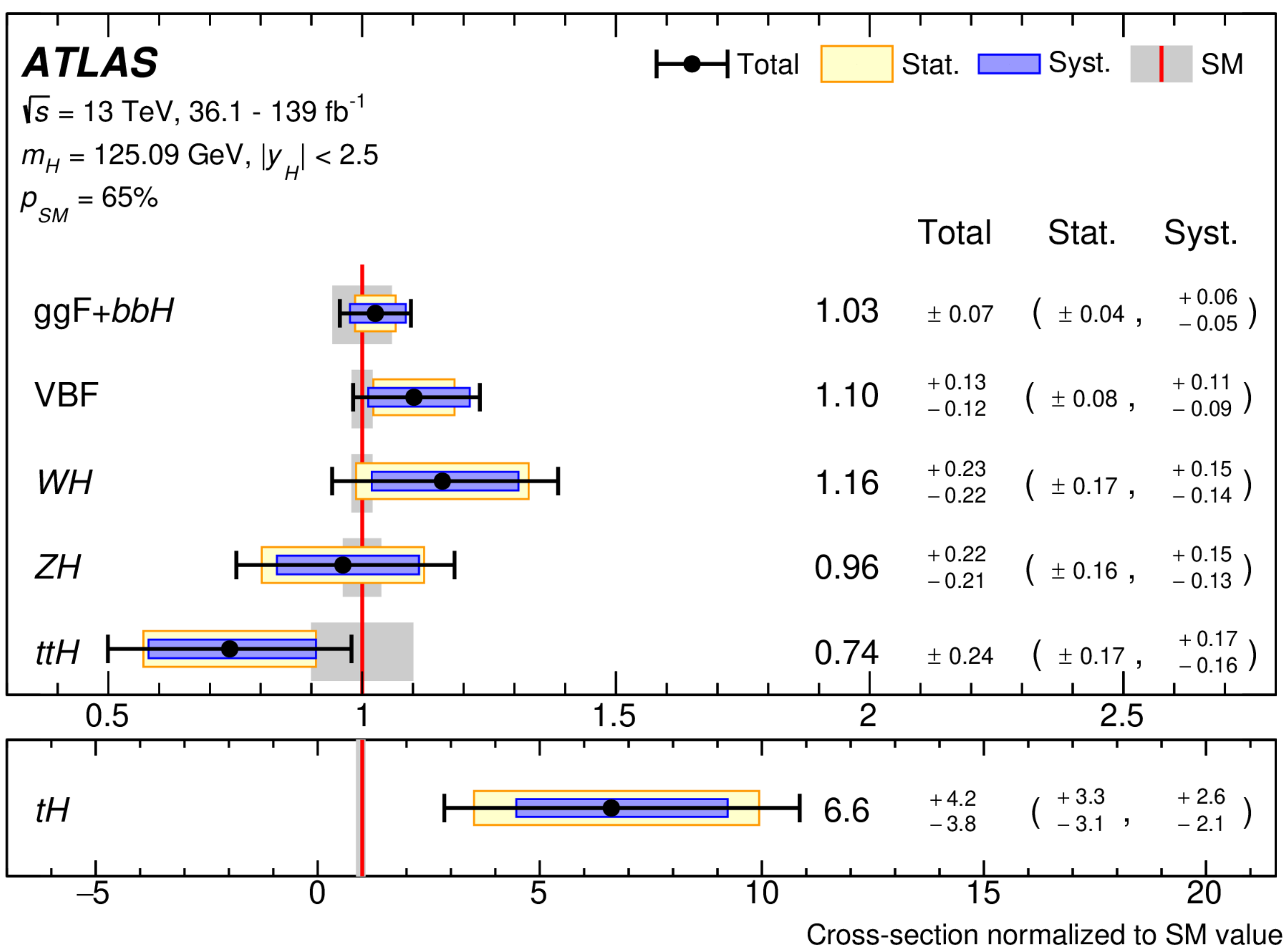
ttH

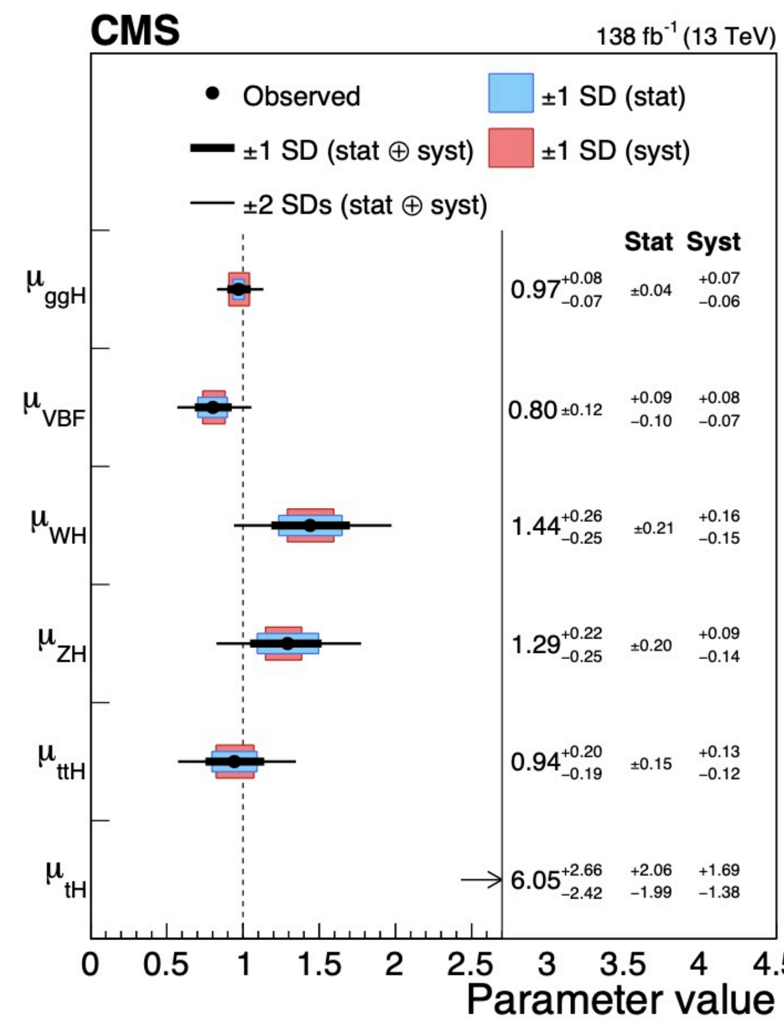
The associated production of the Higgs boson with a top-quark pair is a crucial process at the LHC

It allows a direct extraction of the top Yukawa

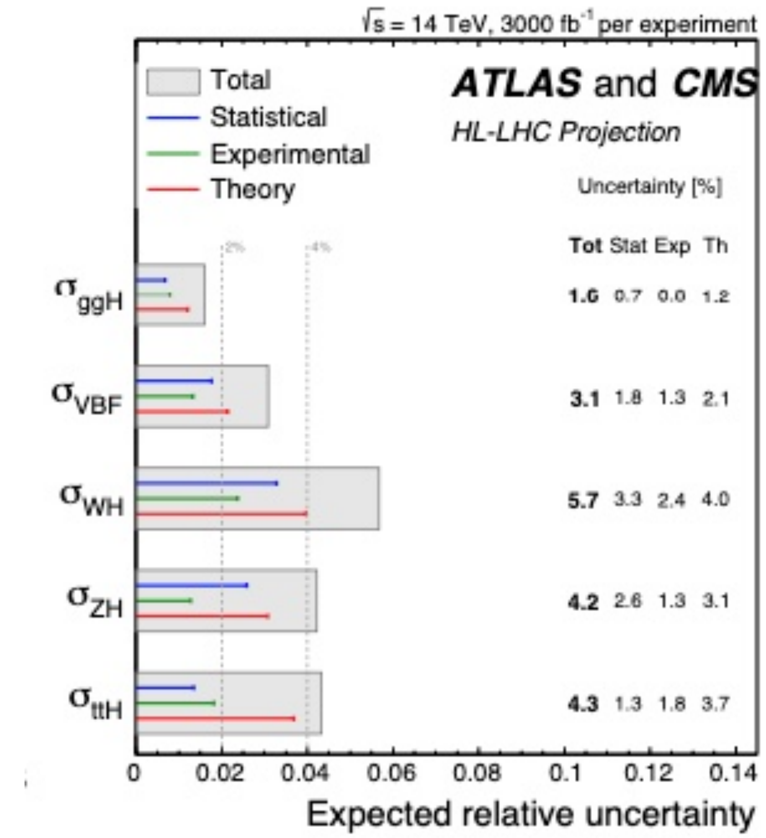
Experimental uncertainties are now at the $\mathcal{O}(20\%)$ level







ttH



Experimental precision expected to get to the $\mathcal{O}(2\%)$ level at the end of HL-LHC

Current predictions based on NLO QCD+EW (+ resummations) and affected by $\mathcal{O}(10\%)$ uncertainty

NNLO QCD needed to bring theory uncertainty down to the $\mathcal{O}(2\%)$ level expected

First step completed by evaluation of the contribution of the off-diagonal partonic channels

Catani, Fabre,Kallweit, MG(2020)

Missing ingredients are the two-loop $gg \to t\bar{t}H$ and $q\bar{q} \to t\bar{t}H$ amplitudes

Massive $2 \to 3$ amplitudes: at the frontier of current techniques (new classes of functions, really charting a new territory…)

The idea: use an approximation for the missing two-loop amplitude

Soft-Higgs radiation

When soft gluons (or soft-photons) are emitted in a high-energy QCD (or QED) process the corresponding amplitudes obey well known factorisation formulae

$$\mathcal{M}(\{p_i\},k) \simeq J(k)\mathcal{M}(\{p_i\})$$

Example: QCD

Bassetto, Ciafaloni, Marchesini (1983)

$$J(k) = g_S \mu^\epsilon \left(J^{(0)}(k) + g_S^2 J^{(1)}(k) + \ldots \right)$$

purely non-abelian

Del Duca et al (1999)
Catani, MG (2000)
Duhr, Gehrmann (2013)
…………..

An analogous formula holds for the emission of a soft scalar off heavy quarks

$$\mathcal{M}(\{p_i\},k) \simeq J(k)\mathcal{M}(\{p_i\})$$

At tree level it is straightforward to show that

heavy-quark mass

$$J(k) = \sum_i \frac{m}{v}\frac{m}{p_i \cdot k}$$

heavy-quark momenta

Soft-Higgs radiation

This formula can be extended to all orders in the QCD coupling α_S

$$\mathcal{M}(\{p_i\},k) \simeq F(\alpha_S(\mu_R);m/\mu_R)\,J(k)\mathcal{M}(\{p_i\})$$

Physical picture: Higgs soft current essentially "abelian": no corrections beyond LO except for over all normalisation

The perturbative function $F(\alpha_S(\mu_R);m/\mu_R)$ can be extracted from the soft limit of the scalar form factor of the heavy quark

Bernreuther et al (2005)
Blümlein et al (2017)

$$F(\alpha_S(\mu_R);m/\mu_R) = 1 + \frac{\alpha_S(\mu_R)}{2\pi}(-3C_F) + \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2\left(\frac{33}{4}C_F^2 - \frac{185}{12}C_FC_A + \frac{13}{6}C_F(n_L+1) - 6C_F\beta_0\ln\frac{\mu_R^2}{m^2}\right) + \mathcal{O}(\alpha_S^3)$$

Alternatively, it can be derived by using Higgs low-energy theorems

See e.g. Kniehl and Spira (1995)

Soft-Higgs radiation

The basic observation is that at the bare amplitude level we have

$$\lim_{k\to 0} \mathscr{M}^{\text{bare}}(\{p_i\},k) = \frac{m_0}{v}\sum_i \frac{m_0}{p_i\cdot k}\mathscr{M}^{\text{bare}}(\{p_i\})$$

The renormalisation of the heavy-quark mass and wave-function induce a modification of the Higgs coupling to the heavy quark

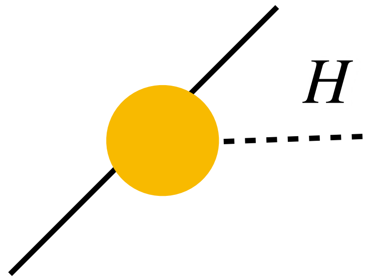
The bare amplitude for the soft-scalar emission is

$$\lim_{k\to 0} \mathscr{M}^{\text{bare}}_{Q\to QH}(p,k) = \frac{1}{v} m_0 \frac{\partial}{\partial m_0}\mathscr{M}^{\text{bare}}_{Q\to Q}(p)\bigg|_{p^2=m^2}$$

By using the results of the $\mathcal{O}(\alpha_S^2)$ contribution to the heavy-quark self energy and carrying out the wave function and mass renormalisation we recover the function $F(\alpha_S(\mu_R); m/\mu_R)$ discussed before

Broadhurst, Gray, Schilcher (1991)
Gray, Broadhurst, Grafe, Schilcher (1990)

Note that intermediate results are gauge dependent: gauge invariance recovered only in the final on-shell limit



Soft-Higgs radiation

We have done several checks of our factorisation formula by assuming a very light and soft Higgs boson

$$\mathcal{M}(\{p_i\}, k) \simeq F(\alpha_S(\mu_R); m/\mu_R)\, J(k) \mathcal{M}(\{p_i\})$$

- We have tested it numerically with Openloops up to one-loop order in the case of $t\bar{t}H$ production ✅
- We have tested it numerically with Recola up to one-loop order in the case of $t\bar{t}t\bar{t}H$ production ✅

The formula can be useful to cross check future exact calculations of QCD amplitudes with heavy quarks and a Higgs boson

Can it be used to complete the NNLO calculation for $t\bar{t}H$ production ?

Remarkably, yes !

Differences with other approaches

The idea of a treating the Higgs as a parton radiating off the top quark was used already in the past

Effective Higgs approximation in early NLO calculations: introduce a function expressing the probability to extract the Higgs boson from the top quark

Dawson and Reina (1997)

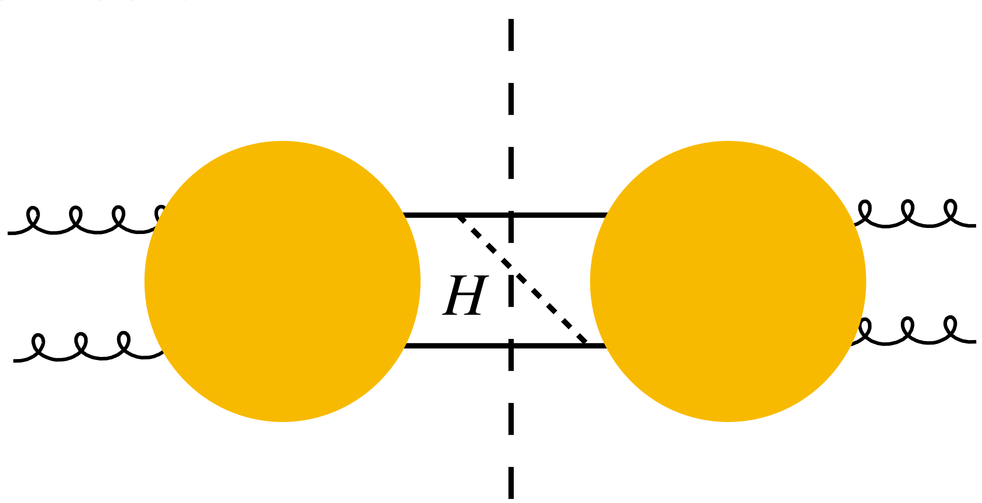
Fragmentation functions $D_{t\to H}$ and $D_{g\to H}$ evaluated at NLO

Brancaccio, Czakon, Gerenet, Krämer (2021)

These approaches are based on a collinear approximation

Our approximation is purely soft (collinear non-soft emissions are neglected but soft quantum interferences are included)

Moreover, we apply it only to the finite part of the two-loop contribution



The computation

The starting point is again the q_T subtraction formula

$$d\sigma = \mathcal{H} \otimes d\sigma_{\rm LO} + \left[d\sigma_{\rm R} - d\sigma_{\rm CT}\right]$$

All the ingredients in this formula for $t\bar{t}H$ are now available and implemented in MATRIX except the two-loop virtual amplitudes entering $\mathcal{H}$

We define

$$\mathcal{H} = H\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H} \qquad H^{(n)} = \frac{2\mathrm{Re}\left(\mathcal{M}_{\rm fin}^{(n)}\mathcal{M}^{(0)*}\right)}{|\mathcal{M}^{(0)}|^2}$$

with

$$H = 1 + \frac{\alpha_S(\mu_R)}{2\pi}H^{(1)} + \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2 H^{(2)} + \ldots . \qquad |\mathcal{M}_{\rm fin}(\mu_{IR})\rangle = \mathbf{Z}^{-1}(\mu_{IR})\,|\mathcal{M}\rangle$$

IR subtraction

For $n=2$ this definition allows us to single out the only missing ingredient in the NNLO calculation, that is, the coefficient $H^{(2)}$

Note that all the remaining terms are computed exactly (including $|\mathcal{M}_{\rm fin}^{(1)}|^2$)

We have used our factorisation formula to construct approximations of the $H^{(1)}$ and $H^{(2)}$ coefficients

Since the Higgs is not at all soft, in order to use the factorisation formula we have to introduce a mapping that from a $t\bar{t}H$ event defines a $t\bar{t}$ event with no Higgs boson

To this purpose we use the q_T recoil prescription

Catani, Ferrera, de Florian, MG (2016)

With this prescription the momentum of the Higgs boson is equally reabsorbed by the initial state partons, leaving the top and antitop momenta unchanged

The required tree-level and one-loop amplitudes are obtained using Openloops

The $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ two-loop amplitudes needed to apply our approximation are those provided by Czakon et al.

Bärnreuther, Czakon, Fiedler (2013)

Setup: NNPDF31 NNLO partons with 3-loop α_S

$m_H = 125\,\mathrm{GeV}$ and $m_t = 173.3\,\mathrm{GeV}$

Central values for factorisation and renormalisation scales

$\mu_F = \mu_R = (2m_t + m_H)/2$

Our first check is on the LO cross sections: we find that the soft approximation overestimates it by

- gg channel: a factor of 2.3 at $\sqrt{s} = 13\,\text{TeV}$ and a factor of 2 at $\sqrt{s} = 100\,\text{TeV}$
- $q\bar{q}$ channel: a factor of 1.11 at $\sqrt{s} = 13\,\text{TeV}$ and a factor of 1.06 at $\sqrt{s} = 100\,\text{TeV}$

These are absolute LO predictions: in our calculation we will actually need to approximate $H^{(1)}$ and $H^{(2)}$ that are normalised to LO matrix elements

$$H^{(n)} = \frac{2\text{Re}\left(\mathcal{M}^{(n)}_{\text{fin}}\mathcal{M}^{0)*}\right)}{|\,\mathcal{M}^{(0)}|^2}$$

We expect this approximation to work better than simply computing $2\text{Re}\left(\mathcal{M}^{(n)}_{\text{fin}}\mathcal{M}^{(0)*}\right)$: effective reweighing of LO cross section

When computing virtual amplitudes we will set the infrared subtraction scale μ_{IR} to the invariant mass of the final state system

	$\sqrt{s} = 13\,\text{TeV}$		$\sqrt{s} = 100\,\text{TeV}$	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO,H}}\vert_{\text{soft}}$	61.98	7.413	5612	206.0

We now move to NLO and compare the exact contribution from $H^{(1)}$ to the one computed in the soft approximation

The hard contribution computed in the soft approximation is underestimated by just 30 % in the gg channel and by 5 % in the $q\bar{q}$

The mismatch that we observe at NLO can be used to estimate the uncertainty of our approximation at NNLO

The quality of our final result will depend on the size of the contribution we approximate

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
$\sigma_{\rm LO}$	261.58	129.47	23055	2323.7
$\Delta\sigma_{\rm NLO,H}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\rm NLO,H}\|_{\rm soft}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\rm NNLO,H}\|_{\rm soft}$	$-2.980(3)$	$2.622(0)$	$-239.4(4)$	$65.45(1)$

At NNLO the hard contribution is about 1% of the LO cross section in the gg channel and 2% in the $q\bar{q}$ channel

We can therefore anticipate that at NNLO the uncertainties due to the soft approximation will be rather small.

But how can we estimate these uncertainties ?

We have carefully studied the stability of our results under variations of the approximation procedure

- We have varied the recoil procedure: reabsorbing the Higgs momentum in just one of the initial state partons leads to negligible differences

- We have repeated our computation by using different subtraction scales at which the finite part of the two-loop virtual amplitude in $H^{(2)}$ is defined

  When varying μ_{IR} from $M/2$ to $2M$ and adding the exact evolution terms from these scales back to M

  - In the gg channel we find ${}^{+164\%}_{-25\%}$ at 13 TeV and ${}^{+142\%}_{-20\%}$ at 100 TeV

  - In the $q\bar{q}$ channel we find ${}^{+4\%}_{-0\%}$ at 13 TeV and ${}^{+3\%}_{-0\%}$ at 100 TeV

To define our uncertainties we start from the NLO result: the hard contribution computed in the soft approximation is underestimated by just 30% in the gg channel and by 5% in the $q\bar{q}$ therefore the NNLO uncertainty cannot be smaller than these values

We multiply these uncertainties by a tolerance factor of 3

We finally combine the gg and $q\bar{q}$ uncertainties linearly ➡ $\pm 0.6\,\%$ on σ_{NNLO}

Results

σ [pb]	$\sqrt{s} = 13\,\mathrm{TeV}$	$\sqrt{s} = 100\,\mathrm{TeV}$
σ_{LO}	$0.3910\,^{+31.3\%}_{-22.2\%}$	$25.38\,^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875\,^{+5.6\%}_{-9.1\%}$	$36.43\,^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)\,^{+0.1\%}_{-2.2\%}$

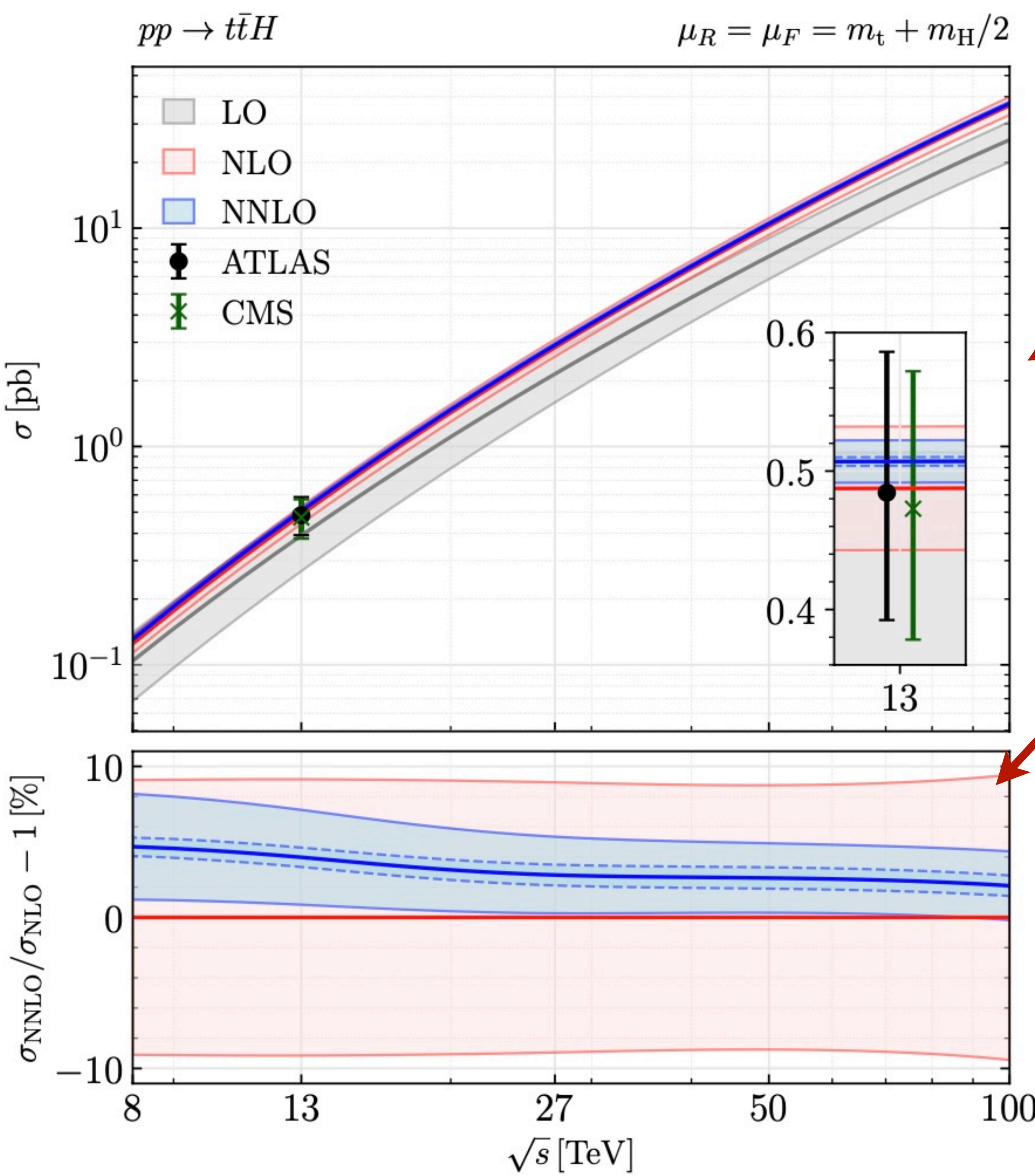
NLO effect is about +25 % at 13 TeV and +44 % at 100 TeV

NNLO effect is about +4 % at 13 TeV and +2 % at 100 TeV

Significant reduction of perturbative uncertainties

Errors in bracket obtained combining uncertainty from the soft approximation and the q_T subtraction systematics (same procedure used in MATRIX)

Results



ATLAS and CMS results from Nature 2022 papers

Perturbative uncertainties estimated by symmetrising the standard 7-point scale variation

Dashed band: residual error from soft approx+systematics

Note that: sensible comparison with data should eventually be done including NLO EW corrections ($+1.7\,\%$ at $\sqrt{s} = 13\,\text{TeV}$)

Summary

- The current and expected precision of LHC data requires NNLO accurate QCD predictions for the most relevant processes
- NNLO results now available for essentially all the relevant $2 \to 1$ and $2 \to 2$ processes and lead to an improved description of the data for many benchmark processes
- The q_T subtraction formalism has been widely used to obtain NNLO predictions for the production of many colourless systems and heavy-quark production
- The production of a heavy-quark pair and a colourless system can be attacked by addressing some additional complications, namely the evaluation of the soft contributions and (in some cases) the computation of three-parton color correlated contributions
- Associated production of the Higgs boson with a top-quark pair is one of the crucial $2 \to 3$ processes in this class

- We have now applied our framework to evaluate NNLO corrections to $t\bar{t}H$
- We have computed the last missing ingredient in the NNLO calculation by using a soft-Higgs approximation
- The approximation is based on a soft-Higgs factorisation formula that has been presented, for the first time, to NNLO accuracy
- Our formula will provide a strong check of future multi-loop computations of amplitudes involving heavy-quarks and a Higgs boson
- This is the first computation for a $2 \to 3$ process with massive coloured particles at this perturbative order
- NNLO corrections are moderate and lead to a significant reduction of perturbative uncertainties

Backup

Stability of the subtraction procedure

$$d\sigma^{F}_{(N)NLO} = \mathcal{H}^{F}_{(N)NLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+\mathrm{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

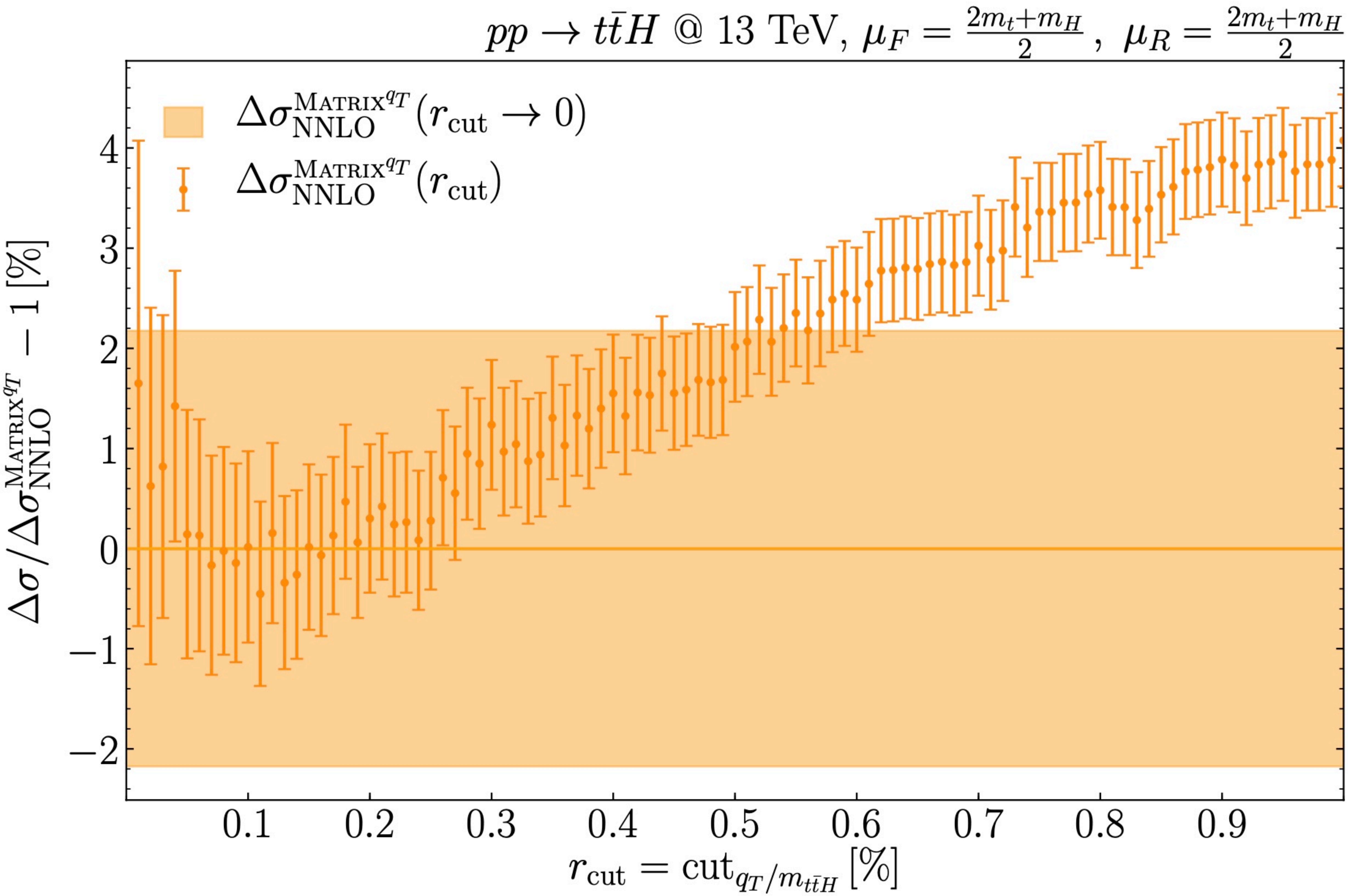
The q_T subtraction counterterm is non-local → the difference in the square bracket is evaluated with a cut-off r_{cut} on the ratio $r = q_T/Q$

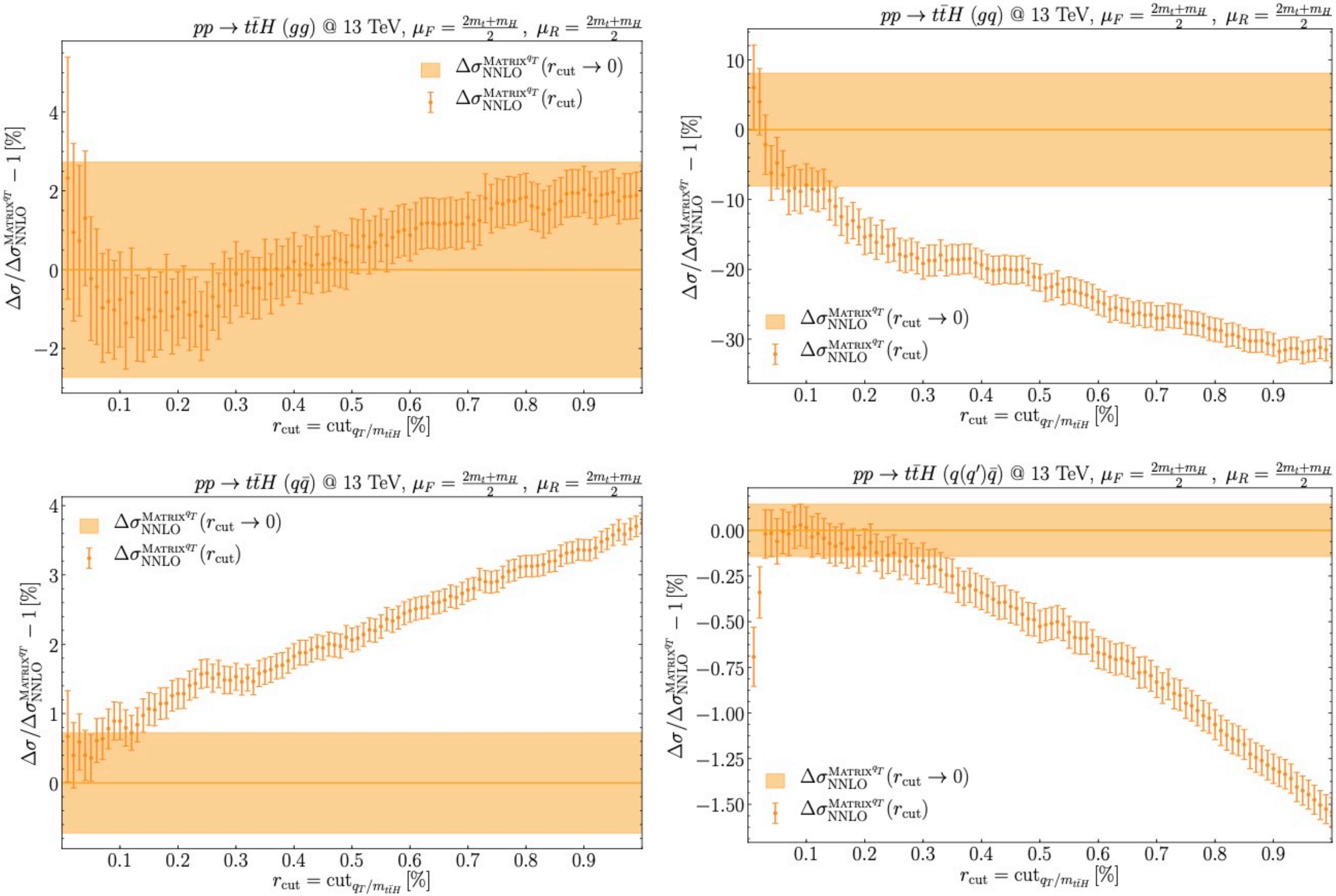
In MATRIX q_T subtraction indeed works as a slicing method

It is important to monitor the dependence of our results on r_{cut}

MATRIX allows for a simultaneous evaluation of the NNLO cross section for different values of r_{cut}

The dependence on r_{cut} is used by the code to provide an estimate of the systematic uncertainty in any NNLO run


$pp \to t\bar{t}H$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$\Delta\sigma^{\text{MATRIX}^{q_T}}_{\text{NNLO}}(r_{\text{cut}} \to 0)$
$\Delta\sigma^{\text{MATRIX}^{q_T}}_{\text{NNLO}}(r_{\text{cut}})$
$\Delta\sigma/\Delta\sigma^{\text{MATRIX}^{q_T}}_{\text{NNLO}} - 1$ [%]
$r_{\text{cut}} = \text{cut}_{q_T/m_{t\bar{t}H}}$ [%]
4
3
2
1
0
−1
−2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


$pp \to t\bar{t}H\ (gg)$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$\Delta\sigma^{\mathrm{MATRIX}^{q_T}}_{\mathrm{NNLO}}(r_{\mathrm{cut}} \to 0)$
$\Delta\sigma^{\mathrm{MATRIX}^{q_T}}_{\mathrm{NNLO}}(r_{\mathrm{cut}})$
$\Delta\sigma/\Delta\sigma^{\mathrm{MATRIX}^{q_T}}_{\mathrm{NNLO}} - 1\ [\%]$
$r_{\mathrm{cut}} = \mathrm{cut}_{q_T/m_{t\bar{t}H}}\ [\%]$
$pp \to t\bar{t}H\ (gq)$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$pp \to t\bar{t}H\ (q\bar{q})$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$
$pp \to t\bar{t}H\ (q(q')\bar{q})$ @ 13 TeV, $\mu_F = \frac{2m_t+m_H}{2}$, $\mu_R = \frac{2m_t+m_H}{2}$

The calculation at NLO

Catani, Torre, MG (2014)

Standard soft current contain the correct soft behaviour
but also additional initial state collinear singularities

$$-\mathbf{J}(k)^2 = \sum_{i,j=1}^{4} \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} \mathbf{T}_i \cdot \mathbf{T}_j$$

These singular contributions are already accounted for in the calculation of colour-singlets

➡ We define a suitably subtracted soft current

$$-\mathbf{J}(k)^2|_{\rm sub} = \sum_{J=3,4} \left[\frac{p_J^2}{(p_J \cdot k)^2} \mathbf{T}_J^2 + \sum_{i=1,2} \left(\frac{p_i \cdot p_J}{p_J \cdot k} - \frac{p_1 \cdot p_2}{(p_1 + p_2) \cdot k} \right) \frac{2\, \mathbf{T}_i \cdot \mathbf{T}_J}{p_i \cdot k} \right] + \frac{2 p_3 \cdot p_4}{(p_3 \cdot k)(p_4 \cdot k)} \mathbf{T}_3 \cdot \mathbf{T}_4$$

final state (heavy-quark) emitters

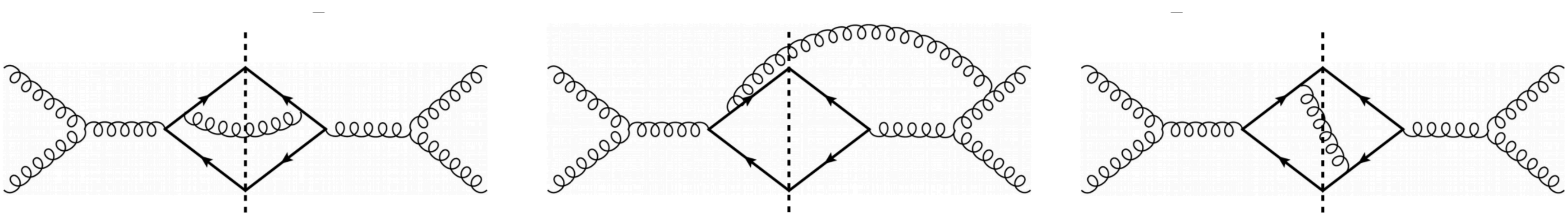
Initial state (massless) emitters

The calculation at NLO

Catani, Torre, MG (2014)

We need to compute the integral of the subtracted soft current over the phase space of the unresolved gluon

$$\int d^d k \, \delta_+(k^2)\, e^{i\mathbf{b}\cdot\mathbf{k}_T}\, \mathbf{J}^2(k)|_{\rm sub}$$



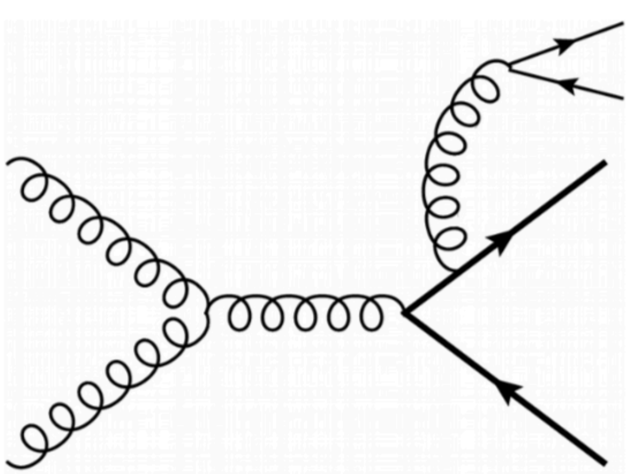
$$\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}\left(\epsilon, \frac{M^2}{\mu_R^2}\right) = -\frac{1}{2}\left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon}\left\{\left(\frac{1}{\epsilon^2}+i\pi\frac{1}{\epsilon}-\frac{\pi^2}{12}\right)(\mathbf{T}_1^2+\mathbf{T}_2^2)+\frac{2}{\epsilon}\gamma_c-\frac{4}{\epsilon}\,\mathbf{\Gamma}_t^{(1)}(y_{34})+\mathbf{F}_t^{(1)}(y_{34})\right\}$$

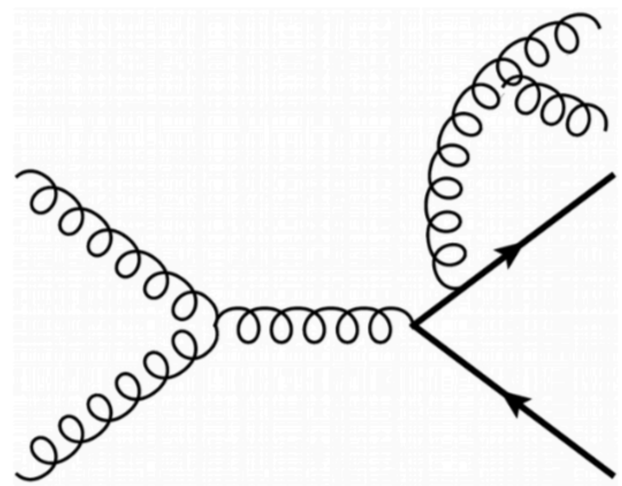
Singular structure from initial state radiation

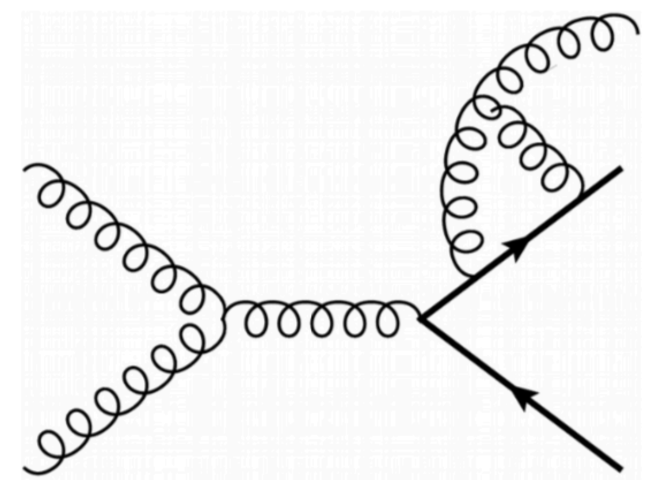
Additional soft contribution obtained from integration of the subtracted soft current

The calculation at NNLO

Catani, Devoto, Mazzitelli, MG , to appear







Three classes of contributions: singular structure fully known

- Emission of a soft quark-antiquark pair — Catani, MG (2000)
- Emission of two soft gluons — Catani, MG (2000); Czakon (2011)
- Soft-gluon emission at one loop — Catani, MG (2000); Bierenbaum, Czakon, Mitov (2011); Czakon, Mitov (2018)

Construct suitably subtracted soft current for each of these contribution

Intermediate results contain $1/\varepsilon^3$ poles ➡ add up to $1/\varepsilon^2$ in the end

Extension to heavy-quark production

Catani, Torre, MG (2014)

$$(\mathbf{H}\,\mathbf{\Delta})_{c\bar{c}} = \frac{\langle\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,|\, \mathbf{\Delta} \,|\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,\rangle}{\alpha_S^2(M^2)\ |\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}(p_1,p_2;p_3,p_4)|^2}$$

$|\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,\rangle$ subtracted virtual amplitude

$$\mathbf{\Delta}(\mathbf{b},M;y_{34},\phi_3) = \mathbf{V}^\dagger(b,M;y_{34})\ \ \mathbf{D}(\alpha_S(b_0^2/b^2);\phi_{3b},y_{34})\ \mathbf{V}(b,M;y_{34})$$

$$\mathbf{V}(b,M;y_{34}) = \overline{P}_q\ \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\, \mathbf{\Gamma}_t(\alpha_S(q^2);y_{34}) \right\}$$

$\alpha_S^n L^m$ terms $\quad n \geq m$

soft anomalous dimension

$\mathbf{\Gamma}_t^{(1)}$ and $\mathbf{\Gamma}_t^{(2)}$ directly related to singular structure of $|\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle$

Mitov, Sterman, Sung (2009)
Neubert et al (2009)

$\mathbf{D}(\alpha_S;\phi_{3b},y_{34})$ embodies azimuthal correlations at scale 1/b

$$\langle\, \mathbf{D}(\alpha_S;\phi_{3b},y_{34})\rangle_{\mathbf{av.}} = 1$$