RECENT PROGRESS IN HIGHER-ORDER CALCULATIONS*

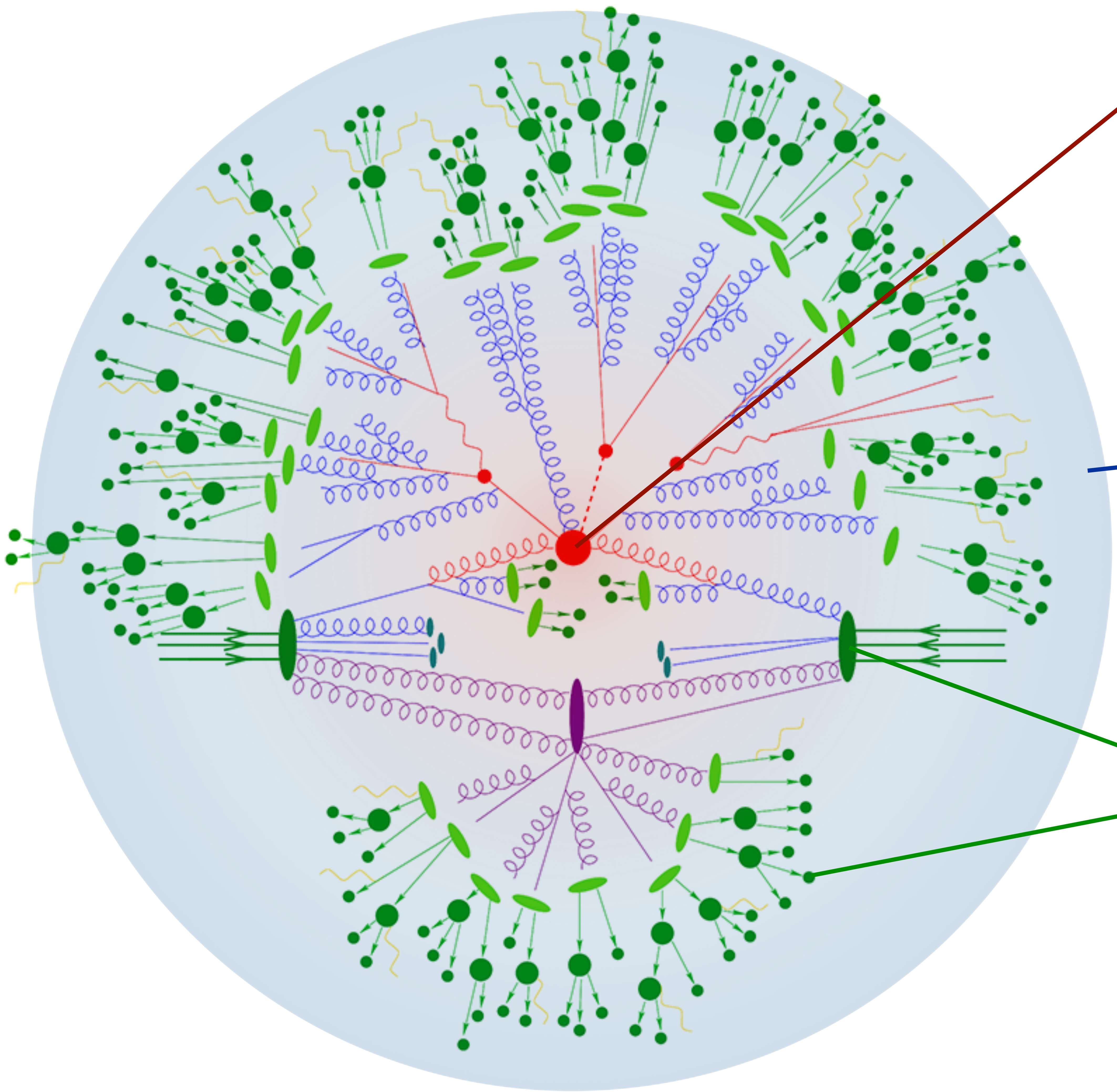
Alexander Huss

CERN

* impossible to cover everything; personal selection

Workshop on Tools for High Precision LHC Simulations — October 31st 2022

Scattering Reactions @ LHC



short distance "hard"

- high scales: $10^2 - 10^3$ GeV

evolution towards a physical observable state

long distance "soft"

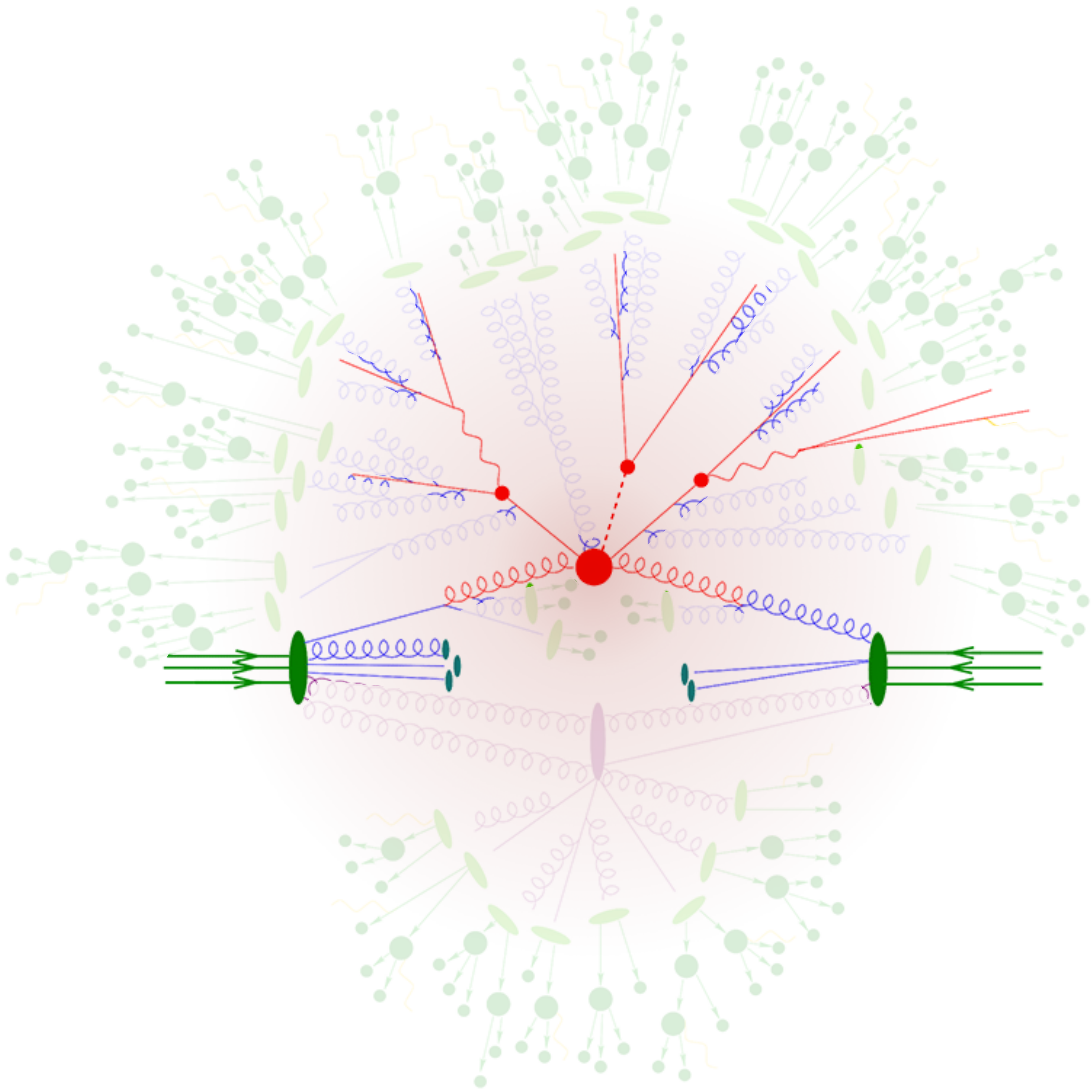
- low scales: $\mathcal{O}$(few GeV)

Monte Carlo & Matching, Parton Shower Sessions

non perturbative *M. Ubiali, P. Nason*

- quarks & gluons ⇝ hadrons

Scattering Reactions @ LHC



- **Focus:** high momentum transfer & clean signatures
- perturbation theory:

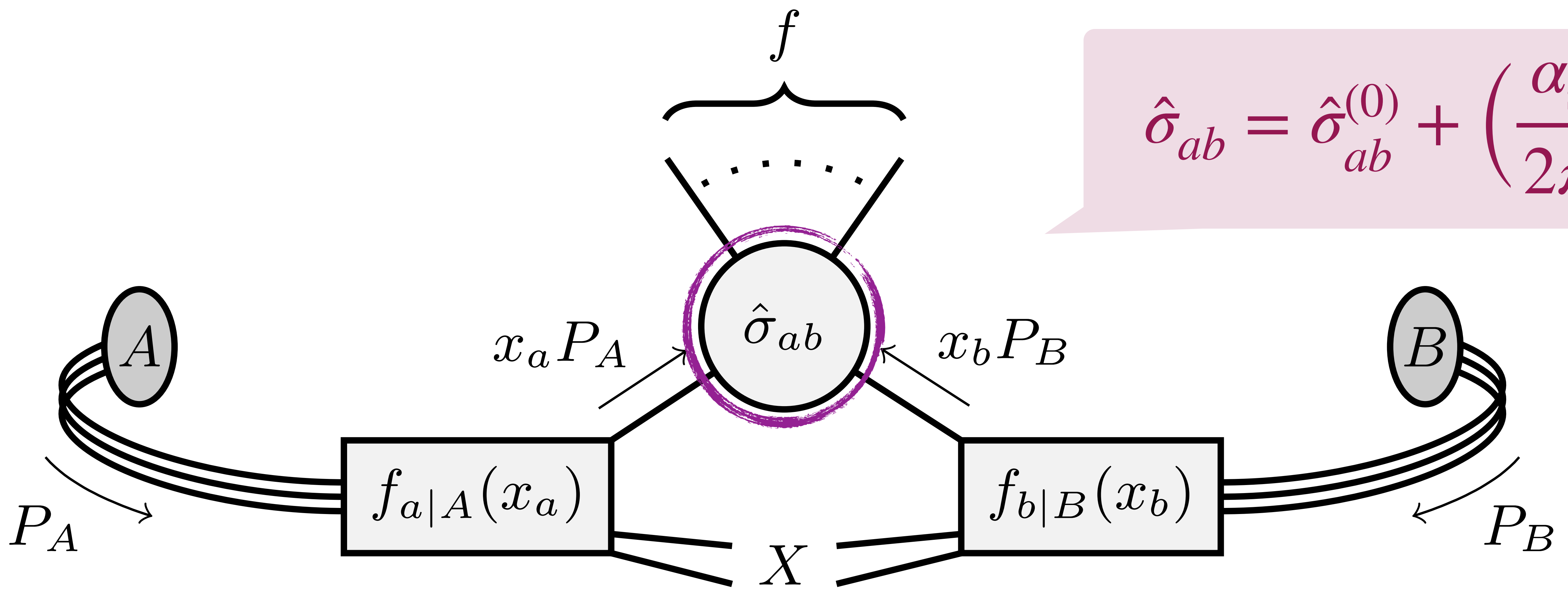
$$\sigma = \sigma_0 \times \left(1 + \alpha_x + \alpha_x^2 + \alpha_x^3 + \ldots\right)$$

fixed order: LO NLO NNLO N³LO …

- $\alpha_s \sim 0.1$ & $\alpha_{ew} \sim 0.01$
  - 1% target $\leftrightarrow$ $\mathcal{O}(\alpha_s{}^2, \alpha_{ew})$ $\rightsquigarrow$ $\mathcal{O}(\alpha_s{}^3, \alpha_s\,\alpha_{ew})$

The Master Formula — Collinear Factorization

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \cdots$$



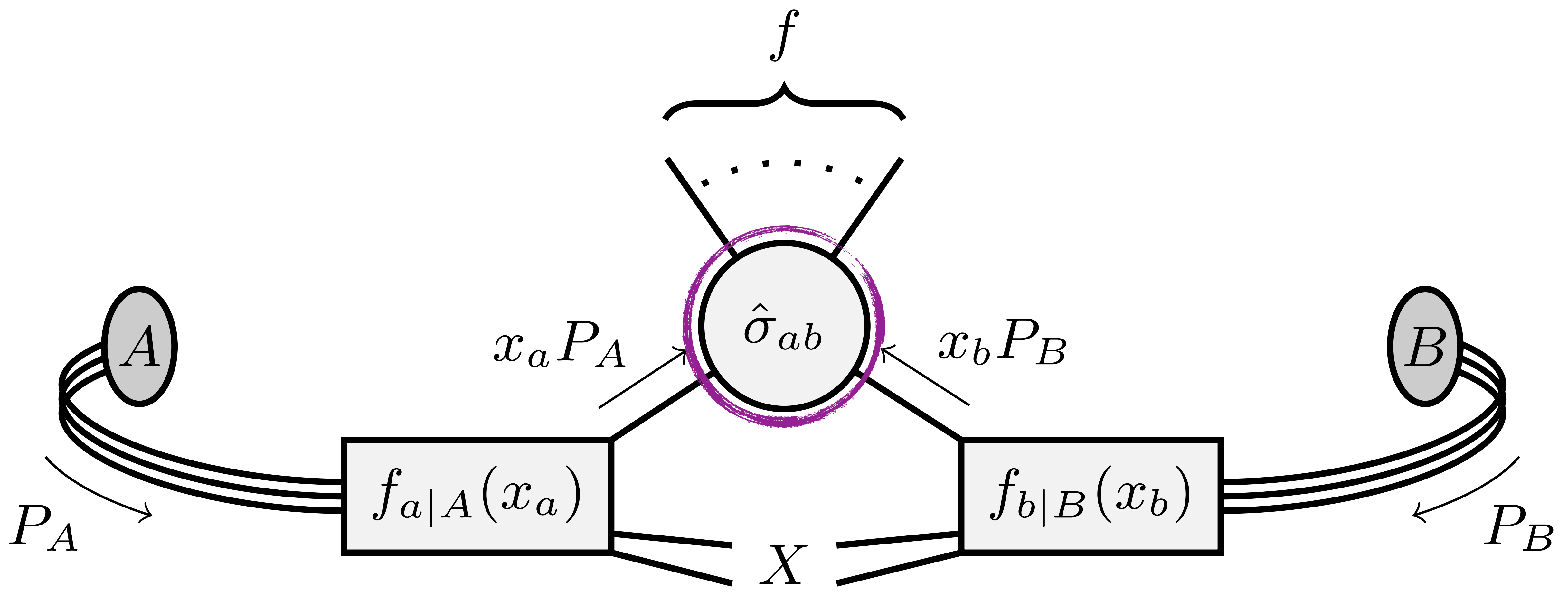
$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; f_{a|A}(x_a)\; f_{b|B}(x_b)\; \hat{\sigma}_{ab}(x_a, x_b)\; \left(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)$$

parton distribution functions
(non-perturbative, universal)
in principle, improvable

hard scattering
(perturbation theory)
systematically improvable

non-perturbative effects
(power suppressed)
ultimately, limiting factor?

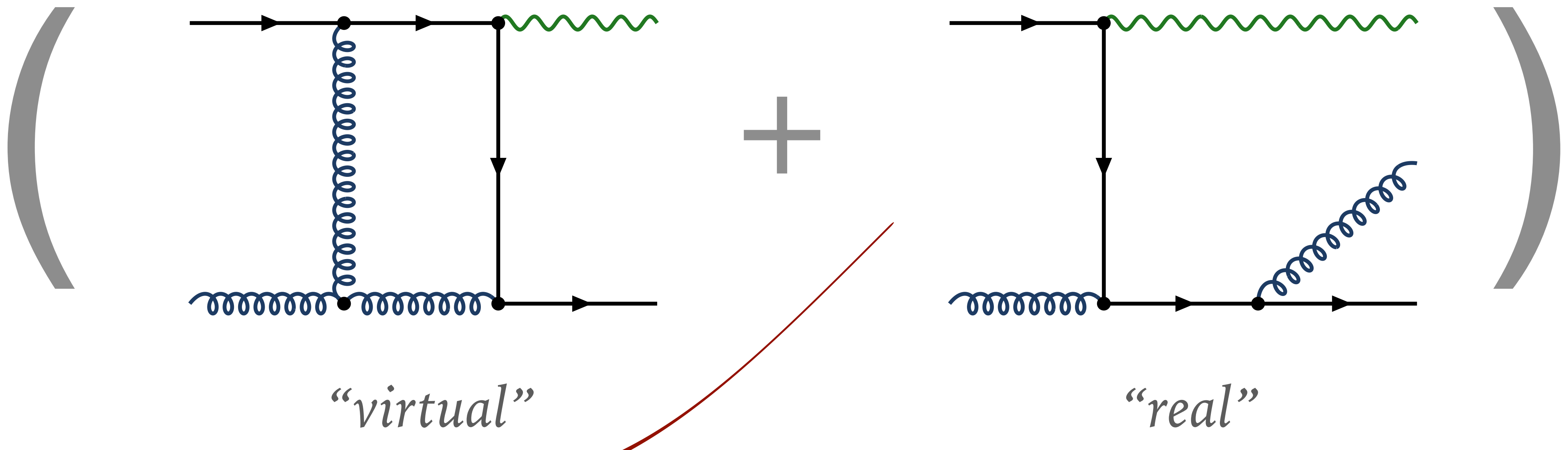
NLO – Conceptually Solved?



automated 1-loop providers

- Gosam [Chiesa et al. '14]
- MadGraph5_aMC@NLO [Frixione et al. '18]
- NLOX [Honeywell et al. '18]
- OpenLoops [Pozzorini et al. '19]
- Recola [Actis et al. '16]

one-loop amplitudes
(all master integrals known, well understood: log, Li2)

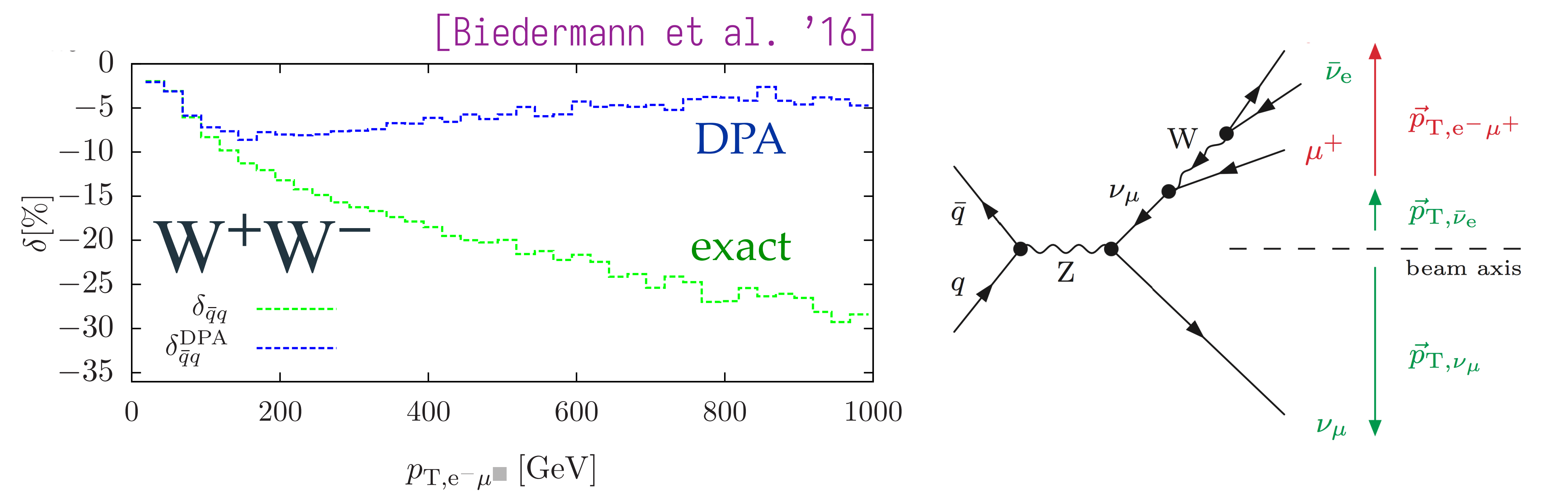


IR subtraction
(involved IR structure, numerical stability, construction)

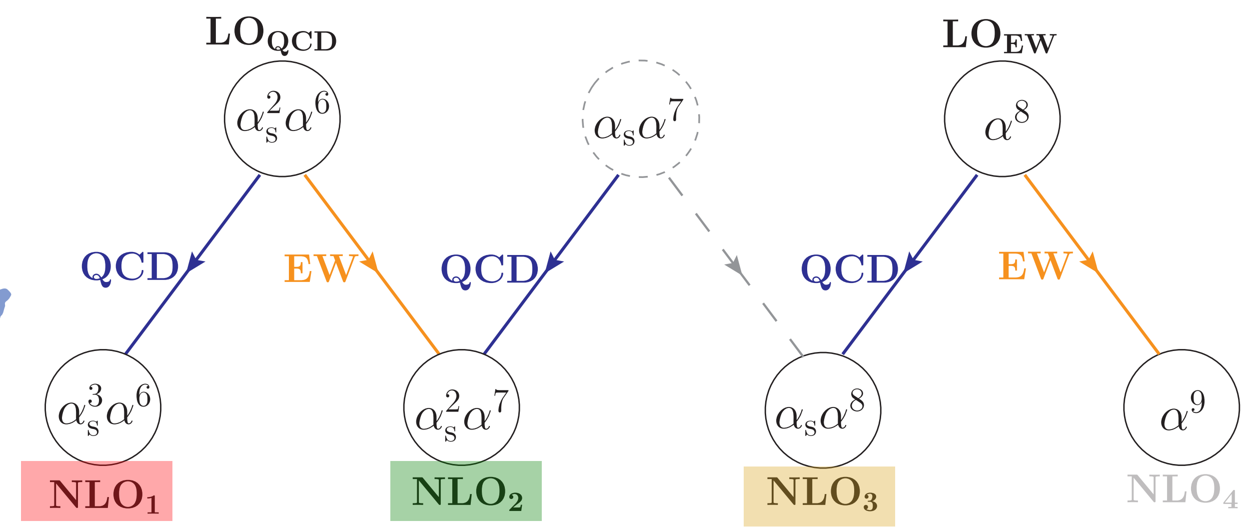
infrared singularities

J. Lindert

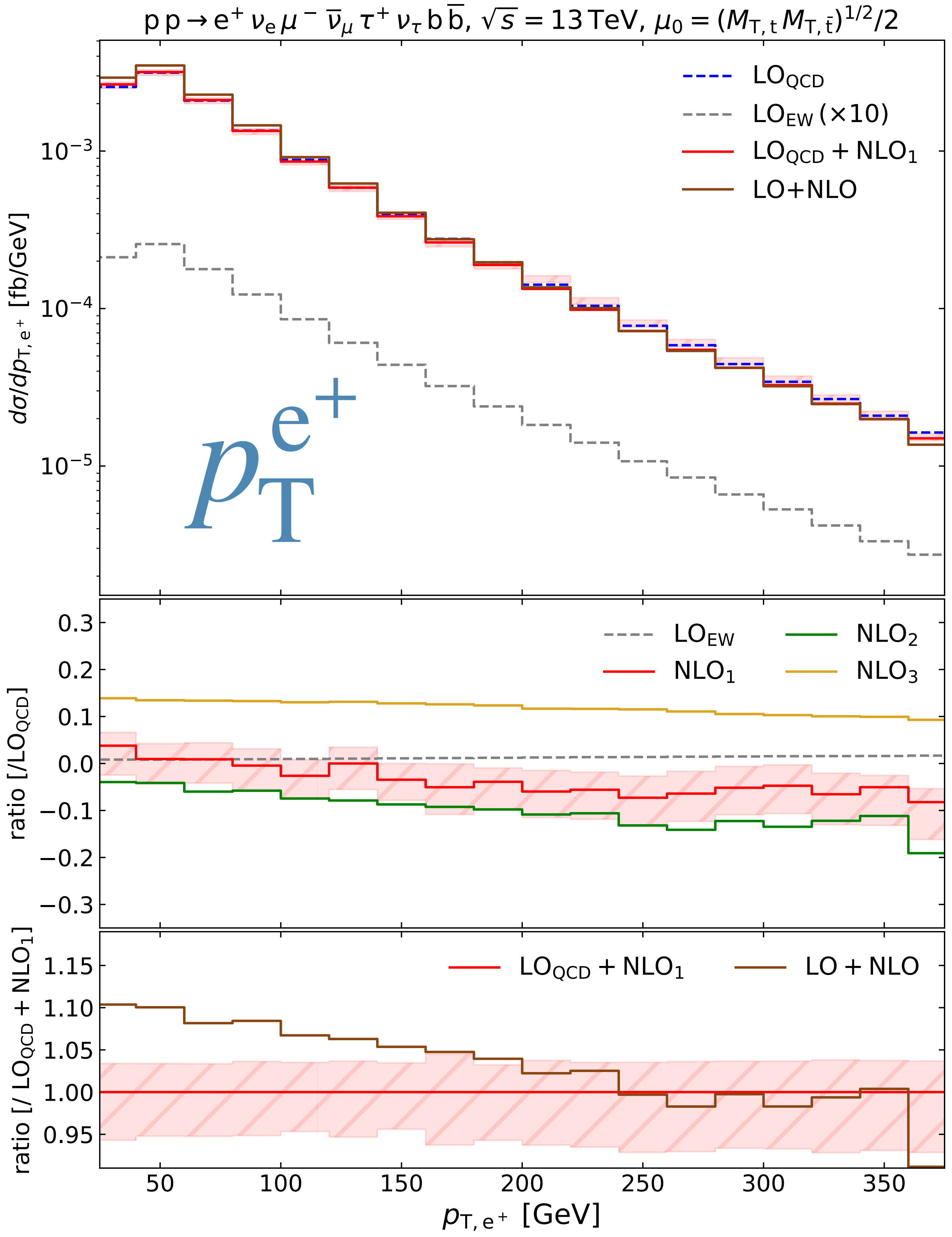
NLO – PUSHING THE LIMIT*



- **off-shell** ⇝ high-multiplicity
  - ↪ large non-resonance effects?
  - ↪ particularly challenging for EW
- **$2 \to 6$ processes**
  - $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ ($t\bar{t}$) [Denner, Pellen '16]
  - $pp \to 4\ell jj$ (VBS) [Denner et al. '16–22]
  - $pp \to \ell_1^-\bar{\nu}_{\ell_1}\ell_2^+\nu_{\ell_2}\ell_2^+\nu_{\ell_3}$ (WWW) [Schönherr '18; Dittmaier et al. '19]
  - $pp \to e^+e^-\mu^+\nu_\mu jj_b$ (tZj) [Denner, Pelliccioli, Schwan '22]
- **$2 \to 7$ processes**
  - $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}H$ ($t\bar{t}H$) [Denner, Lang, Pellen, Uccirati '16]
- **$2 \to 8$ processes**
  - $pp \to e^+\nu_e\tau^+\nu_\tau\mu^-\bar{\nu}_\mu b\bar{b}$ ($t\bar{t}W$) [Denner, Pelliccioli '21]

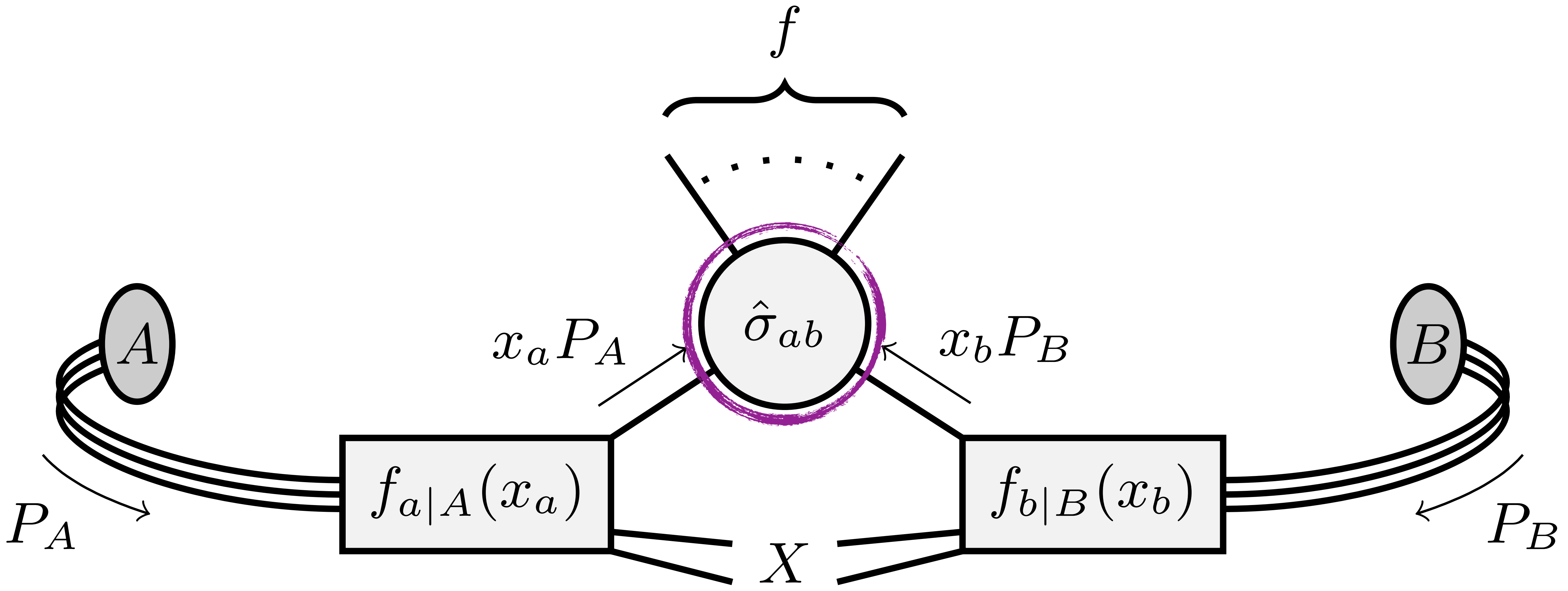


- NLO_1 ∼ QCD corr. ±5 % (scl)
- NLO_2 ∼ EW corr. −(3–15) %
- NLO_3 ∼ QCD to LO_{EW} (>NLO_2) +(10–14) %
- NLO_4 ∼ negligible
- EW corr. beyond NLO_{QCD} > Δ_{scl}



* another frontier: *G. Pellicioli*
loop-induced, polarization, …

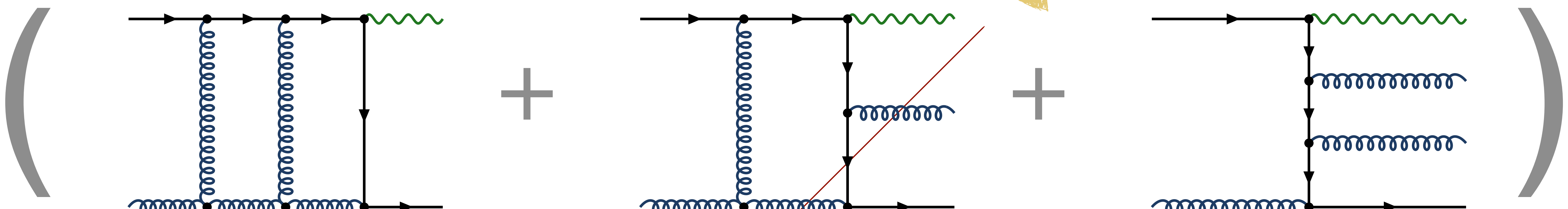
NNLO — The Building Blocks & Challenges



two-loop amplitudes
(new class of functions, combinatoric & algebraic complexity)

L. Tancredi

one-loop amplitudes
(evaluation in singular & unstable regions)



"double virtual" *"real-virtual"* *"double real"*

IR subtraction
(involved IR structure, numerical stability, construction)

infrared singularities

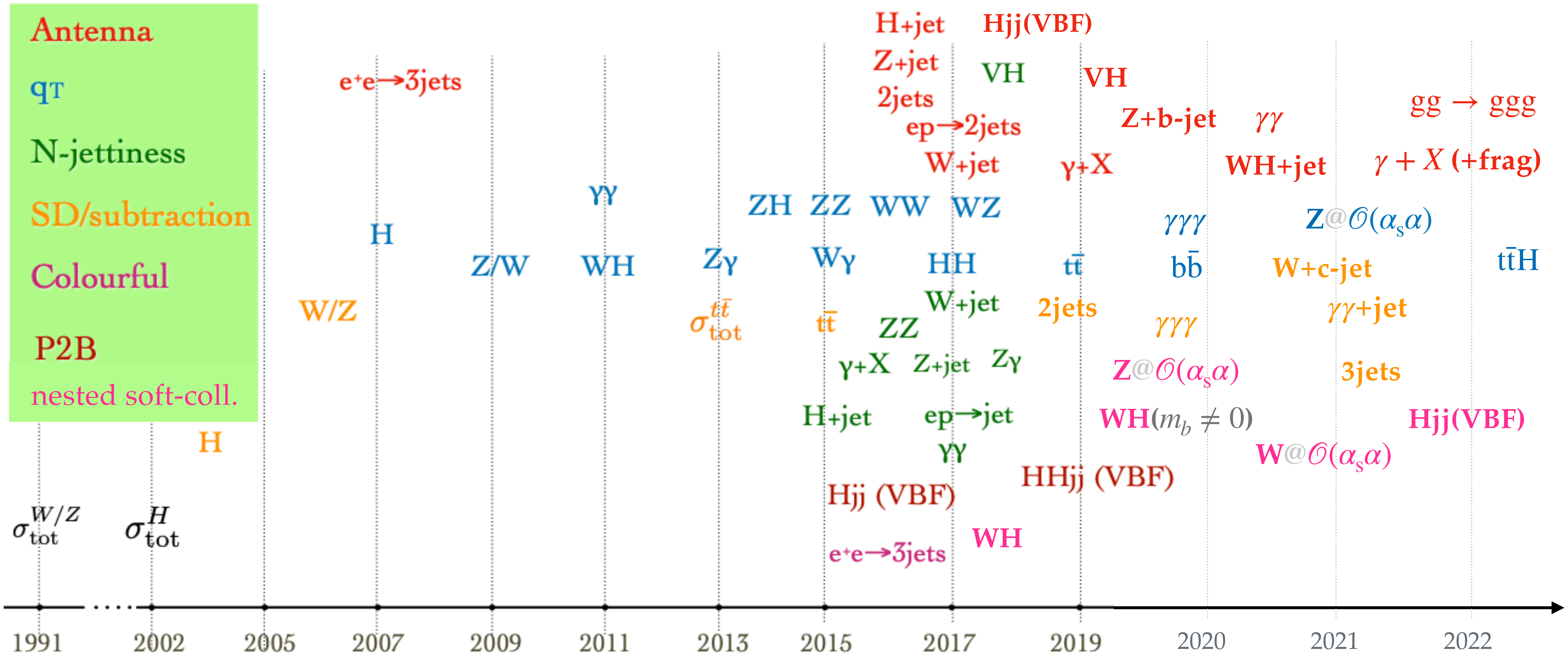
WHAT CAN WE DO TODAY? — THE NNLO TIMELINE

[timeline adapted from M. Grazzini]

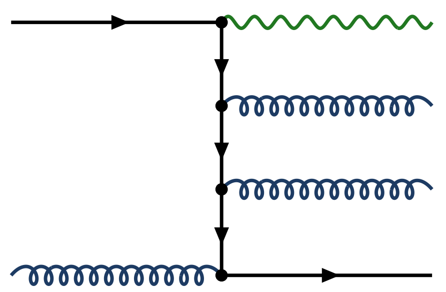
Tremendous progress in the past ~ 5–10 years!

↪ $2 \to 2$ under good control; $2 \to 3$ good progress

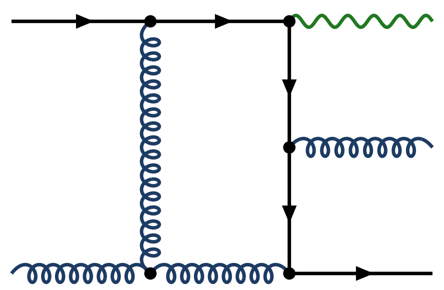
K. Ellis (colour-less), D. Lombardi (VV), A. Ratti ($b\bar{b}$), J. Mazzitelli ($t\bar{t}$), M. Grazzini ($t\bar{t}H$), L. Buonocore (massive f.s.), L. Rottoli (f.s. jet)



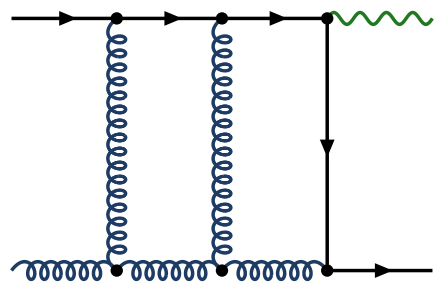
$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} d\sigma^{RR}_{\text{NNLO}}$$



$$+ \int_{\Phi_{Z+2}} d\sigma^{RV}_{\text{NNLO}}$$



$$+ \int_{\Phi_{Z+1}} d\sigma^{VV}_{\text{NNLO}}$$



$$\sum \quad \text{finite}$$

- in general: measurement function
  - fiducial cross sections
  - differential distributions
  - reconstruction (jets, γ, …)

  } $\mathscr{F}^{(n)}_{\text{obs}}$

➡ massage expression to render intermediate objects finite (suitable for MC integration)

Different Methods*

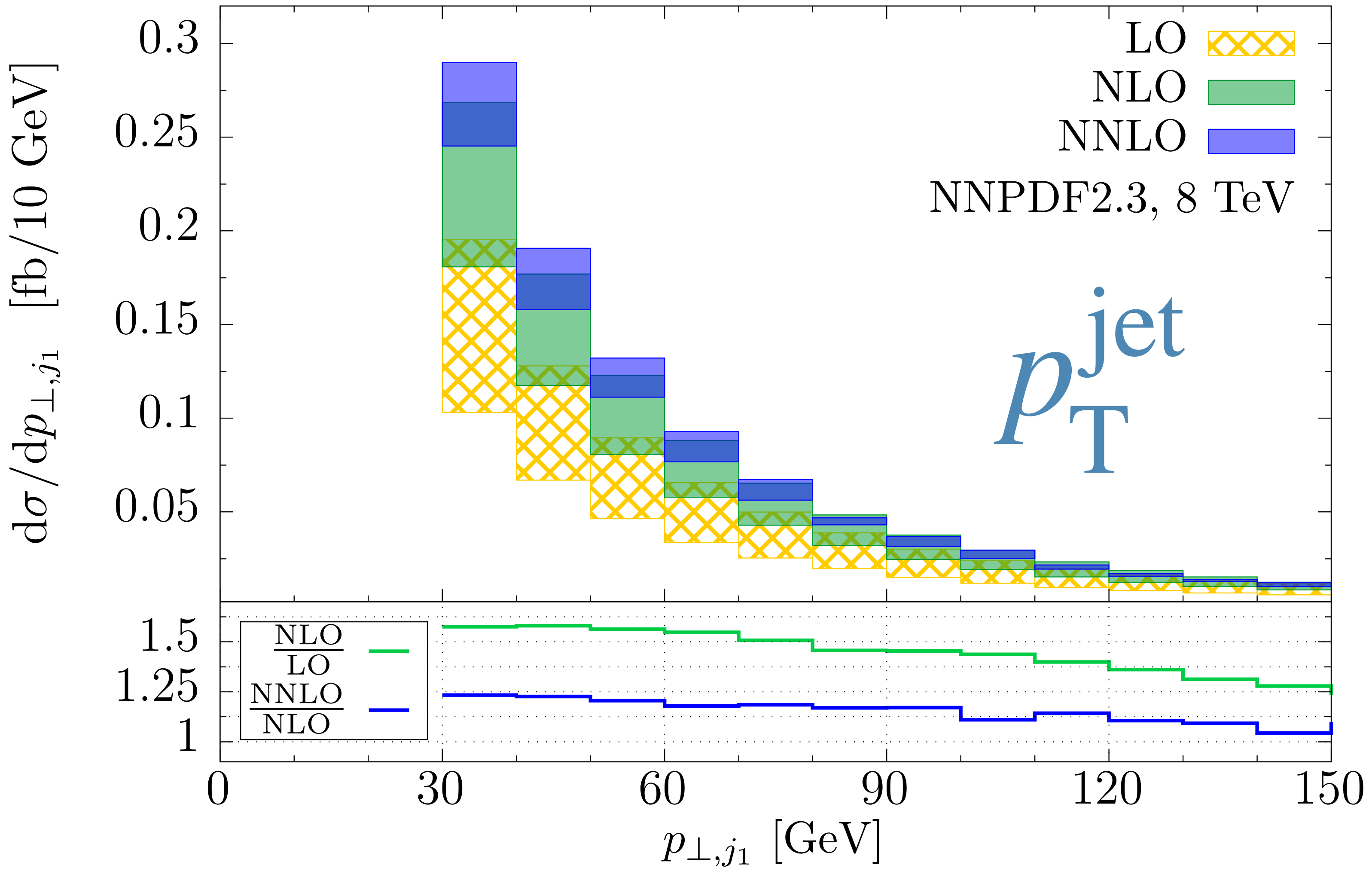
- Antenna [Gehrmann–De Ridder, Gehrmann, Glover ’05]
- CoLorFul [Del Duca, Somogyi, Trocsanyi ’05]
- qT-subtraction [Catani, Grazzini ’07; MATRIX]
- STRIPPER (sector-improved residues) [Czakon ‘10]
- nested soft-collinear [Caola, Melnikov, Röntsch ’17]
- N-jettiness [Gaunt, Stahlhofen, Tackmann, Walsh ’15] [Boughezal, Focke, Liu, Petriello ’15; MCFM]
- Projection-to-Born [Cacciari, et al. ’15]
- Geometric, Local analytical Sectors
  [Herzog ’18] [Magnea et al. ‘18]

* *Subtraction & Slicing*

Independent Calculations – H + jet ×3!

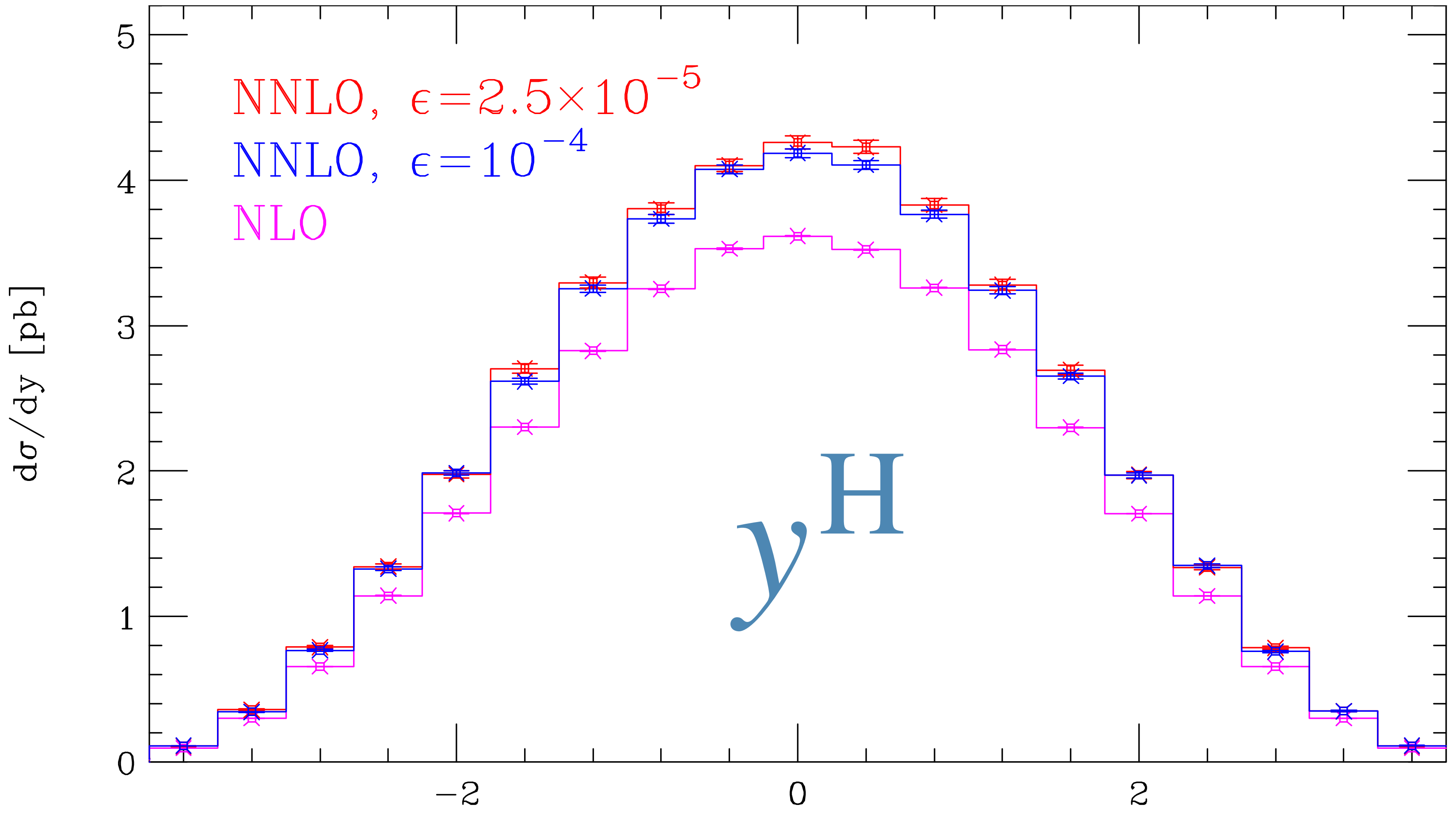
residue subtraction

[Caola, Melnikov, Schulze '15]



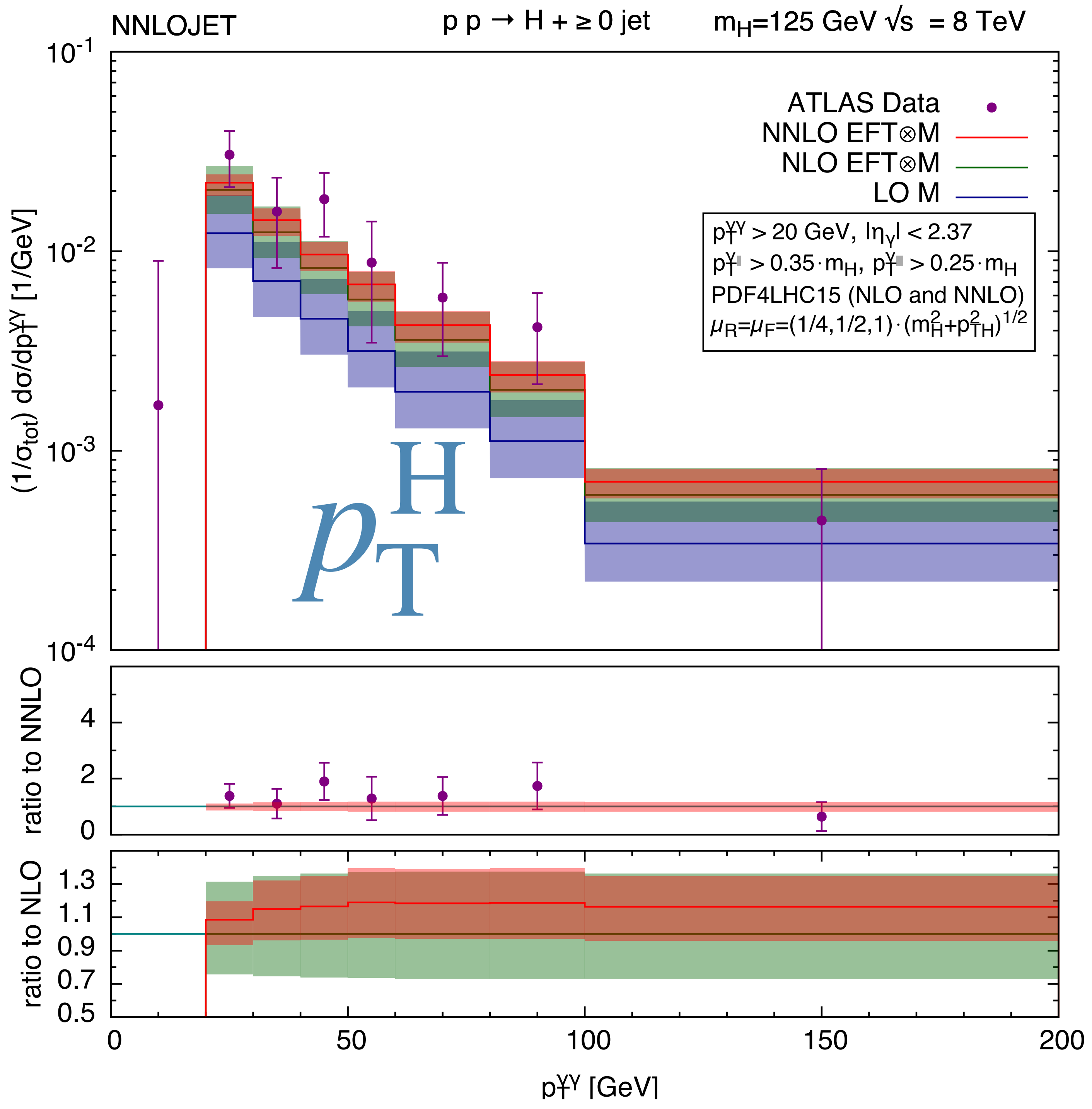
τ_1 jettiness subtraction

[Boughezal, Focke, Giele, Liu, Petriello '15]
[Campbell, Ellis, Seth '19]



antenna subtraction

[Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '16]



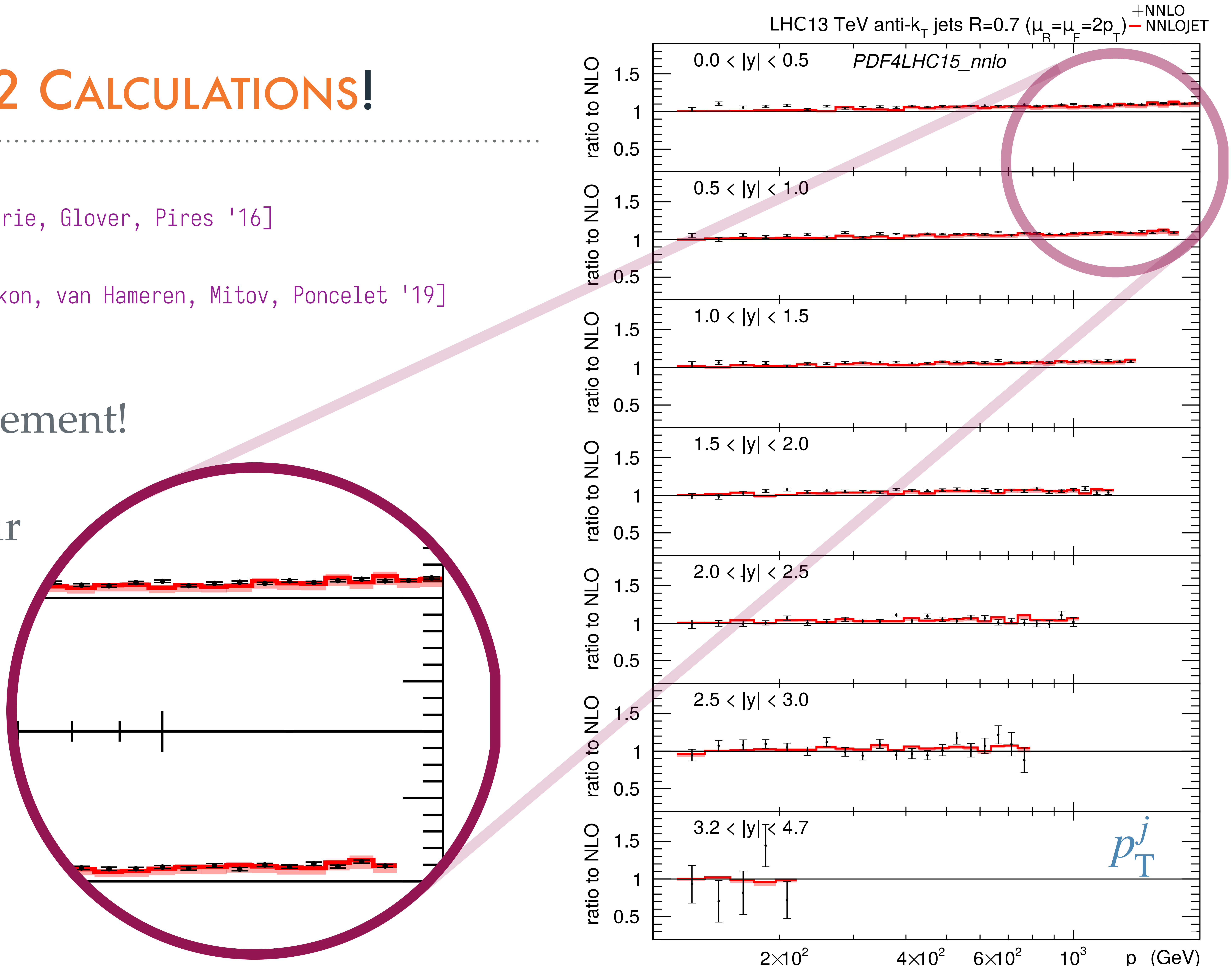
- very complex calculations ⇝ validation!
  - long-standing [~'15] discrepancy in H + jet
    ↪ only resolved in ['19]
- benchmark approaches

Inclusive Jets – 2 Calculations!

NNLOJET [Currie, Glover, Pires '16]

STRIPPER [Czakon, van Hameren, Mitov, Poncelet '19]

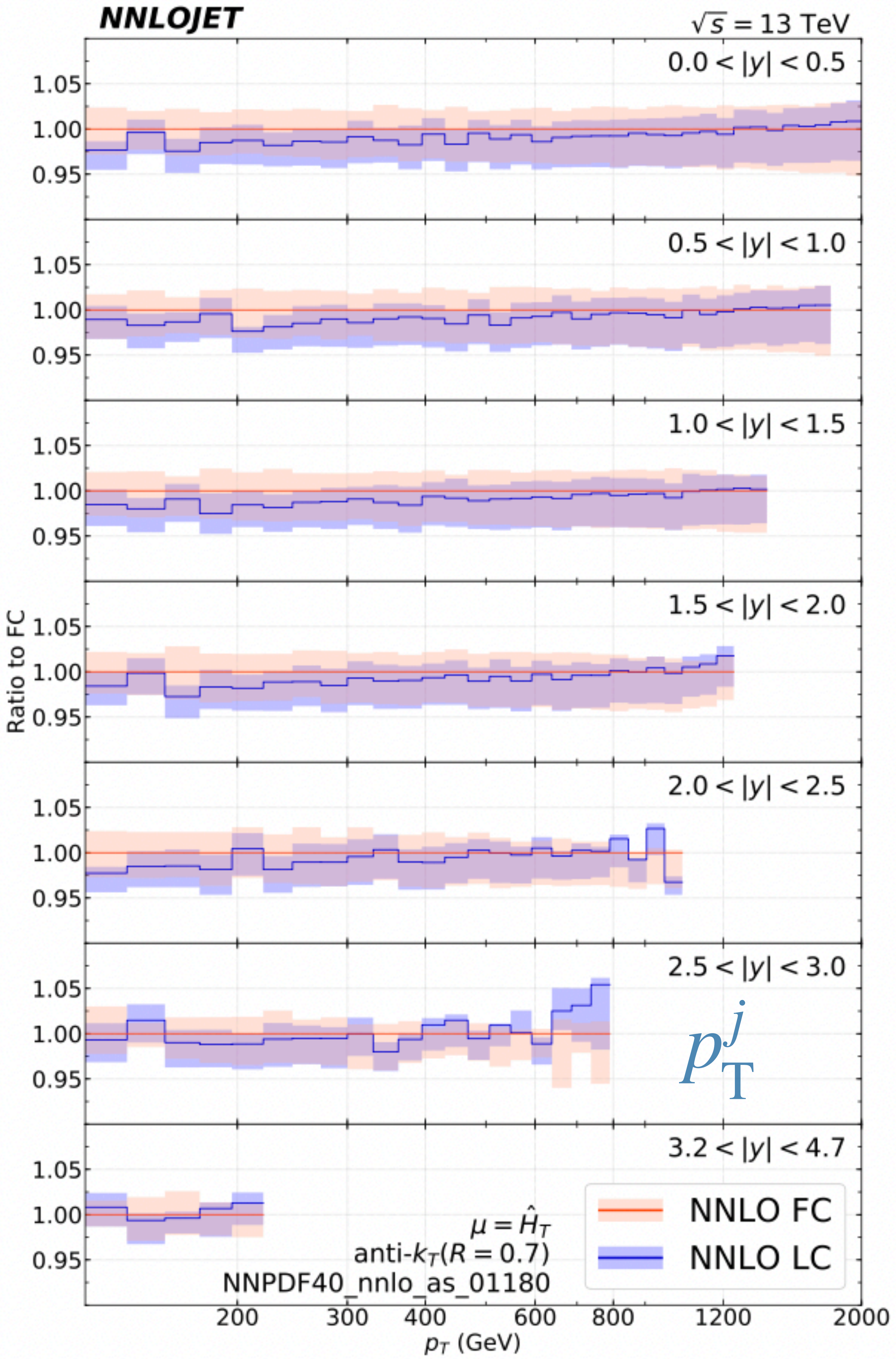
- in very good agreement!
- sub-leading colour negligible!(?)



How Good is Leading Colour?

sub-leading colour: SLC
leading colour: LC
full colour (LC+SLC): FC

[Chen, Gehrmann, Glover, AH, Mo '22]



✓ **Inclusive Jets** ($R = 0.7$)

SLC corrections positive
↪ up to 20% on $\delta\sigma^{\mathrm{NNLO}}$
↪ largest @ low-p_{T}

impact on NNLO
↪ $\lesssim 2\,\%$ & within Δ_{scl}

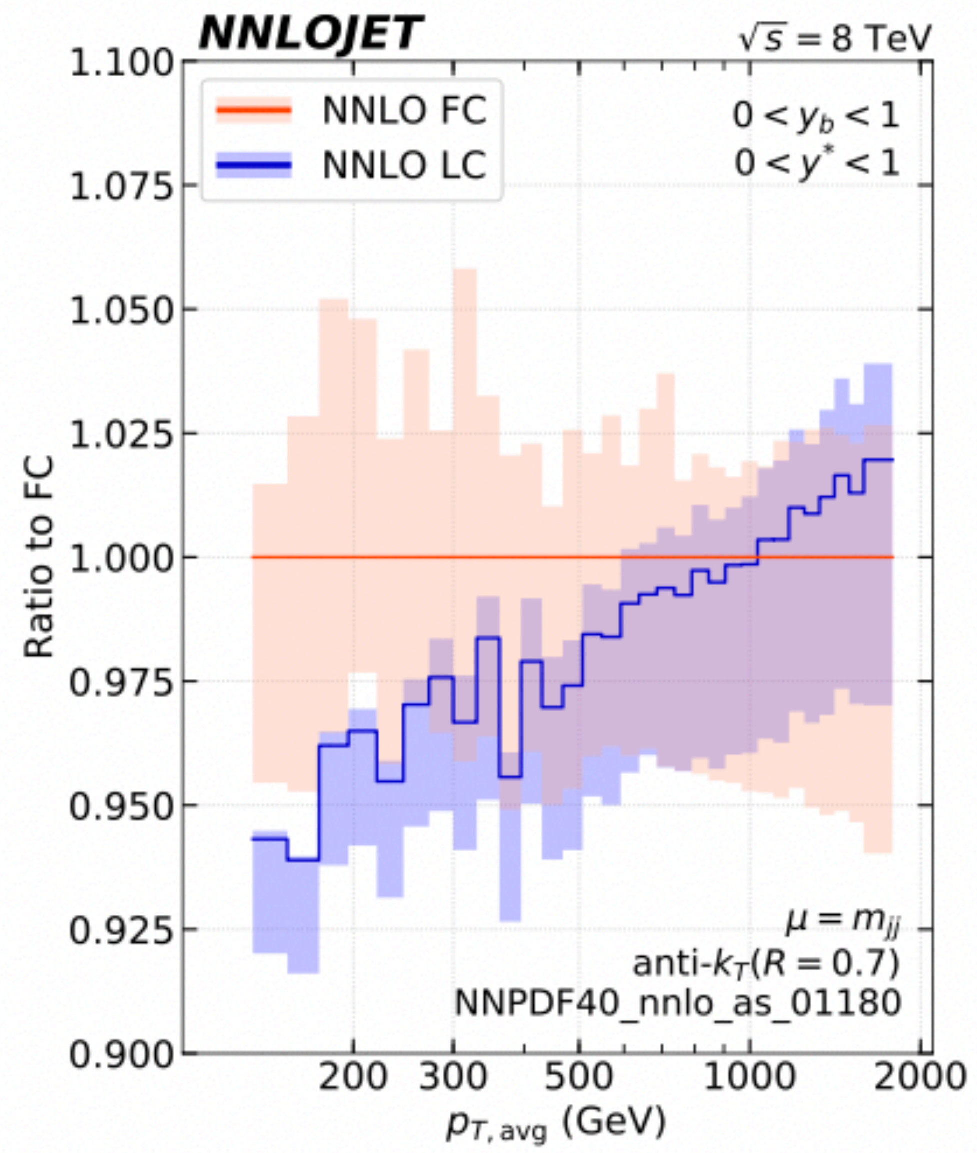
✓ **Incl. jets** ($R = 0.4$)

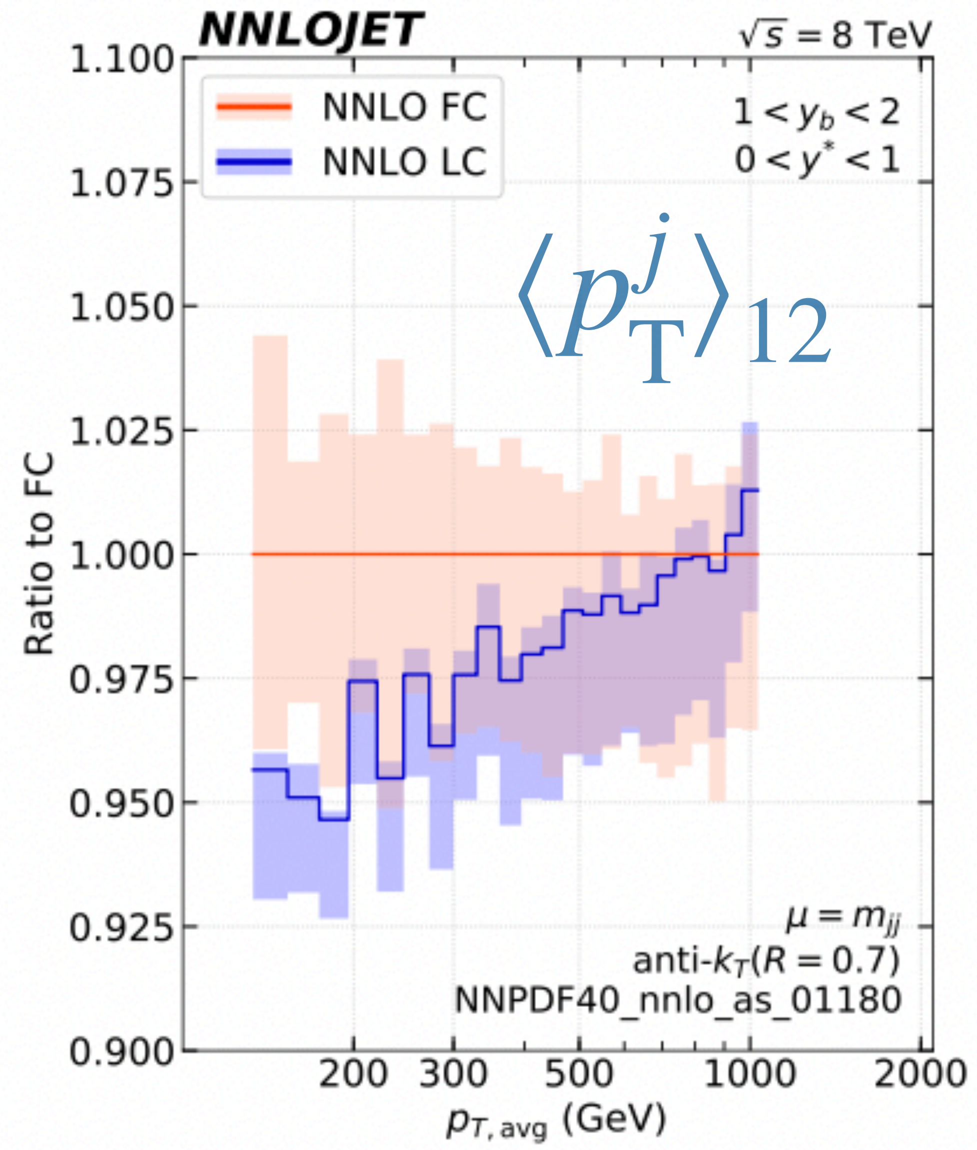
✓ **Dijet** $\mathrm{d}\sigma/\mathrm{d}m_{jj}$ ($R = 0.4$)

✗ **Dijet 3D** ($R = 0.7$)

large SLC corrections
↪ low-$p_{\mathrm{T,avg}}$ ↭ 30–60%
↪ med-$p_{\mathrm{T,avg}}$ ↭ small $|\cdot|$
↪ high-$p_{\mathrm{T,avg}}$ ↭ $-20\,\%$

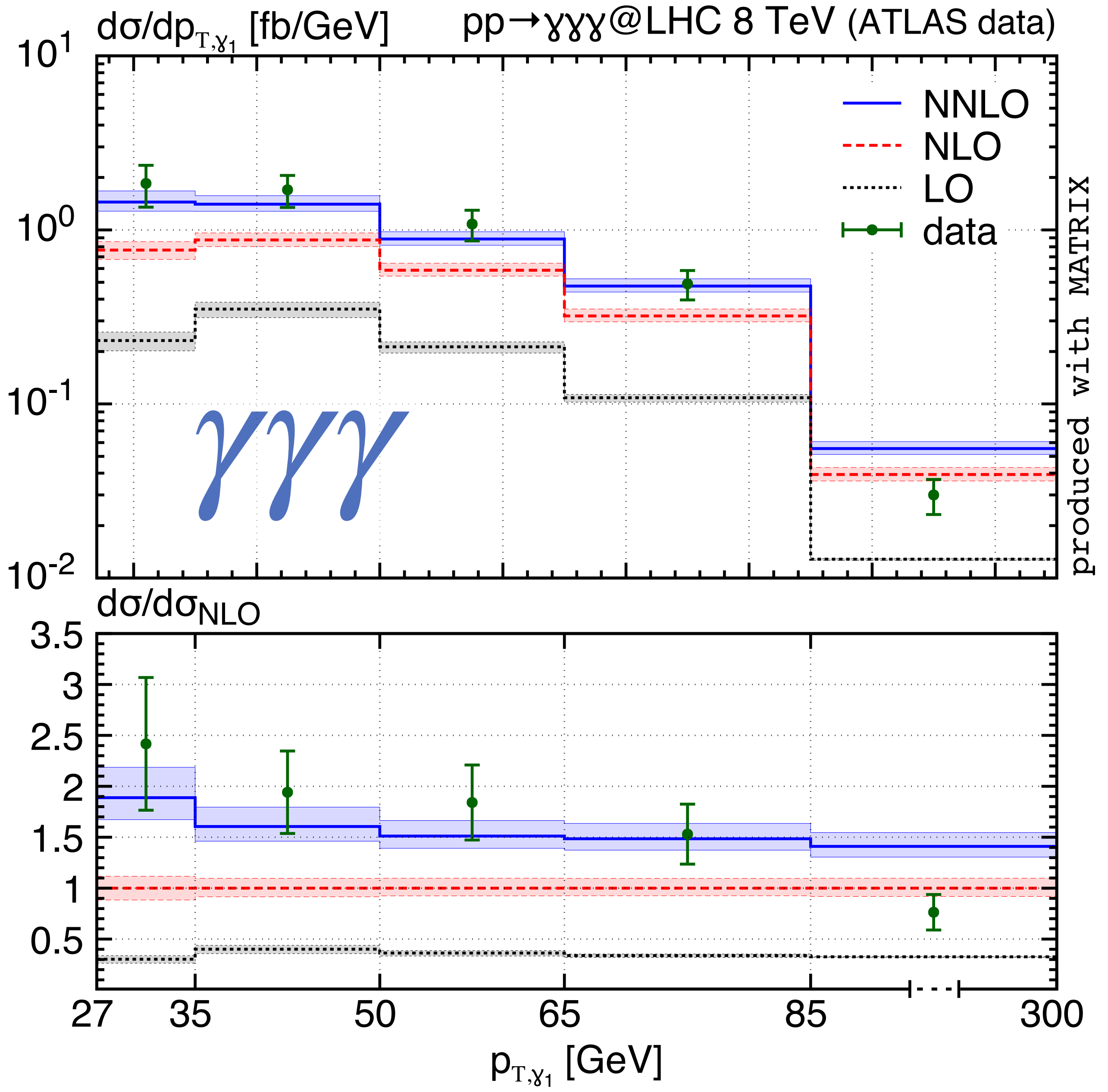
LC → FC
↪ 5 % enhancement



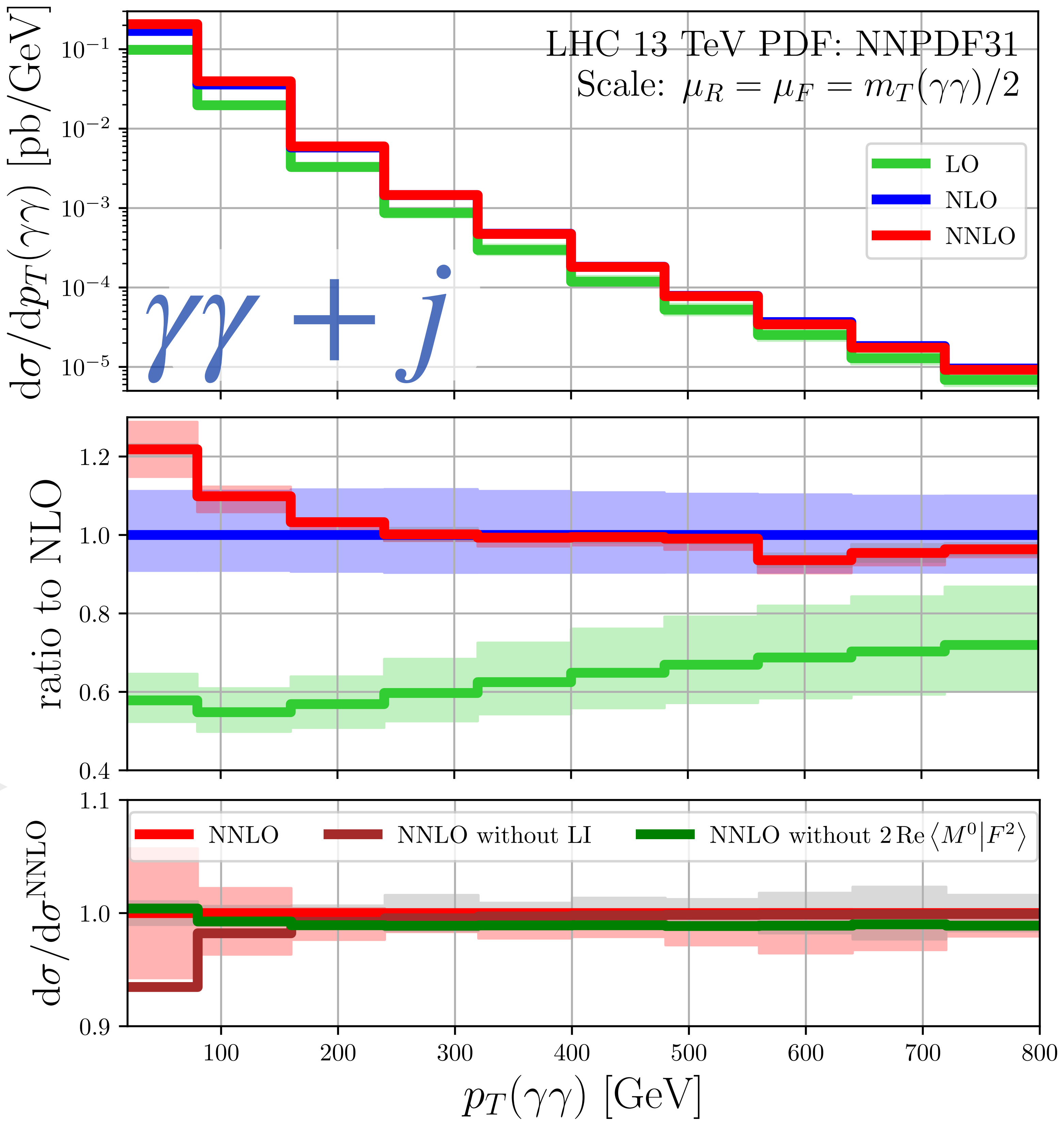


CURRENT FRONTIER: $2 \to 3$

- $\mathrm{pp} \to \gamma\gamma\gamma$
  [Chawdhry, Czakon, Mitov, Poncelet '19]
  [Kallweit, Sotnikov, Wiesemann '20]
- $\mathrm{pp} \to \gamma\gamma + j$
  [Chawdhry, Czakon, Mitov, Poncelet '21]
  (gluon-fusion @ NLO ≃ N3LO)
  ↪ [Badger, Gehrmann, Marcoli, Moodie '21]
- $\mathrm{pp} \to jjj$
  [Czakon, Mitov, Poncelet '21]
  ($gg \to ggg$; antenna automation)
  ↪ [Chen, Gehrmann, Glover, Huss, Marcoli '22]
- $\mathrm{pp} \to \mathrm{Wb\bar{b}}$
  [Hartanto, Poncelet, Popescu, Zoia '22]
- $\mathrm{pp} \to \mathrm{t\bar{t}H}$
  [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini '22]



LO → NLO (× 2.8)
NLO → NNLO (× 1.6)
pert. convergence?
(c.f. di-gamma)



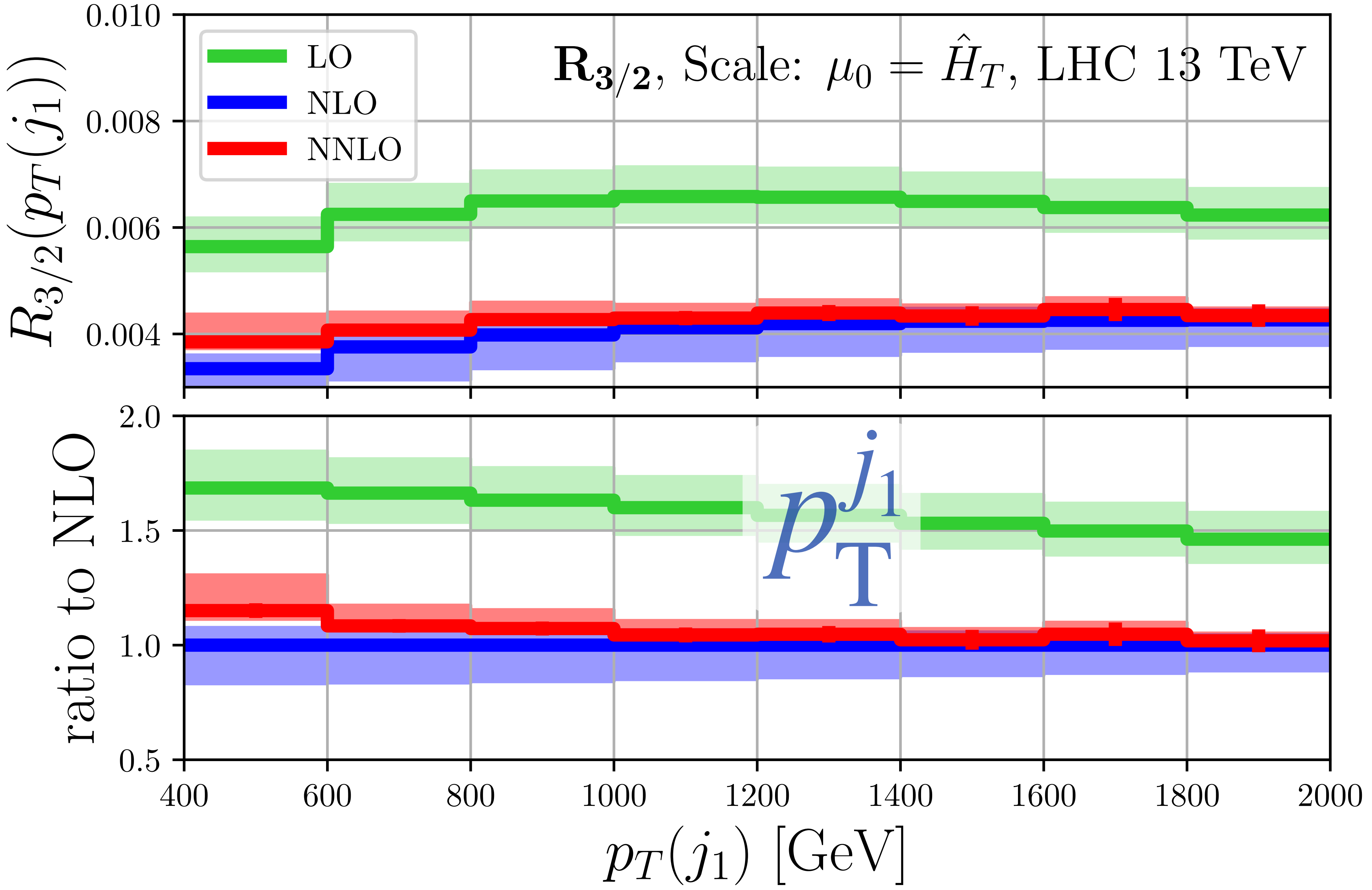
good convergence @ NNLO
except: low-$p_T^{\gamma\gamma}$, $\cos\phi_{CS}$, …

⇜ loop-induced large δNLO (100%)

$2 \to 3$ Results at the Frontier

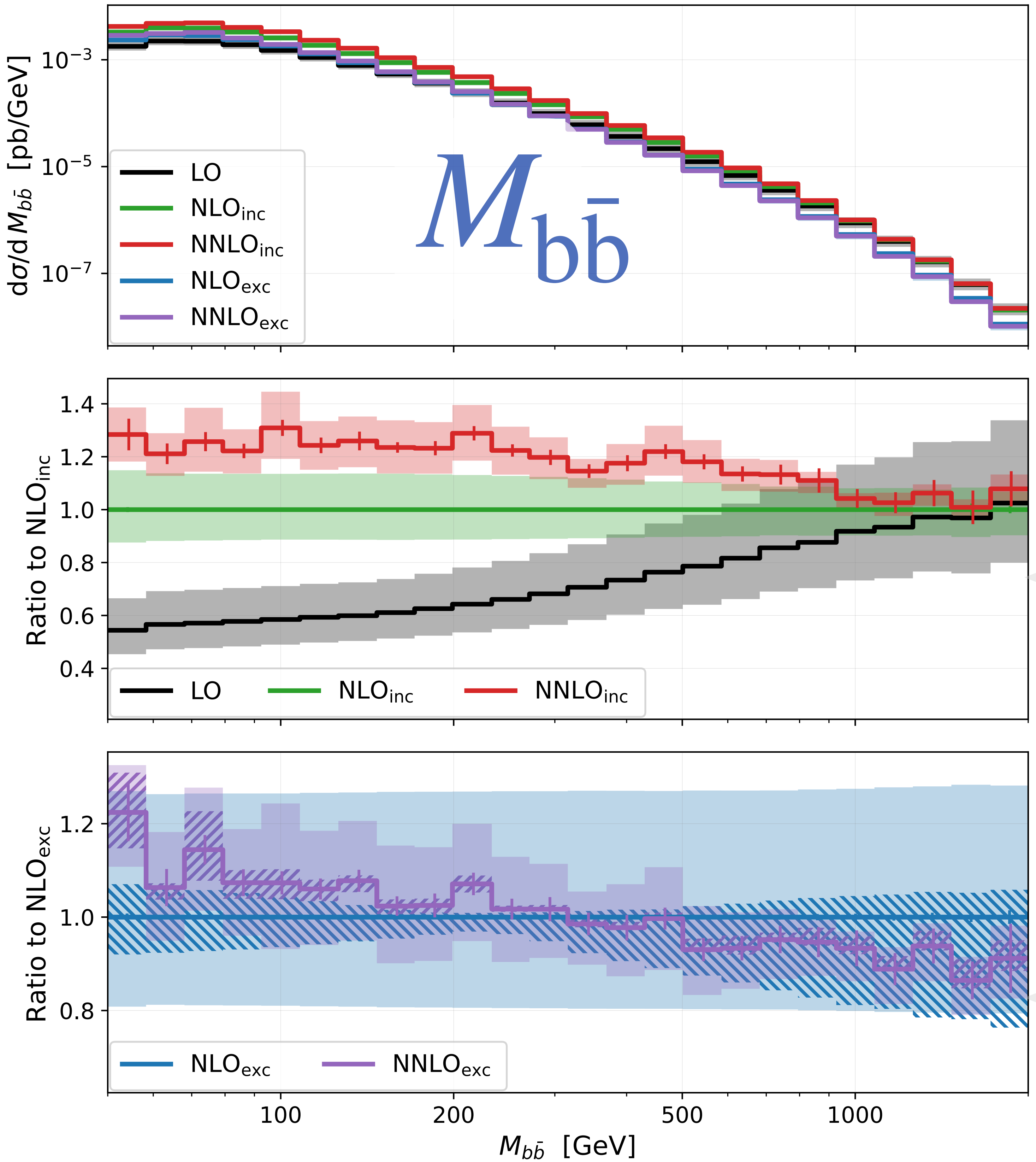
Tri-jet Production [Czakon, Mitov, Poncelet '21]

- *most complex* subtraction for $2 \to 3$
- non-degenerate Born kinematics
  ↪ event shapes, TEEC, azimuth. decorr.
- $R_{3/2} = \mathrm{d}\sigma_{3j}/\mathrm{d}\sigma_{2j}$ ⇝ α_s



$pp \to Wb\bar{b}$ [Hartanto, Poncelet, Popescu, Zoia '22]

- $2 \to 3$ with *external mass*
  ⇝ bottleneck: 2-loop amplitudes
- study bottom, BG to WH & single-top …

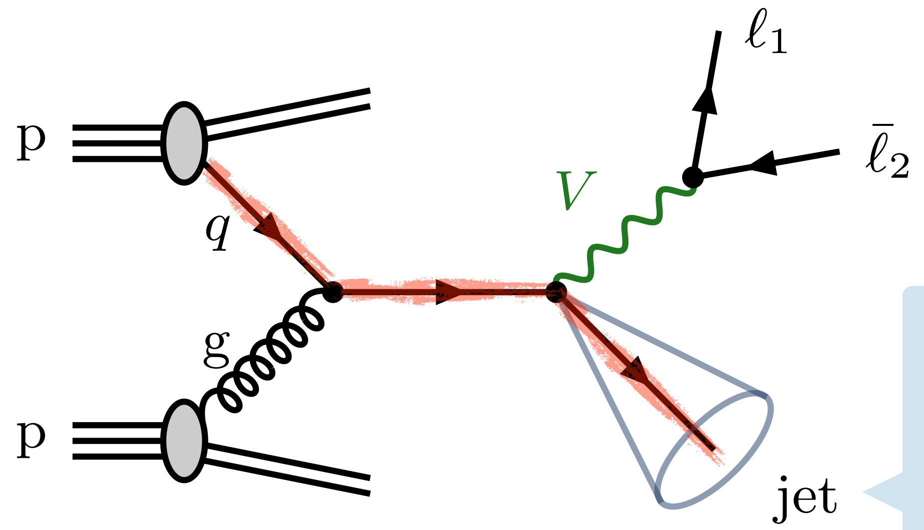


inc.: $Wb\bar{b} + \geq 0$ jets
sensitive to soft $b\bar{b}$?

exc.: $Wb\bar{b} + 0$ jets
hashed: scales
solid fill: $\Delta^2_{0j} = \Delta^2_{\geq 0j} + \Delta^2_{\geq 1j}$

Beyond "Standard" Calculations

- adding flavour
  - Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, AH, Majer '20]
  - W+c-jet [Czakon, Mitov, Pellen, Poncelet '20]



R. Gauld

last year:
IRC safe "anti-k_T"

- adding masses
  - $pp \to WH\ (H \to b\bar{b})$ [Behring, Bizoń, Caola, Melnikov, Röntsch '20]
  - $pp \to b\bar{b}$ [Catani, Devoto, Grazzini, Kallweit, Mazzitelli '21]

A. Ratti, J. Mazzitelli
L. Buonocore

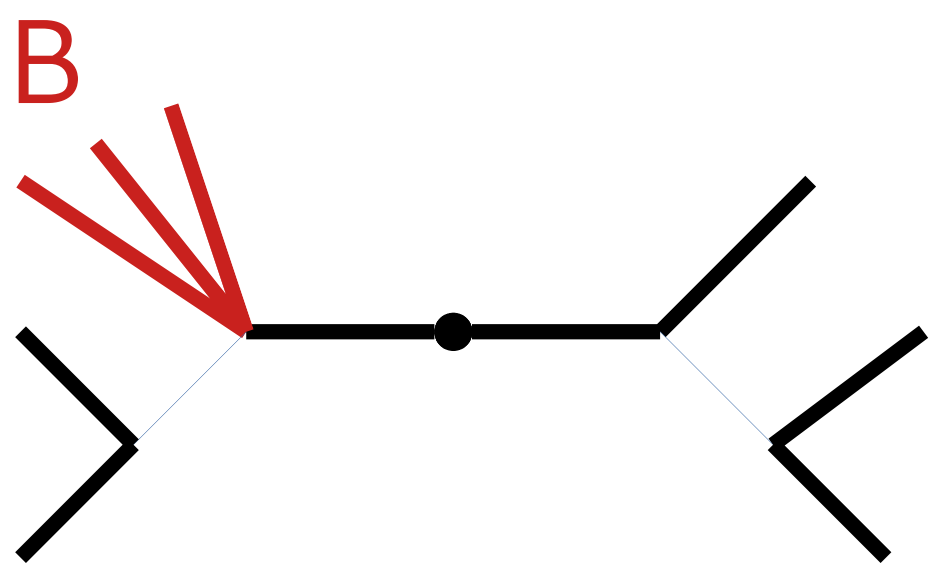
- identified particles / fragmentation
  - hadron fragmentation [Catani, Devoto, Grazzini, Kallweit, Mazzitelli '21]
  - isolated photons [Gehrmann, Schürmann '22; + Chen, Glover, Höfer, AH '22]
- beyond approximations
  - non-factorizable corrections
    - VBF [Liu, Melnikov, Penin '19]; [Dreyer, Karlberg, Tancredi '20];
    - single-t [Brønnum-Hansen, Melnikov, Quarroz, Signorile-Signorile, Wang '22]

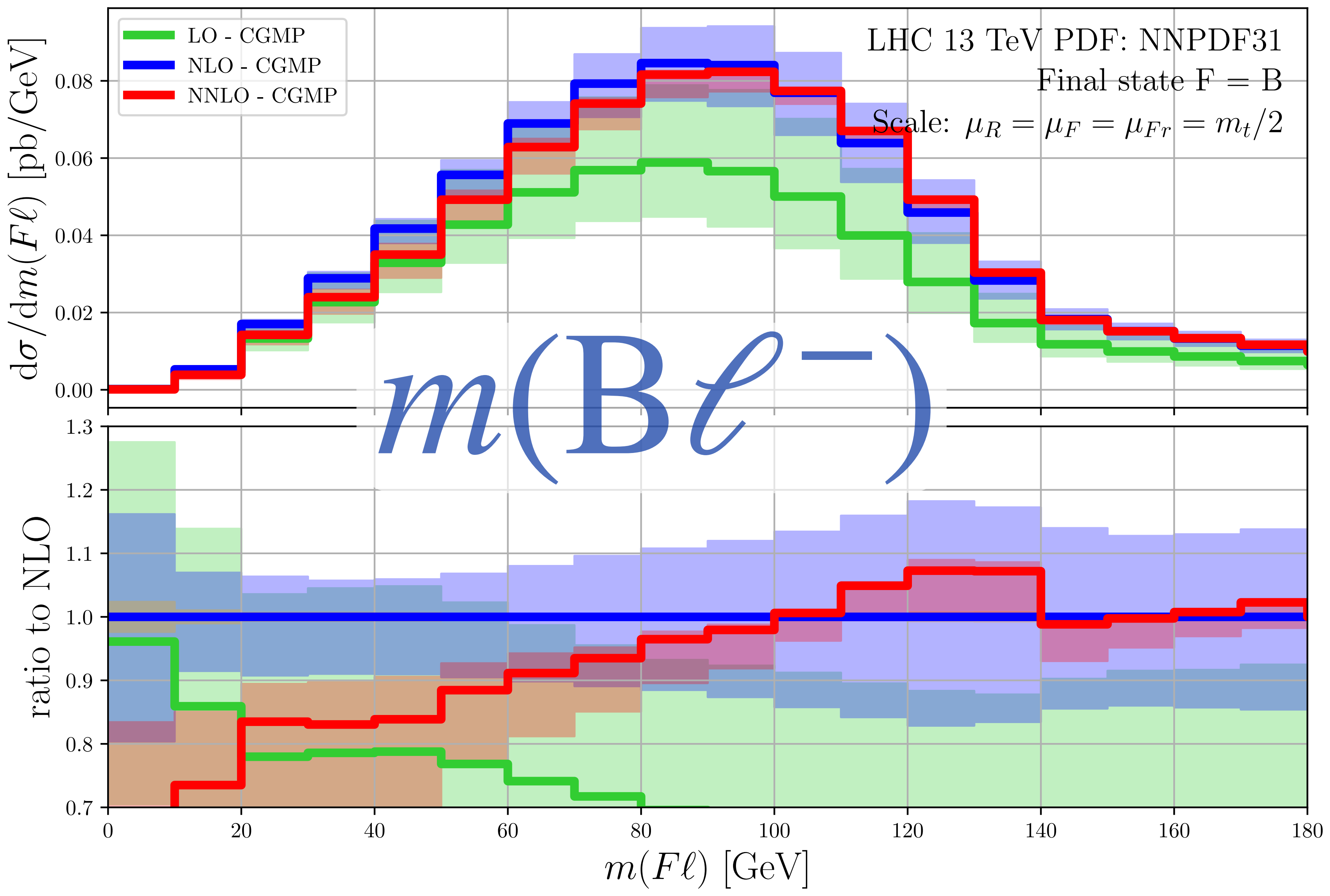
C.Y. Wang

  - Higgs beyond HTL ($m_t \to \infty$) [Czakon, Harlander, Klappert, Niggetiedt '20]
- NNLO ♡ PS *separate session*

B-Hadron in $t\bar{t}$

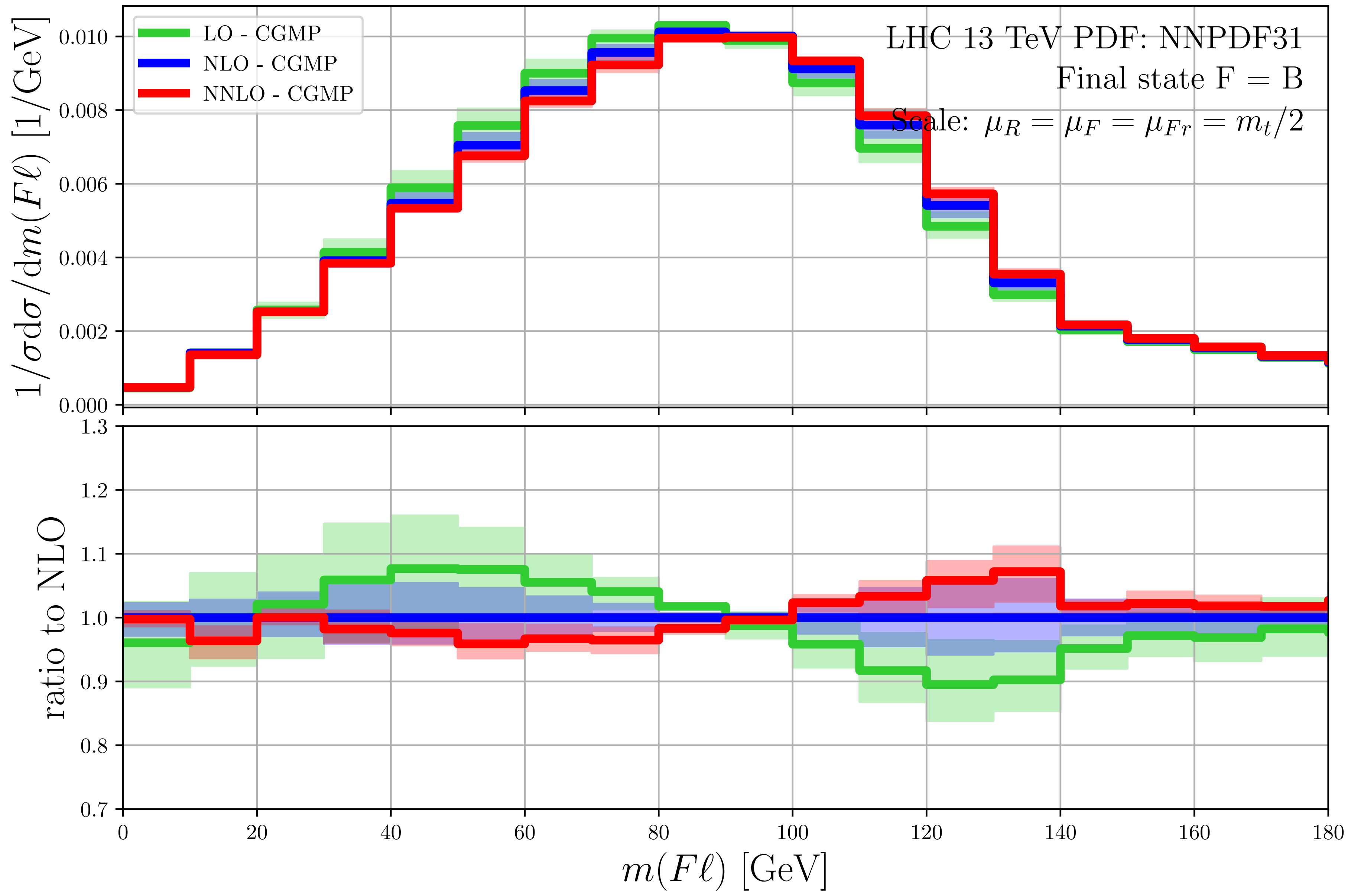
[Czakon, Generet, Mitov, Poncelet '21,'22]





non-overlap
$m(B\ell) \lesssim 50\,\mathrm{GeV}$

normalized
⊖ jet cuts



- $t\bar{t}$ ⇝ high purity & statistics
- B-hadrons measured precisely
  ↪ precise m_t extraction?
- $m_t \gg m_b$
  ↪ small power corrections
- extract $D_{i\to B}$ from e^+e^- data

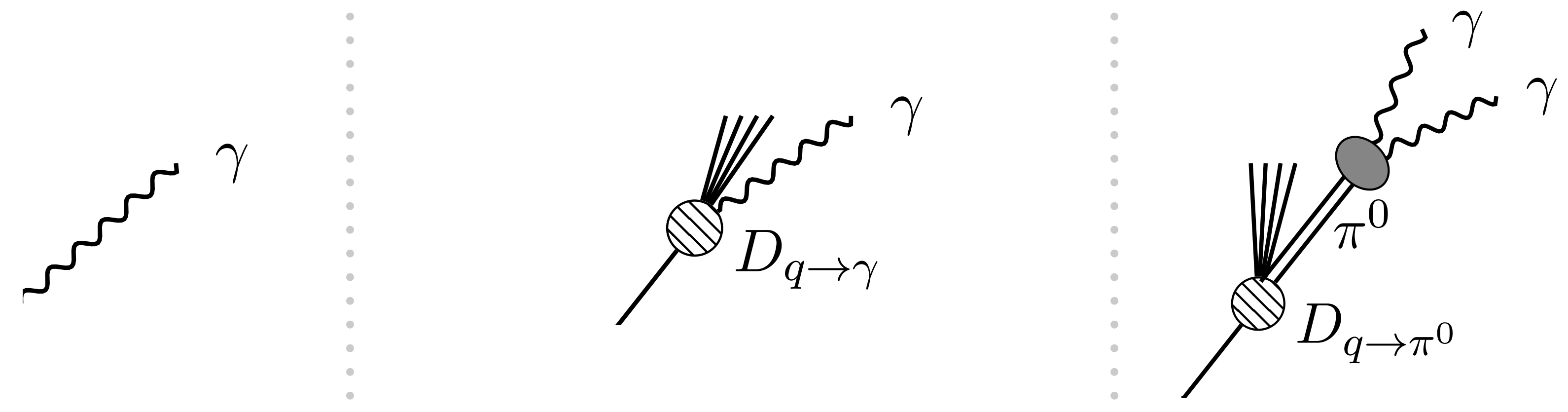
shape sensitive to m_t

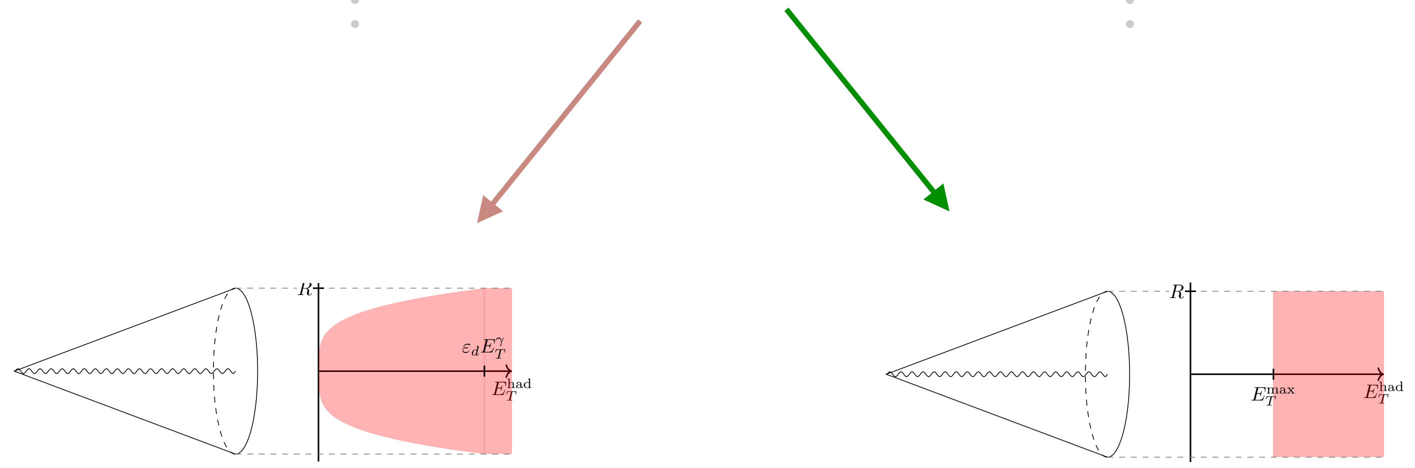
δ_{NNLO} shape distortion
⇝ $\Delta m_t \sim 1\,\mathrm{GeV}$

ISOLATED PHOTONS $\gamma + \text{jet}$

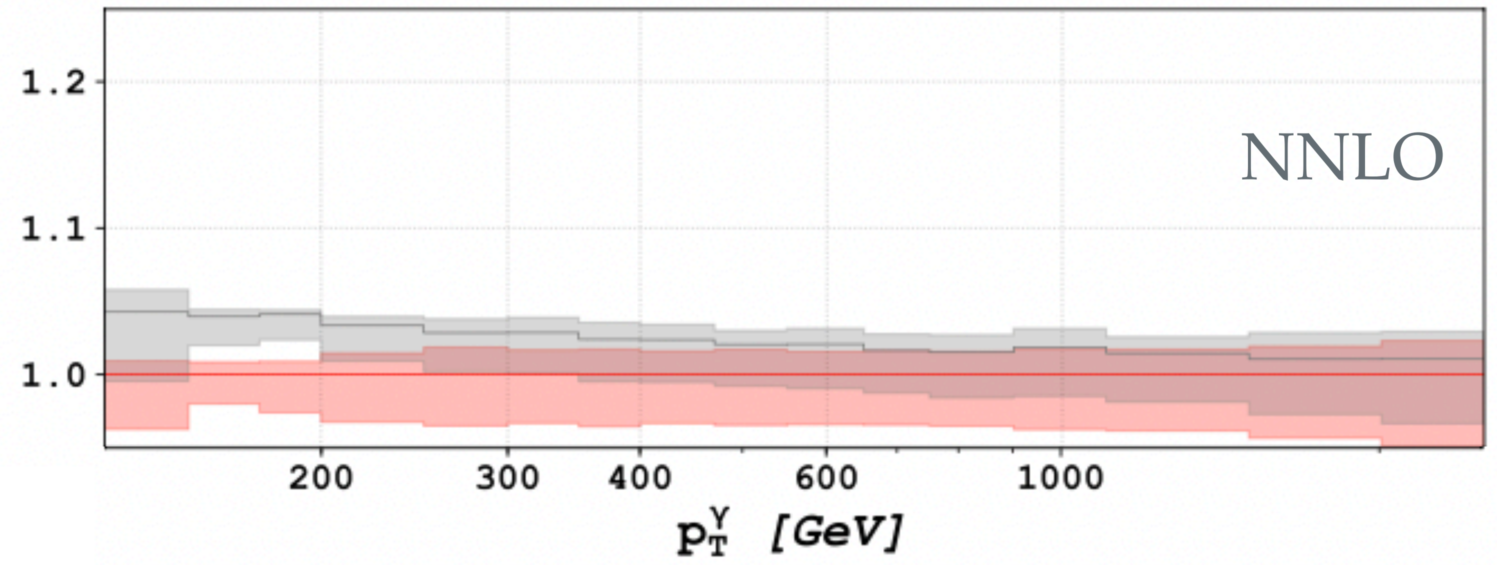
[Chen, Gehrmann, Glover, Höfer, AH, Schürmann '22]

photons:	prompt	fragmentation	decays
EXP	✓	✓	✗
TH	✓	✗ (✓)	✗

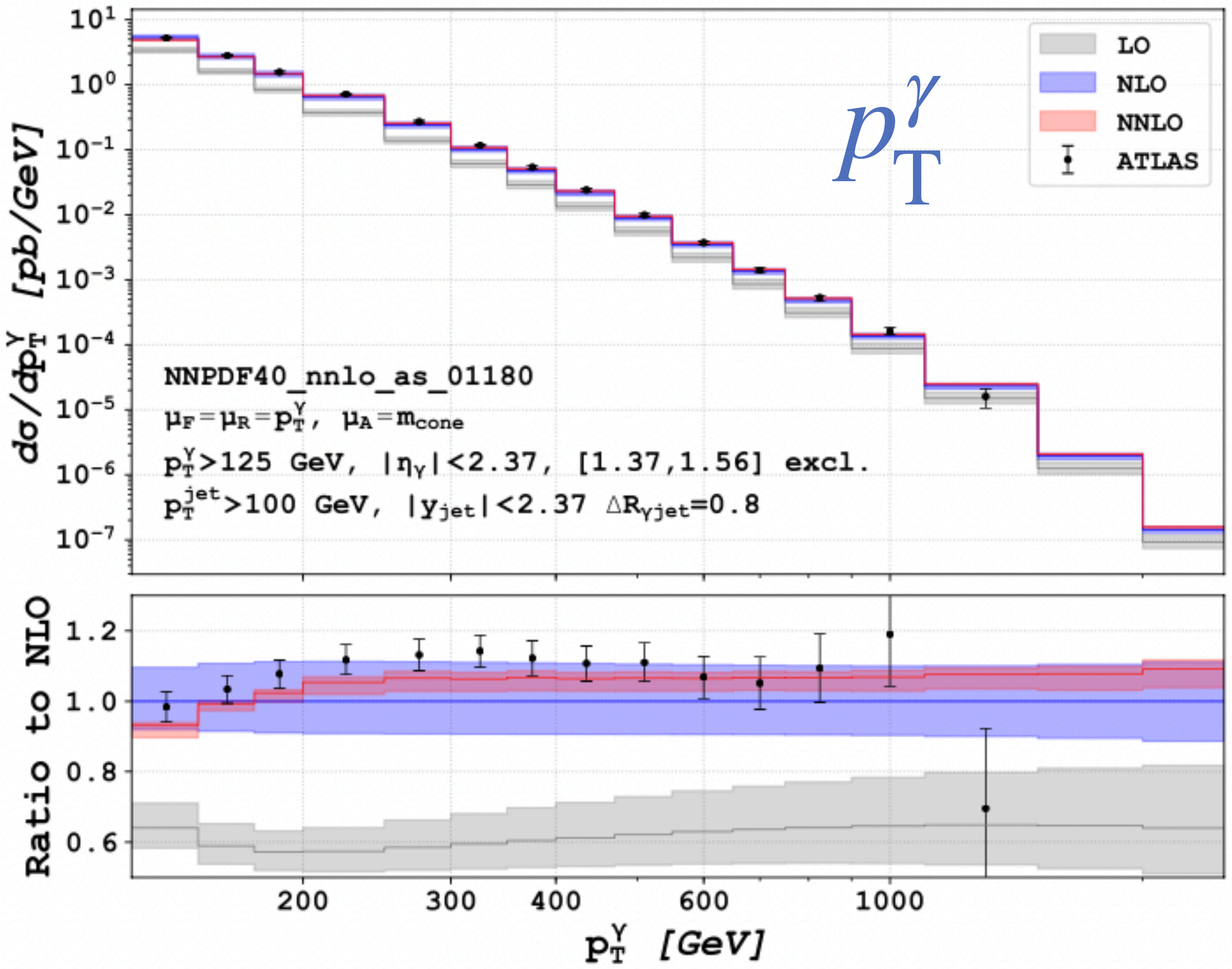




smooth cone ⟿ fixed cone



maybe ok at NLO; **matters** at NNLO



Mixed QCD–EW Corrections for Drell–Yan

$$d\sigma = d\sigma_{LO}\left(1 + \left(\frac{\alpha_s}{2\pi}\right)\delta^{(1,0)} + \left(\frac{\alpha}{2\pi}\right)\delta^{(0,1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\delta^{(2,0)} + \left(\frac{\alpha_s}{2\pi}\right)\left(\frac{\alpha}{2\pi}\right)\delta^{(1,1)} + \cdots\right)$$

notation

$$\sigma^{\mathrm{NLO}_{s\oplus ew}} \sim 1,\ \delta^{(1,0)},\ \delta^{(0,1)}$$

$$\sigma^{\mathrm{NNLO}_{s\otimes ew}} \sim 1,\ \delta^{(1,0)},\ \delta^{(0,1)},\ \delta^{(1,1)}$$

$$\sigma^{\mathrm{NNLO}_{s\otimes ew}}_{\text{naive prod.}} \sim 1,\ \delta^{(1,0)},\ \delta^{(0,1)},\ \delta^{(1,0)} \times \delta^{(0,1)}$$

resonant / on-shell

- pole expansion [Dittmaier, Huss, Schwinn '14,'15]
- on-shell Z (QCD×QED) [Delto, Jaquier, Melnikov, Röntsch '19]
- σ_Z^{tot} [Bonciani, Buccioni, Rana, Vicini '20]
- on-shell
  [Buccioni, Caola, Delto, Jaquier, Melnikov, Roentsch '20]
  [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

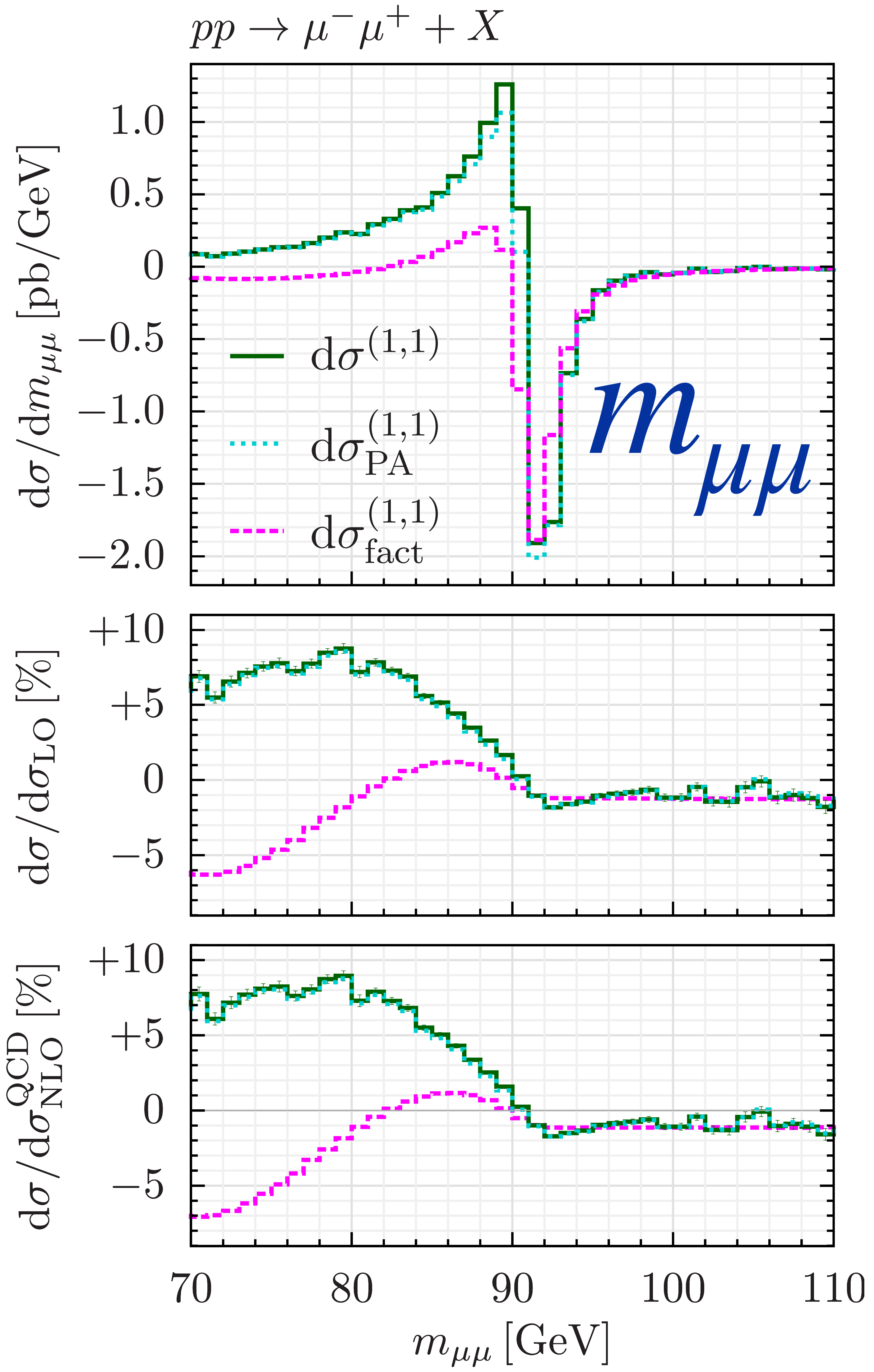
off-shell

- W
  [Buonocore, Grazzini, Kallweit, Savoini, Tramontano '21]
- Z
  [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '21]
  [Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Rontsch, Signorile-Signorile '22]

$\mathcal{O}(\alpha_s\,\alpha)$ — Resonance Region

bare muons
("dressing" ⇝ × 1/2)

[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '21]

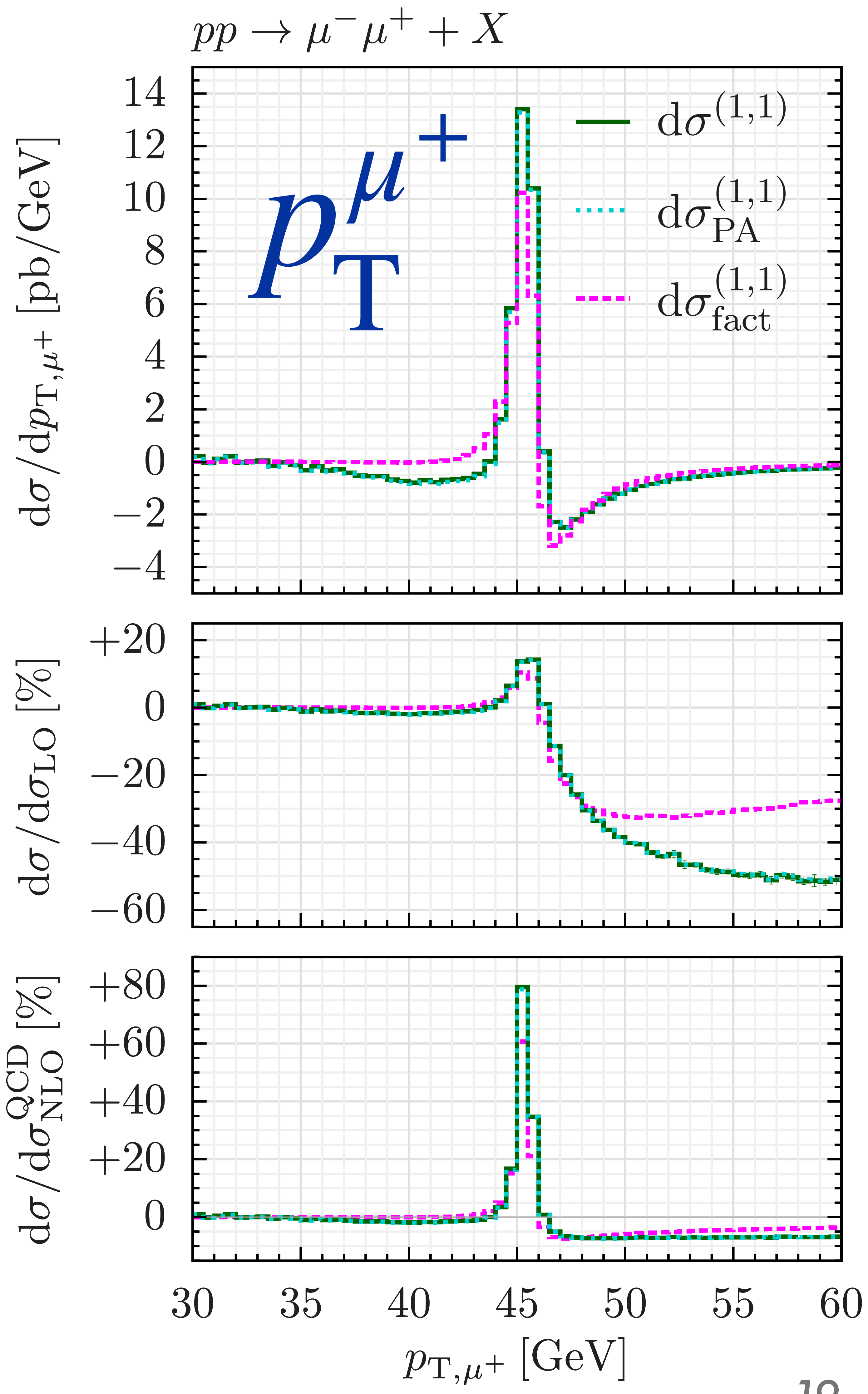


$\sigma^{\mathrm{NNLO}_{s\otimes ew}}$

$\sigma^{\mathrm{NNLO}_{s\otimes ew}}_{\mathrm{PA}}$

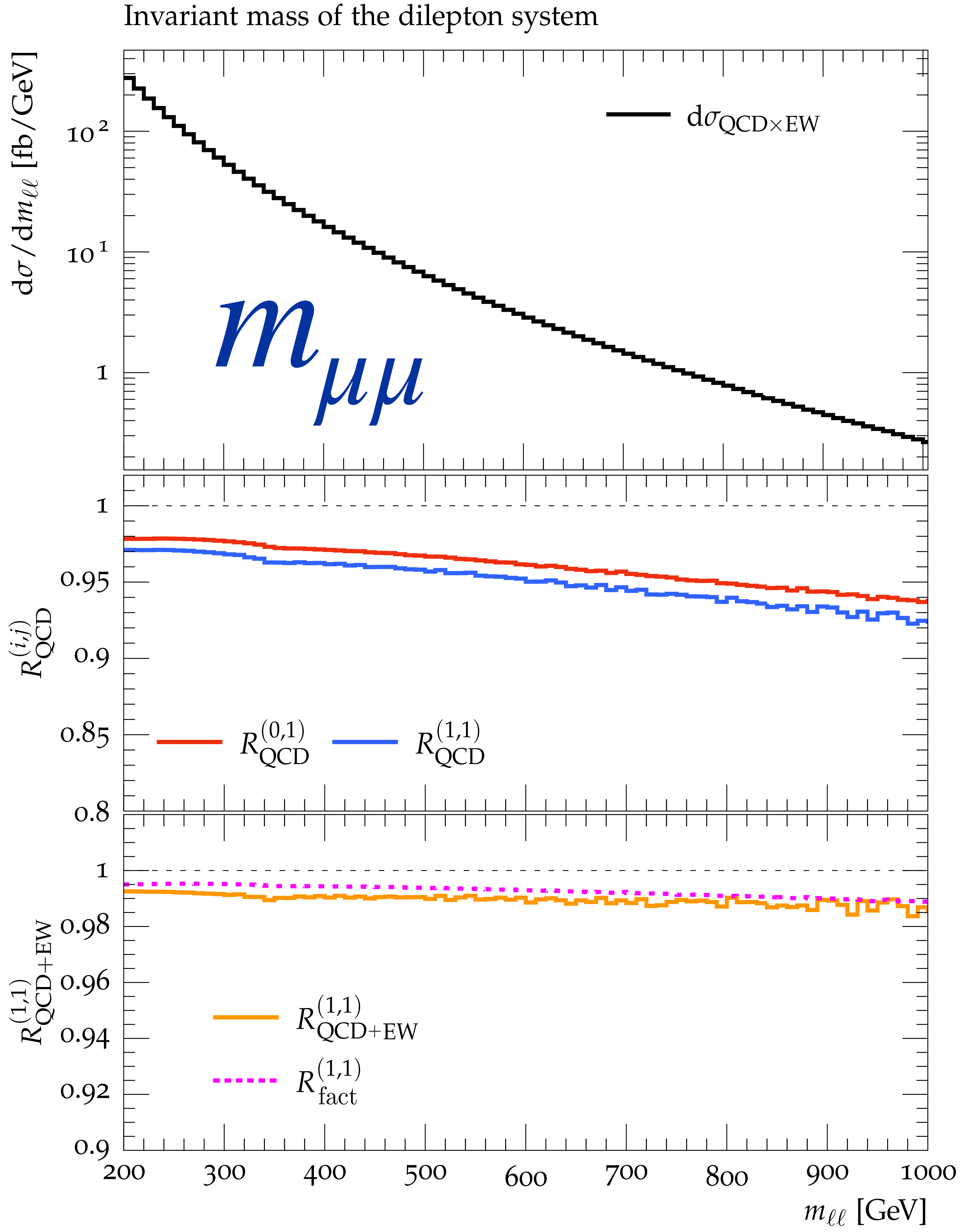
$\sigma^{\mathrm{NNLO}_{s\otimes ew}}_{\text{naive prod.}}$

- naive product not able to capture kinematic effects
  - ↪ fails below resonance ($m_{\ell\ell}$)
  - ↪ fails away from shoulder (p_T^μ)
- pole approximation (PA)
  - ↪ well-captures full result here



$\mathcal{O}(\alpha_s\,\alpha)$ — High-Energy Tails

[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Rontsch, Signorile-Signorile '22]



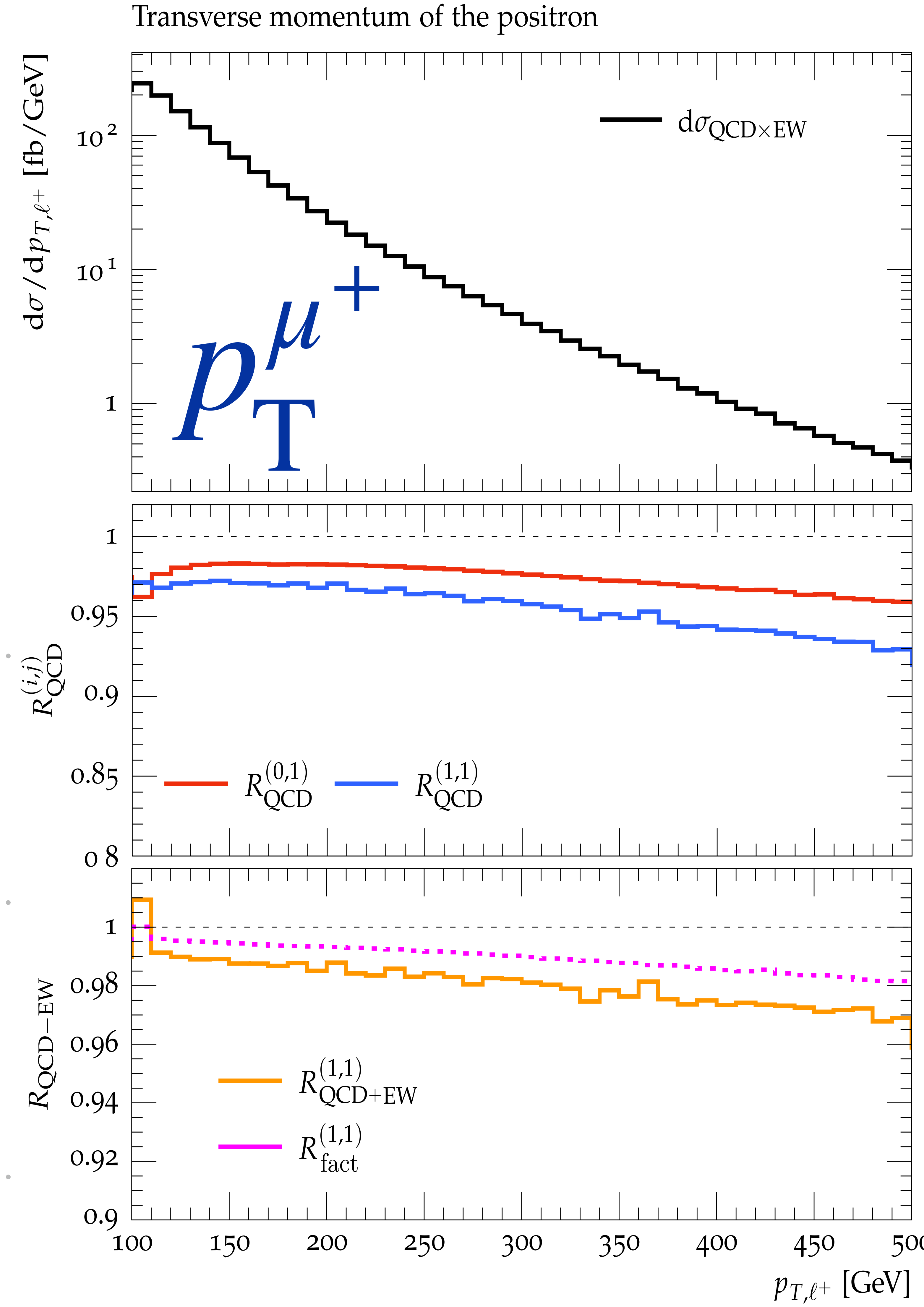
- naive product
  - ↪ works well at high-$m_{\ell\ell}$
  - ↪ differences in p_T^μ spectrum
- tails ⇝ Sudakov (non flat)
  - ↪ QCD×EW $\sim -3\,\%$

$\sigma^{\mathrm{NNLO}_{s\otimes ew}}$ (blue)

$\sigma^{\mathrm{NLO}_{s\oplus ew}}$ (red)

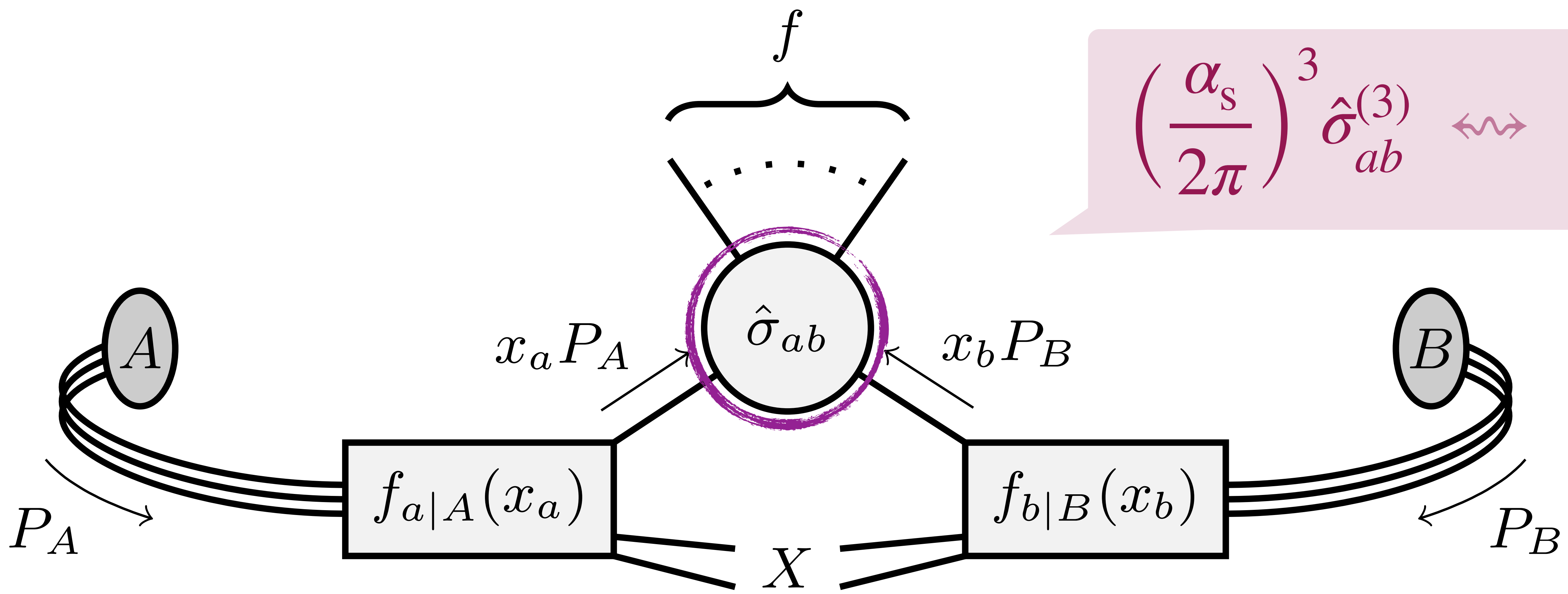
$\sigma^{\mathrm{NNLO}_{s\otimes ew}}$ (orange)

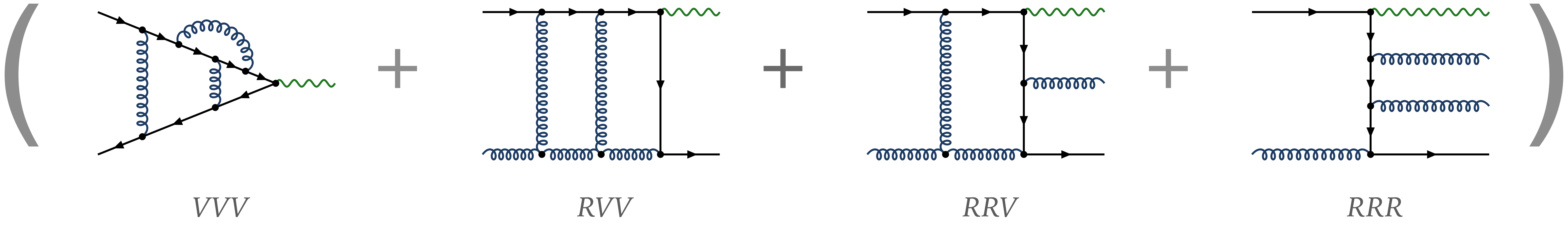
$\sigma^{\mathrm{NNLO}_{s\otimes ew}}$ naive prod. (magenta, dashed)



N³LO — A New Frontier

$\left(\frac{\alpha_s}{2\pi}\right)^3 \hat{\sigma}_{ab}^{(3)}$ ⇝ largely limited to "$2 \to 1$"

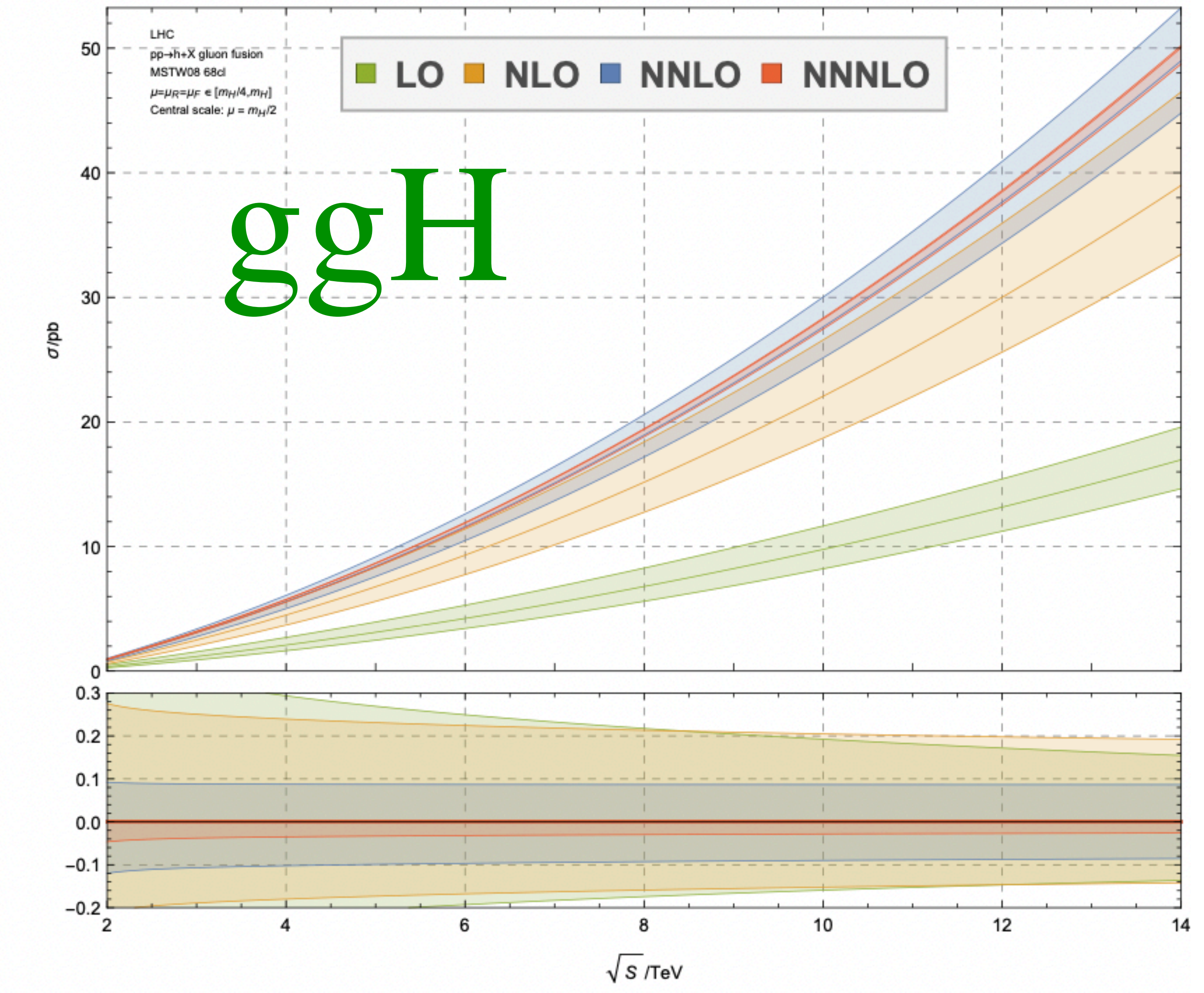


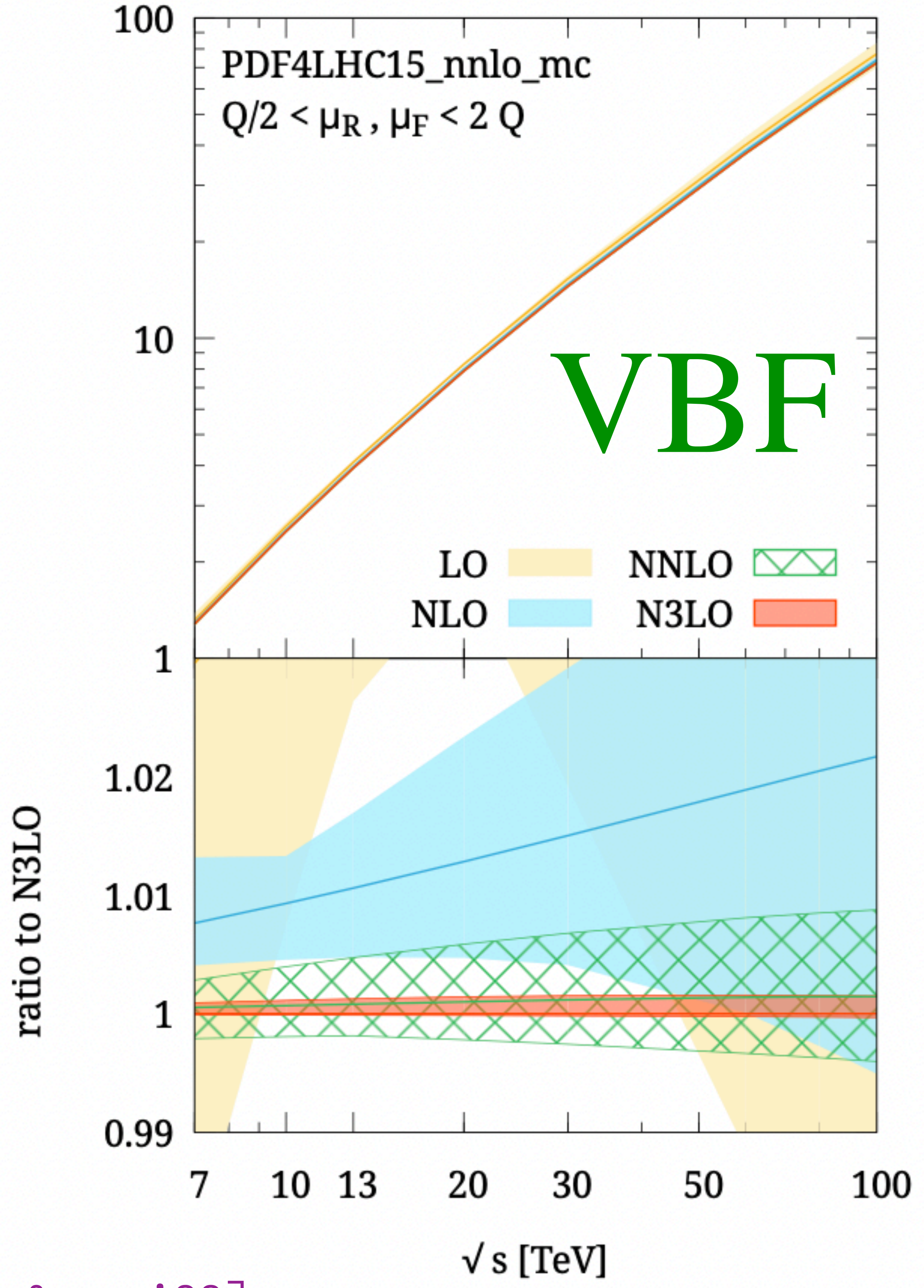


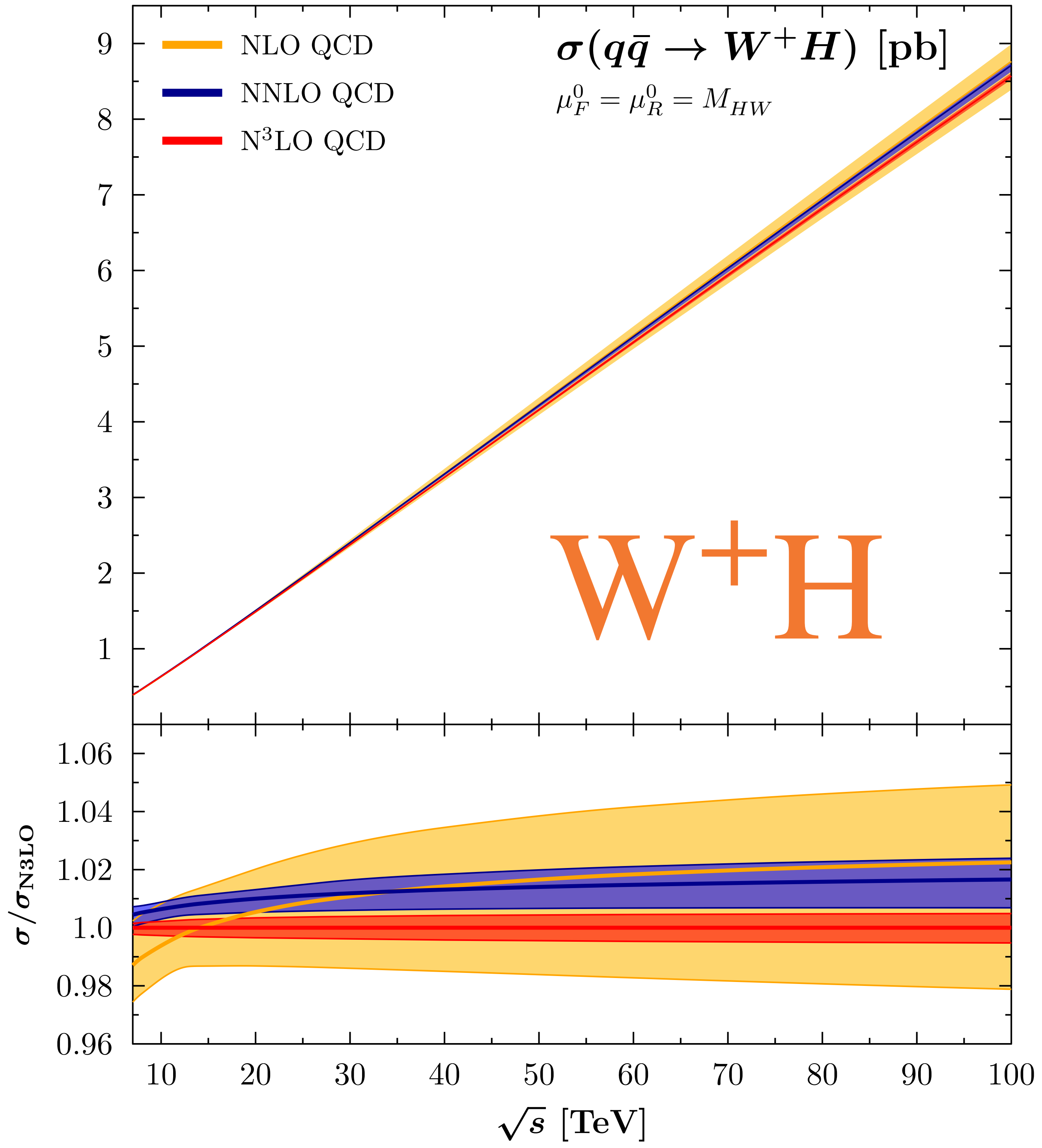
N3LO Inclusive Cross Sections

- $gg \to H$ ✓
  [C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger '15]
- VBF-H ✓, VBF-HH ✓
  [F. Dreyer, A. Karlberg '16, '18]
- $b\bar{b} \to H$ ✓
  [C. Duhr, F. Dulat, B. Mistlberger '19]
- $pp \to \gamma^*$?, $pp \to W^\pm$?
  [C. Duhr, F. Dulat, B. Mistlberger '20]
- $gg \to HH$ ✓
  [L. Chen, H. Li, H. Shao, J. Wang '20]
- $pp \to \gamma^*/Z$?
  [C. Duhr, B. Mistlberger '21]
- $pp \to VH$?
  [J. Baglio, C. Duhr, B. Mistlberger, R. Szafron '22]

✓ nice convergence of pert. expansion



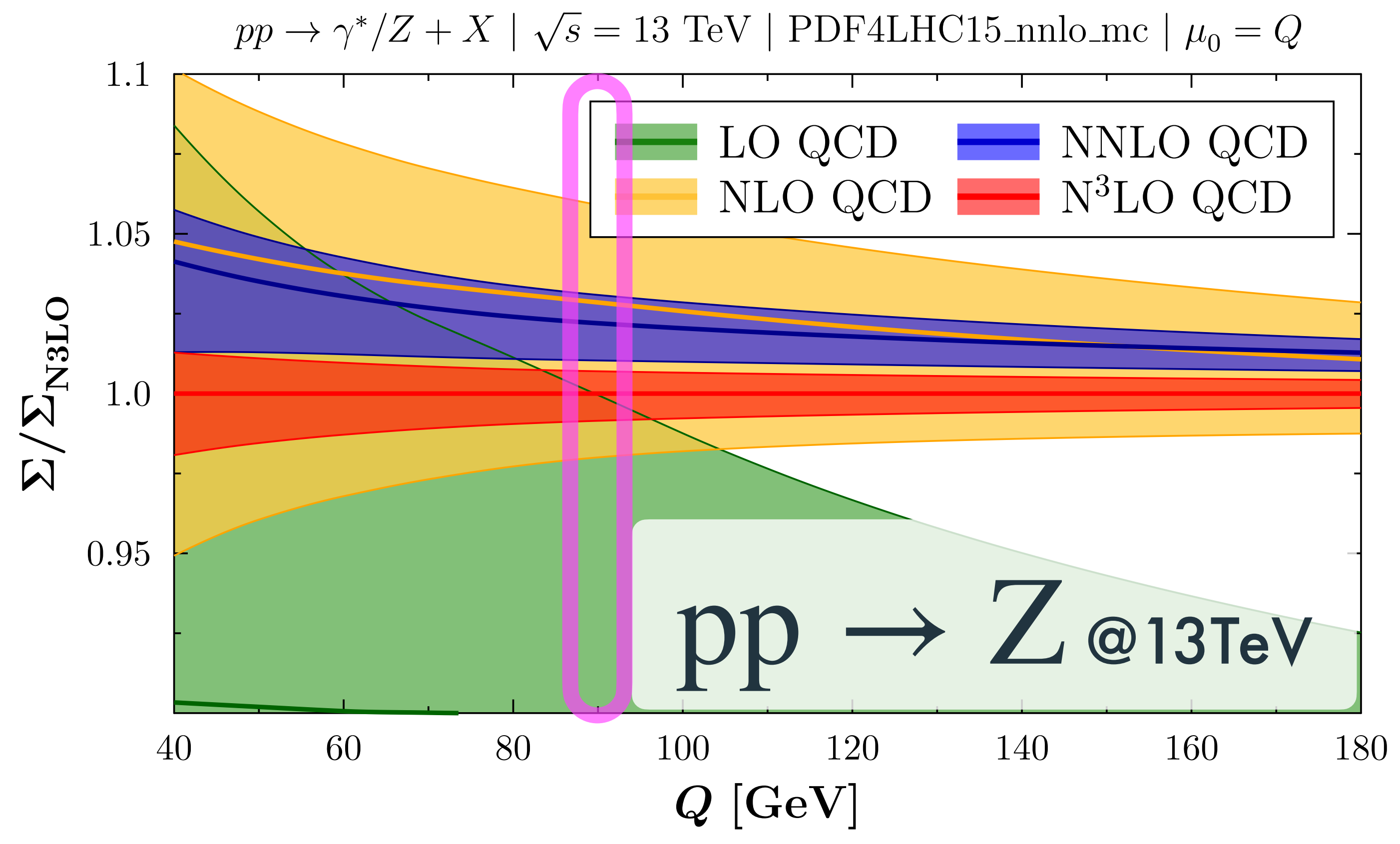


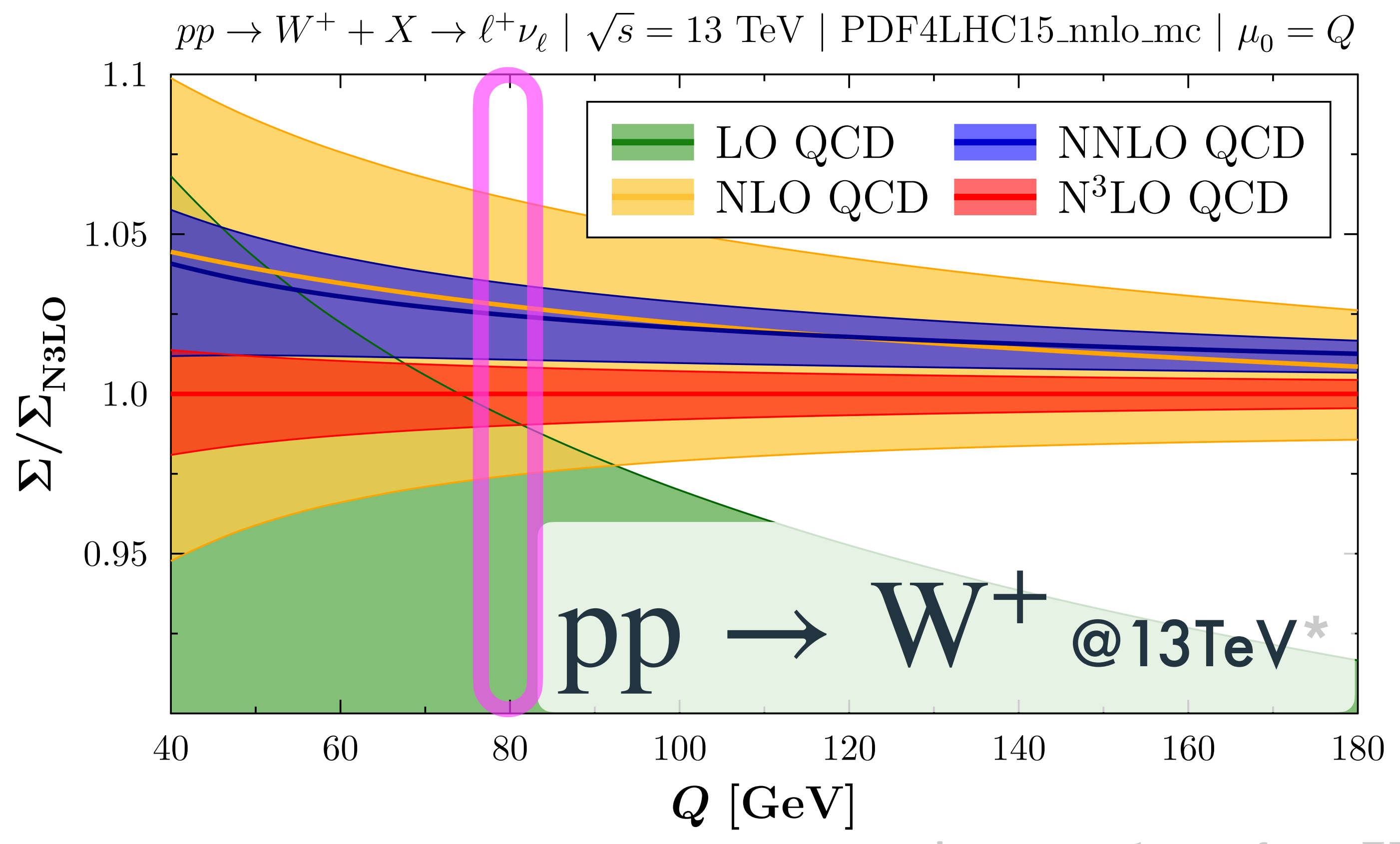


? poor convergence inherited from Drell-Yan

Drell–Yan @ N3LO — Scale & PDF Dependence [Baglio, Duhr, Mistlberger, Szafron '22]

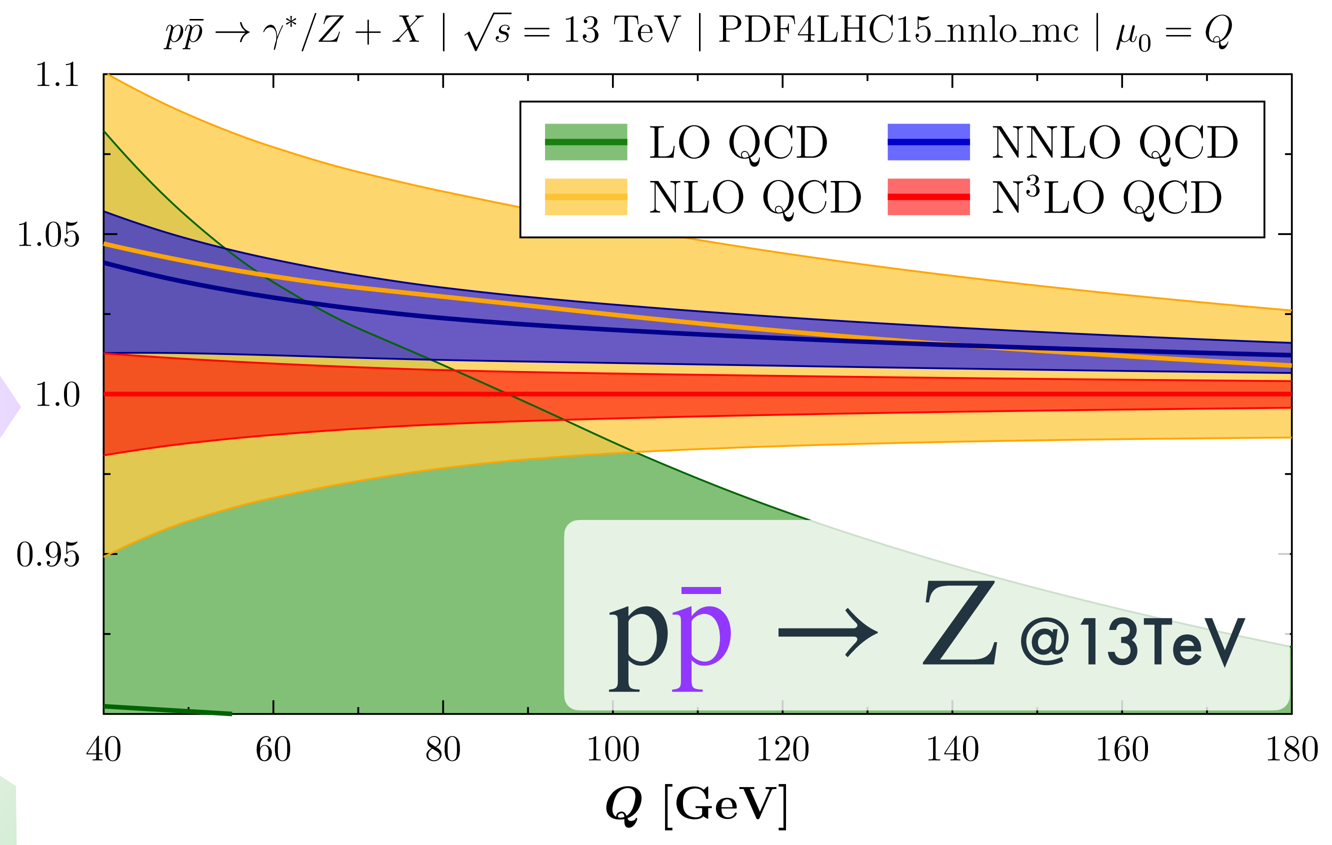
DY Processes

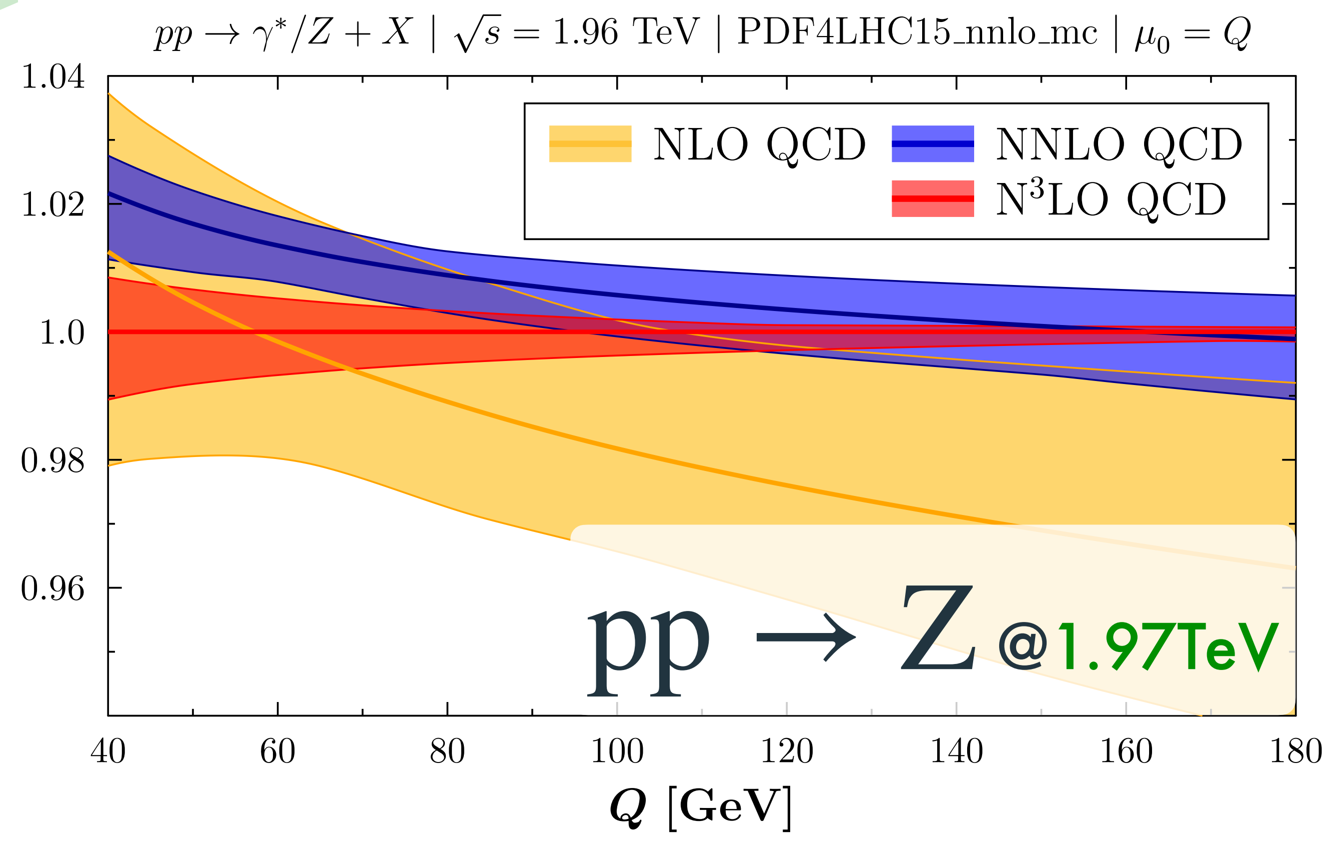




* similar for W^-

Coll. / Engy

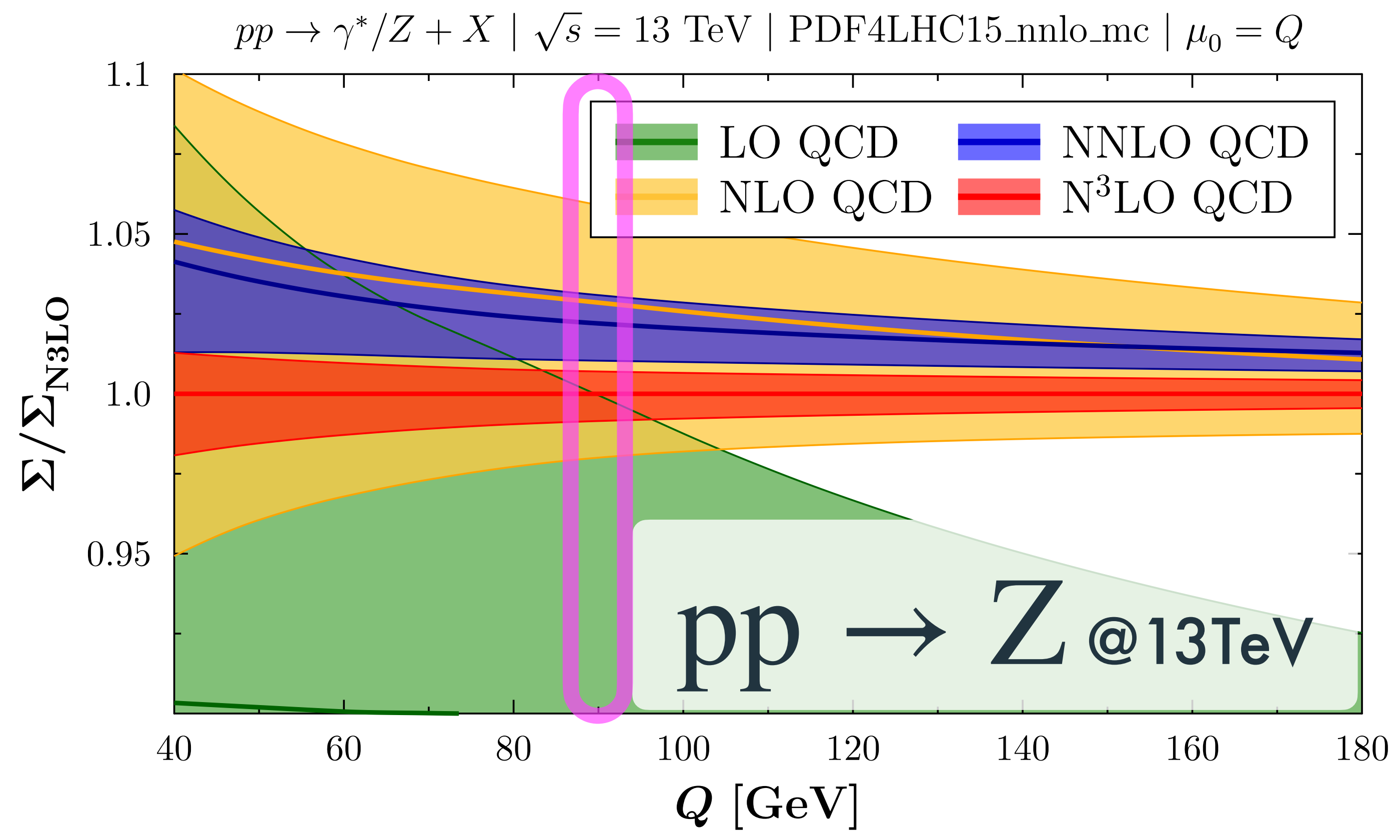


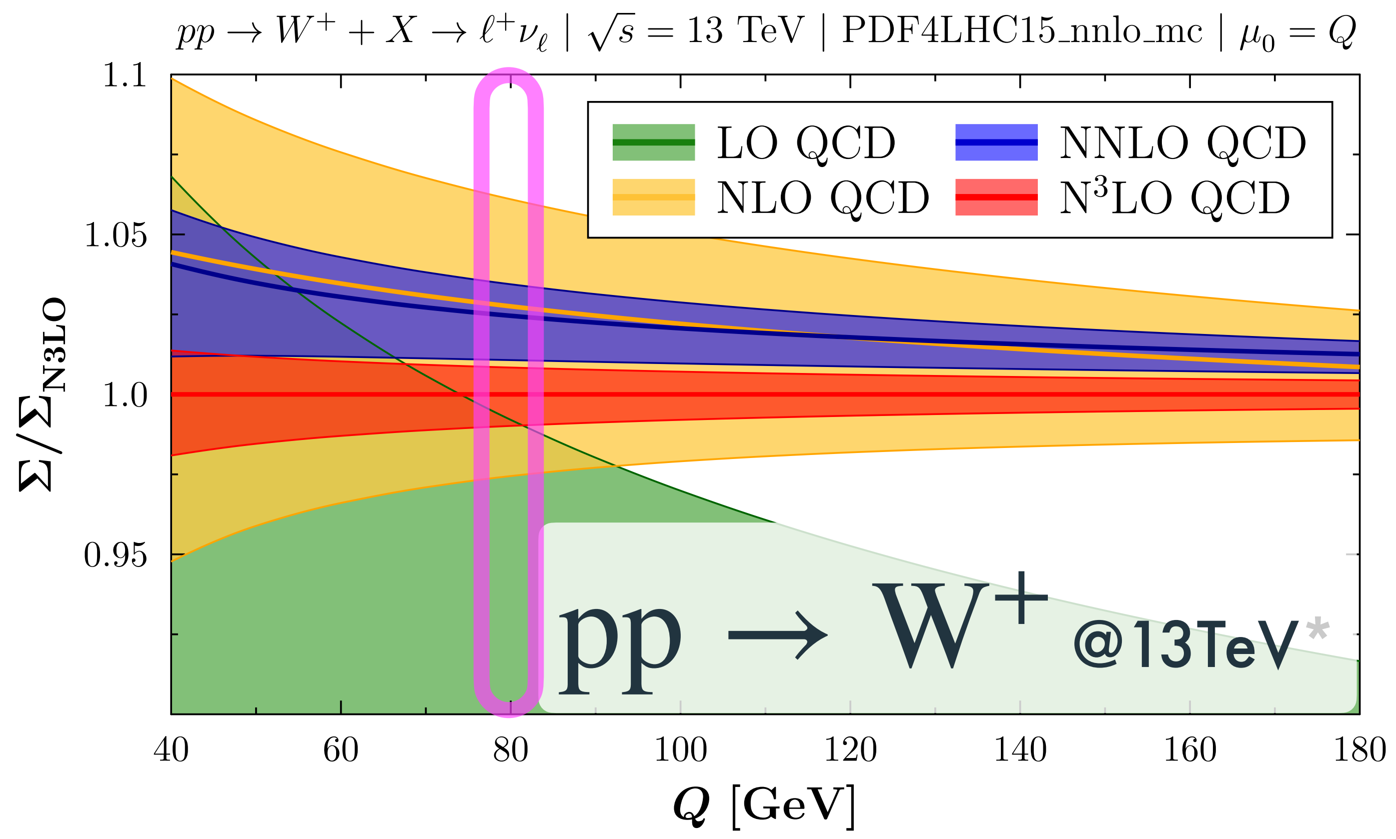


resonance region ⇝ non-overlapping bands; $\Delta^{\rm NNLO}_{\rm scl} \simeq \Delta^{\rm N^3LO}_{\rm scl}$?!

Drell–Yan @ N3LO — Scale & PDF Dependence [Baglio, Duhr, Mistlberger, Szafron '22]

DY Processes





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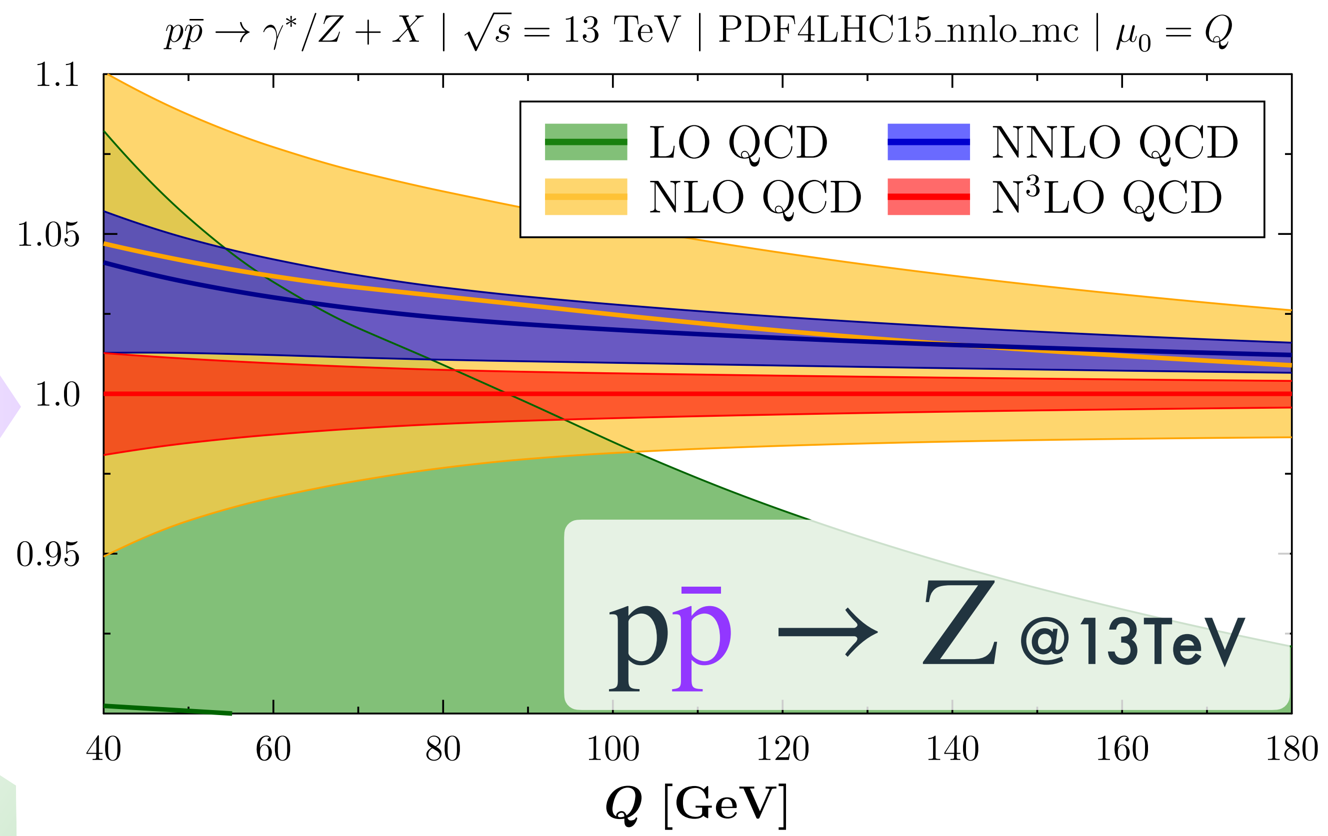
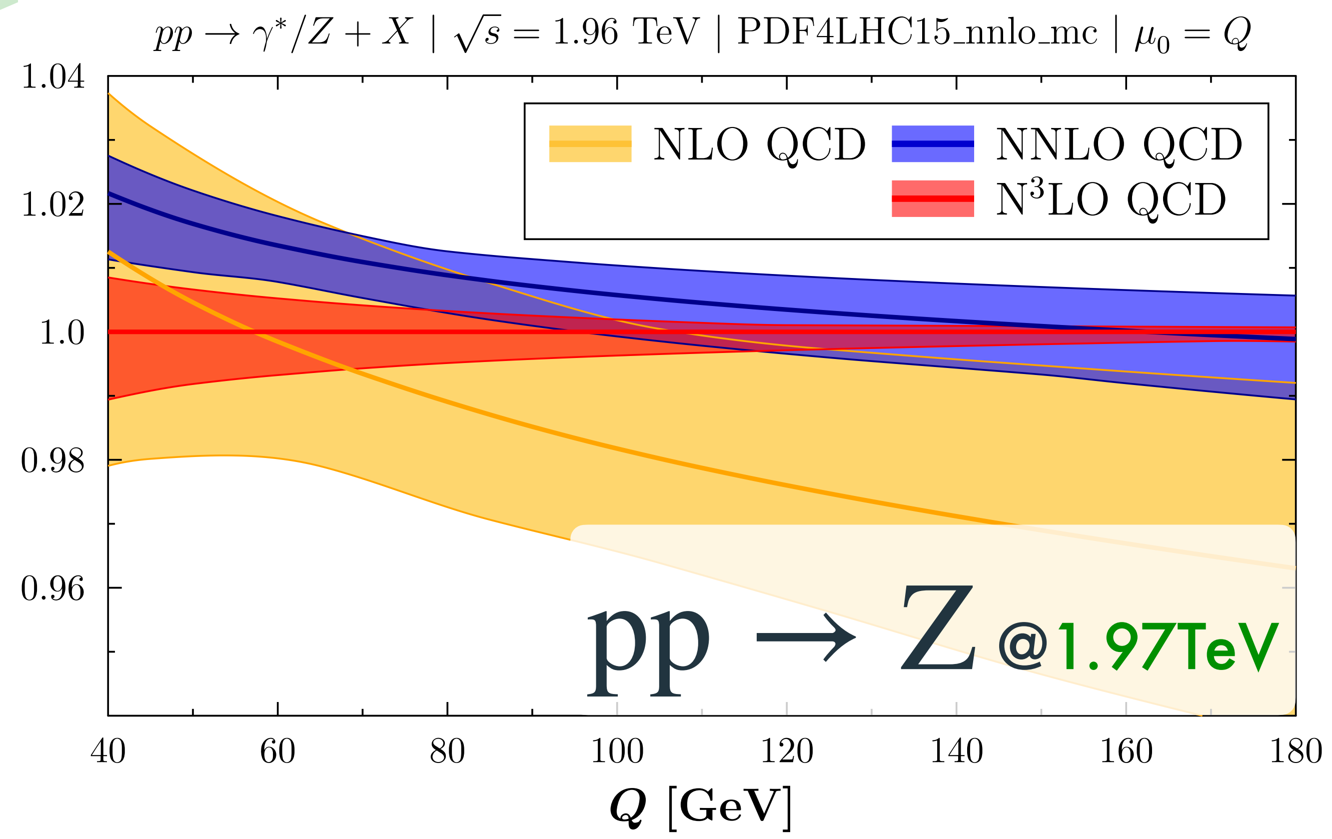
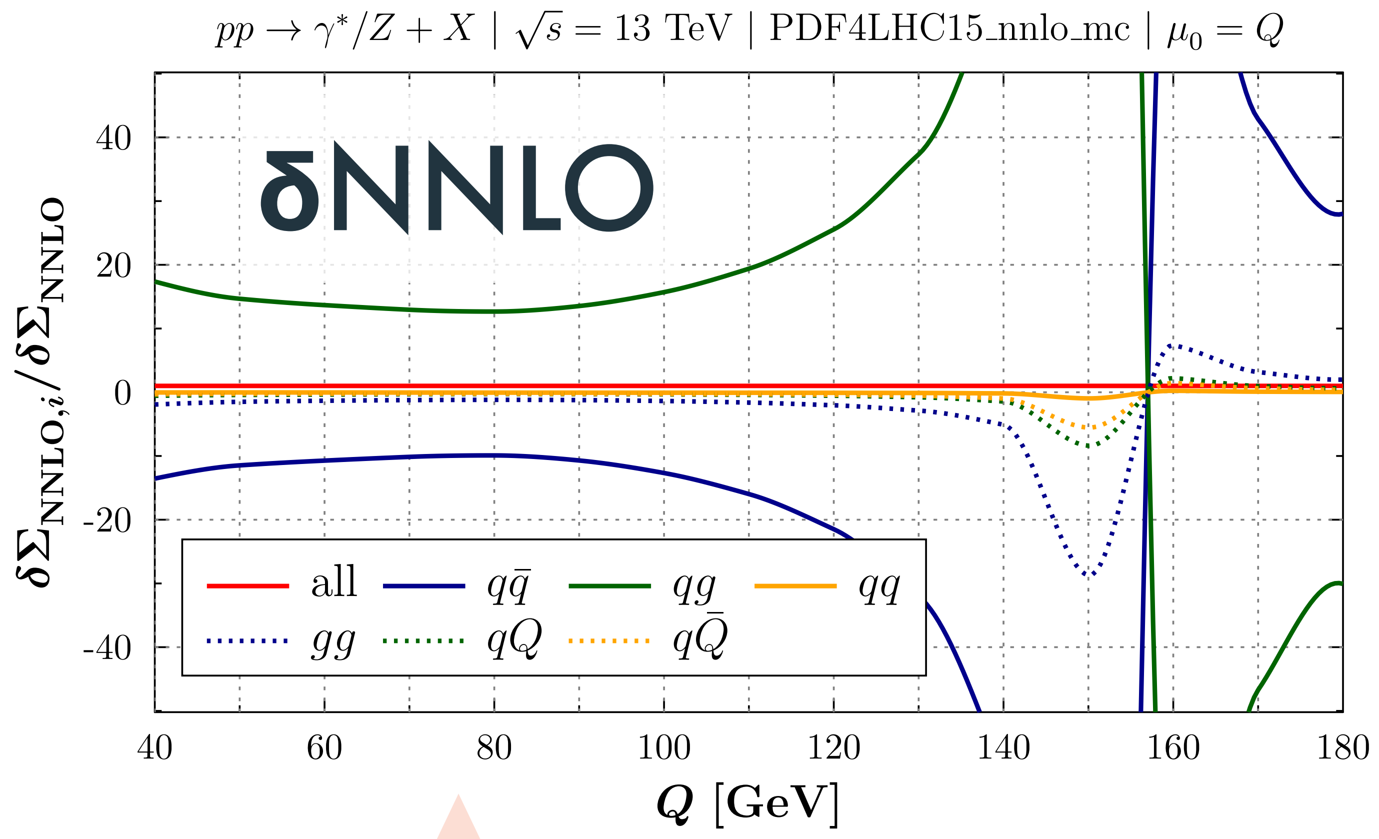
Coll. / Engy

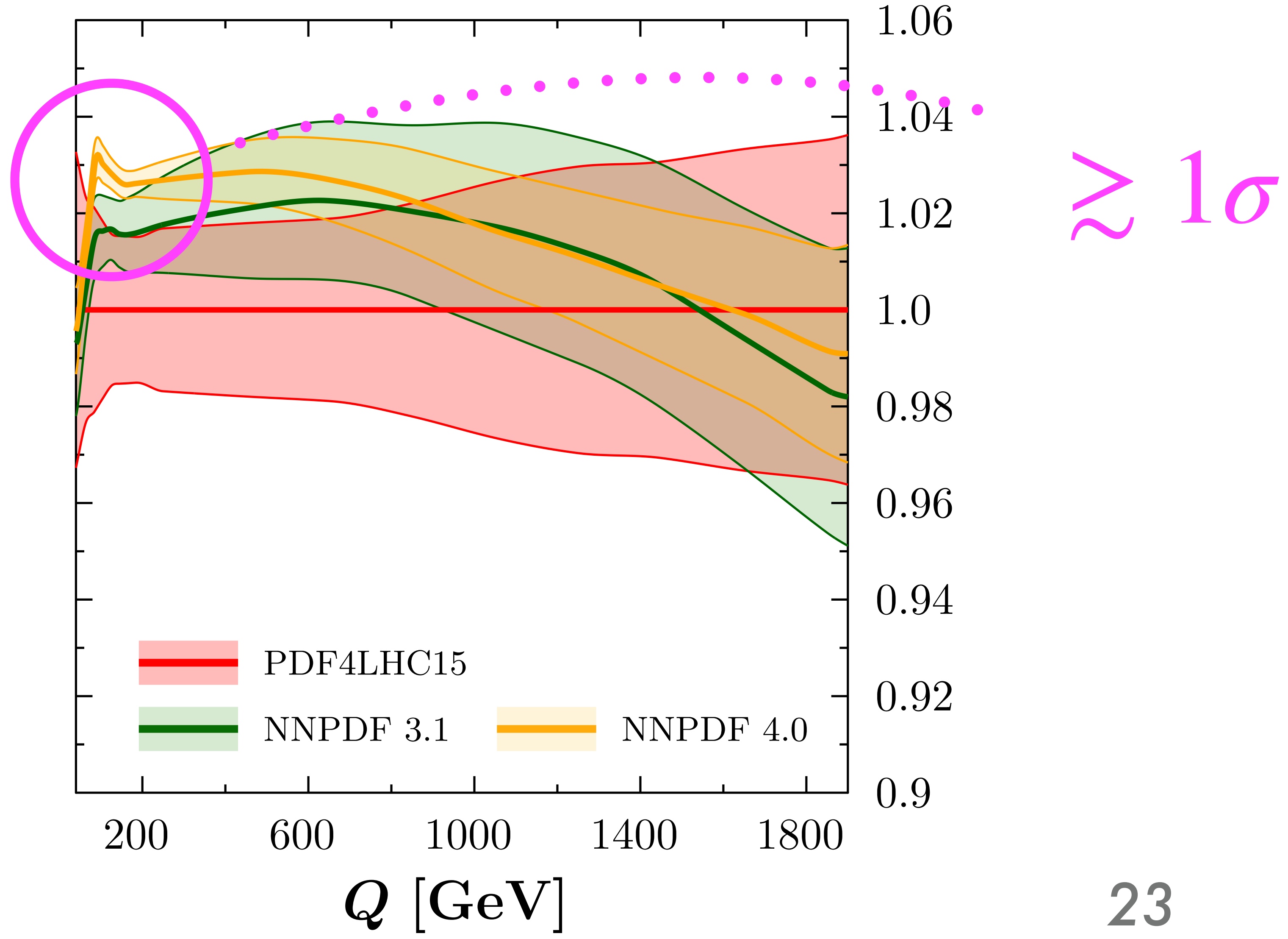






large cancellations ± 20

N3LO: reduced to ± 2



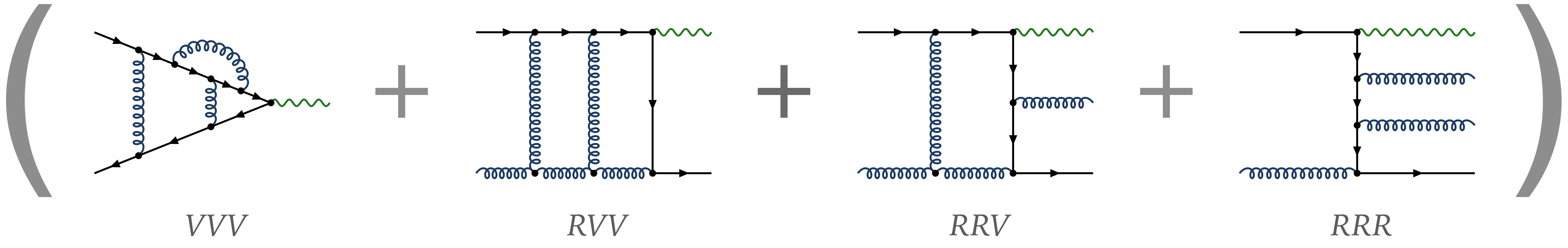
resonance region ⇝ non-overlapping bands; $\Delta_{\text{scl}}^{\text{NNLO}} \simeq \Delta_{\text{scl}}^{\text{N}^3\text{LO}}$?!

Summary of Inclusive "$2 \to 1$" Predictions

	Q [GeV]	$\delta\sigma^{\mathrm{N^3LO}}$	$\delta\sigma^{\mathrm{NNLO}}$	δ(scale)	$\delta(\mathrm{PDF}+\alpha_S)$	δ(PDF-TH)
$gg \to$ Higgs	m_H	3.5%	30%	$^{+0.21\%}_{-2.37\%}$	$\pm 3.2\%$	$\pm 1.2\%$
$b\bar{b} \to$ Higgs	m_H	-2.3%	2.1%	$^{+3.0\%}_{-4.8\%}$	$\pm 8.4\%$	$\pm 2.5\%$
NCDY	30	-4.8%	-0.34%	$^{+1.53\%}_{-2.54\%}$	$^{+3.7\%}_{-3.8\%}$	$\pm 2.8\%$
	100	-2.1%	-2.3%	$^{+0.66\%}_{-0.79\%}$	$^{+1.8\%}_{-1.9\%}$	$\pm 2.5\%$
CCDY(W^+)	30	-4.7%	-0.1%	$^{+2.5\%}_{-1.7\%}$	$\pm 3.95\%$	$\pm 3.2\%$
	150	-2.0%	-0.1%	$^{+0.5\%}_{-0.5\%}$	$\pm 1.9\%$	$\pm 2.1\%$
CCDY(W^-)	30	-5.0%	-0.1%	$^{+2.6\%}_{-1.6\%}$	$\pm 3.7\%$	$\pm 3.2\%$
	150	-2.1%	-0.6%	$^{+0.6\%}_{-0.5\%}$	$\pm 2\%$	$\pm 2.13\%$

- *K*-factors (N³LO / NNLO) ~ 2–5% ⇝ important for %-level target!
- Δ_{scl} ~ few %
- PDF uncertainties ~ 2–9% (+ few % from missing N³LO PDFs)

Going Differential



- $1/\varepsilon^6$, $1/\varepsilon^5$, …

- $1/\varepsilon^4$, $1/\varepsilon^3$, …
- *single unresolved*

- $1/\varepsilon^2$, $1/\varepsilon$
- *single unresolved*
- *double unresolved*

- *single unresolved*
- *double unresolved*
- *triple unresolved*

two methods for "$2 \to 1$"

isolate "radiating" part

double unresolved $\simeq$ V+jet @ NNLO

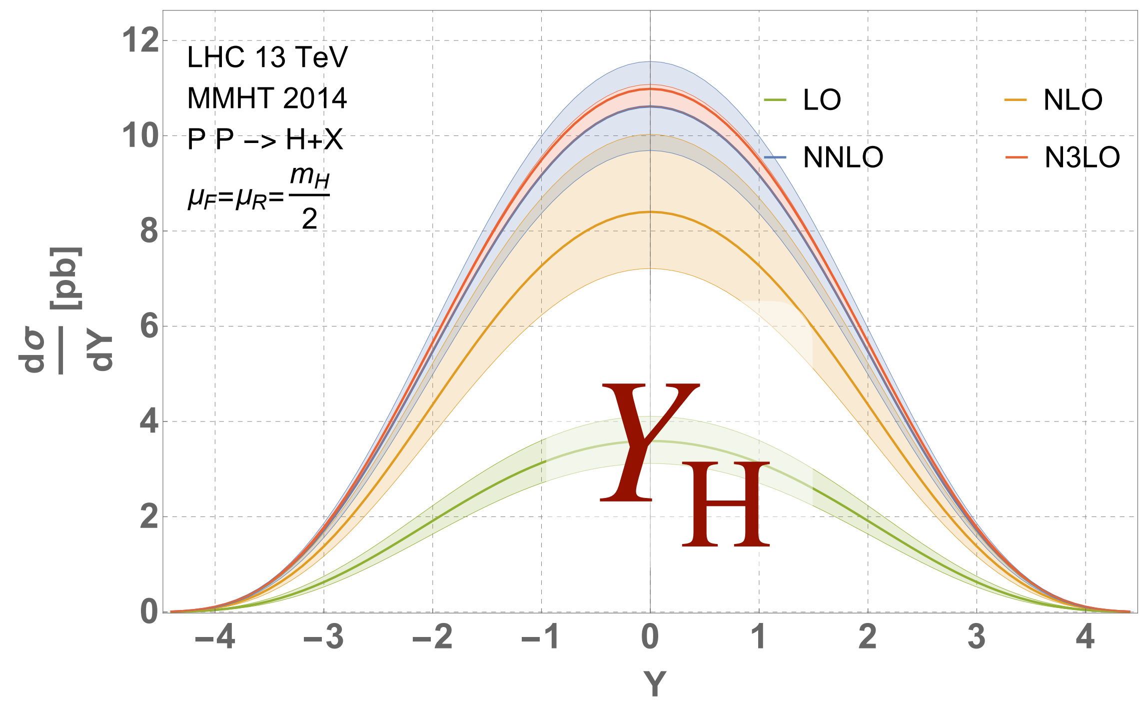
fully unresolved ($\leftrightsquigarrow$ $q_T \to 0$) $\simeq$ V @ N³LO

- **Projection-to-Born**
  - $gg \to H$
    [Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]
- **q_T subtraction**
  - $gg \to H$
    [Billis, Dehnadi, Ebert, Michel, Tackmann '21]
  - $pp \to \gamma^*$
    [Chen, Gehrmann, Glover, AH, Yang, Zhu, '21]
  - $pp \to Z$
    [Camarda, Cieri, Ferrera '21]
    [NNLOJET + RadISH '22]
    [Neumann, Campbell '22]
  - $pp \to W$
    [Chen, Gehrmann, Glover, AH, Yang, Zhu '22]

FULLY DIFFERENTIAL ggH @ N3LO

[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

- only non-trivial observable:

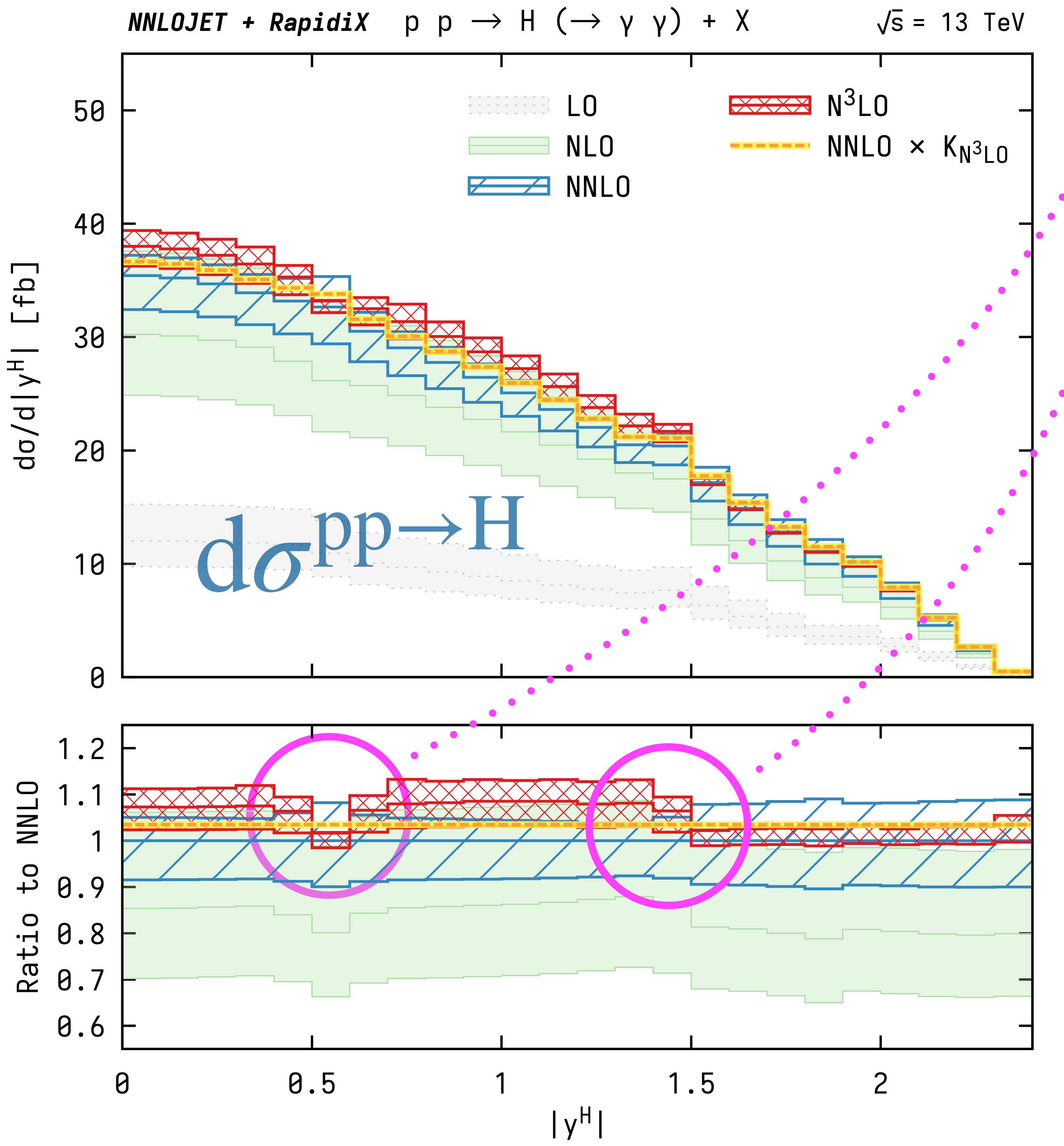


- idea: restore *differential* info

$$\frac{\mathrm{d}\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}} = \frac{\mathrm{d}\sigma_{F,\,\mathrm{inc.}}^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}_B} + \left\{ \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}} - \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}}\bigg|_{\mathcal{O}\to\mathcal{O}_B} \right\}$$

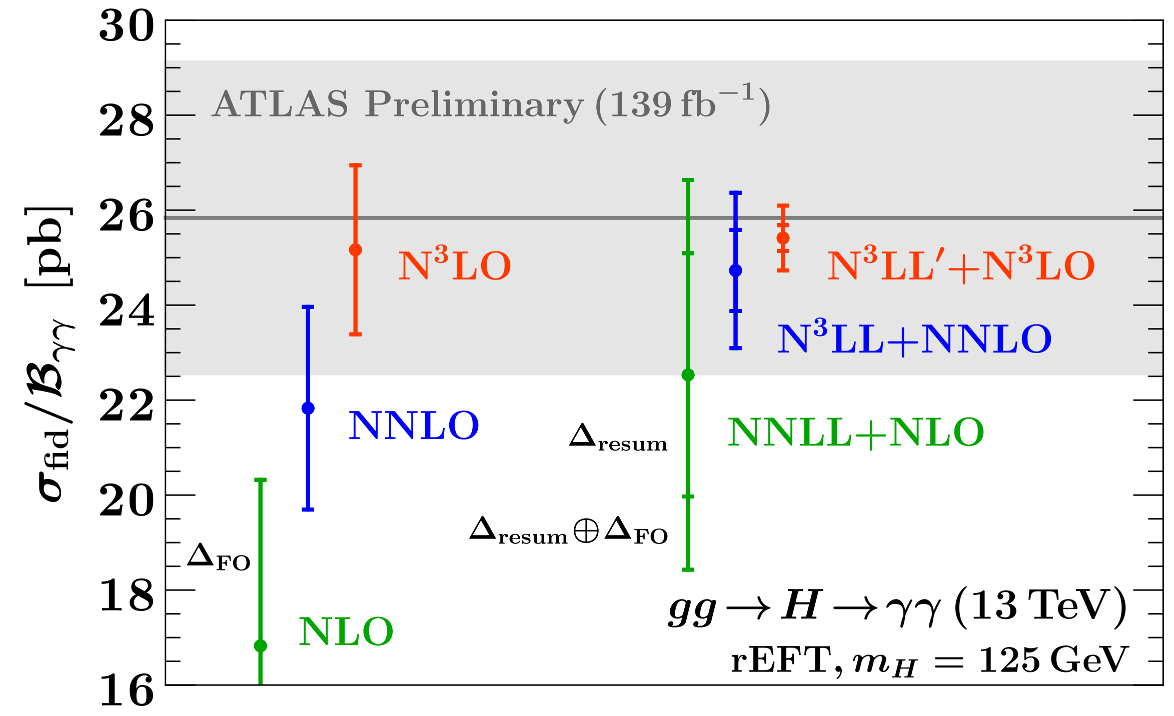
projection to Born

ATLAS CUTS



- linear fiducial power corrections ⇒ instabilities

[Billis, Dehnadi, Ebert, Michel, Tackmann '21]



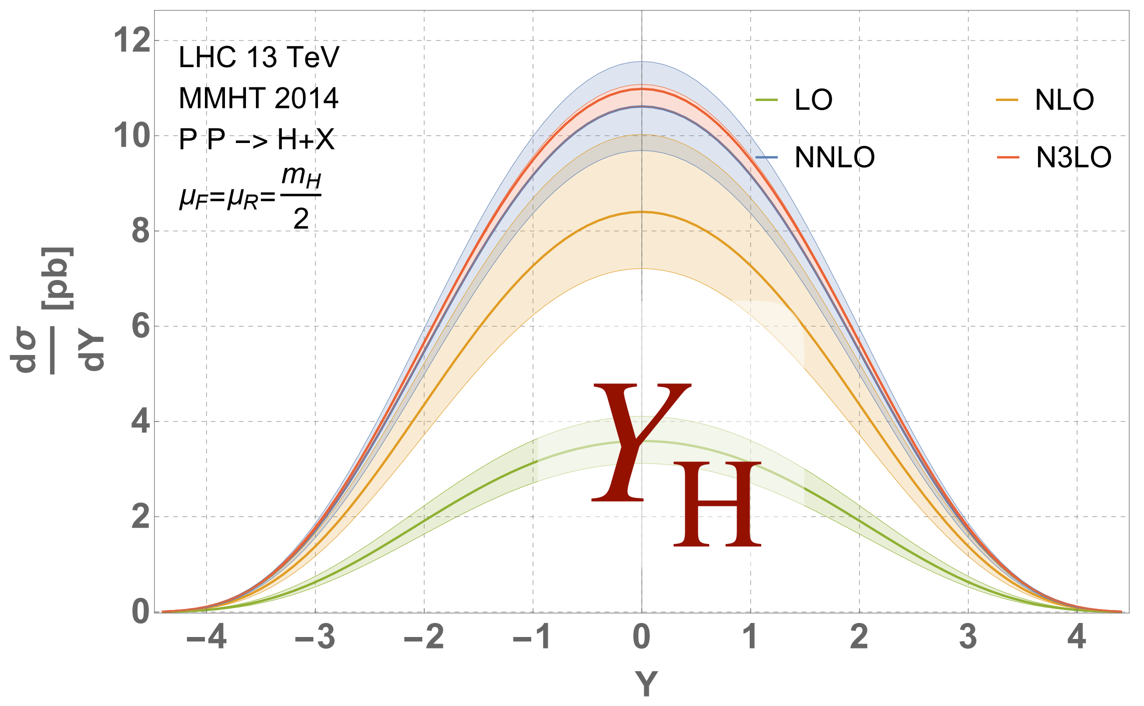
⊕ can be cured by resummation

⊖ hard $\sigma^{\mathrm{fid.}}$ should not need resummation

Fully Differential ggH @ N3LO

[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

- only non-trivial observable:



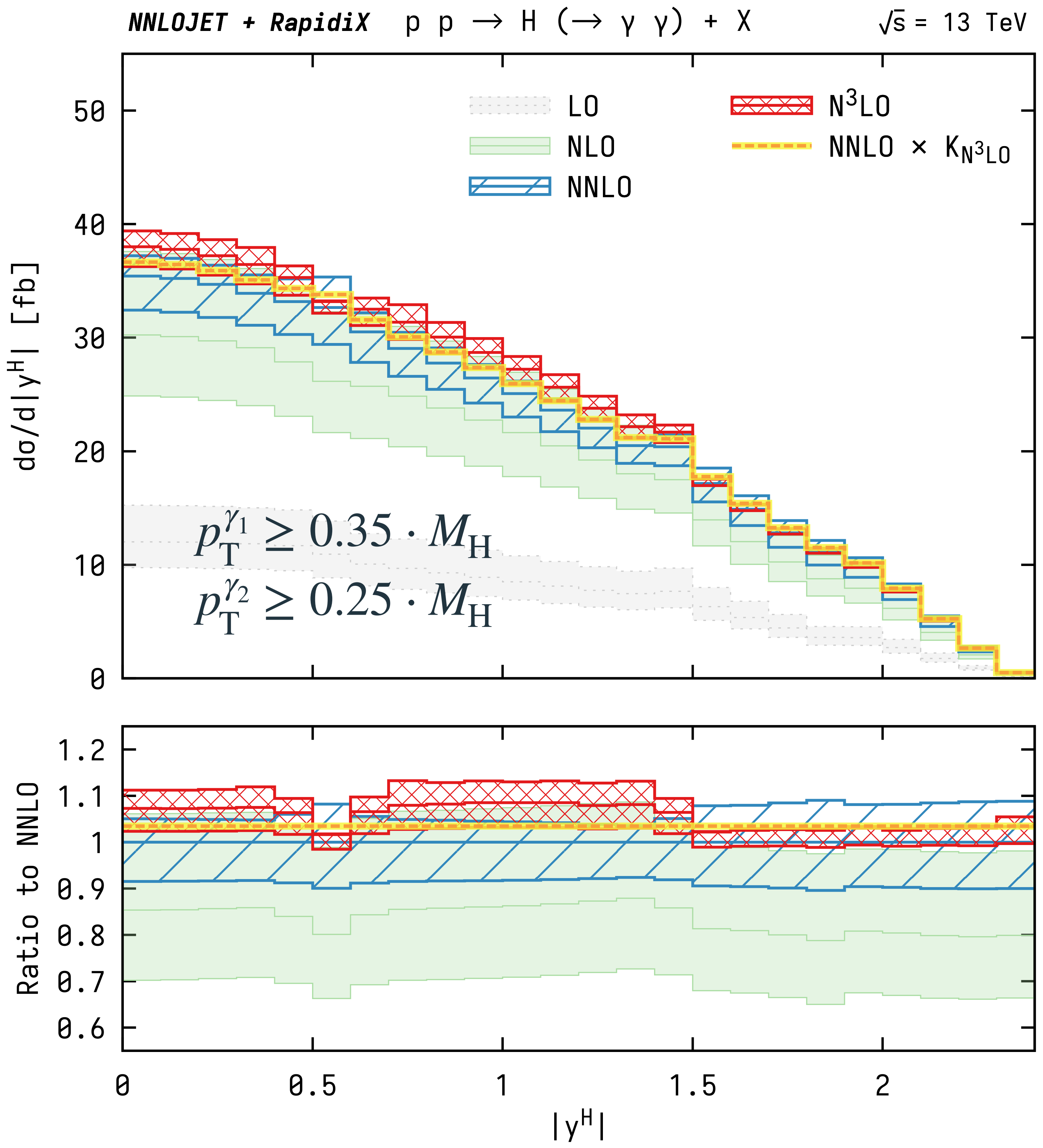
- idea: restore *differential* info

$$\frac{d\sigma_F^{N^kLO}}{d\mathcal{O}} = \frac{d\sigma_{F,\,\text{inc.}}^{N^kLO}}{d\mathcal{O}_B} + \left\{ \frac{d\sigma_{F+\text{jet}}^{N^{k-1}LO}}{d\mathcal{O}} - \frac{d\sigma_{F+\text{jet}}^{N^{k-1}LO}}{d\mathcal{O}}\bigg|_{\mathcal{O}\to\mathcal{O}_B} \right\}$$

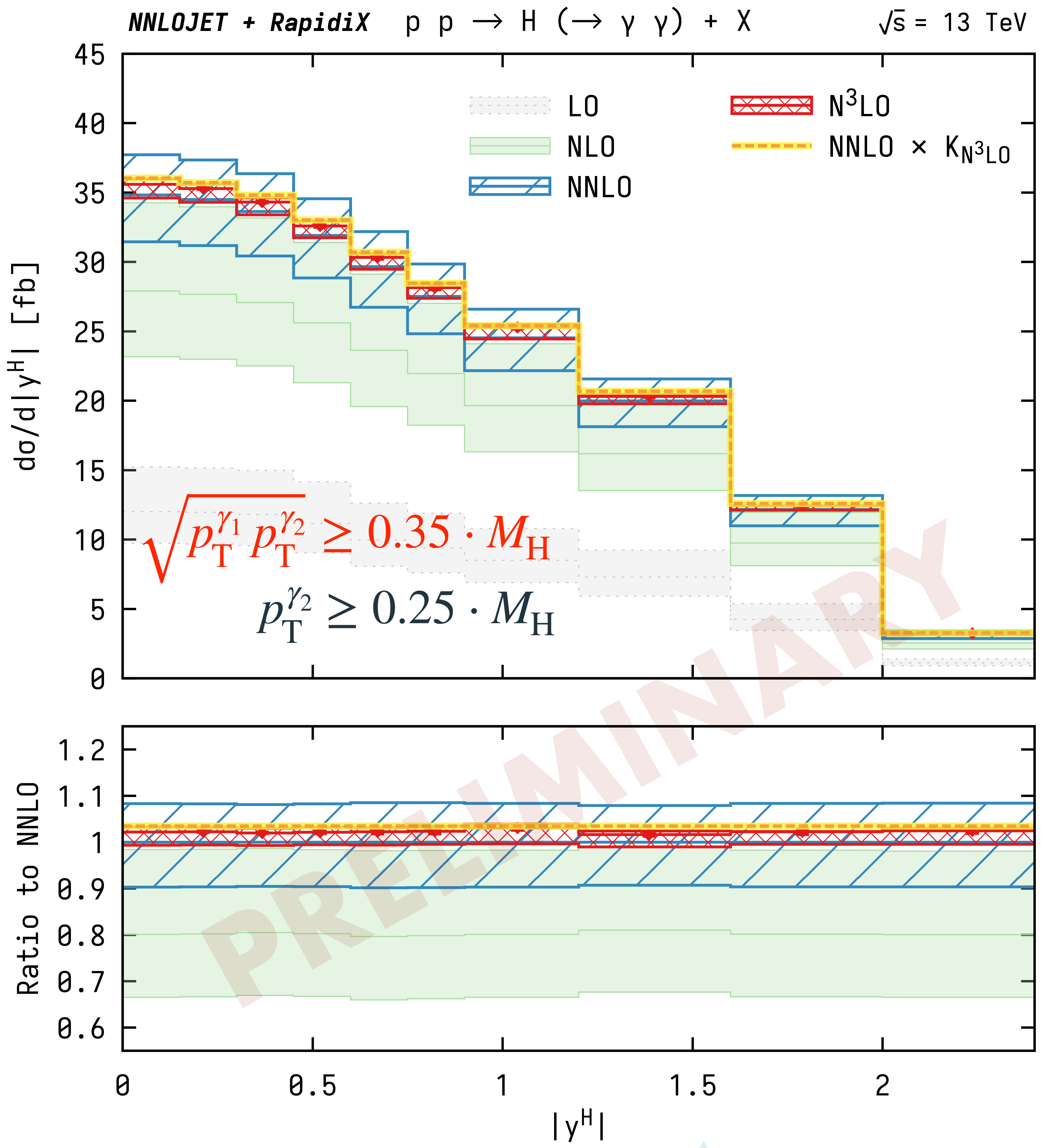
projection to Born

lin. fid. power corr.
[Salam, Slade '21]

ATLAS Cuts



Product Cuts

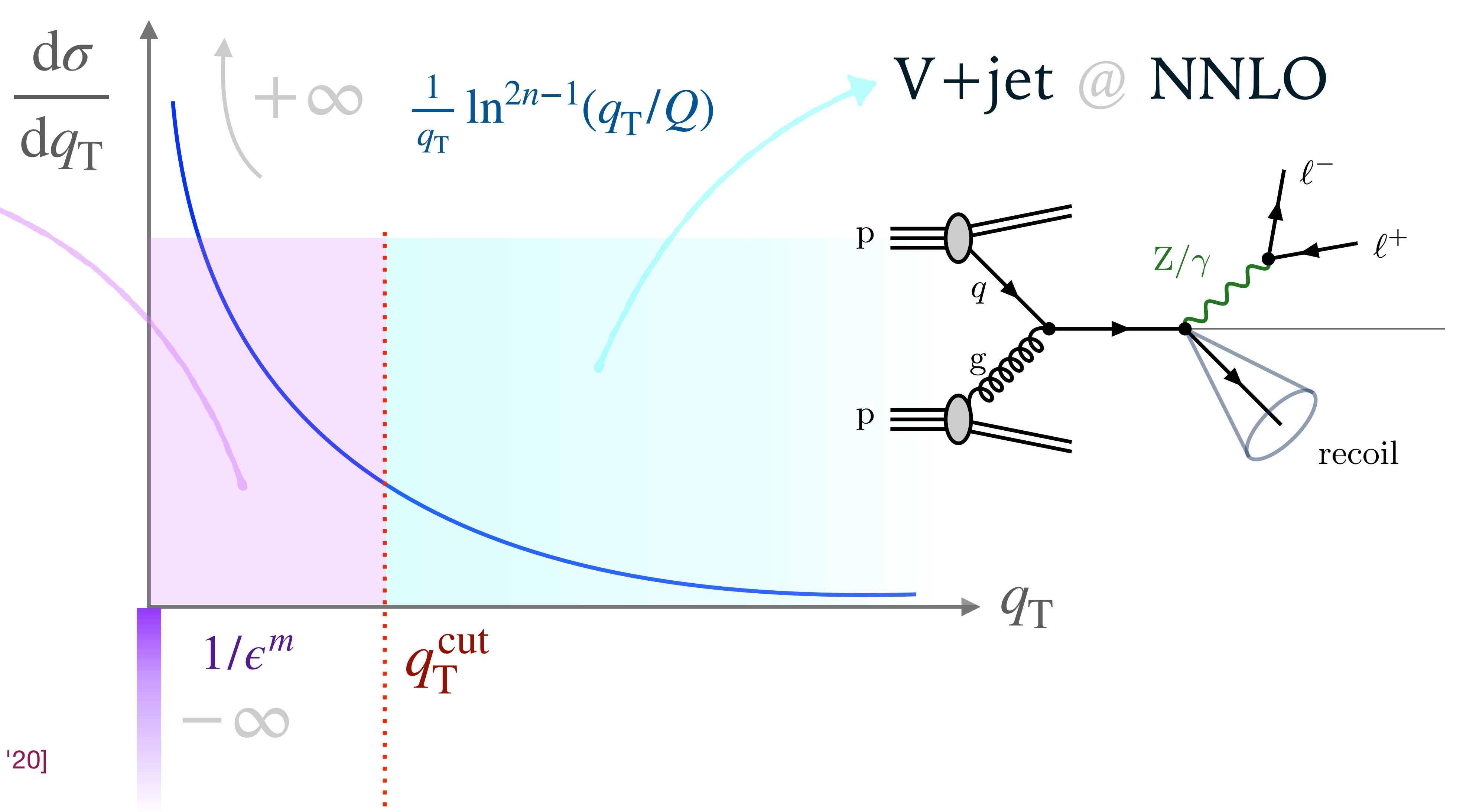


no instabilities & flat K-factor: $\text{N}^3\text{LO} \simeq \text{NNLO} \times K_{\text{N}^3\text{LO}}$

q_T Subtraction @ N3LO

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients:
  - hard function $H_{q\bar{q}}$
    [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
  - soft function $S(\mathbf{b}_\perp)$
    [Li, Zhu '16]
  - beam function $B_q(\mathbf{b}_\perp)$
    [Luo, Yang, Zhu, Zhu '19] [Ebert, Mistlberger, Vita '20]

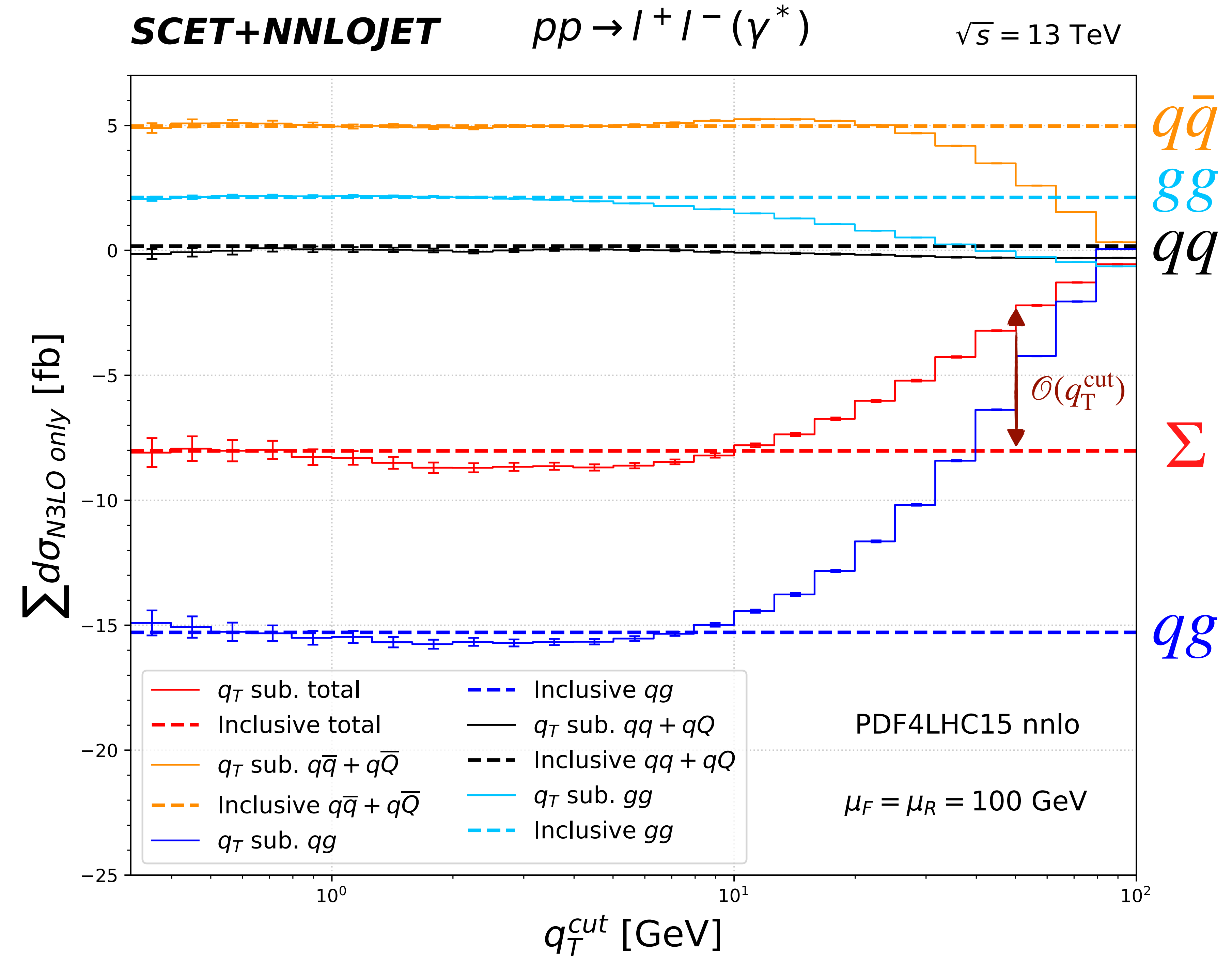


[Catani, Grazzini '07]

$$\mathrm{d}\sigma^V_{\mathrm{N^3LO}} = \mathrm{d}\sigma^V_{\mathrm{N^3LO}}\Big|_{q_T<q_T^{\mathrm{cut}}} + \mathrm{d}\sigma^V_{\mathrm{N^3LO}}\Big|_{q_T>q_T^{\mathrm{cut}}}$$

$$= \mathcal{H}^V_{\mathrm{N^3LO}} \otimes \mathrm{d}\sigma^V_{\mathrm{LO}} + \left[\mathrm{d}\sigma^{V+\mathrm{jet}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{V,\mathrm{CT}}_{\mathrm{N^3LO}}\right]_{q_T>q_T^{\mathrm{cut}}} + \mathcal{O}\left((q_T^{\mathrm{cut}}/Q)^n\right)$$

[Chen, Gehrmann, Glover, AH, Yang Zhu '21]



validation against *analytic result* (– – –)
[Duhr, Dulat, Mistlberger '20]

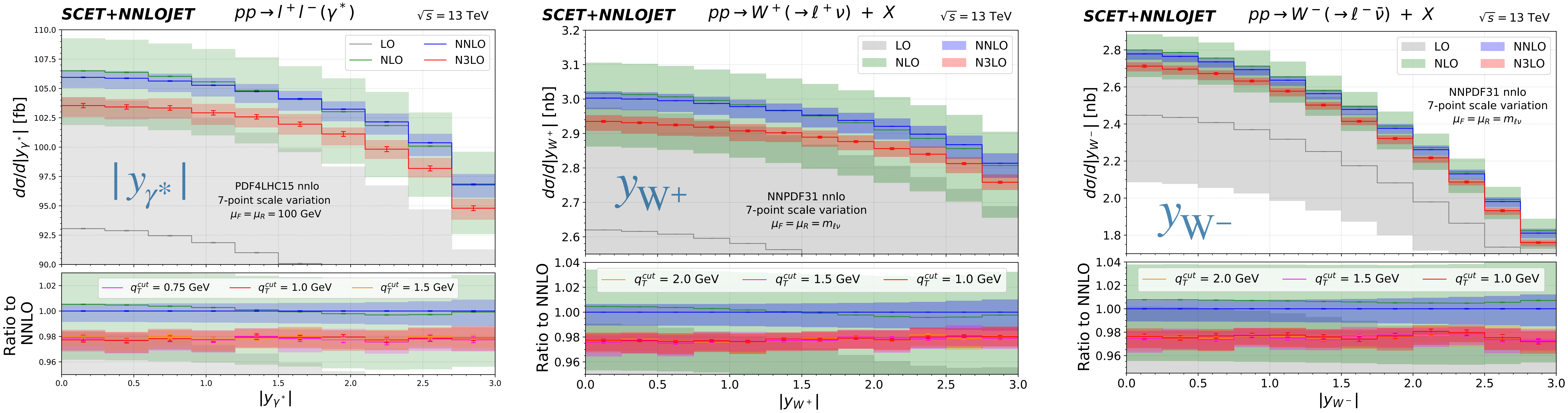
plateau $\lesssim 1\,\mathrm{GeV}$ & fully independent cal.

Competing interests: q_T^{cut} as small as possible ↭ q_T^{cut} as large as possible

↪ suppress power corrections ↪ numerical stability & efficiency

Drell–Yan @ N3LO — Y_V Distributions

[Chen, Gehrmann, Glover, AH, Yang, Zhu '21,'22]

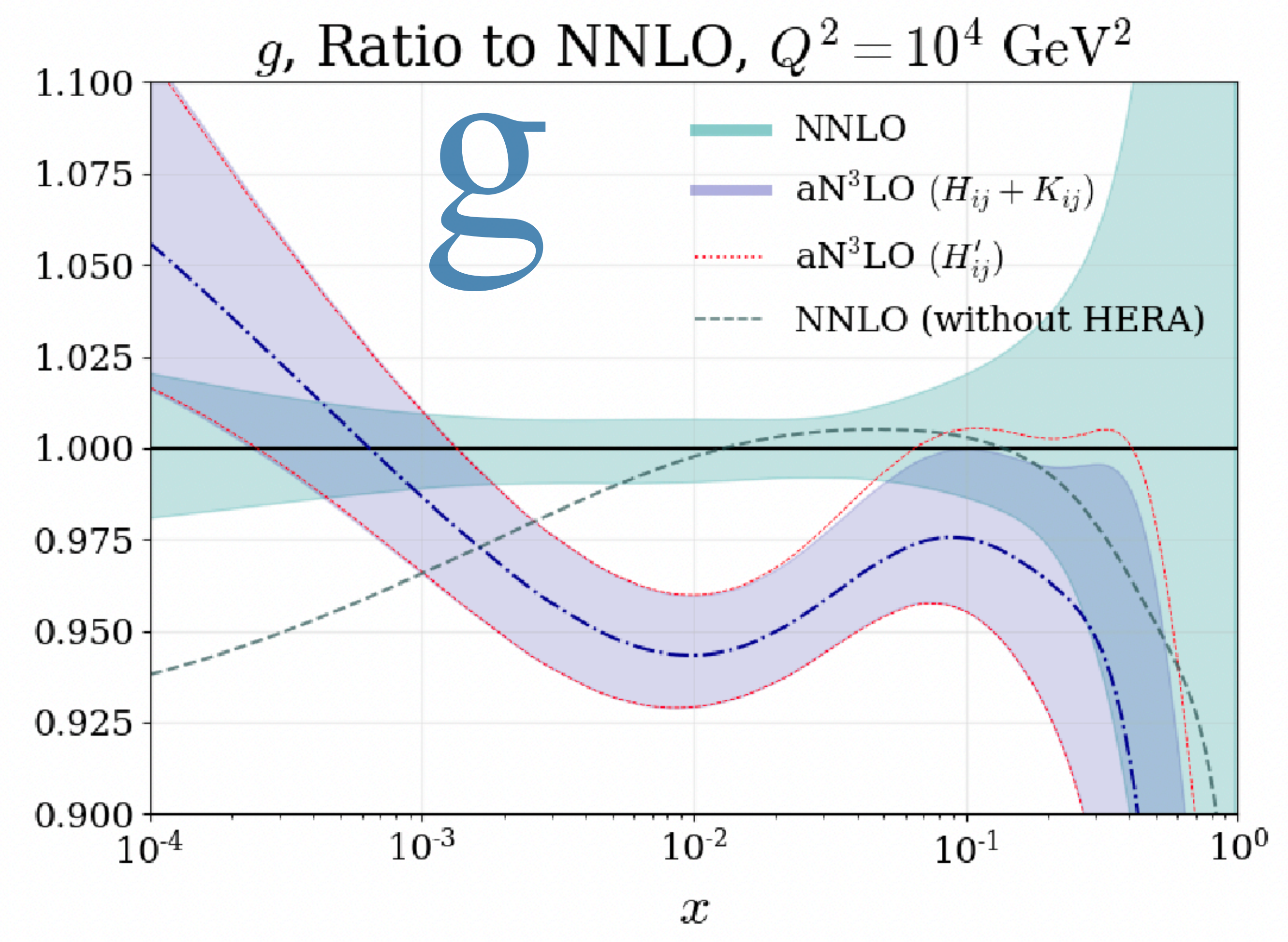


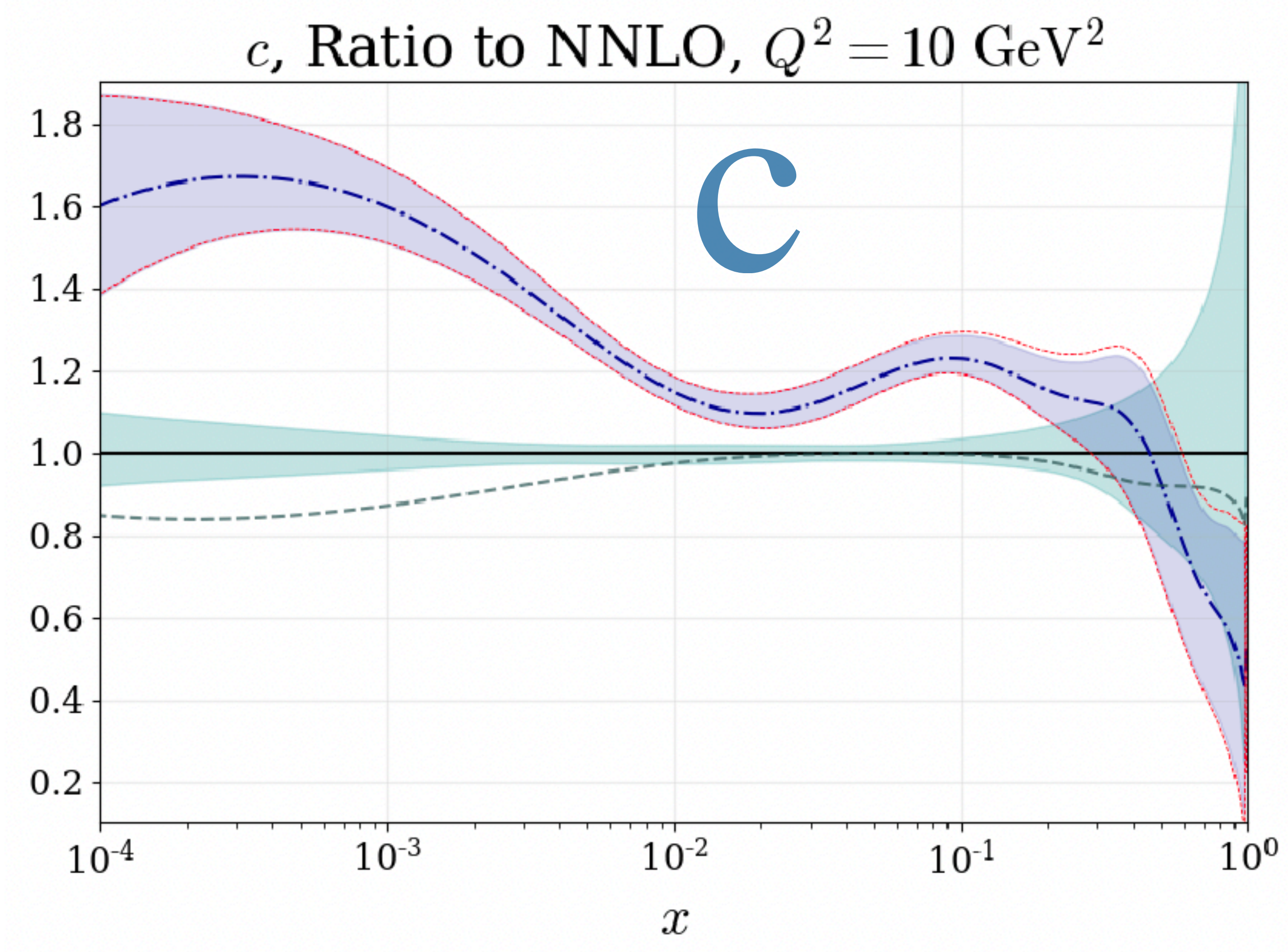
- same collider @ 13 TeV ⇝ almost universal NNLO → N³LO corrections!
- NC & CC$^{\pm}$ processes probe different parton content across Y_V (valence u vs. d, …)

N3LO Parton Distribution Functions

- N3LO evolution
  ⇝ 4-loop splitting functions
  [Moch, Ruijl, Ueda, Vermaseren, Vogt '17,'18,'22, in progress]

- aN3LO PDFs (MSHT)
  [McGowan, Cridge, Harland-Lang, Thorne ‘22]

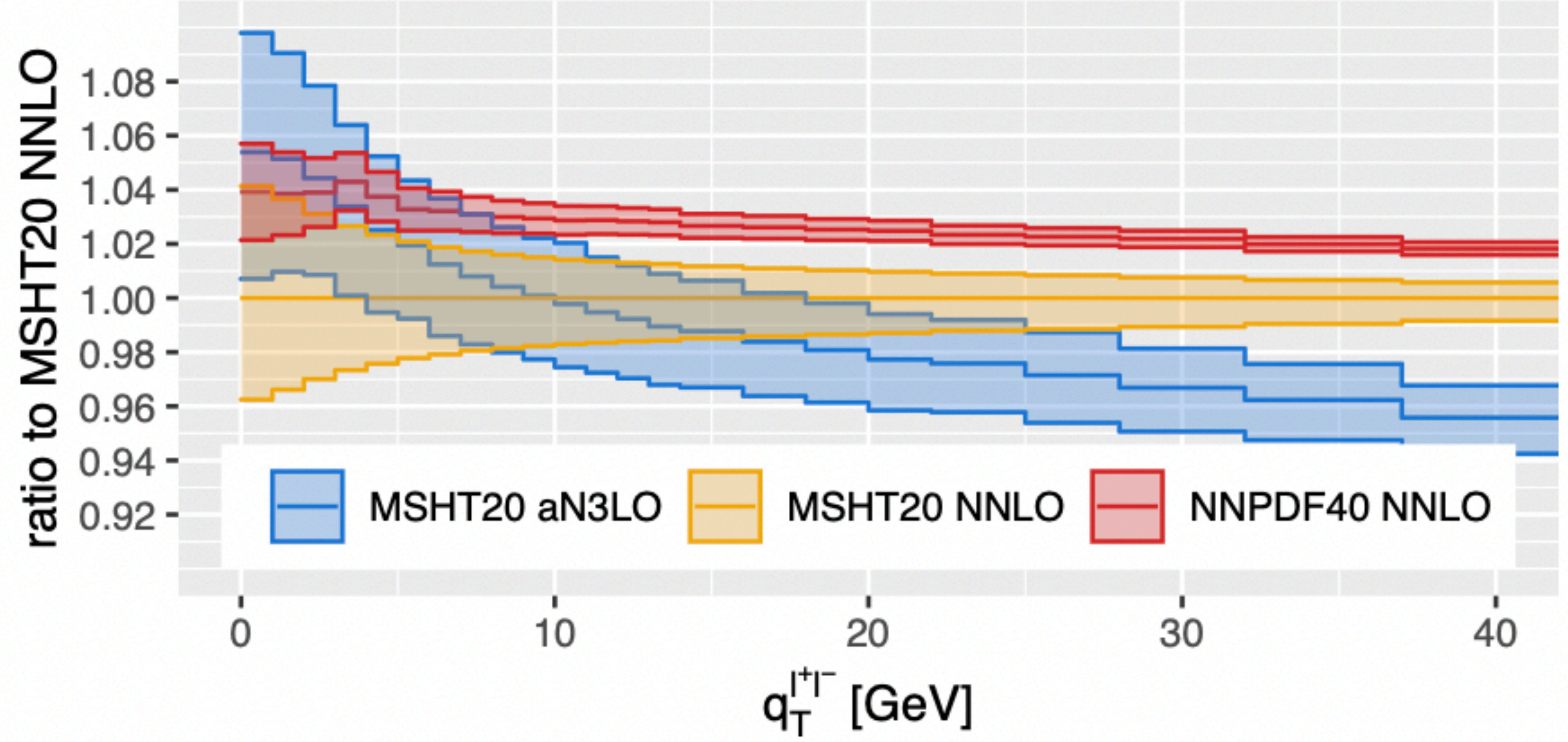




ggH: $\delta\sigma^{\mathrm{N^3LO}}$ ↘ VBF: $\delta\sigma^{\mathrm{N^3LO}}$ ↗

- purely resummed p_T^Z spectrum
  ⇝ PDF uncertainties

[Neumann, Campbell ’22]



syst. differences between PDFs

PDF (NNLO → N^{3}LO) $\delta\sigma^{\mathrm{N^3LO}}$ ↗ (?)

Conclusions & Outlook Part 1

- NNLO QCD calculations in good shape
  - $2 \to 2$ essentially solved
  - $2 \to 3$ new frontier ⇜⇝ methods reaching maturity
  - *loop amplitudes* becoming a bottleneck again
  - in the quest for percent-level theory ⇜⇝ mixed QCD×EW important
- dissemination of results
  - public codes (MCFM, Matrix), nTuples, …
  - fast interpolation grids ⇜⇝ `APPLgrid` `fastNLO` `PineAPPL` (anyway needed in fitting)
- identified objects ⇜⇝ mismatch in TH vs. Exp/NNLO
  - photon isolation, flavour tagging, hadron fragmentation, …

Conclusions & Outlook Part 2

- N^{3}LO predictions are key to reach percent-level accuracy
  - computation of *inclusive* $2 \to 1$ processes very mature ⇝ ggH, DY, VBF, VH, …
  - differential predictions for pp → "colour neutral" appearing
    ⇝ relies on very stable NNLO "+jet" calculation
  - *but:* performance of slicing methods very poor ⇝ $\mathcal{O}$(10M) CPU core hours
- Fiducial cuts ⇝ linear power corrections (other processes?)
  ↪ crucial for practicability of slicing approaches
- Inadequacies in traditional scale variations ⇝ DY @ N3LO
  ↪ effect from missing N^{3}LO PDFs?
  ↪ more robust TH uncertainties desirable

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Thank you!

BACKUP