Estimating QCD-EW and EW higher-order corrections¶

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Workshop on Tools for High Precision LHC Simulations

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Perturbative expansion

aMC@NLO, Sherpa, Herwig… &
Recola, Madloop, Gosam, OpenLoops

dedicated MC's: Matrix, MCFM,
NNLOjet, …

$$d\sigma = d\sigma_{\mathrm{LO}} + \alpha_S\, d\sigma_{\mathrm{NLO}}$$

NLO QCD O(10%)

$$+\alpha_S^2\, d\sigma_{\mathrm{NNLO}}$$

NNLO QCD O(1%)

$$+\alpha_S^3\, d\sigma_{\mathrm{NNLO}} \overset{?}{+} \dots$$

N3LO QCD O(0.1%)

scale variation at NNLO

Perturbative expansion

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$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_S\, \mathrm{d}\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, \mathrm{d}\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD   NLO EW

$$+\alpha_S^2\, \mathrm{d}\sigma_{\mathrm{NNLO}}$$

NNLO QCD

$$+\alpha_S^3\, \mathrm{d}\sigma_{\mathrm{NNLO}} \overset{?}{+} \ldots$$

N3LO QCD

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$$d\sigma = d\sigma_{\rm LO} + \alpha_S\, d\sigma_{\rm NLO} + \alpha_{\rm EW}\, d\sigma_{\rm NLO\,EW}$$

NLO QCD — NLO EW

$$+\alpha_S^2\, d\sigma_{\rm NNLO} + \alpha_{\rm EW}^2\, d\sigma_{\rm NNLO\,EW} \; ? \; + \alpha_S \alpha_{\rm EW}\, d\sigma_{\rm NNLO\,QCDxEW} \; ?$$

NNLO QCD — NNLO EW — NNLO QCD-EW

$$+\alpha_S^3\, d\sigma_{\rm NNLO} \; ? \; + \ldots$$

N3LO QCD

only known for DY

scale variation at NNLO

Perturbative expansion

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$$d\sigma = d\sigma_{\mathrm{LO}} + \alpha_S\, d\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD — NLO EW

$$+\alpha_S^2\, d\sigma_{\mathrm{NNLO}} + \alpha_{\mathrm{EW}}^2\, d\sigma_{\mathrm{NNLO\,EW}} \;? + \alpha_S \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NNLO\,QCDxEW}} \;?$$

NNLO QCD — NNLO EW — NNLO QCD-EW

$$+\alpha_S^3\, d\sigma_{\mathrm{NNLO}} \;? + \ldots$$

N3LO QCD

scale variation at NNLO

scheme variation, e.g. Gmu vs. a(mZ)?

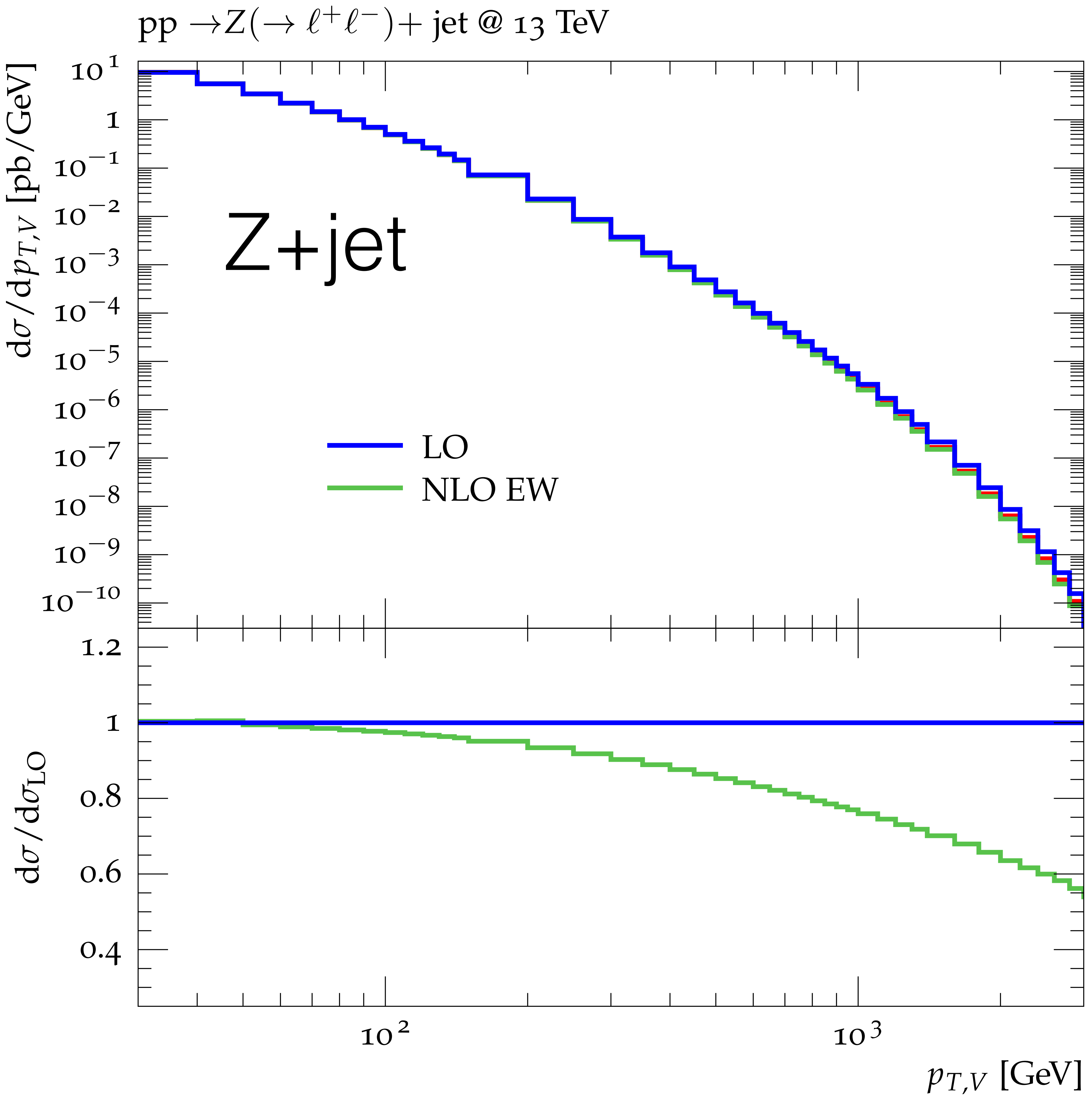
+

in case of EW Sudakov dominance: exponentiation

NLO QCD + EW
vs.
NLO QCD x EW

- sufficient?
- reliable?

EW uncertainties: Sudakov

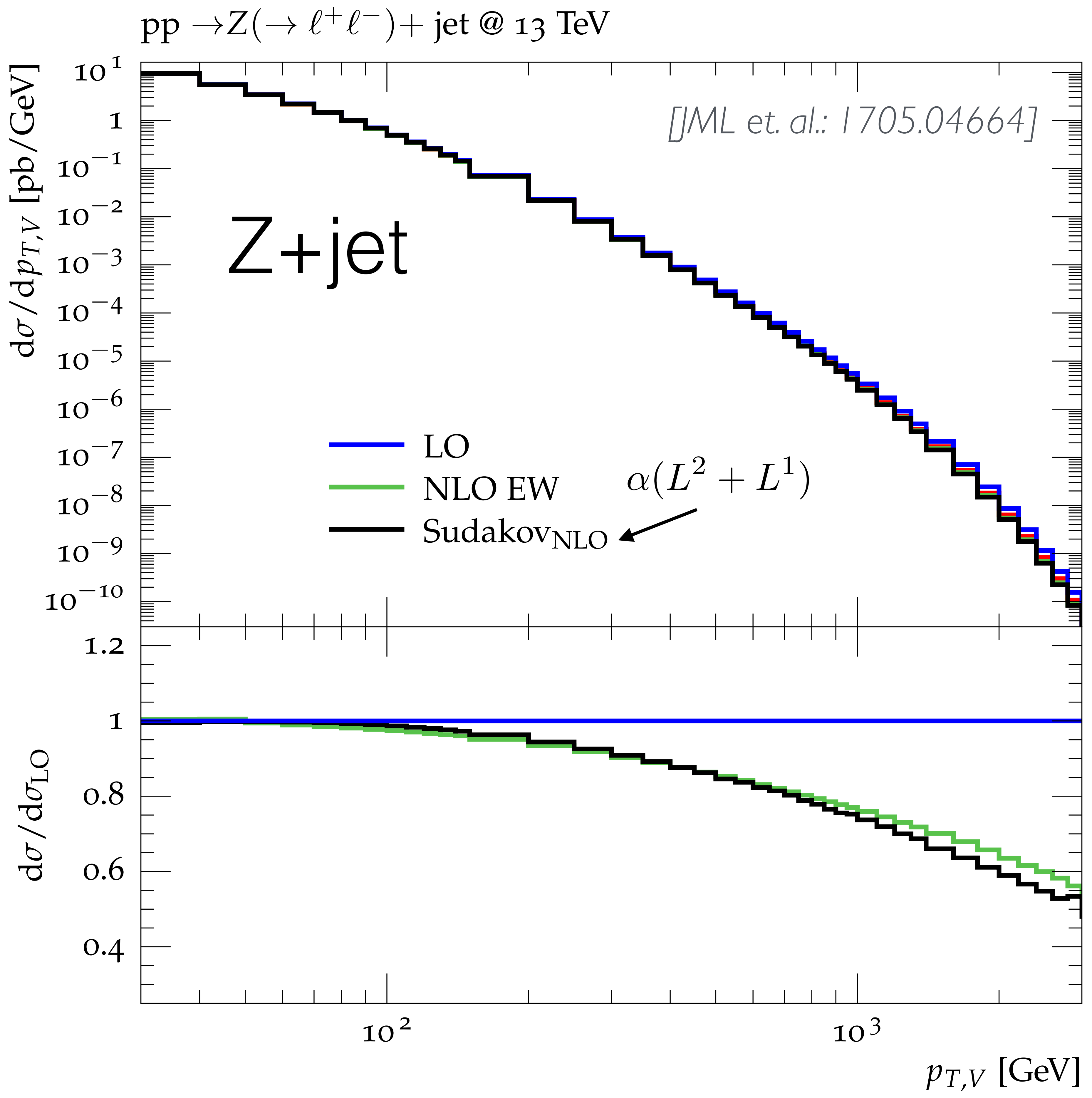


EW corrections become sizeable at large $p_{T,V}$: -30% @ 1 TeV

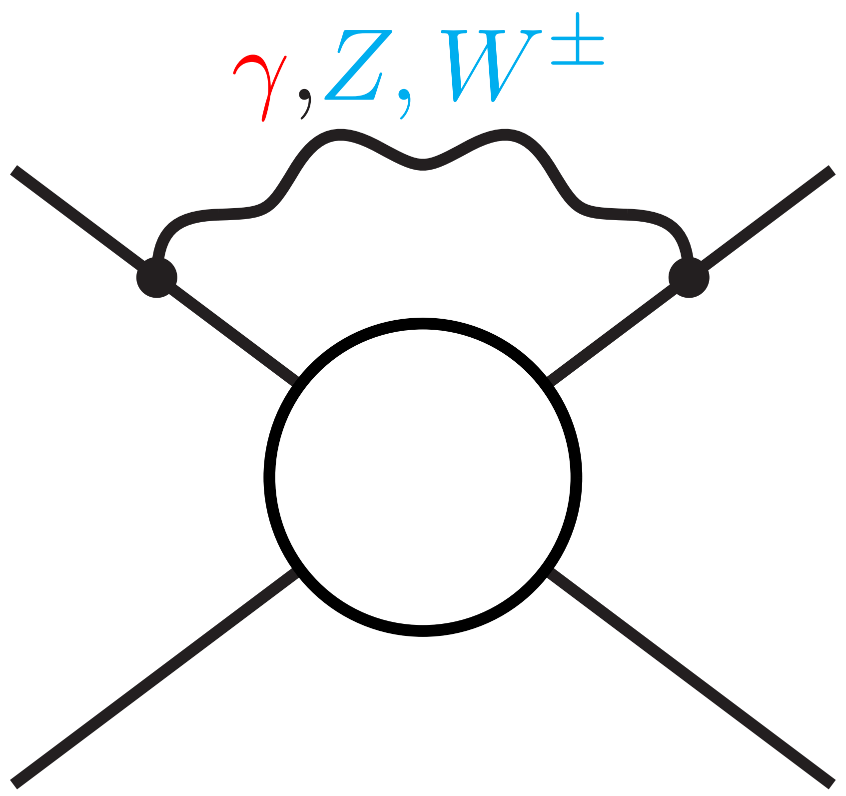
Origin: virtual EW Sudakov logarithms

How to estimate corresponding pure EW uncertainties of relative $\mathcal{O}(\alpha^2)$?

EW uncertainties: Sudakov



Large EW corrections dominated by Sudakov logs

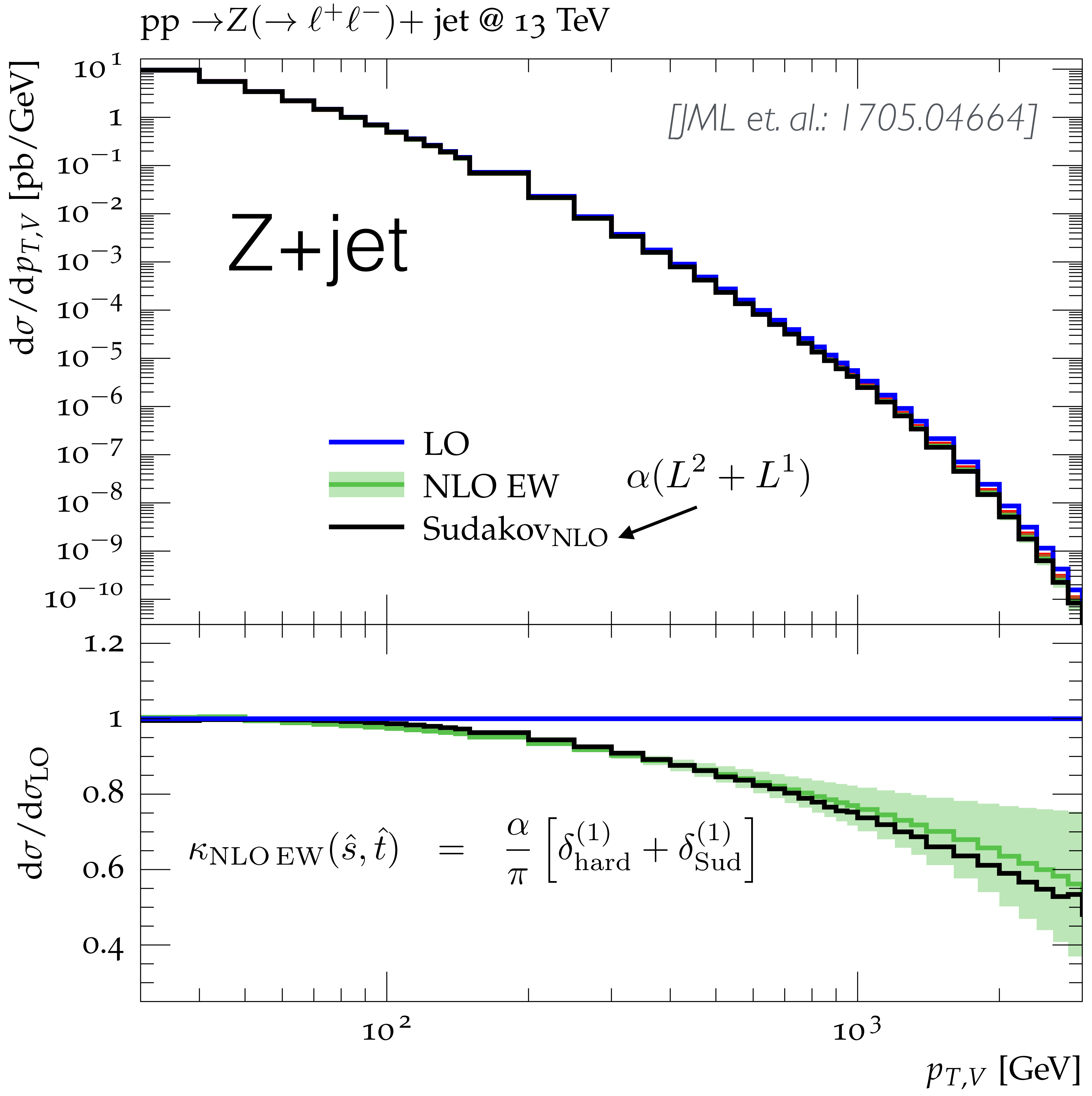


[Ciafaloni, Comelli,'98; Lipatov, Fadin, Martin, Melles, '99; Kuehen, Penin, Smirnov, '99; Denner, Pozzorini, '00]

Universality and factorisation: *[Denner, Pozzorini; '01]*

$$\delta\mathcal{M}^{1-\text{loop}}_{\text{LL+NLL}} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\left\{\frac{1}{2}\sum_{l\neq k}\sum_{a=\gamma,Z,W^\pm} I^a(k)I^{\bar{a}}(l)\ln^2\frac{\hat{s}_{kl}}{M^2} + \gamma^{\text{ew}}(k)\ln\frac{\hat{s}}{M^2}\right\}\mathcal{M}_0$$

EW uncertainties: Sudakov



Large EW corrections dominated by Sudakov logs

↓

Uncertainty estimate of (N)NLO EW from naive exponentiation x 2:

$$\Delta^{\mathrm{Sud}}_{\mathrm{EW}} \simeq \frac{2}{k!}\left(\kappa_{\mathrm{NLO,EW}}\right)^k$$

$$\mathrm{d}\sigma_{\mathrm{EW}} = \exp\left\{\int_{M_W^2}^{Q^2}\frac{\mathrm{d}t}{t}\left[\int_{M_W^2}^{t}\mathrm{d}\tau\mathrm{d}\frac{\gamma(\alpha(\tau))}{\tau} + \chi(\alpha(t)) + \xi\left(\alpha(M_W^2)\right)\right]\right\}\mathrm{d}\sigma_{\mathrm{hard}},$$

$$= \left(1+\frac{\alpha}{\pi}\delta^{(1)}_{\mathrm{Sud}} + \left(\frac{\alpha}{\pi}\right)^2\delta^{(2)}_{\mathrm{Sud}} + \ldots\right)\left(1+\frac{\alpha}{\pi}\delta^{(1)}_{\mathrm{hard}} + \left(\frac{\alpha}{\pi}\right)^2\delta^{(2)}_{\mathrm{hard}} + \ldots\right)\mathrm{d}\sigma_{\mathrm{Born}}$$

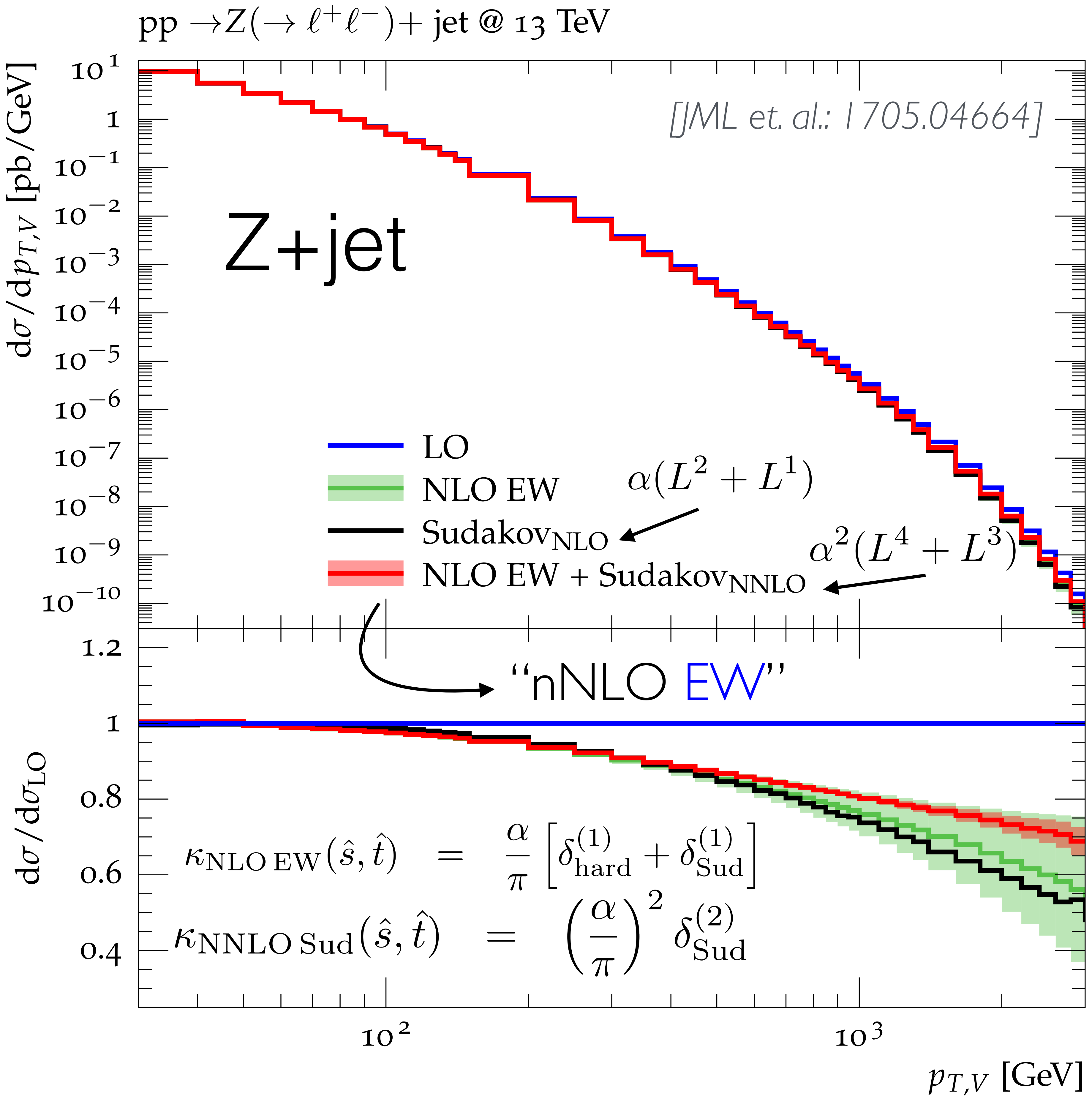
with

$$\delta^{(1)}_{\mathrm{Sud}} = \sum_{i,j} C^{(1)}_{2,ij}\ln^2\left(\frac{Q^2_{ij}}{M^2}\right) + C^{(1)}_1\ln^1\left(\frac{Q^2}{M^2}\right),$$

$$\delta^{(2)}_{\mathrm{Sud}} = \sum_{i,j} C^{(2)}_{4,ij}\ln^4\left(\frac{Q^2_{ij}}{M^2}\right) + C^{(2)}_3\ln^3\left(\frac{Q^2}{M^2}\right) + \mathcal{O}\left[\ln^2\left(\frac{Q^2}{M^2}\right)\right]$$

→ $\alpha^m \ln^n\left(Q^2/M_W^2\right)$ → finite in limit $Q^2/M_W^2 \to \infty$.

EW uncertainties: Sudakov



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↓

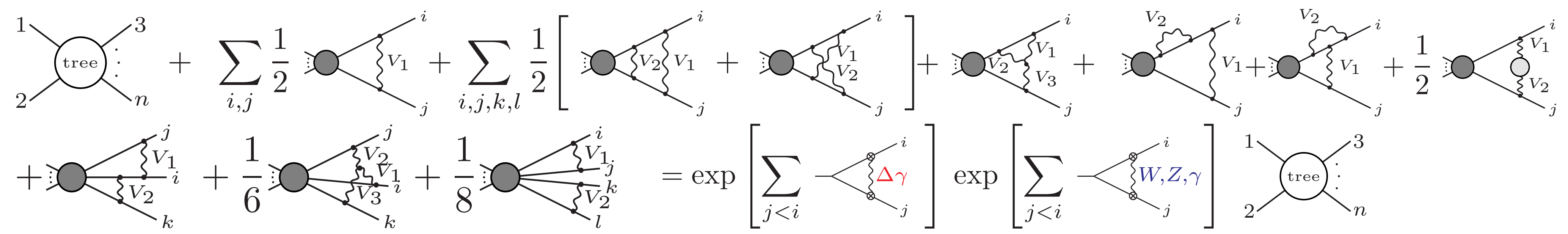
Uncertainty estimate of (N)NLO EW from naive exponentiation x 2:

$$\Delta^{\rm Sud}_{\rm EW} \simeq \frac{2}{k!}\left(\kappa_{\rm NLO,EW}\right)^k$$

↓

check against two-loop Sudakov logs

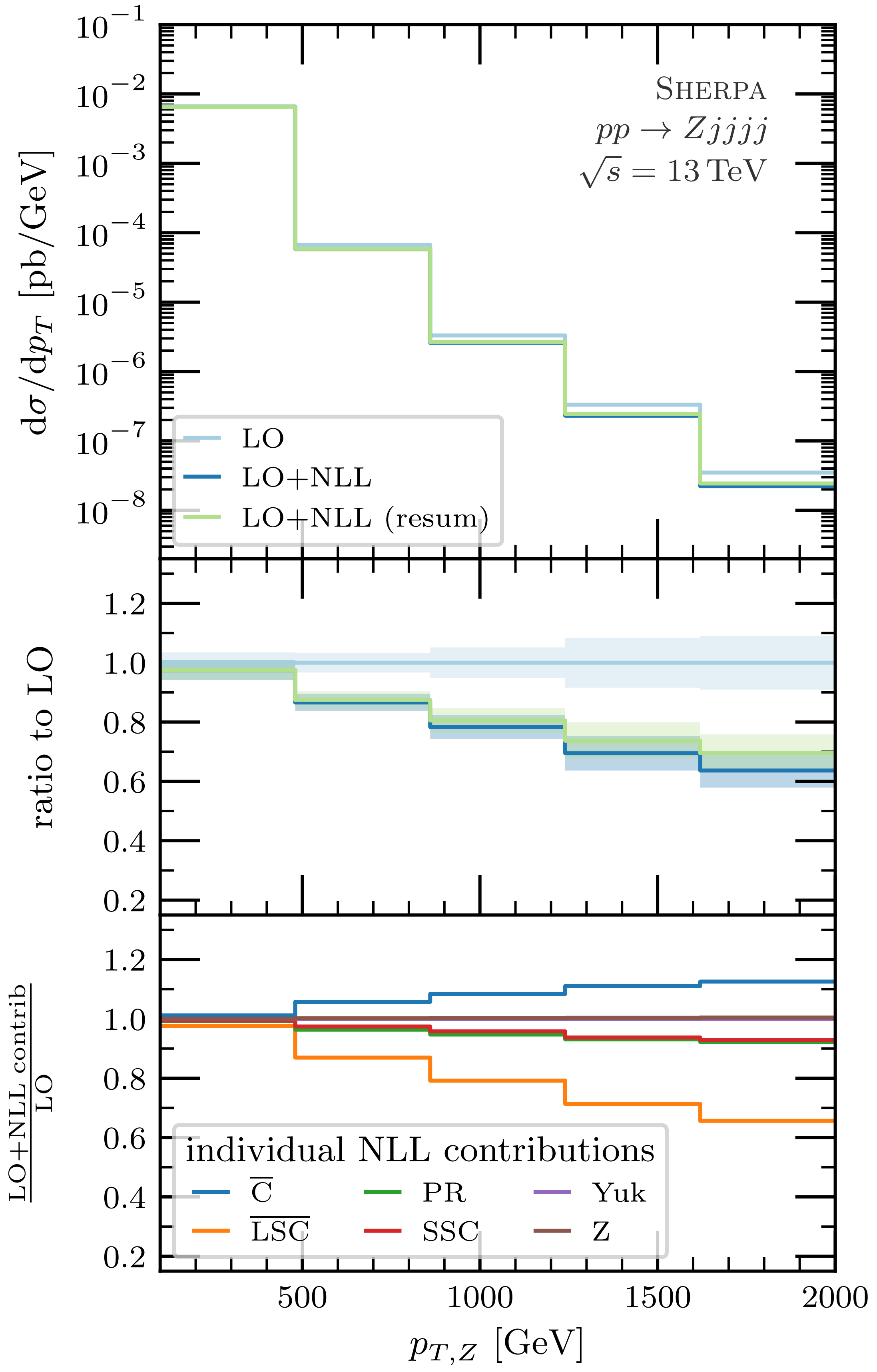
[Kühn, Kulesza, Pozzorini, Schulze; 05-07]



Tools for EW Sudakov corrections

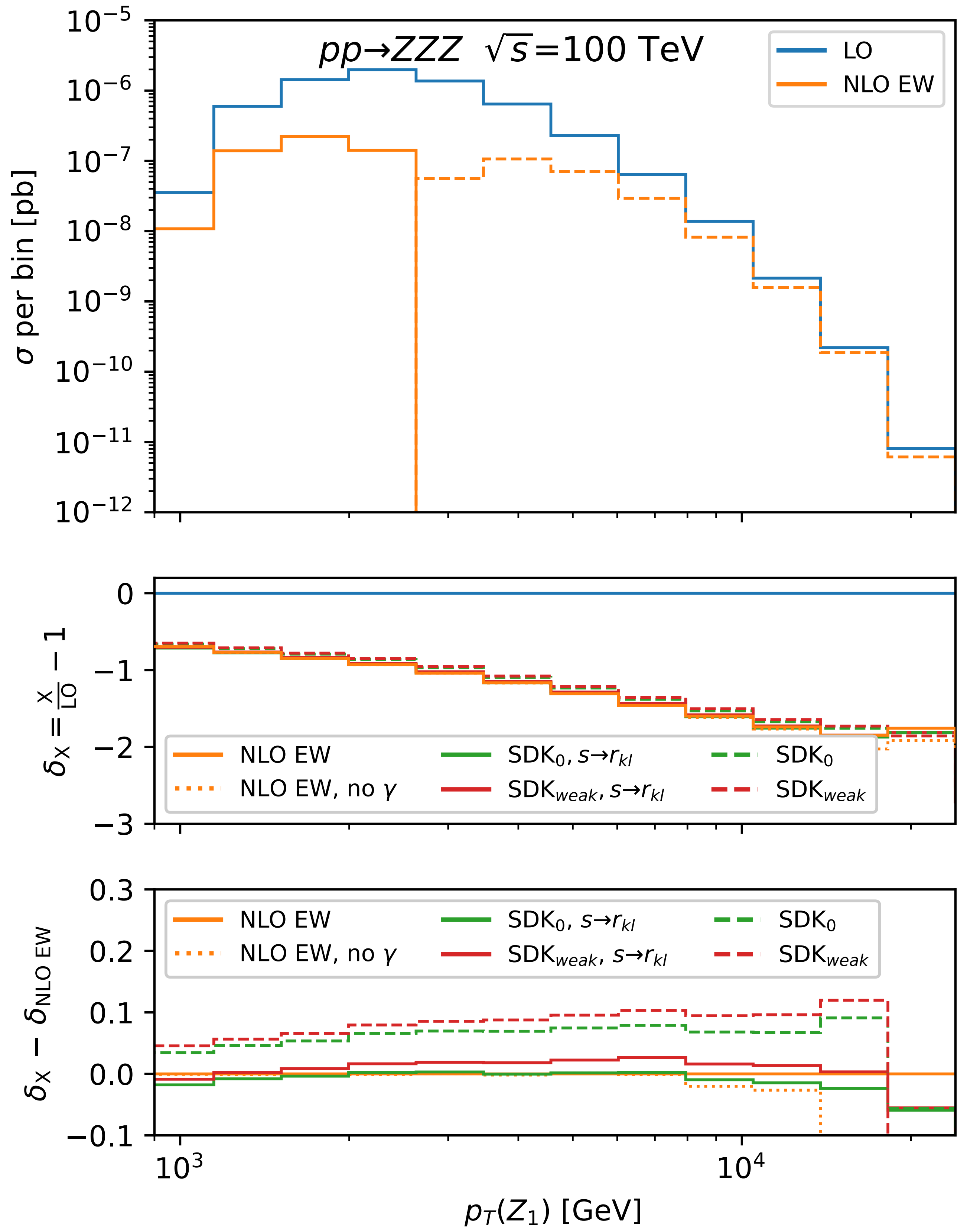
Sherpa

[Bothmann, Napoletano, '20]



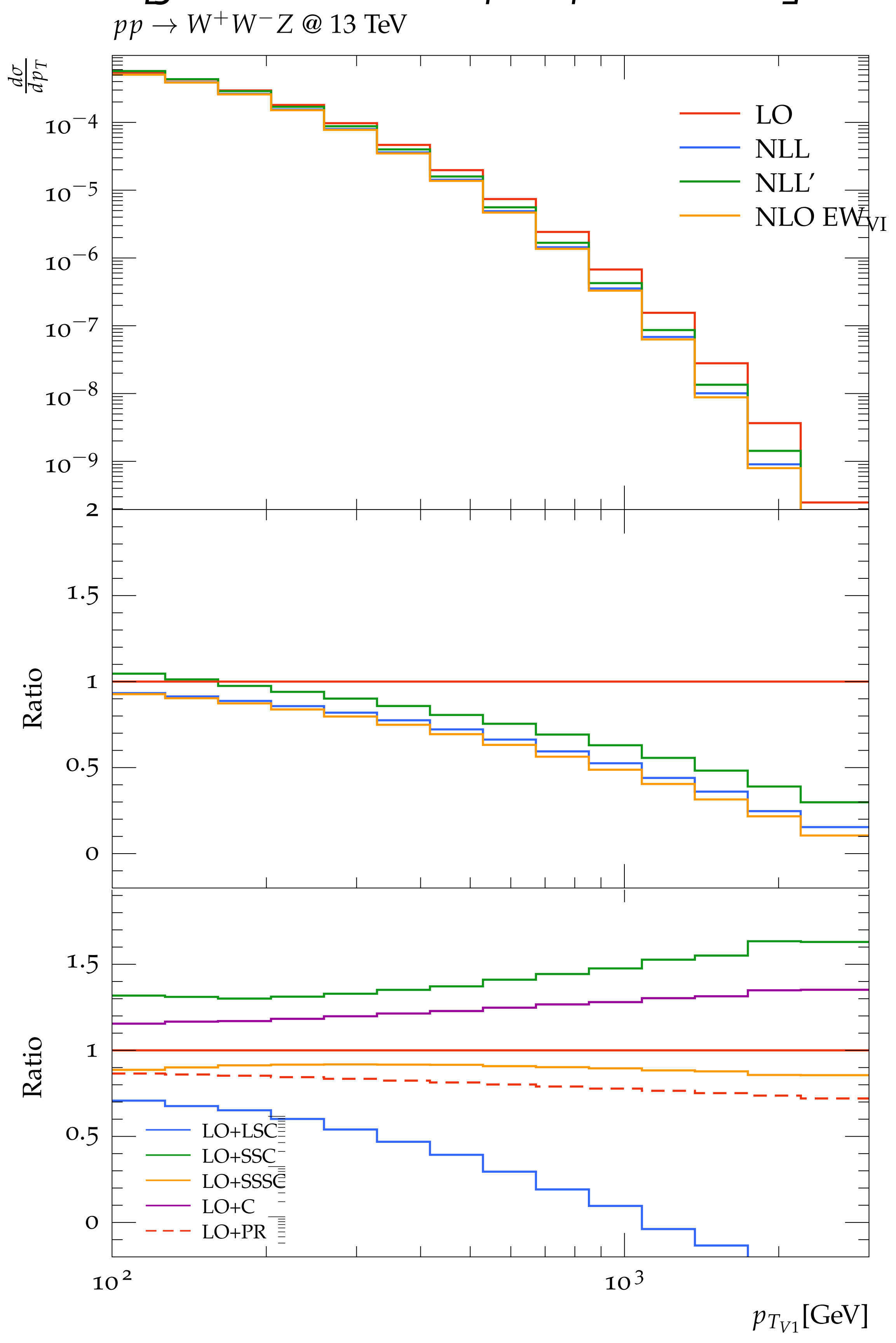
MadGraph5_aMC@NLO

[Pagani, Zaro, '21]

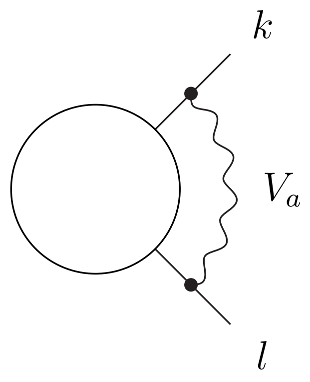


OpenLoops

[JML, Mai, in preparation]



- all based on *[Denner, Pozzorini, '00, '01]*



$$\overset{\text{LA}}{\sim} L(s, M^2) + 2l(s, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

$$L(|r_{kl}|, M^2) := \frac{\alpha}{4\pi}\log^2\frac{r_{kl}}{M^2}, \qquad l(r_{kl}, M^2) := \frac{\alpha}{4\pi}\log\frac{r_{kl}}{M^2}$$

Tools for EW Sudakov corrections

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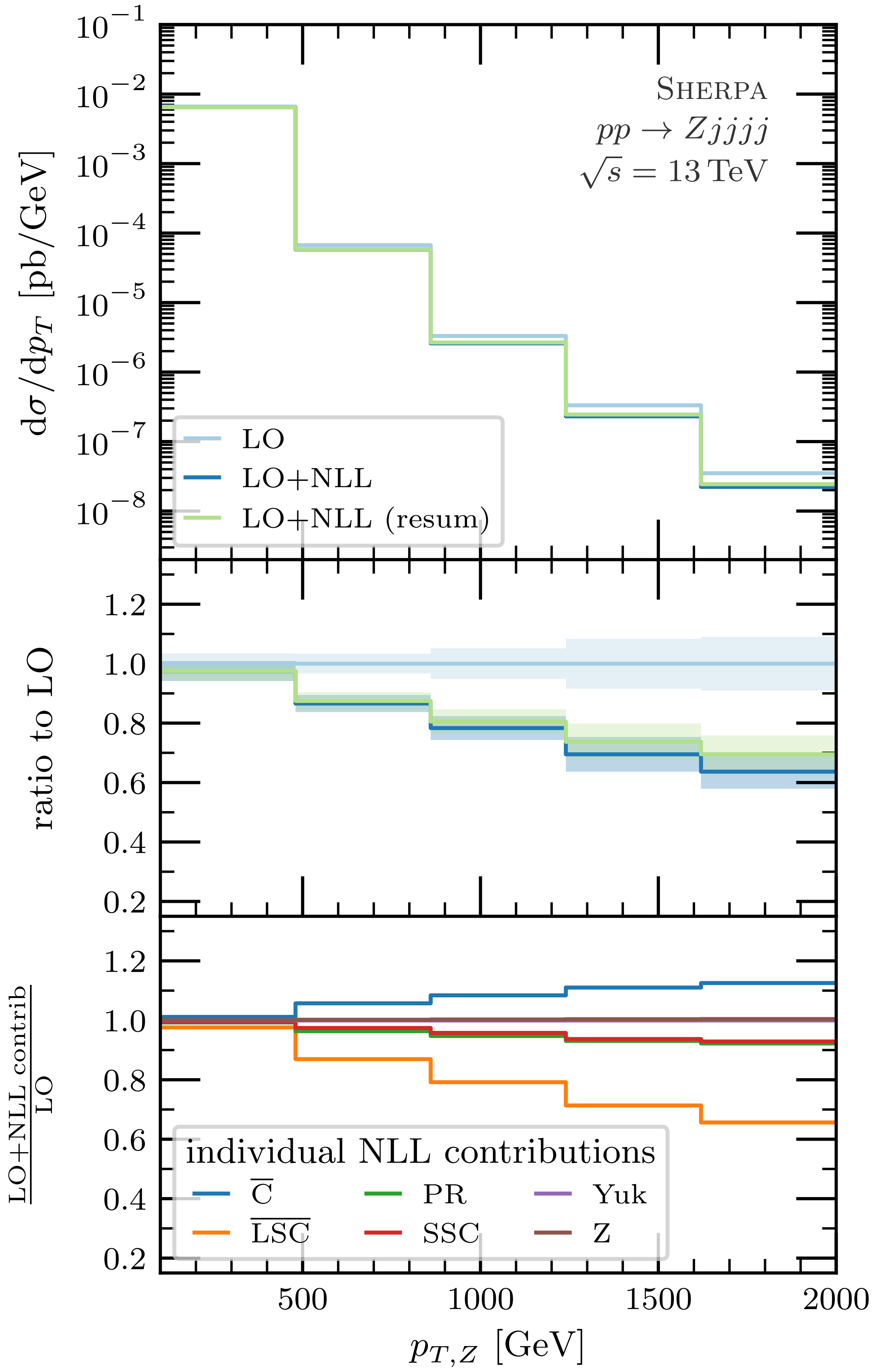

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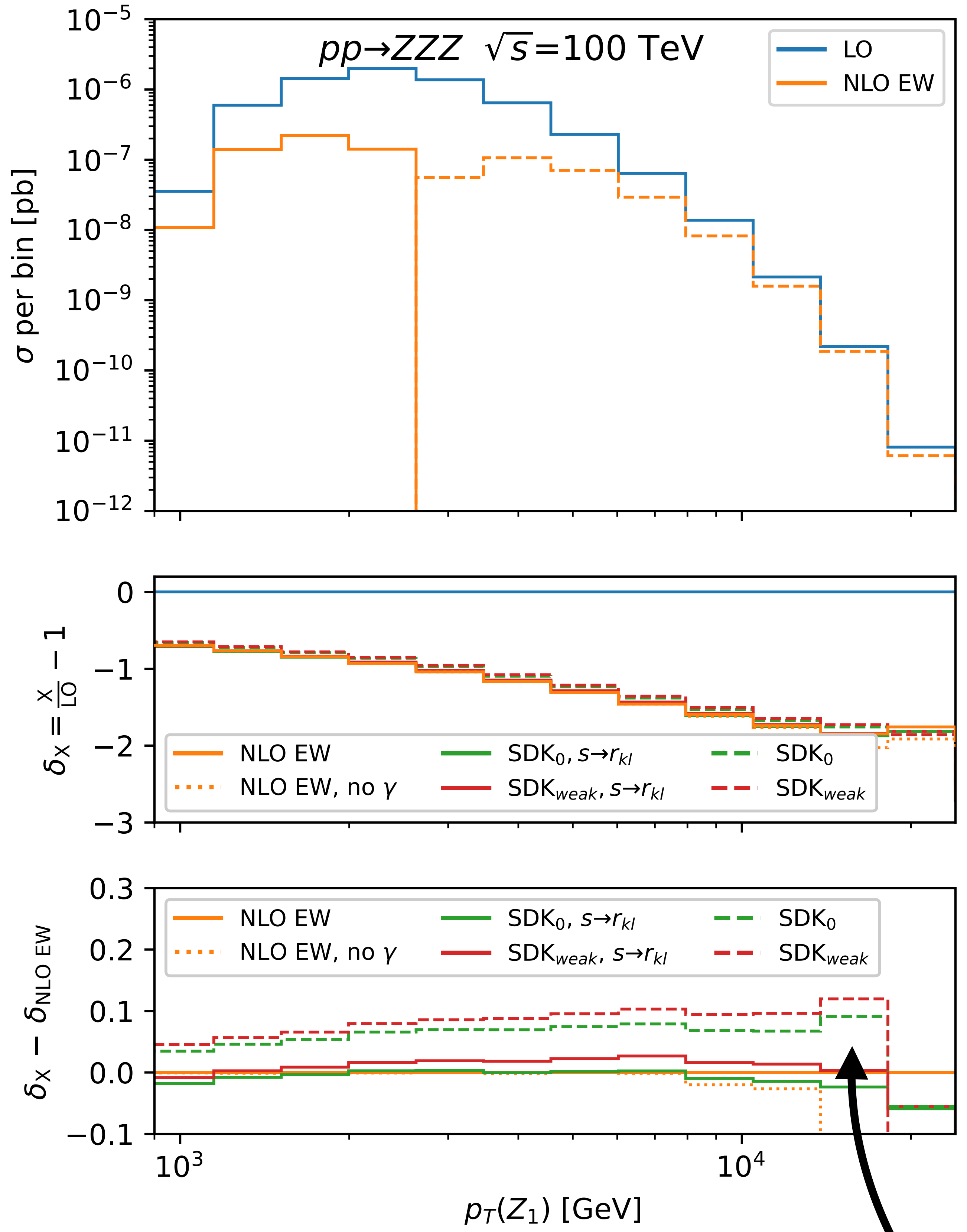

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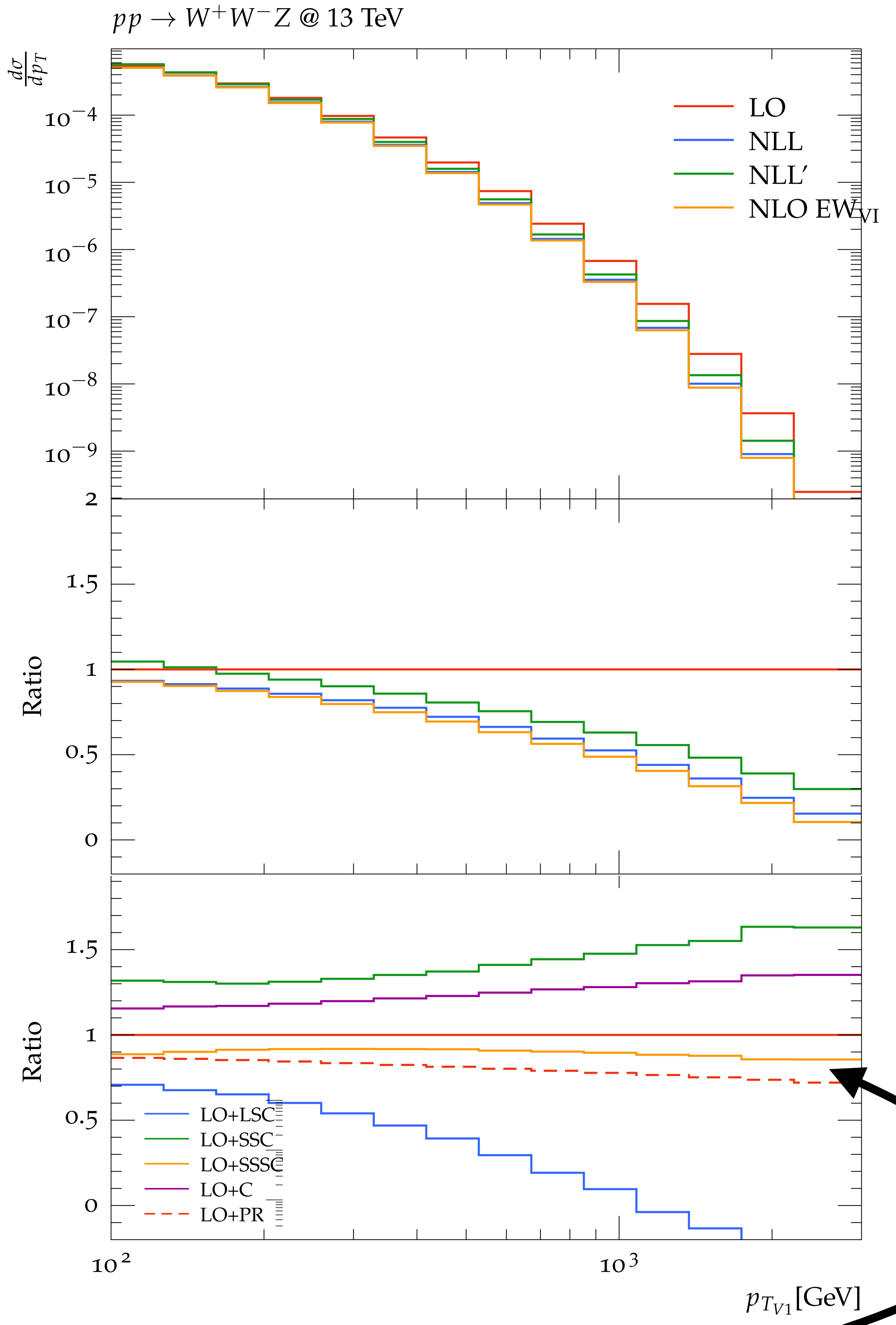

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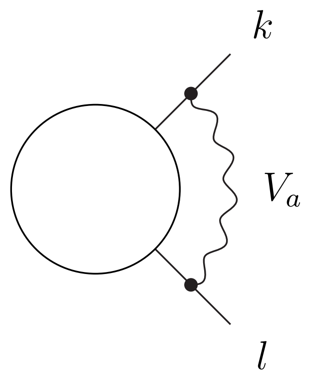

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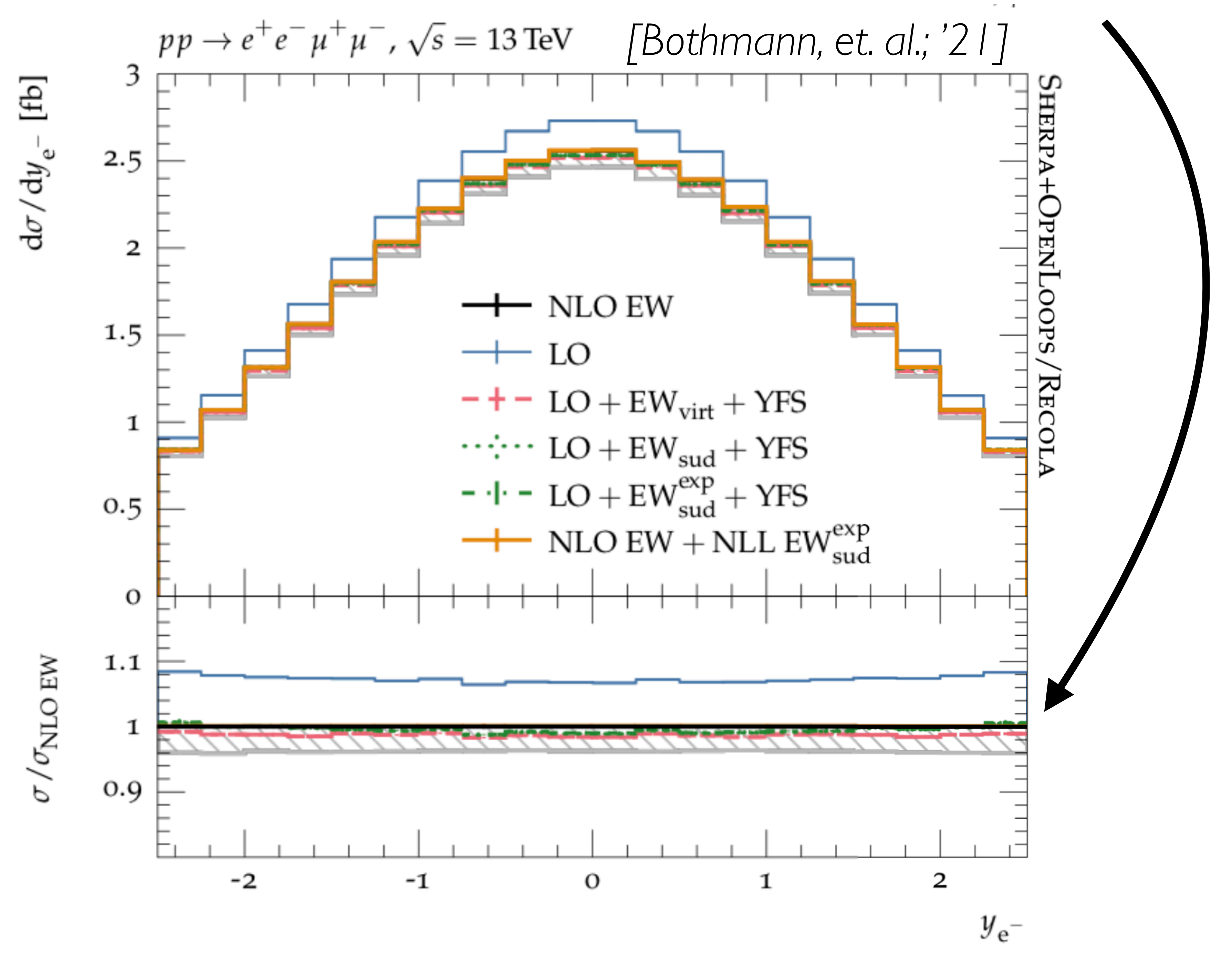
Last term relevant when strict Sudakov limit $r_{kl} = (p_k + p_l)^2 \sim 2p_k p_l \gg M_W^2$ is violated.
However: no control on these terms!

EW uncertainties: hard-coefficient

Scheme variations

e.g. $\{G_\mu, m_W, m_Z\}$ vs. $\{\alpha(m_Z), m_W, m_Z\}$



However: scheme variations mix perturbative and parametric uncertainties!

Estimate hard coefficient

Typical size of hard EW corrections: 2%

$$\left(\frac{\alpha}{\pi}\right)\delta^{(1)}_{\text{hard}} = 2\% \leftrightarrow \delta^{(1)}_{\text{hard}} = 10$$

Require: $\delta^{(2)}_{\text{hard}} \leq 100\,\delta^{(1)}_{\text{hard}}$

$$\Delta^{\text{hard}}_{\text{EW}} = 1000 \times \left(\frac{\alpha}{\pi}\right)^2 = 0.6\%$$

EW uncertainties: QED radiation

Conservative estimate of higher-order QED radiation:

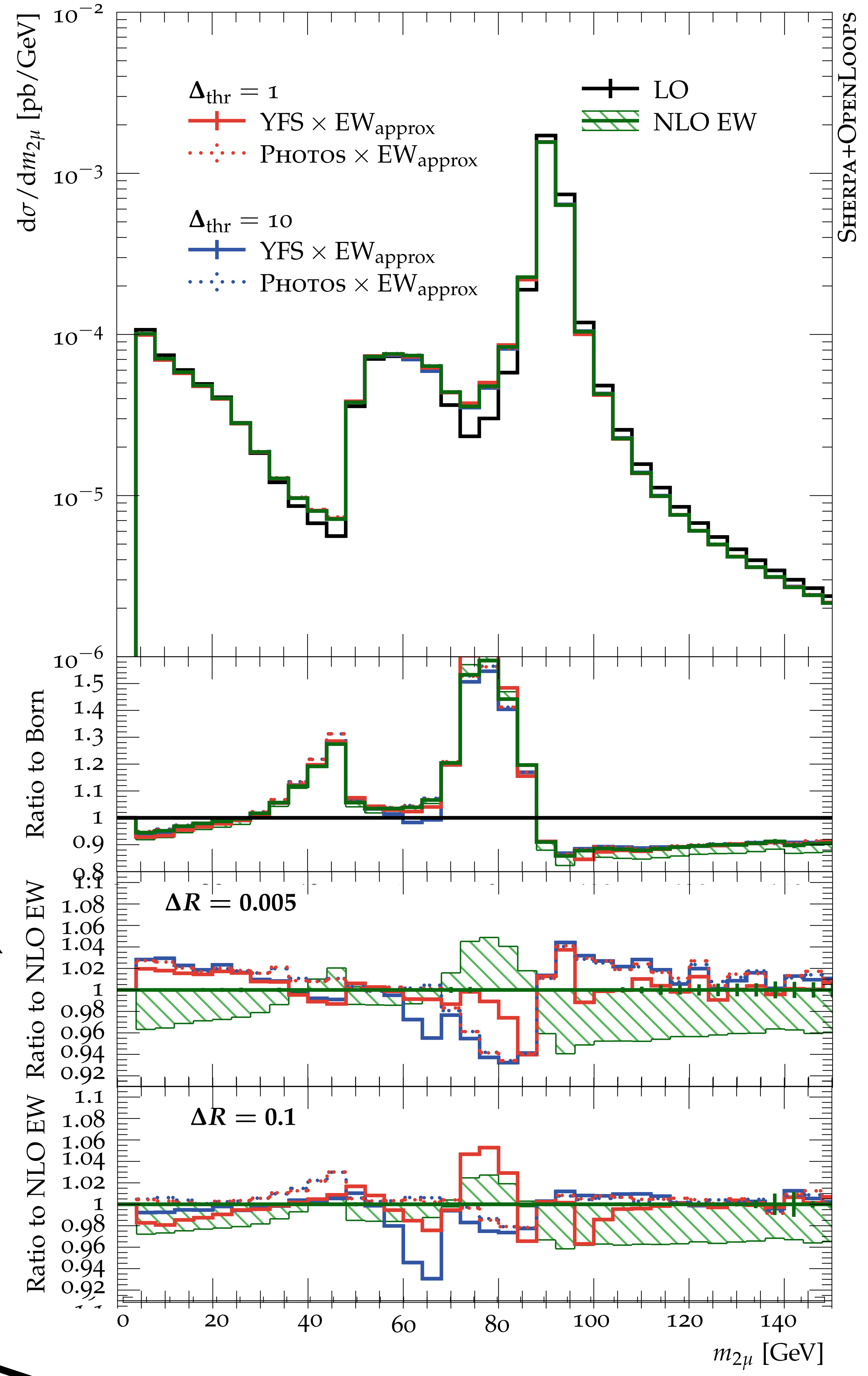
NLO EW

vs.

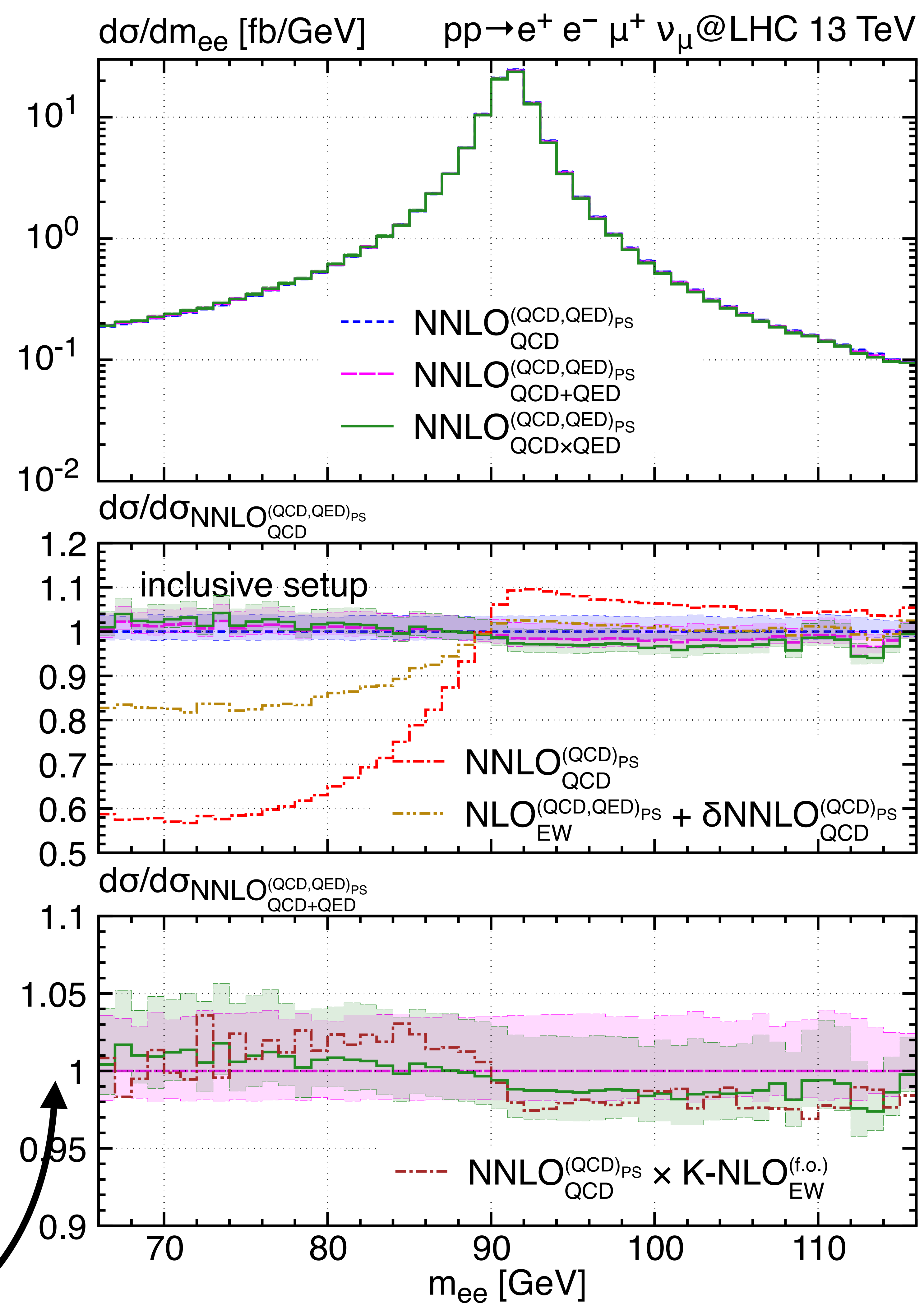
multi-photon radiation (YFS)
or
QED-PS

$$\Delta_{\mathrm{EW}}^{\mathrm{QED}} = |\delta_{\mathrm{EW}} - \delta_{\mathrm{EW+PS/YFS}}|$$

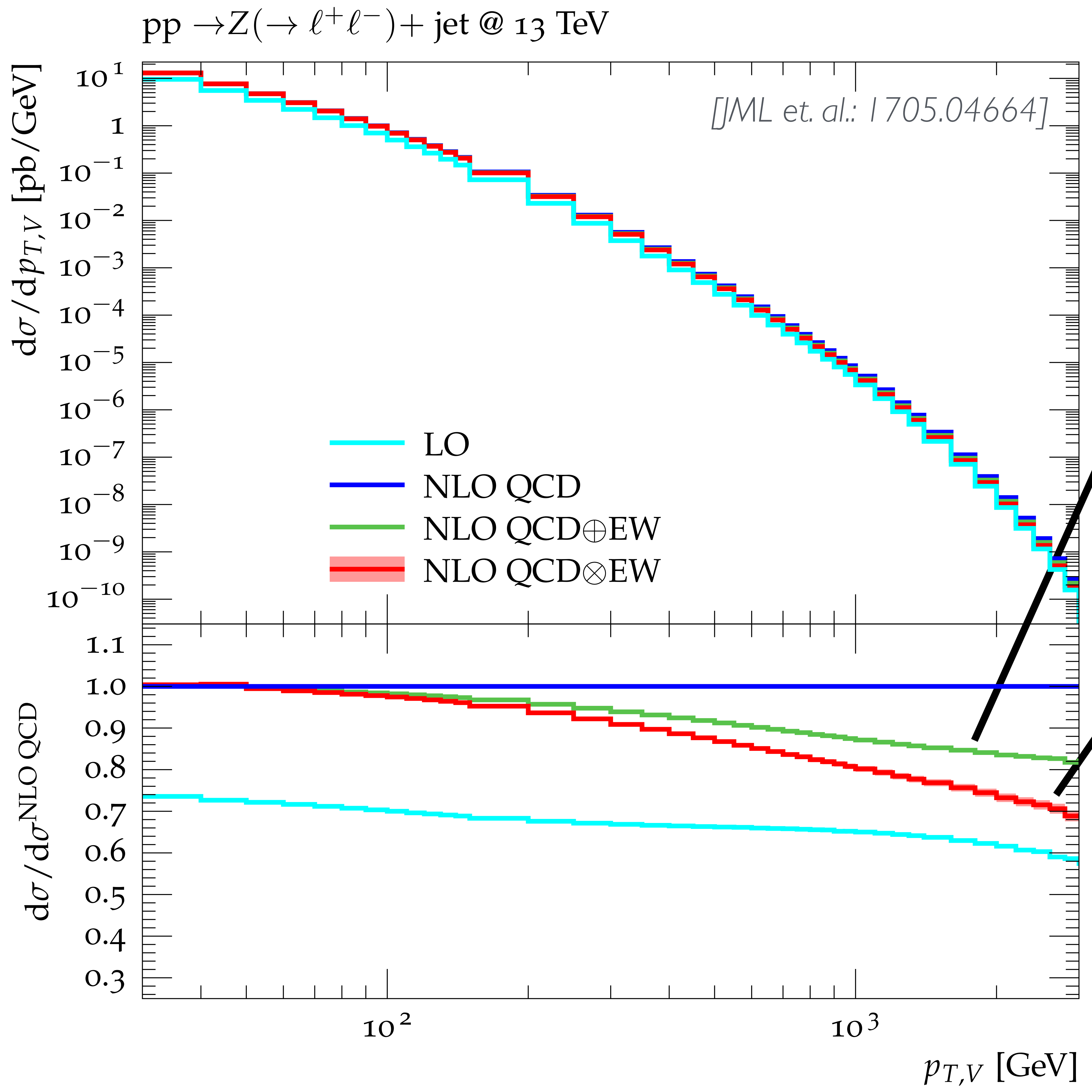
[Gütschow, Schönherr, '20]



[JML, Lombardi, Wiesemann, Zanderighi, Zanoli, '22]



Mixed QCD-EW uncertainties



Additive combination:

$$\sigma^{\rm NLO}_{\rm QCD+EW} = \sigma^{\rm LO} + \delta\sigma^{\rm NLO}_{\rm QCD} + \delta\sigma^{\rm NLO}_{\rm EW}$$

(no $\mathcal{O}(\alpha\alpha_s)$ contributions)

Multiplicative combination

$$\sigma^{\rm NLO}_{\rm QCD\times EW} = \sigma^{\rm NLO}_{\rm QCD}\left(1+\frac{\delta\sigma^{\rm NLO}_{\rm EW}}{\sigma^{\rm LO}}\right)$$

(try to capture some $\mathcal{O}(\alpha\alpha_s)$ contributions,
e.g. EW Sudakov logs × soft QCD)

Difference between these two approaches indicates size of missing mixed EW-QCD corrections.

$$\Delta_{\rm QCD-EW} = \delta_{\rm QCD}\,\delta_{\rm EW} \sim 10\% \quad \text{at 1 TeV}$$

Too conservative!?

For dominant Sudakov EW logarithms factorization should be exact!

→alternative: $\Delta_{\rm QCD-EW} = \delta_{\rm QCD}\,(\delta^{\rm SL}_{\rm EW} + \delta^{\rm hard}_{\rm EW})$

Mixed QCD-EW uncertainties

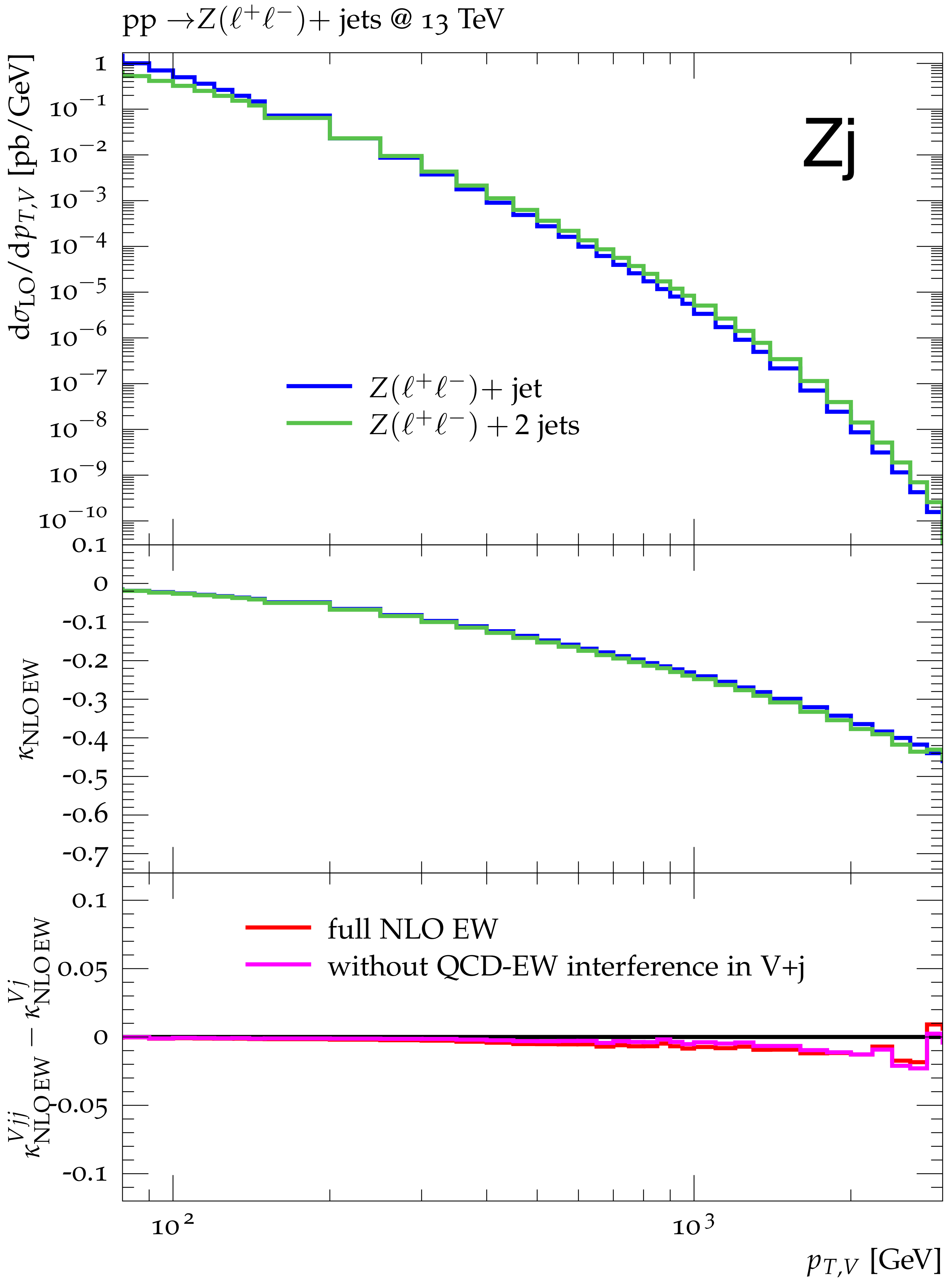
Bold estimate:

Consider real $\mathcal{O}(\alpha\alpha_s)$ correction to X production $\simeq$ NLO EW to X+1jets

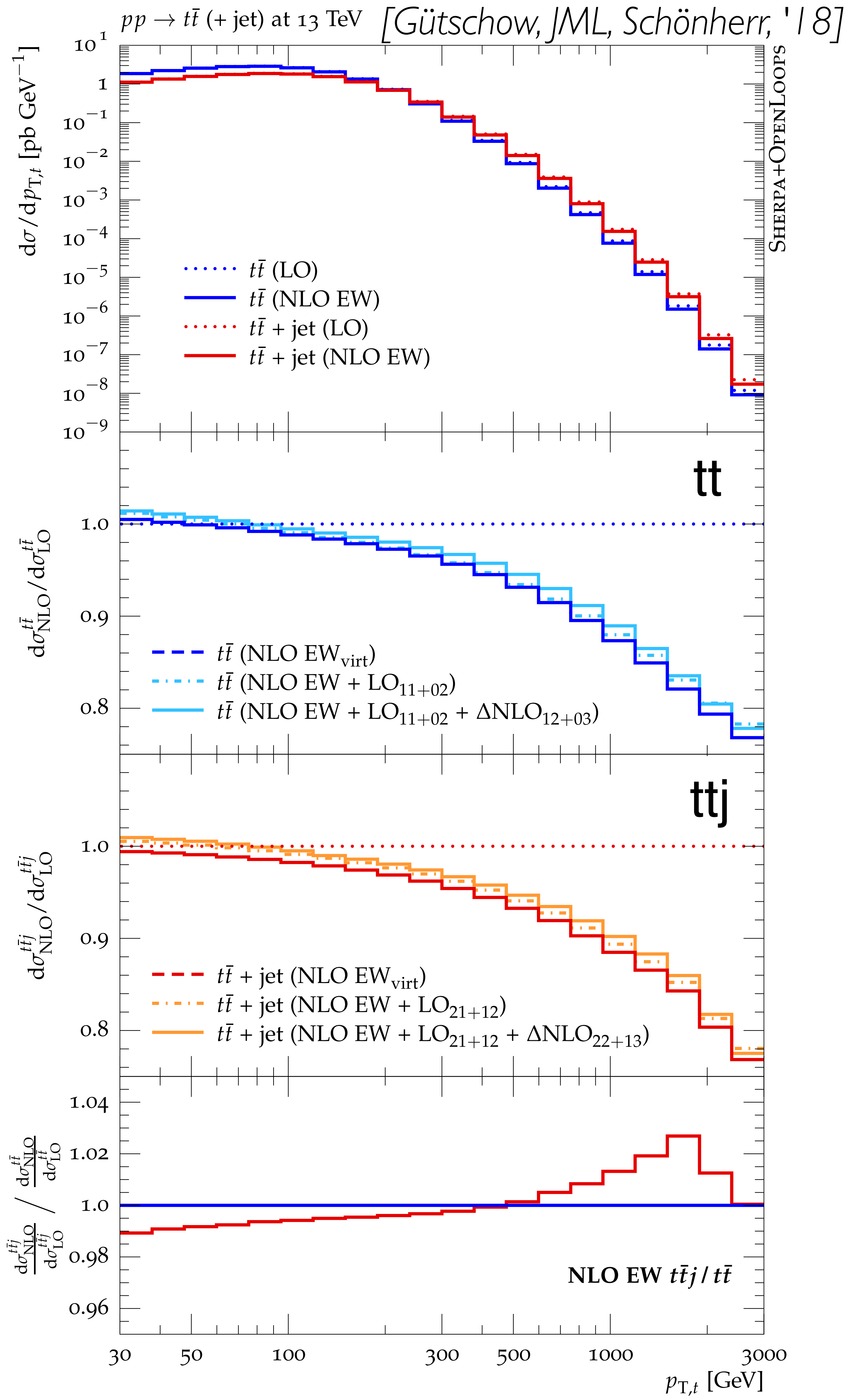
and we often observe

$$\left.\frac{d\sigma_{\mathrm{NLO\,EW}}}{d\sigma_{\mathrm{LO}}}\right|_{X+\mathrm{jet}} - \left.\frac{d\sigma_{\mathrm{NLO\,EW}}}{d\sigma_{\mathrm{LO}}}\right|_{X} \lesssim 1\%$$

In these cases strong support for
- factorisation
- multiplicative QCD x EW combination
- Consider only such non-factorising effects as uncertainty!?



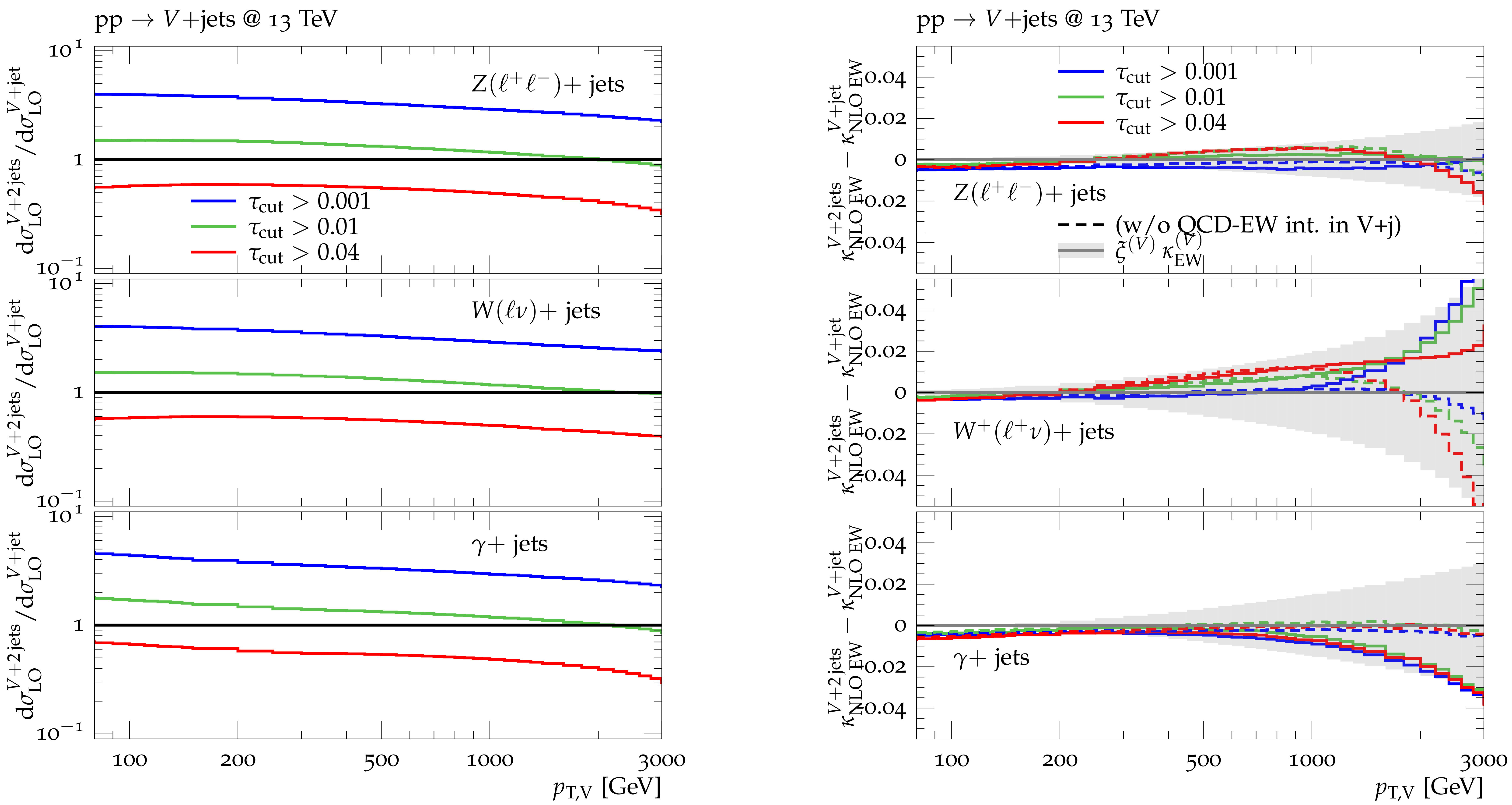
$pT_{j,2} > 30$ GeV



$pT_j > 30$ GeV

Mixed QCD-EW uncertainties

Estimate of non-factorising contributions



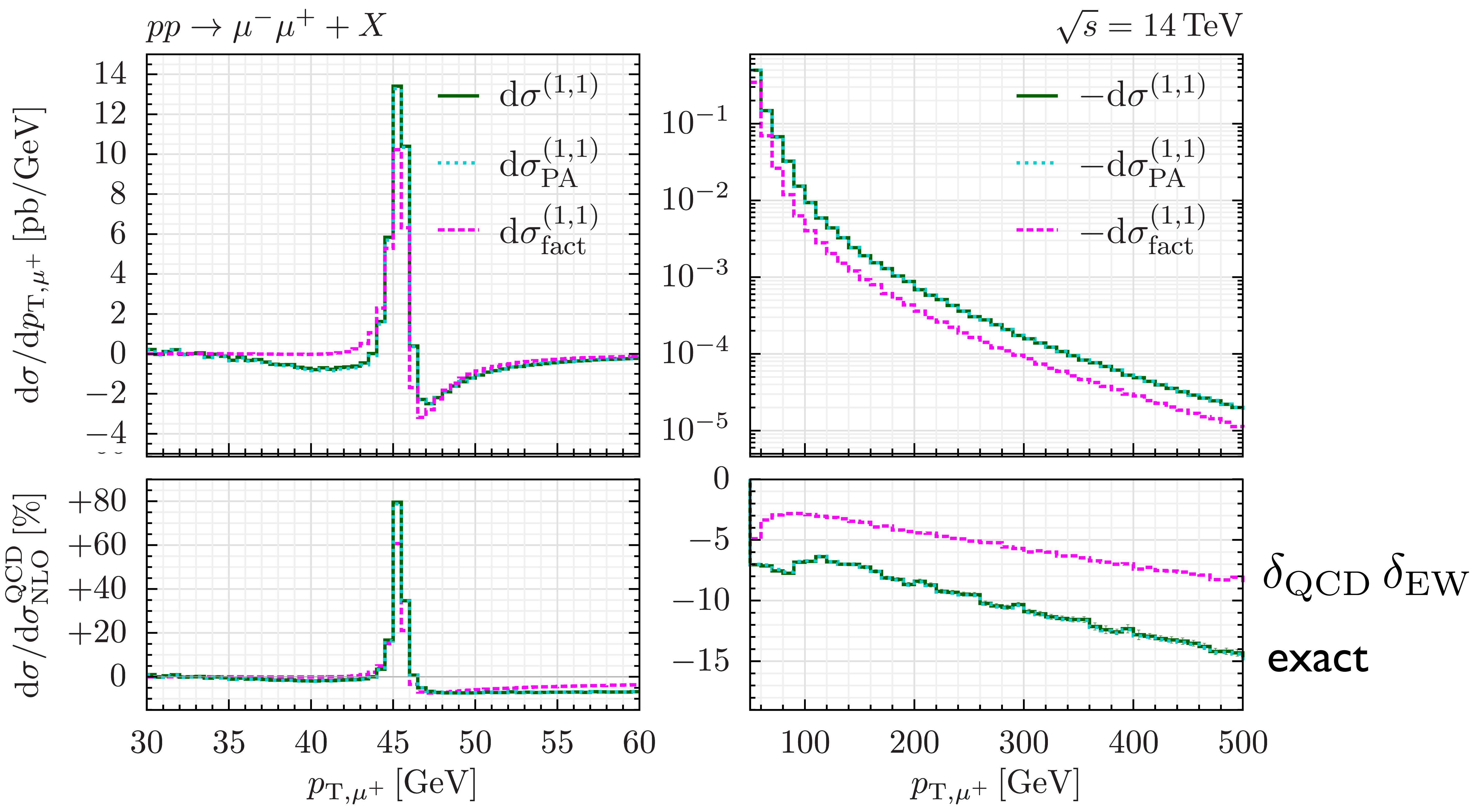
N-jettiness cut ensures approx. constant ratio V+2jets/V+jet

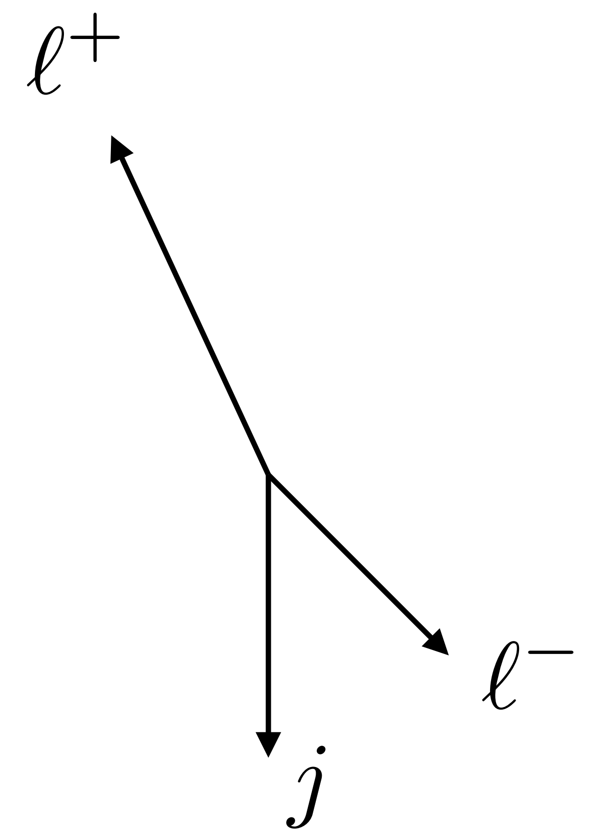
$$\tau_1 = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i \sqrt{\hat{s}}} \right\}$$

Exact mixed QCD-EW for DY

[Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch, '20]
[Behring, Buccioni, Caola, et. al. '20]

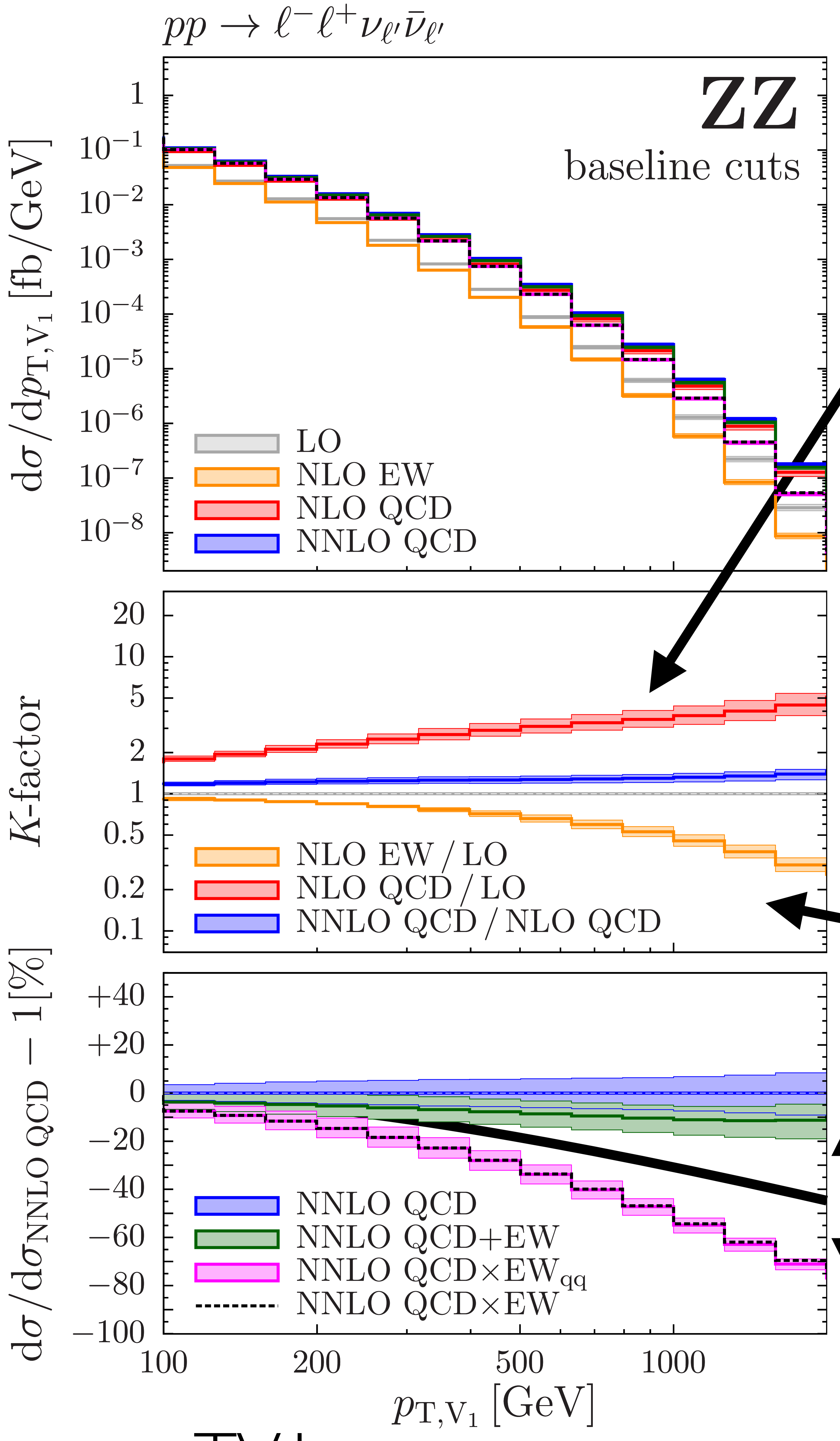
[Bonciani, Buonocore, Grazzini, Kallweit et. al. 2 x '21]





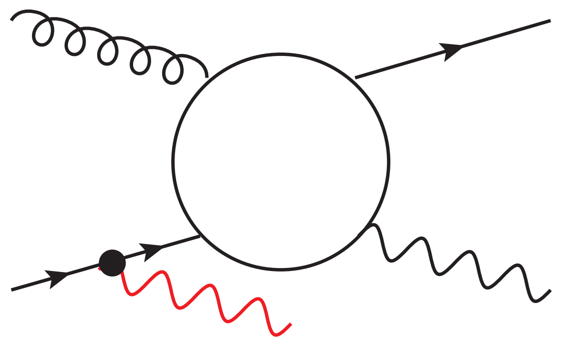
- pole approximation vs. full computation: agree below the percent level
- Comparison against naive factorised NLO QCD x NLO EW ansatz: fail at the 5-10% level
- At large p_{T,μ^+} in DY: sizeable contributions from $pp \to Vj$ which receives larger EW corrections

Mixed QCD-EW uncertainties



pTV1

- NLO QCD/LO=2-5! ("giant K-factor")
  - at large pTV1: VV phase-space is dominated by V+jet (w/ soft V radiation)



$$\frac{d\sigma^{V(V)j}}{d\sigma^{\rm LO}_{VV}} \propto \alpha_S \log^2\left(\frac{Q^2}{M_W^2}\right) \simeq 3 \qquad \text{at} \quad Q = 1\,\text{TeV}$$

- NNLO / NLO QCD moderate and NNLO uncert. 5-10%
- NLO EW/LO=-(40-50)%
- Very large difference $\mathrm{d}\sigma_{\rm NNLO\,QCD+EW}$ vs. $\mathrm{d}\sigma_{\rm NNLO\,QCD\times EW}$
- Problems:
  1. In additive combination dominant Vj topology does not receive any EW corrections
  2. In multiplicative combination EW correction for VV is applied to Vj hard process
- Pragmatic solution I: **take average as nominal and spread as uncertainty**
- Pragmatic solution II: **apply jet veto to constrain Vj toplogoies**

Mixed QCD-EW uncertainties

- More rigorous solution: merge VVj incl. approx. EW corrections with VV with Sherpa's MEPS@NLO QCD + EWvirt

[Kallweit, JML, et. al.; '15]

$$\overline{\mathrm{B}}_{n,\mathrm{QCD+EW_{virt}}}(\Phi_n) = \overline{\mathrm{B}}_{n,\mathrm{QCD}}(\Phi_n) + \mathrm{V}_{n,\mathrm{EW}}(\Phi_n) + \mathrm{I}_{n,\mathrm{EW}}(\Phi_n) + \mathrm{B}_{n,\mathrm{mix}}(\Phi_n)$$

exact virtual contribution

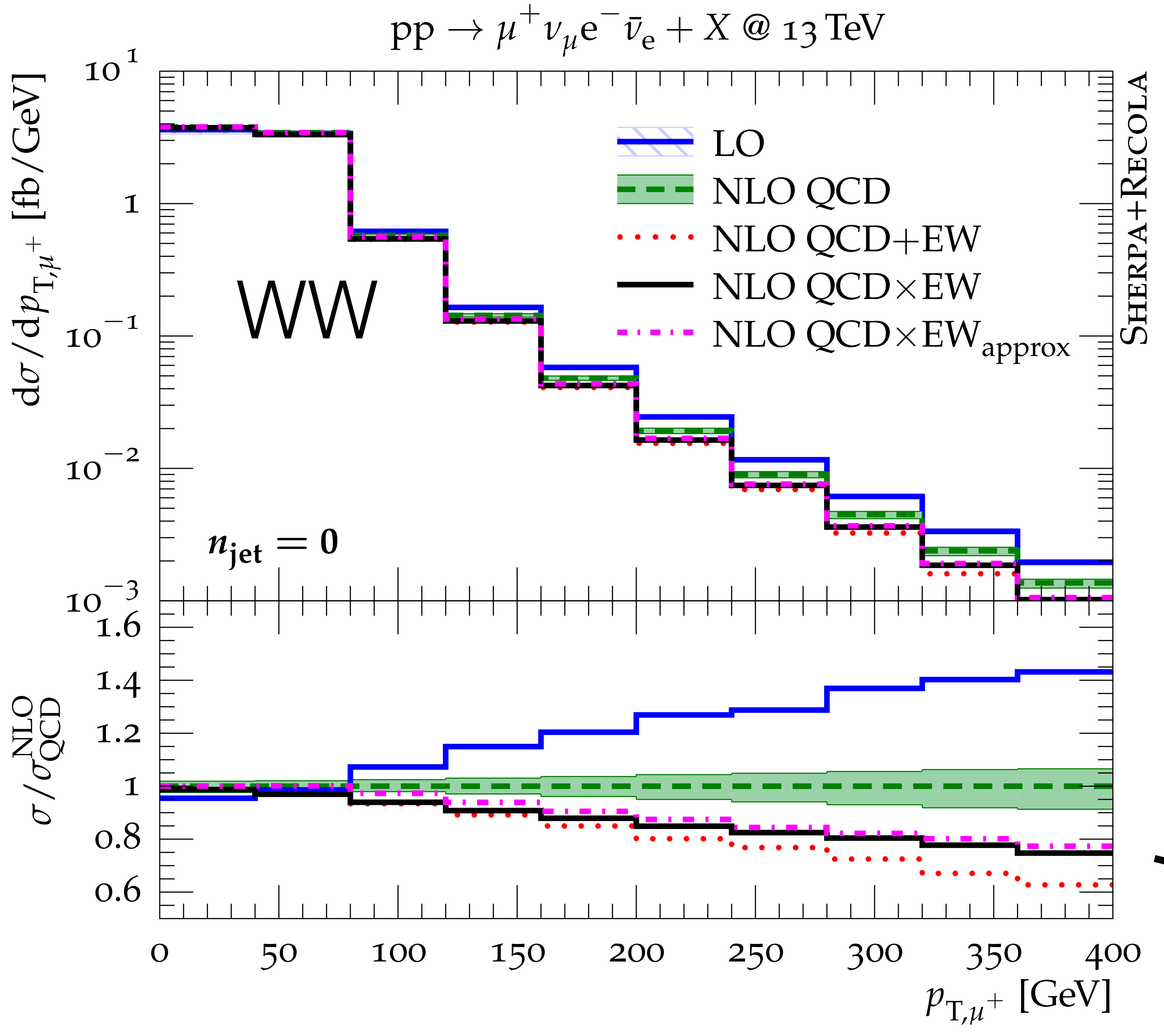
approximate integrated real contribution

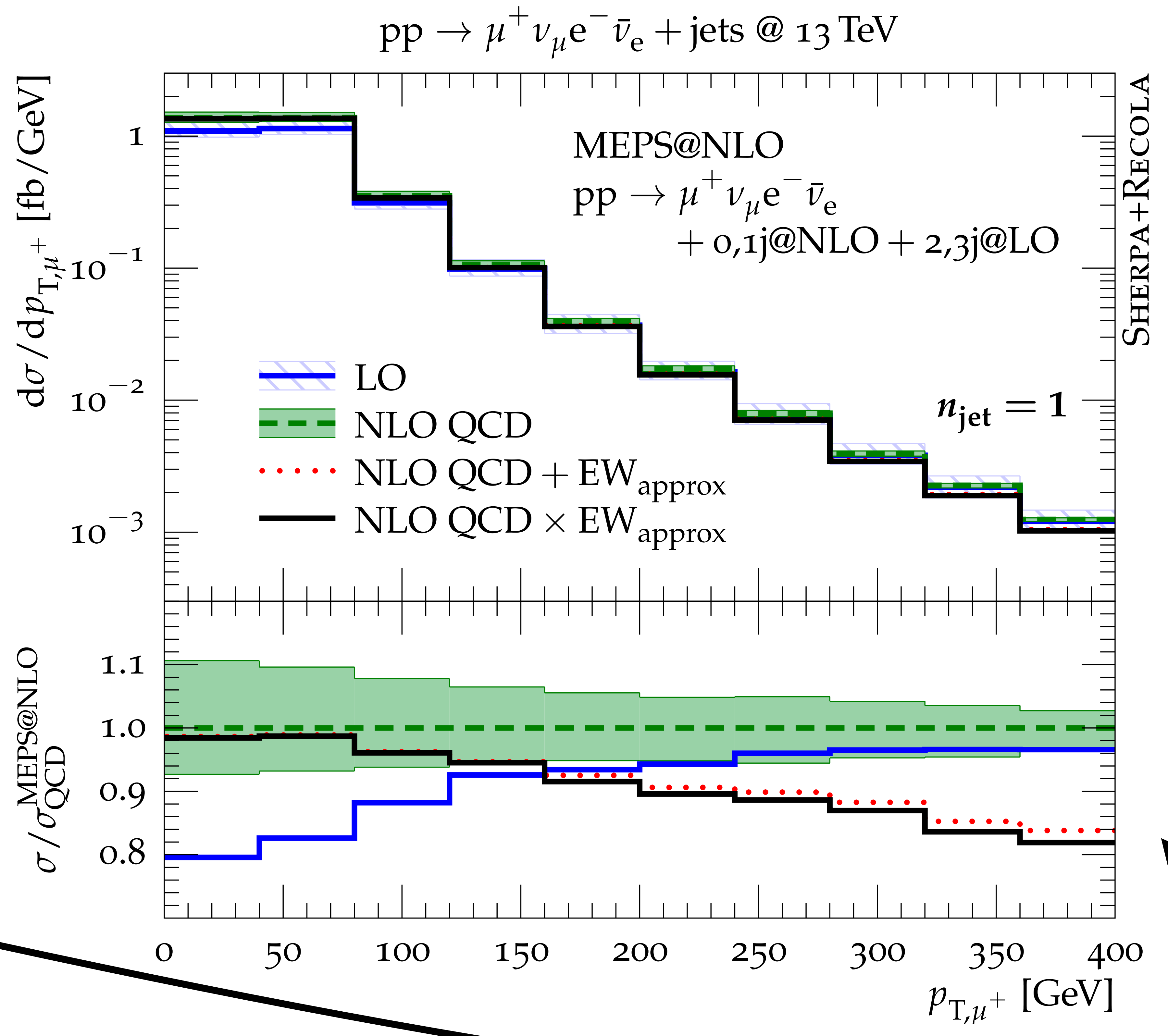
Used in many ATLAS modern multi-purpose samples: V+jets, VV+jets, tt+jets

FO

WW(+jet): [Bräuer, et. al.; '20]
ZZ(+jet): [Bothmann, et. al. '21]

MEPS@NLO QCD + EWvirt

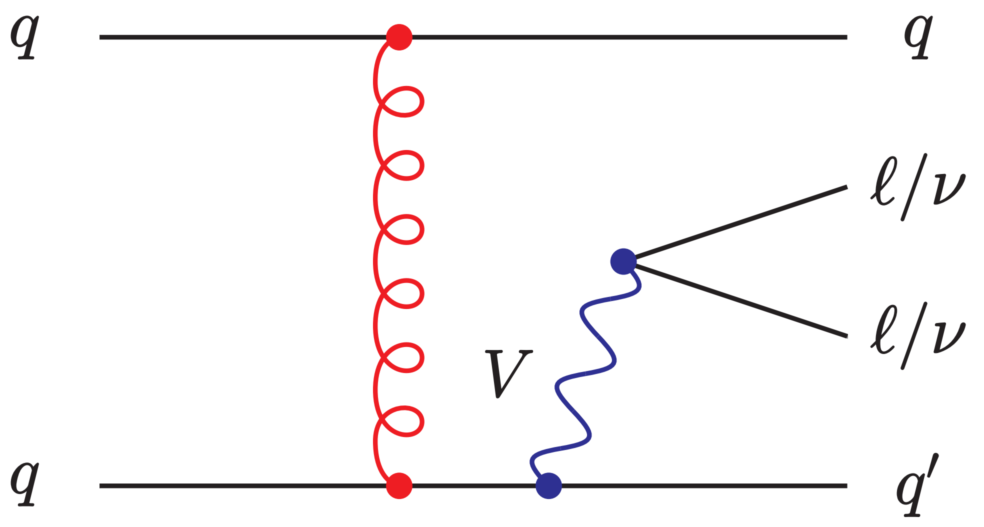


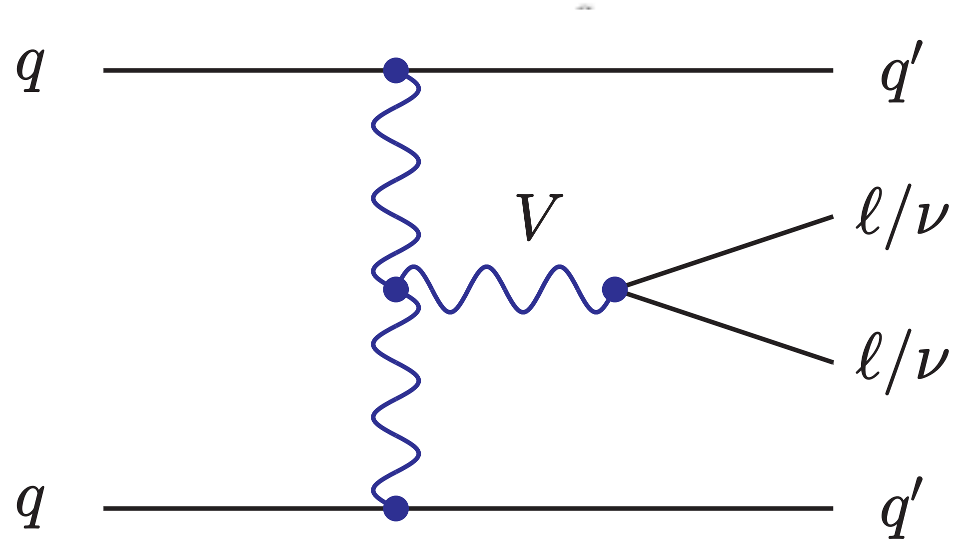


Perturbative expansion: tower of contributions

- For processes with at least 4q there is a tower of LO(NLO) contributions.
- E.g.: multijets, $t\bar{t} + X$, V+jets (VBF-V), VV+jets (VBS-VV),

V+2 jets:





$$\mathrm{d}\sigma = \mathrm{d}\sigma(\alpha_S^2\alpha^2) + \mathrm{d}\sigma(\alpha_S\alpha^3) + \mathrm{d}\sigma(\alpha^4) + \ldots$$

LO

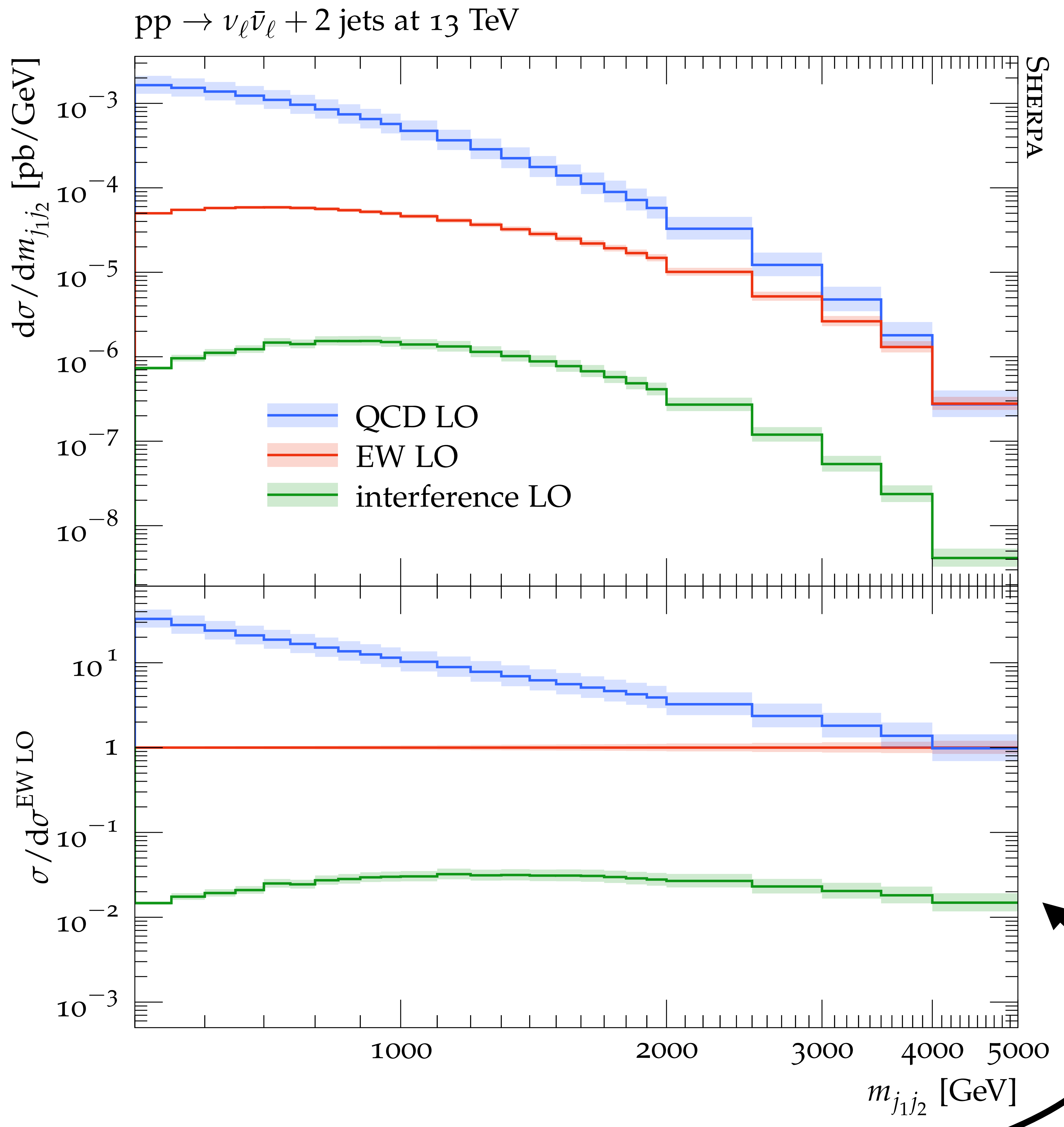
QCD-mode $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$ interference $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$ VBF-mode

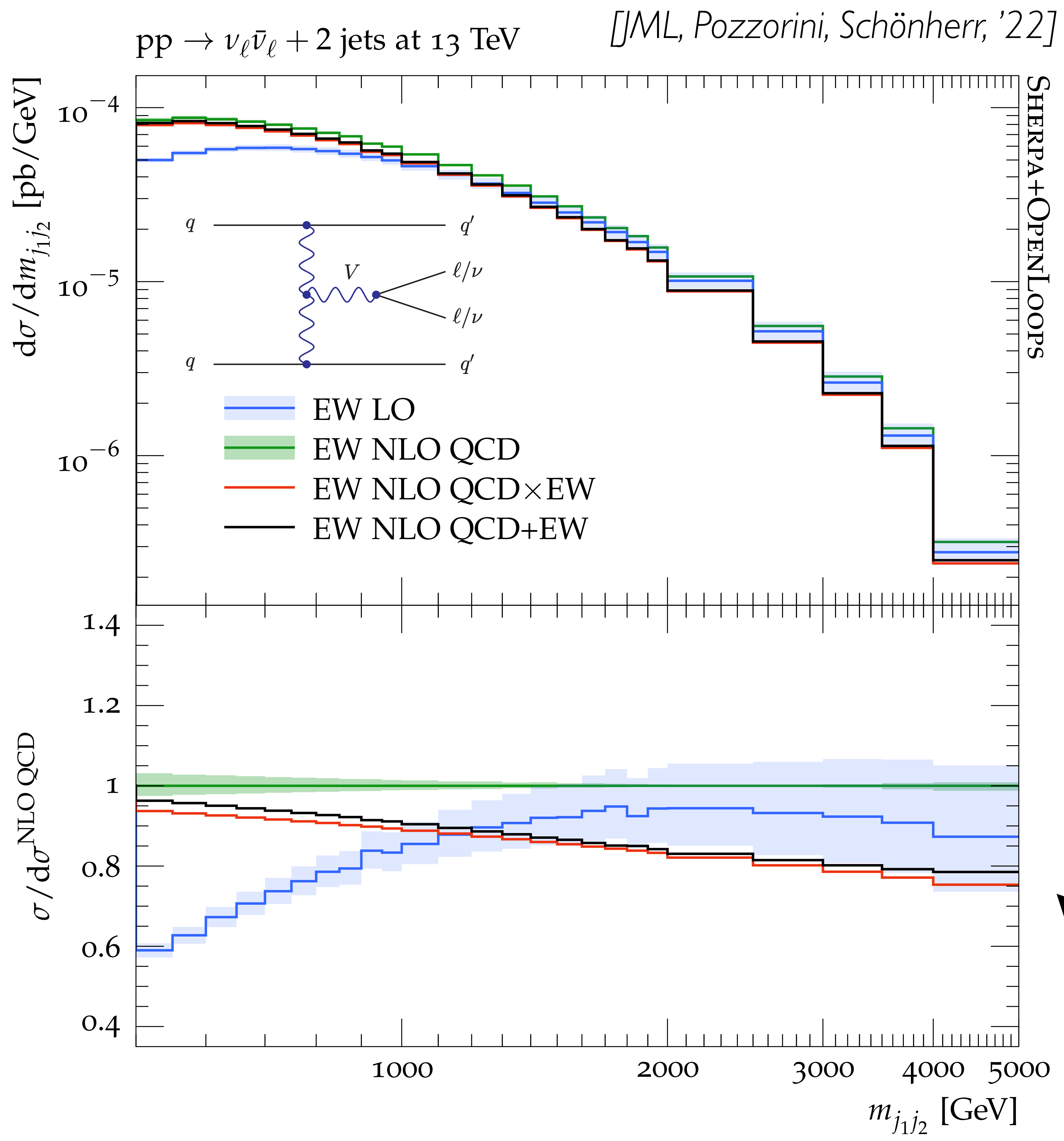
$$\cdots + \mathrm{d}\sigma(\alpha_S^3\alpha^2) + \mathrm{d}\sigma(\alpha_S^2\alpha^3) + \mathrm{d}\sigma(\alpha\alpha^4) + \sigma(\alpha^5)$$

NLO

"NLO QCD" "NLO EW" "NLO QCD" "NLO EW"

Mixed QCD-EW uncertainties





- If LO interference is small: possible to consider QCD and EW production modes as independent and factorise QCD and EW corrections to the respective processes
- Otherwise, still factorise but consider QCD+EW combination as nominal (and QCD×EW as uncertainty)

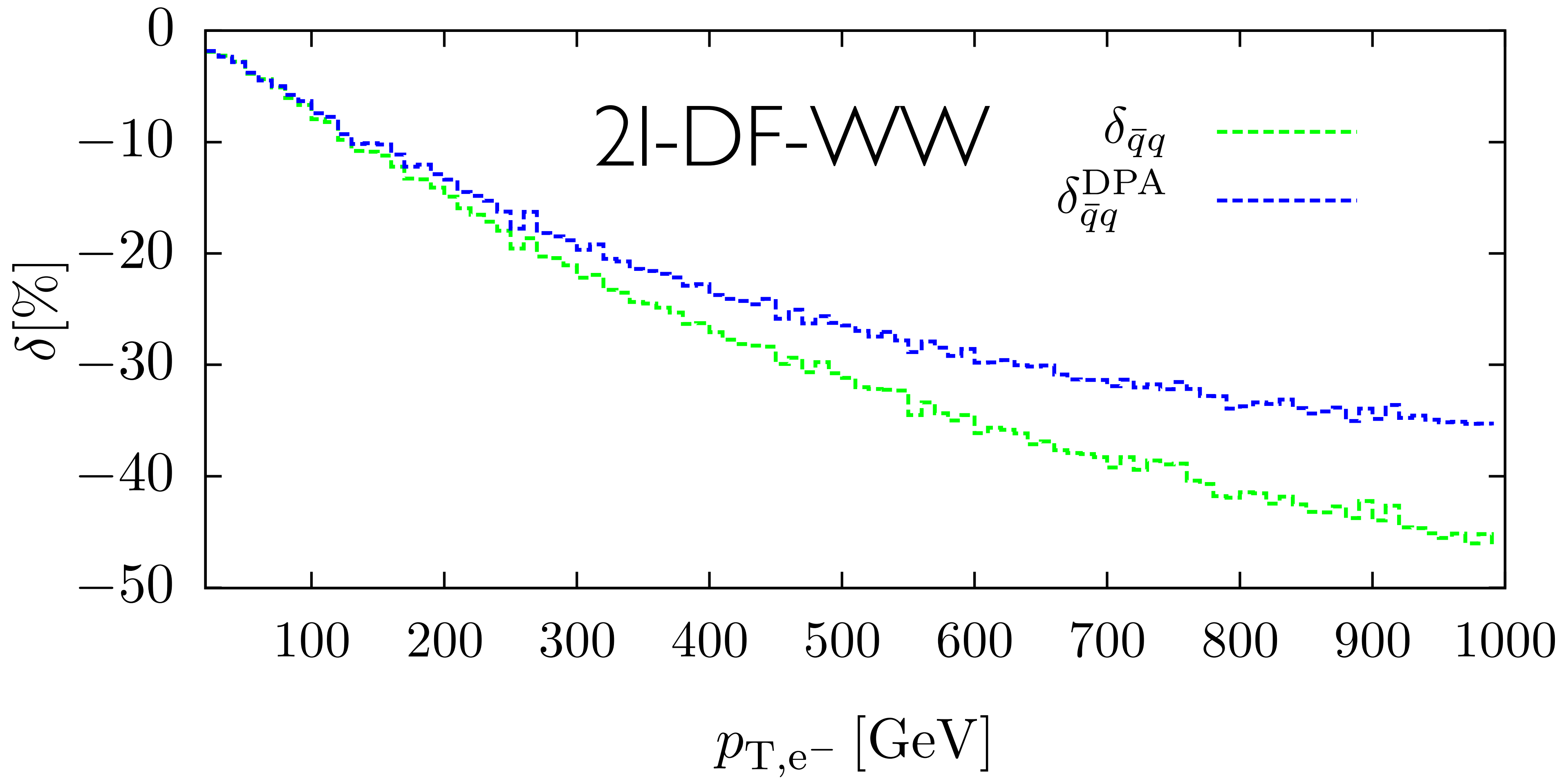
Conclusions

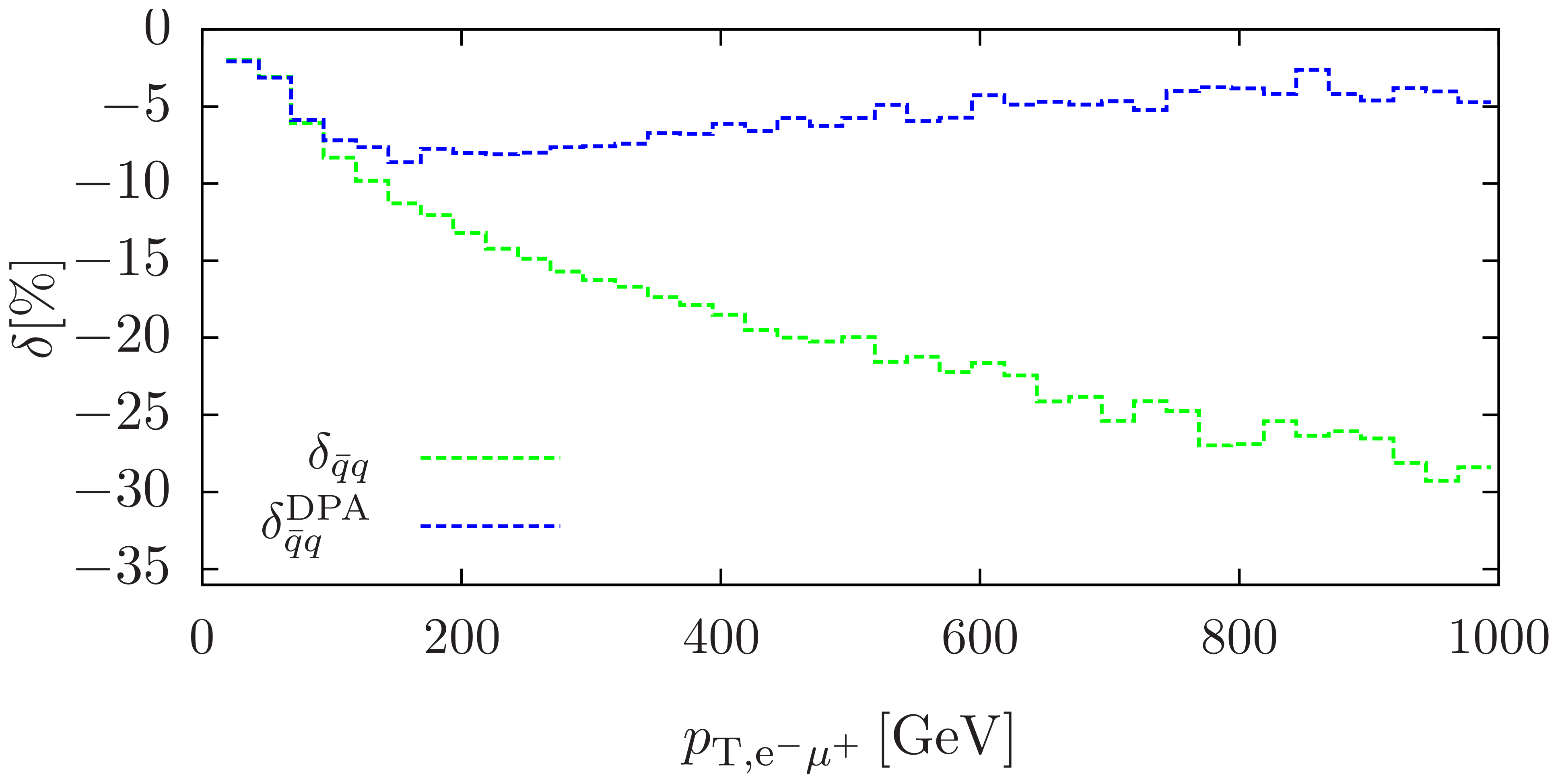
- At the precision frontier reliable estimates of EW and QCD-EW uncertainties are becoming mandatory
- EW uncertainties:
  - Higher-order Sudakov corrections: $\Delta_{\rm EW}^{\rm Sud} = \left(\delta_{\rm Sud}^{(1)}\right)^2$
  - Higher-order hard corrections: $\Delta_{\rm EW}^{\rm hard} \approx 1\%$
  - Higher-order QED radiation: $\Delta_{\rm EW}^{\rm QED} = |\delta_{\rm EW} - \delta_{\rm EW+PS/YFS}|$
- QCD-EW uncertainties:
  - Conservative: difference between add. and multipl. combination: $\Delta_{\rm QCD-EW} = \delta_{\rm QCD}\,\delta_{\rm EW}$
  - More aggressive: $\Delta_{\rm QCD-EW} = \delta_{\rm QCD}\,(\delta_{\rm EW}^{\rm SL} + \delta_{\rm EW}^{\rm hard})$ (applicable when $\delta_{\rm EW} \sim \delta_{\rm EW}^{\rm DL}$)
  - For processes subject to significant QCD radiation: $\Delta_{\rm QCD-EW}^{\rm multi-jet\ merged} = \delta_{\rm QCD}\,\delta_{\rm EW}$
  - X+j @ NLO EW computations might allow for estimate of non-factorising effects
  - Factorisation feasible for processes with small interferences of born orders
- Necessary tools are available:
  - NLO EW in MG5_aMC@NLO / Sherpa / POWHEG
  - NLL EW in Sherpa / MG5_aMC@NLO / OpenLoops
  - NLOPS EW in POWHEG / MEPS NLO EW + YFS in Sherpa

Backup

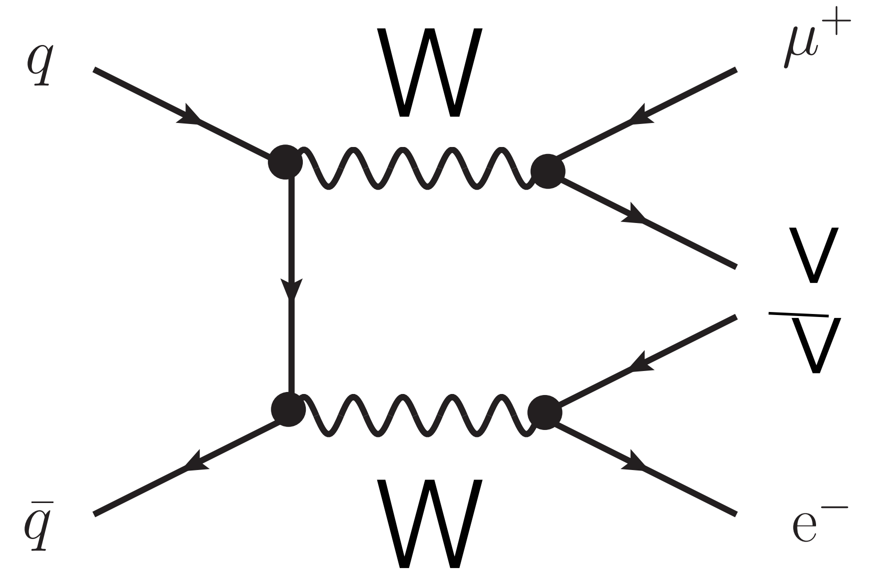
The need for off-shell computations: VV

[Biedermann, M. Billoni, A. Denner, S. Dittmaier, L. Hofer, B. Jäger, L. Salfelder ;'16]

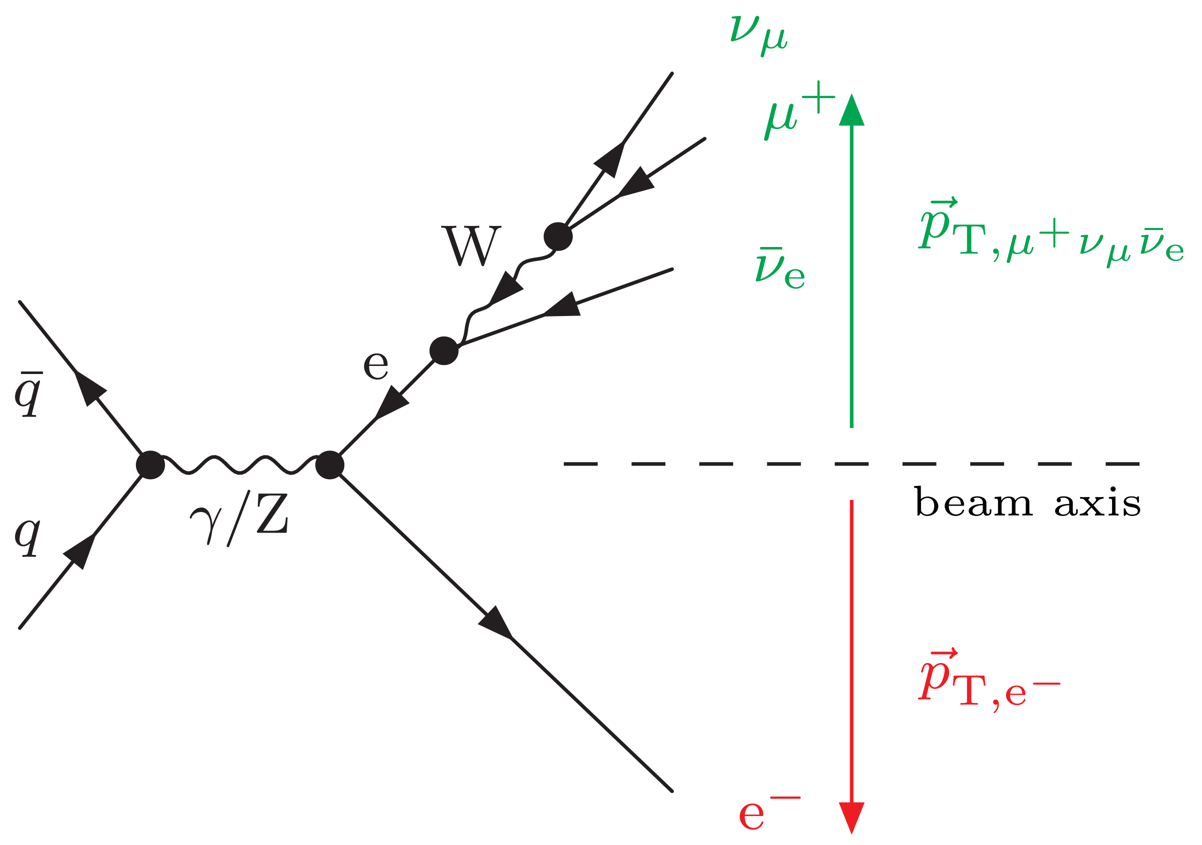




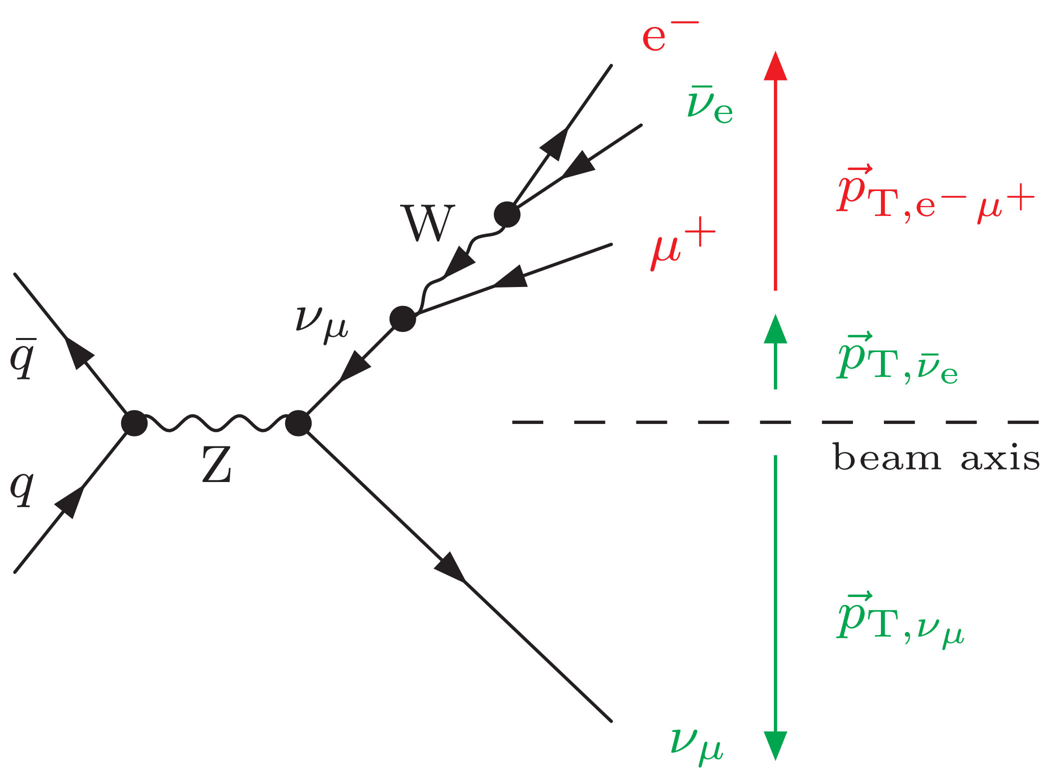
$$pp \to V(\to \ell\bar{\ell})V'(\to \ell'\bar{\ell}')$$



vs.



$$pp \to \ell\bar{\ell}\ell'\bar{\ell}'$$

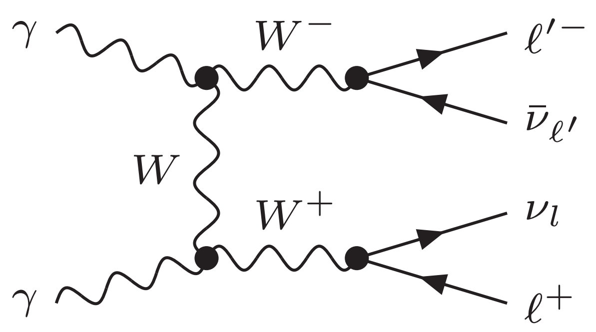


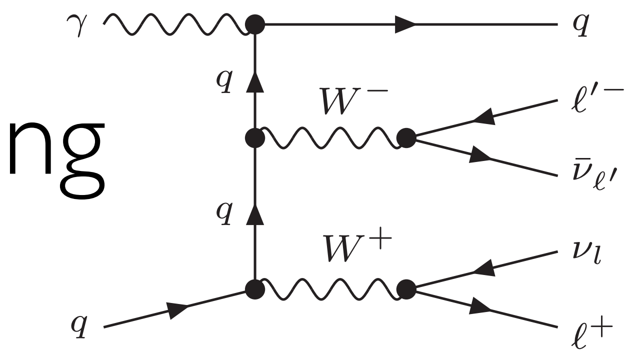
➡ sizeable differences in fully off-shell vs. double-pole approximation in tails

Relevance of EW higher-order corrections: photon-induced channels

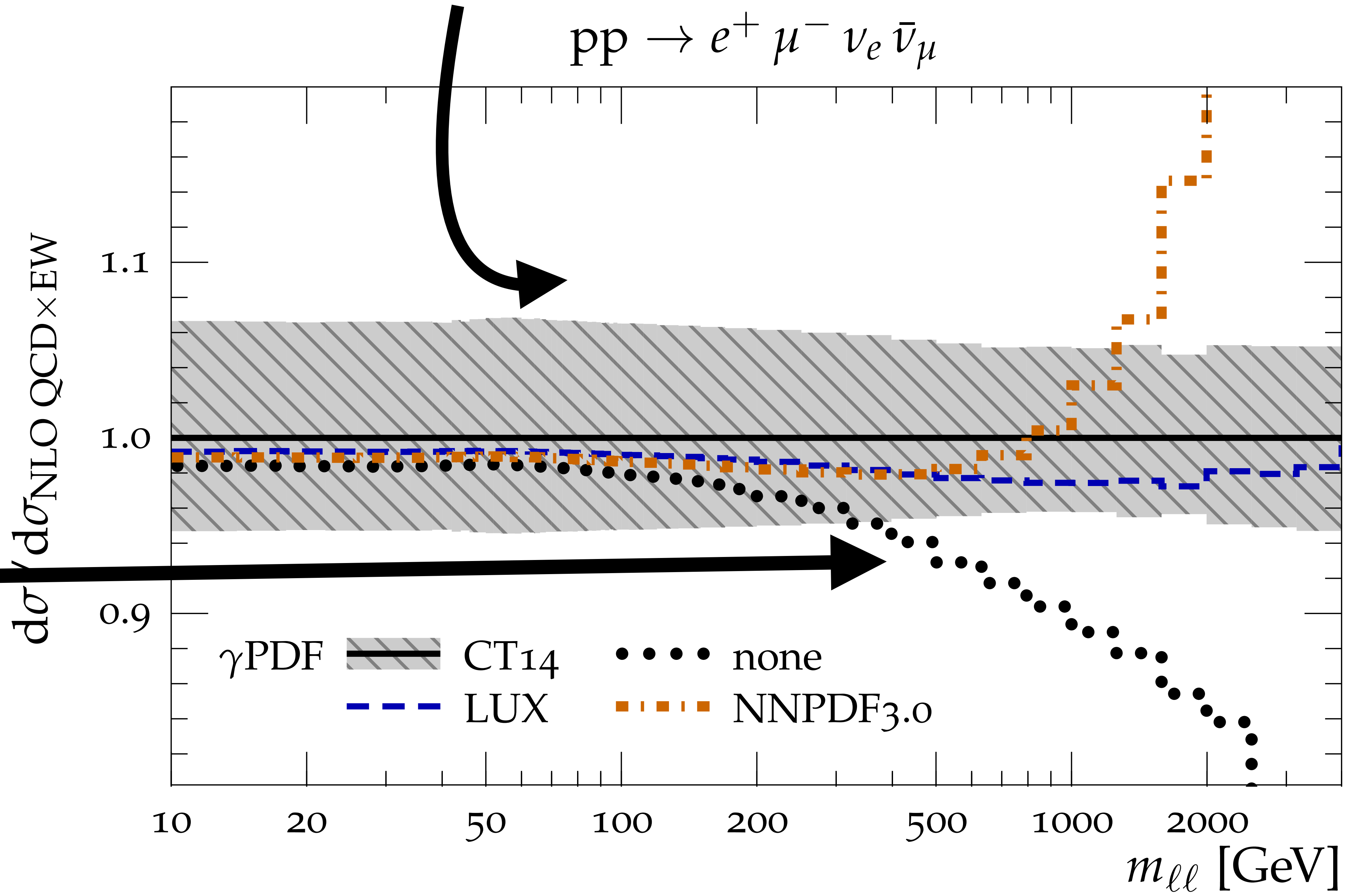
III. QED factorisation and thus photon luminosities needed to absorb IS photon singularities.

➜ Possible large enhancement due to photon-induced channels in the tails of kinematic distributions,

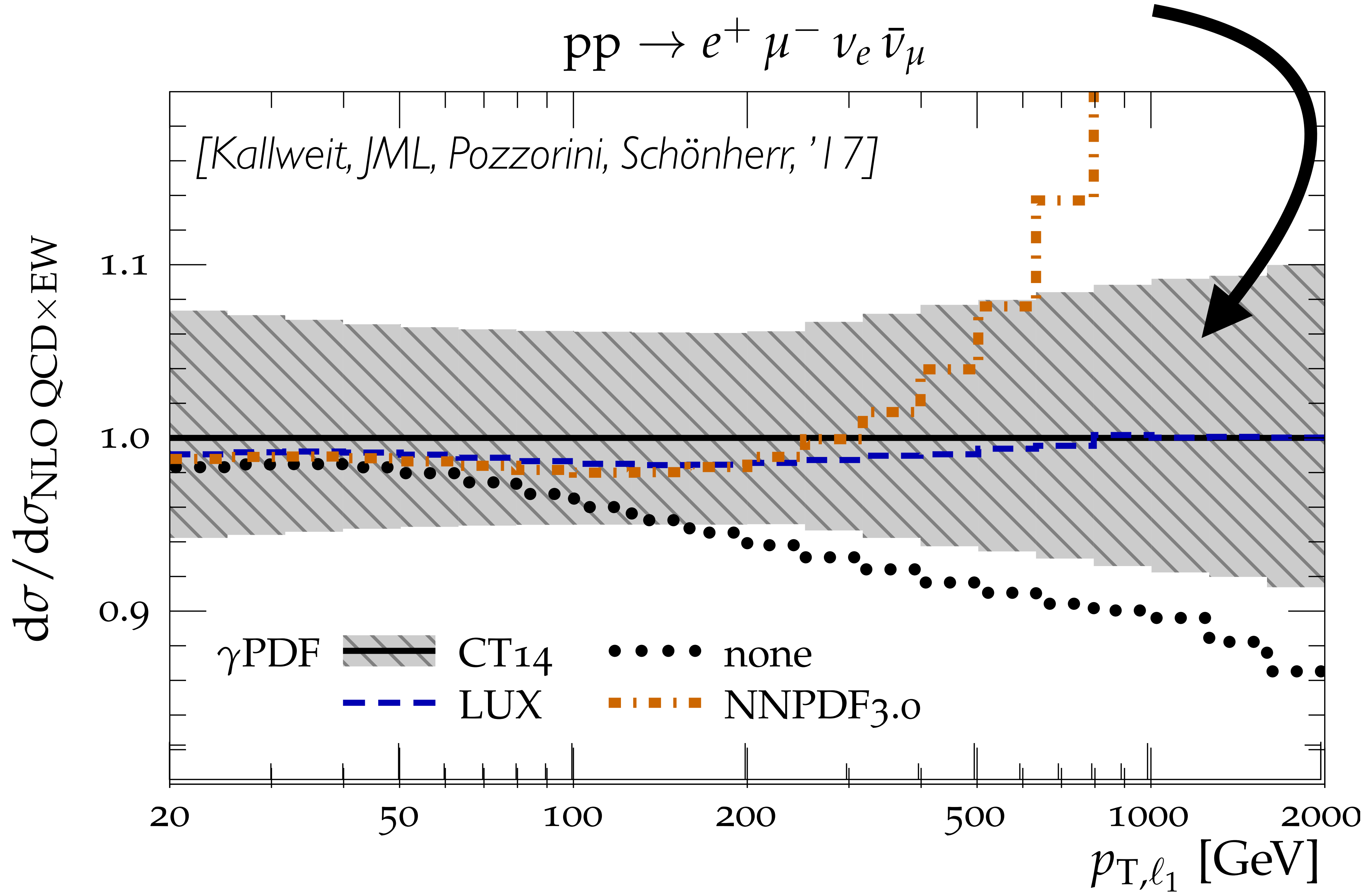
in particular in WW: 

(t-channel enhancement), but also in Bremsstrahlung 

$$d\sigma_{LO} = d\sigma_{LO}^{q\bar{q}} + d\sigma_{LO}^{\gamma\gamma}$$



no γ



➜ **large differences between different photon descriptions. Now settled: LUXqed superior**

➜ **O(10%) contributions from photon-induced channels**

Combination of QCD and EW corrections

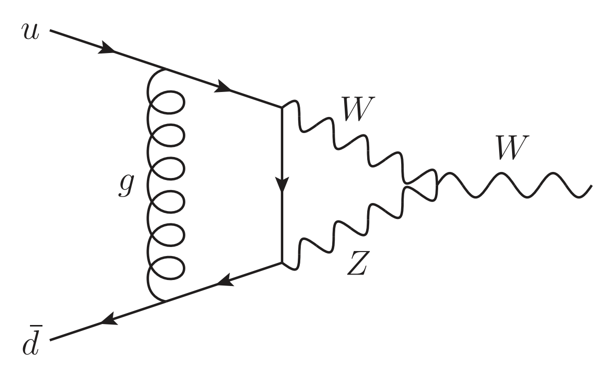
- full calculations of $\mathcal{O}(\alpha\alpha_s)$ out of reach
- Approximate combination: MEPS@NLO including (approximate) EW corrections
- key: QCD radiation receives EW corrections!
- strategy: modify MC@NLO B-function to include NLO EW virtual corrections and integrated approx. real corrections = VI

$$\overline{\mathrm{B}}_{n,\mathrm{QCD+EW_{virt}}}(\Phi_n) = \overline{\mathrm{B}}_{n,\mathrm{QCD}}(\Phi_n) + \mathrm{V}_{n,\mathrm{EW}}(\Phi_n) + \mathrm{I}_{n,\mathrm{EW}}(\Phi_n)$$

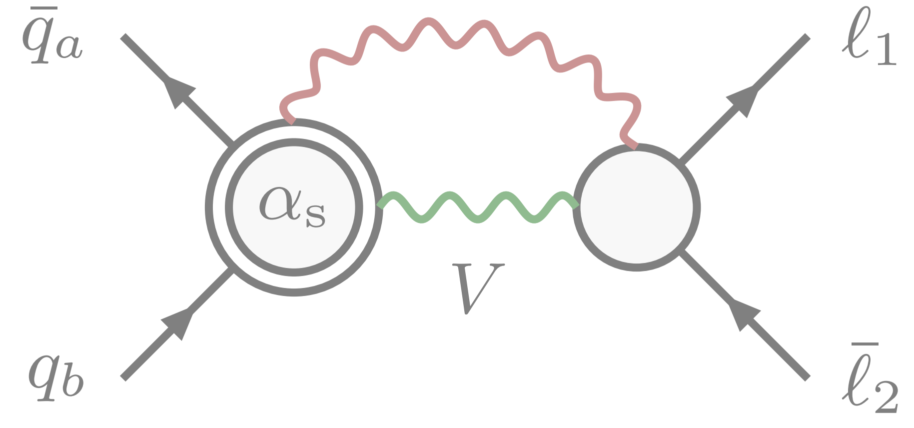
exact virtual contribution ($\mathrm{V}_{n,\mathrm{EW}}$)

approximate integrated real contribution ($\mathrm{I}_{n,\mathrm{EW}}$)

Mixed QCD-EW corrections to DY production: NC



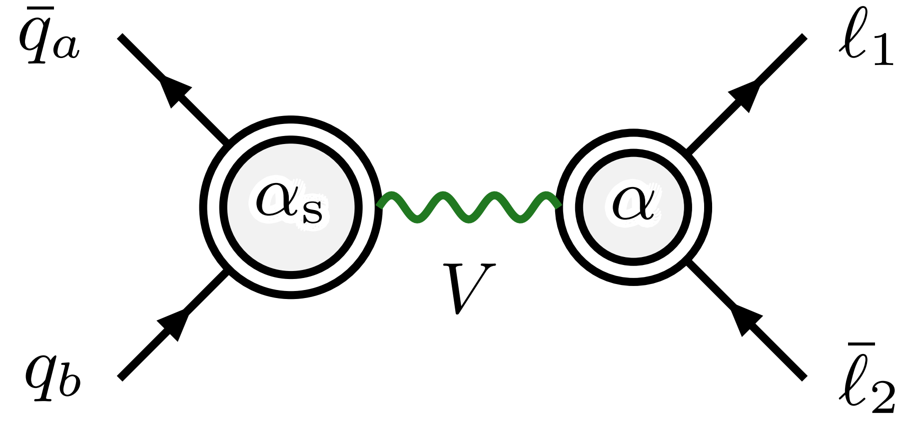
- For precision in resonant region: expand around M^2



non-factorizable

[Dittmaier, Huss, Schwinn, '14]

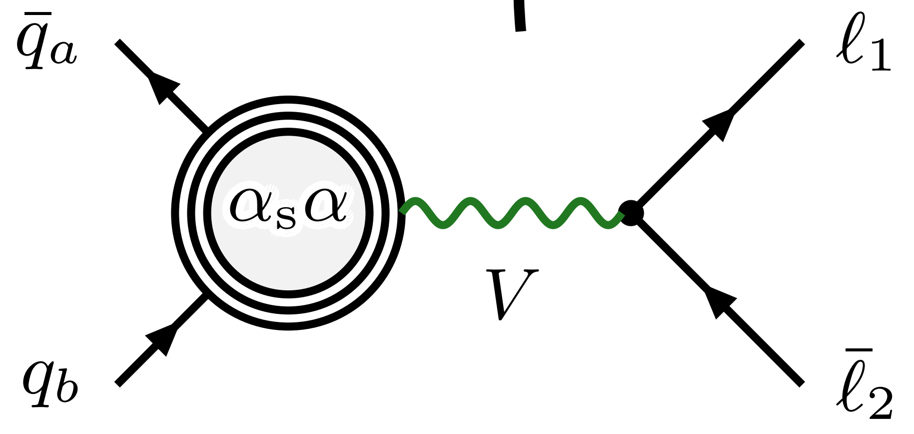
negligible



prod x decay

[Dittmaier, Huss, Schwinn, '15]

dominant



genuine QCD-EW in prod

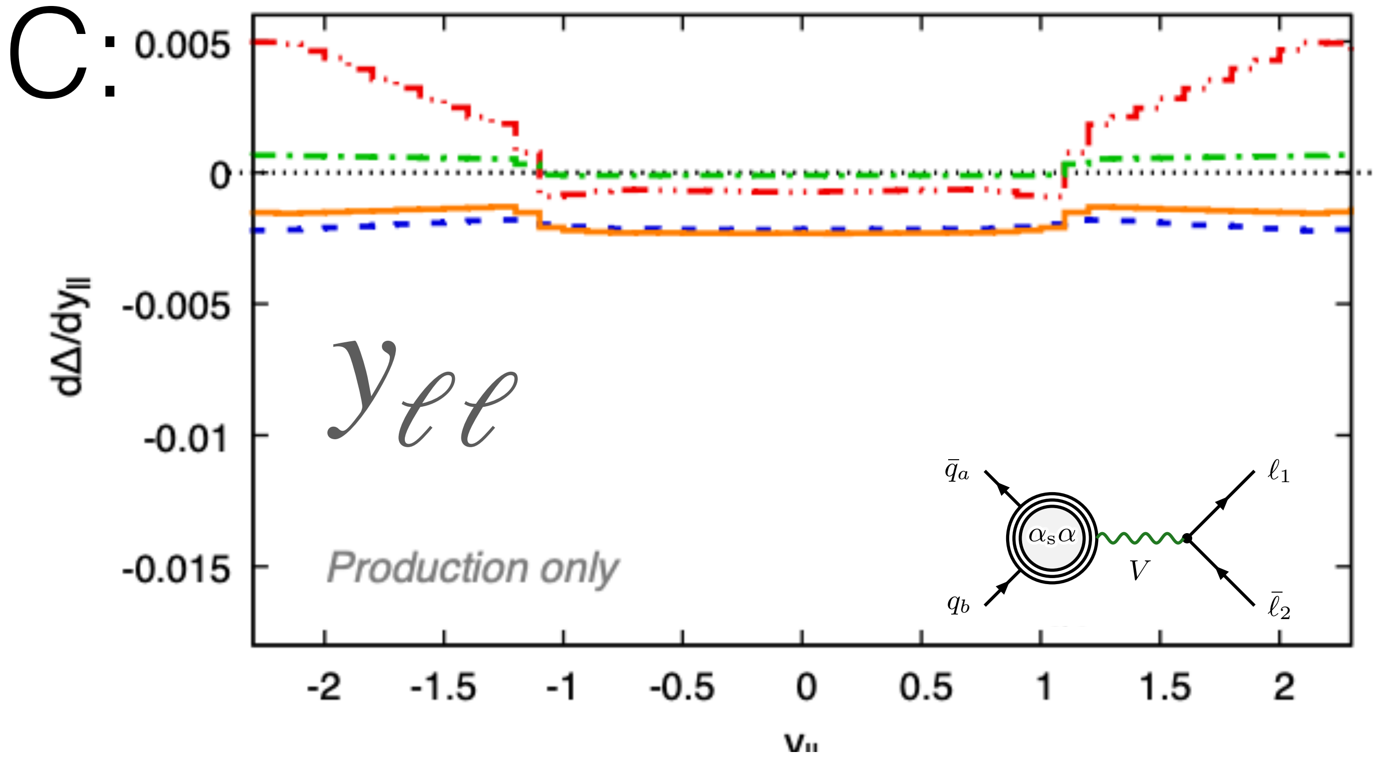
[Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch, '20]
[Behring, Buccioni, Caola, et. al. '20]

last missing piece

For production only

- QCD×weak dominant over QCD×QED
- net effect: few per-mille

NC:



CC+NC:

$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}} = \frac{\delta \langle p_\perp^{l,Z} \rangle^{\text{th}}}{\langle p_\perp^{l,Z} \rangle^{\text{th}}} - \frac{\delta \langle p_\perp^{l,W} \rangle^{\text{th}}}{\langle p_\perp^{l,W} \rangle^{\text{th}}}$$

$$\delta m_W^{\text{meas}} = -17 \pm 2 \text{ MeV}$$

[Behring, Buccioni, Caola, et. al. '21]

QCD² / 10 — QCD×QED — QCD×weak — { + }