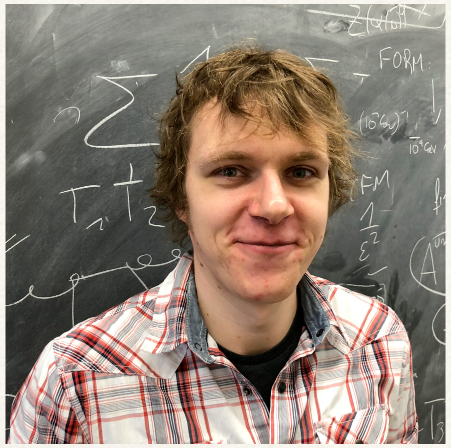
  

Ringberg, October 2022

Resummation in MCFM

R. Keith Ellis,
IPPP, Durham

Fiducial qT resummation of color-singlet processes at N^3LL+NNLO, CuTe-MCFM 2009.11437, Becher and Neumann
Transverse momentum resummation at N^3LL+NNLO for diboson processes,Campbell, RKE, Neumann and Seth, 2210.10724
Results on Jet Veto in Colour singlet production, Campbell et al, in preparation)

MCFM

- MCFM contains about 350 processes at hadron-colliders evaluated at NLO.
- Since matrix elements are calculated using analytic formulae, one can expect better performance, in terms of stability and computer speed, than fully numerical codes.
- In addition MCFM contains several process evaluated at NNLO using both the jetti-ness and the q_T slicing schemes.
- NNLO results for $pp \to X$, require process $pp \to X + 1$ parton at NLO, and two loop matrix elements for$pp \to X$
- Recent(ish) additions to virtual matrix elements
    - H+4 partons with full mass effects at one-loop (2002.04018)
    - Vector boson pair production at one loop: simplified analytic results for the process $q\bar{q}\ell\bar{\ell}\ell'\bar{\ell}'g$ (2203.17170) (work with Giuseppe de Laurentis)

NNLO results

- In a recent paper (2202.07738) we tried to document all the processes calculated at NNLO.
- About 50% are available in MCFM.
- We use both q_T slicing and jetti-ness slicing.
- However I should note that in some cases N^{3}LO is now the start of the art (e.g. 1811.07906 ,2102.07607 2203.01565, 2209.06138)

Process	MCFM	Process	MCFM
$H+0$ jet [8–14]	✓ [15]	$W^{\pm}+0$ jet [16–18]	✓ [15]
$Z/\gamma^{*}+0$ jet [11, 17–19]	✓ [15]	ZH [20]	✓ [21]
$W^{\pm}\gamma$ [18, 22, 23]	✓ [24]	$Z\gamma$ [18, 25]	✓ [25]
$\gamma\gamma$ [18, 26–28]	✓ [29]	single top [30]	✓ [31]
$W^{\pm}H$ [32, 33]	✓ [21]	WZ [34, 35]	✓
ZZ [1, 18, 36–40]	✓	$W^{+}W^{-}$ [18, 41–44]	✓
$W^{\pm}+1$ jet [45, 46]	[3]	$Z+1$ jet [47, 48]	[4]
$\gamma+1$ jet [49]	[5]	$H+1$ jet [50–55]	[6]
$t\bar{t}$ [56–61]		$Z+b$ [62]	
$W^{\pm}H$+jet [63]		ZH+jet [64]	
Higgs WBF [65, 66]		$H\to b\bar{b}$ [67–69]	
top decay [31, 70, 71]		dijets [72–74]	
$\gamma\gamma$+jet [75]		$W^{\pm}c$ [76]	
$b\bar{b}$ [77]		$\gamma\gamma\gamma$ [78]	
HH [79]		HHH [80]	

Examples of NNLO results from MCFM

Process		target		MCFM		
	$\sigma_{NLO^\star}$	σ_{NNLO}	δ_{NNLO}	σ_{NNLO}	δ_{NNLO}	
$pp \to H$	29.78(0)	39.93(3)	10.15(3)	39.91(5)	10.13(5)	nb
$pp \to Z$	56.41(0)	55.99(3)	−0.42(3)	56.03(3)	−0.38(3)	nb
$pp \to W^-$	79.09(0)	78.33(8)	−0.76(8)	78.41(6)	−0.68(6)	nb
$pp \to W^+$	106.2(0)	105.8(1)	−0.4(1)	105.8(1)	−0.4(1)	nb
$pp \to \gamma\gamma$	25.61(0)	40.28(30)	14.67(30)	40.19(20)	14.58(20)	pb
$pp \to e^-e^+\gamma$	2194(0)	2316(5)	122(5)	2315(5)	121(5)	pb
$pp \to e^-\bar{\nu}_e\gamma$	1902(0)	2256(15)	354(15)	2251(2)	349(2)	pb
$pp \to e^+\nu_e\gamma$	2242(0)	2671(35)	429(35)	2675(2)	433(2)	pb
$pp \to e^-\mu^-e^+\mu^+$	17.29(0)	20.30(1)	3.01(1)	20.30(2)	3.01(2)	fb
$pp \to e^-\mu^+\nu_\mu\bar{\nu}_e$	243.7(1)	264.6(2)	20.9(3)	264.9(9)	21.2(8)	fb
$pp \to e^-\mu^-e^+\bar{\nu}_\mu$	23.94(1)	26.17(2)	2.23(3)	26.18(3)	2.24(2)	fb
$pp \to e^-e^+\mu^+\nu_\mu$	34.62(1)	37.74(4)	3.12(5)	37.78(4)	3.16(3)	fb
$pp \to ZH$	780.0(4)	846.7(5)	66.7(6)	847.3(7)	67.3(6)	fb
$pp \to W^\pm H$	1446.5(7)	1476.1(7)	29.6(10)	1476.7(8)	30.2(4)	fb

Table 4. NLO results, computed using MCFM with NNLO PDFs (denoted $\sigma_{NLO^\star}$), total NNLO cross sections from vh@nnlo ($W^\pm H$ and ZH only) and MATRIX (remaining processes, using the extrapolated result from Table 6 of Ref. [24]) and the target NNLO coefficients (δ_{NNLO}, with $\delta_{NNLO} = \sigma_{NNLO} - \sigma_{NLO^\star}$). The result of the MCFM calculation (0-jettiness, fit result b_0 from Eq. (3.9)) is shown in the final column.

Comparative study of jettiness and q_T slicing

- Leading log behavior of a color singlet cross section integrated up to a small cutoff value

$$\Sigma_T = \sigma_0 \exp\Big[-\frac{\alpha_s C_F}{2\pi}\ln^2((q_T^{cut})^2/Q^2)\Big]$$

$$= \sigma_0 \exp\Big[-\frac{2\alpha_s C_F}{\pi}\ln^2(q_T^{cut}/Q)\Big]$$

- Corresponding LL formula for zero-jettiness

$$\Sigma_\tau = \sigma_0 \exp\Big[-\frac{\alpha_s C_F}{\pi}\ln^2\frac{\tau^{cut}}{Q}\Big]$$

- Similar size for the above cut region

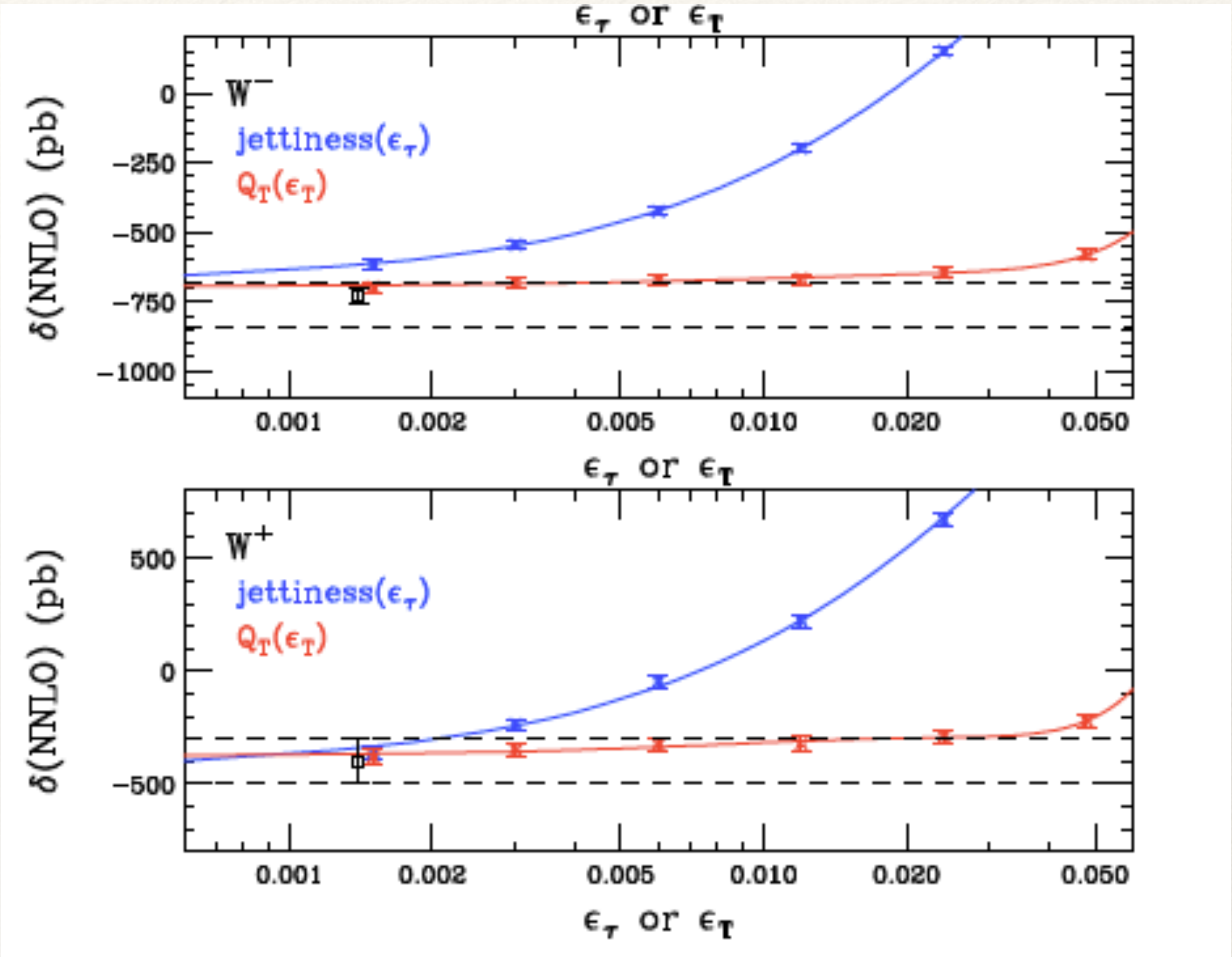
$$\frac{\tau^{\rm cut}}{Q} \simeq \left(\frac{q_T^{\rm cut}}{Q}\right)^{\sqrt{2}}$$

For q_T: $\epsilon_T = q_T^{\rm cut}/Q$

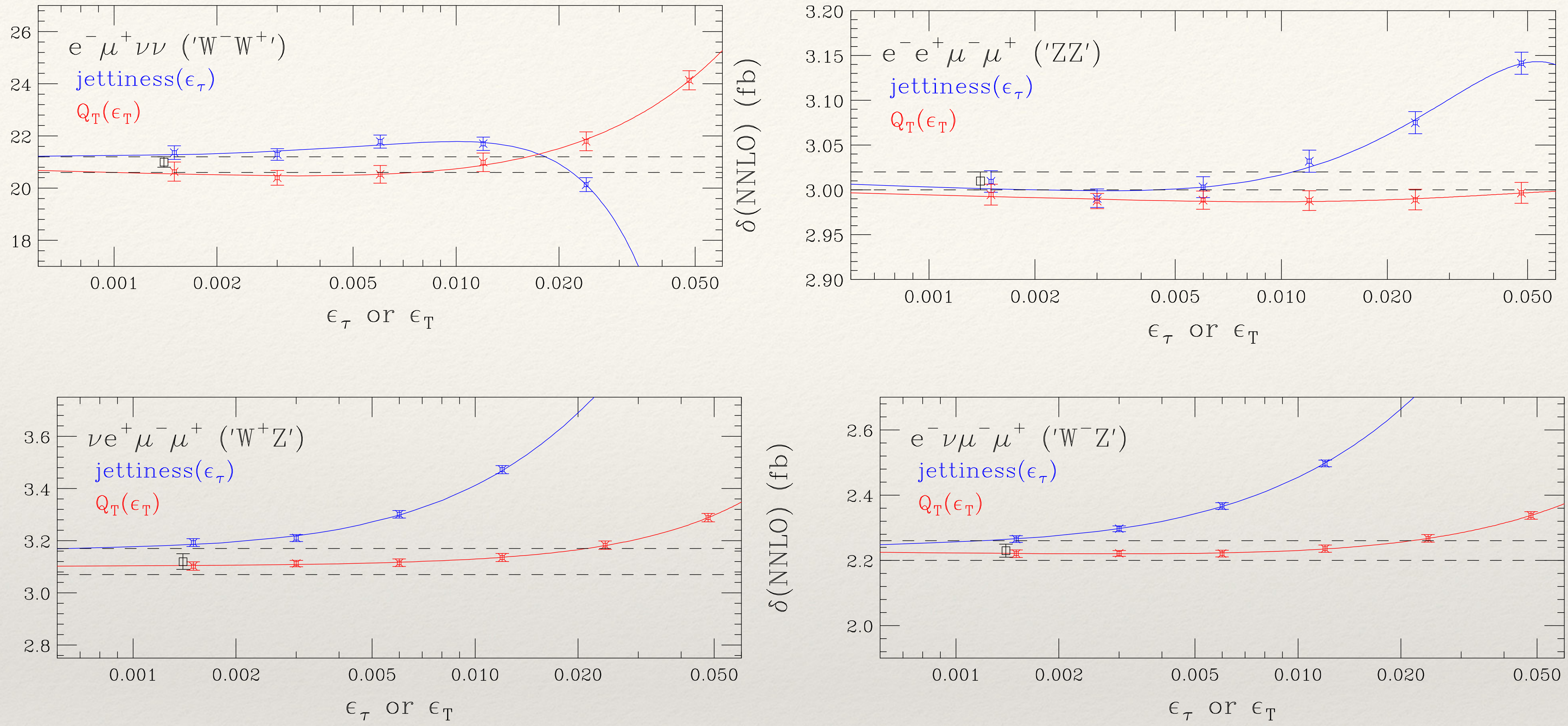
For jettiness: $\epsilon_\tau = (\tau^{\rm cut}/Q)^{\frac{1}{\sqrt{2}}}$

Comparison of NNLO slicing methods

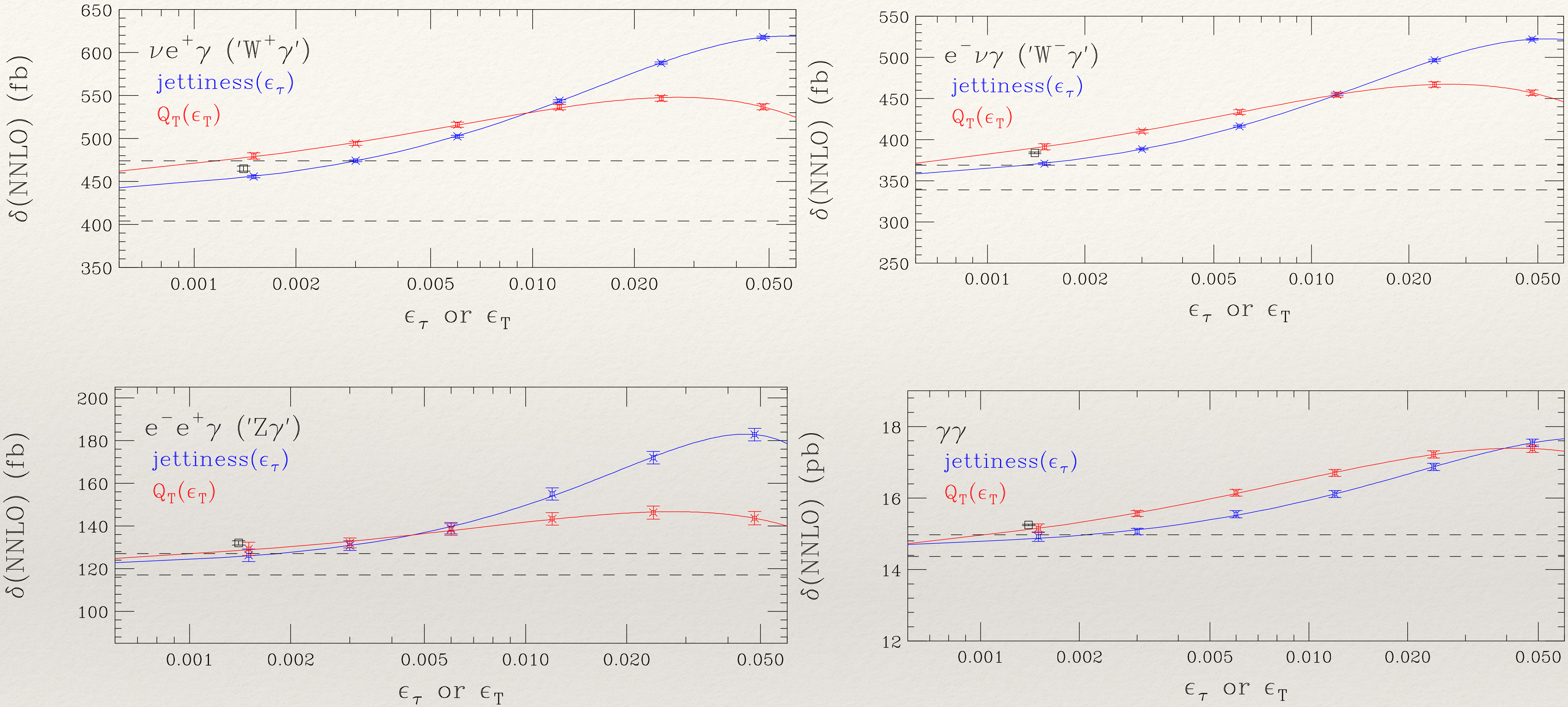
- The jettiness method divides phase space on basis of jettiness;
- q_T slicing method appears to have smaller power corrections in most cases for equal computational burden, but not for all (viz. $W\gamma, Z\gamma, \gamma\gamma$);
- However jettiness has the proven ability to deal with W+jet (1504.02131), Z+jet (1512.01291), Higgs+jet, (1906.01020).



$$\epsilon_T = q_T/Q, \quad \epsilon_\tau = \left(\tau_{\rm cut}/Q\right)^{1/\sqrt{2}}$$



Open square shows the MATRIX result 1711.06631 for $\epsilon_T = 0.15\,\%$



Jetti-ness appears to be comparable or slightly better for processes involving photons.

Transverse momentum resummation at $N^3LL+NNLO$ for color singlet processes

All orders result for q_T distribution

- At small q_T we must sum large logarithms.
- Procedure for color singlet final states based on Collins, Sterman and Soper (1984)

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi\alpha^2}{9Q^2 s} \int d^2b \ \exp(iq_T . b) \sum_j e_j^2$$
$$\times \sum_a \int_{x_a}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_a; 1/b) \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_b; 1/b)$$
$$\times \exp\left\{ -\int_{1/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln \frac{Q^2}{\bar{\mu}} A(\alpha_S(\bar{\mu}) + B(\alpha_S(\bar{\mu})\right] \right\}$$
$$+ \frac{4\pi^2\alpha^2}{9Q^2 s} Y(q_T; Q, x_a, x_b)$$

$$A(\alpha_S(\mu) = \sum_{n=0}^{\infty} A^{(n)} \left(\frac{\alpha_S}{2\pi}\right)^n, \quad A^{(1)} = C_F, A^{(2)} = 2C_F\left\{C_A(\frac{67}{18} - \frac{\pi^2}{6}) - \frac{10T_F n_f}{9}\right\}$$

$$B(\alpha_s(\mu)) = = \sum_{n=0}^{\infty} B^{(n)} \left(\frac{\alpha_S}{2\pi}\right)^n, \quad B^{(1)} = -3C_F,$$

$$B^{(2)} = C_F\left[C_F(\pi^2 - \frac{3}{4} - 12\zeta_3) + C_A(\frac{11\pi^2}{9} - \frac{193}{12} + 6\zeta_3) + T_R n_f(\frac{17}{3} - \frac{4\pi^2}{9})\right]$$

To "b" or not to "b"

$$\frac{d\sigma}{dQ^2dydq_T^2} = \frac{4\pi\alpha^2}{9Q^2s}\int d^2b\, e^{iq_T.b}\sum_j e_j^2 \sum_a \int_{x_a}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_a;1/b)\frac{d\xi_B}{\xi_B} f_{b/B}(\xi_b;1/b)$$

$$\times \exp\left\{-\int_{1/b^2}^{Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{Q^2}{\bar{\mu}}A(\alpha_S(\bar{\mu}) + B(\alpha_S(\bar{\mu})\right]\right\}$$

- b-space, (Fourier conjugate to q_T)
- Advantages
    - Elegant inclusion of transverse momentum conservation.
    - Perturbative predictions for intercept $d\sigma/dq_T^2\Big|_{q_T=0}$
- Disadvantages
    - b-integral extends to infinity; integrate over Landau pole in the coupling.
    - Handled by $b \to b_* = \dfrac{b}{\sqrt{1+(b/b_{lim})^2}}$ so $b_* < b_{lim}$; however this substitution changes prediction even at large q_T where fixed order perturbation theory should work.
    - Difficulties with matching onto fixed order perturbation theory.

Small q_T in SCET language

$$\frac{d^2\sigma}{dq_T^2 dy} = \sigma_0 |C_V(M_Z^2,\mu^2)|^2 \sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1 \frac{dz_1}{z_1}\int_{\xi_2}^1 \frac{dz_2}{z_2}\left[C_{q\bar{q}}(z_1,z_1,q_T^2,M_Z^2,\mu)\ \phi_{i/N_1}(\xi_1/z_1,\mu)\ \phi_{j/N_2}(\xi_2/z_2,\mu)\right]$$

$$C_{q\bar{q}}(z_1,z_1,q_T^2,M_Z^2,\mu) = \frac{1}{4\pi}\int d^2x_\perp e^{-iq_\perp\cdot x_\perp}\left(\frac{x_T^2 M_Z^2}{b_0^2}\right)^{F_{q\bar{q}}} e^{2h_F(L_\perp,a_s)}\ \bar{I}(z_1,L_\perp,a_s)\ \bar{I}(z_2,L_\perp,a_s)$$

$$= \frac{1}{2}\int_0^\infty dx_T x_T\ J_0(x_T q_T)\ \exp(g_F(M_Z^2,\mu,L_\perp,a_s)) \qquad L_\perp = \ln\frac{x_T^2\mu^2}{b_0^2},\ \ x_T^2 = -x_\perp^2$$

$$g_F = -\eta L_\perp - a_s\left[(\Gamma_0+\eta\beta_0)\frac{L_\perp^2}{2} + O(L_\perp)\right] \qquad \eta = C_F\frac{\alpha_s}{\pi}\ln\frac{M_Z^2}{\mu^2}$$

$$\frac{1}{2}\int_0^\infty dx_T\ x_T\ J_0(x_T q_T)e^{-\eta L_\perp} = \frac{1}{q_T^2}\left(\frac{q_T^2}{\mu^2}\right)^\eta \frac{\Gamma(1-\eta)}{\Gamma(\eta)}\left(\frac{b_0}{2}\right)^{2\eta} \longleftarrow$$

Confirms the expected scaling $x_T \sim 1/q_T$

Language of Becher and Neubert, see for example, Becher, Broggio, Ferroglia, 1410.1892

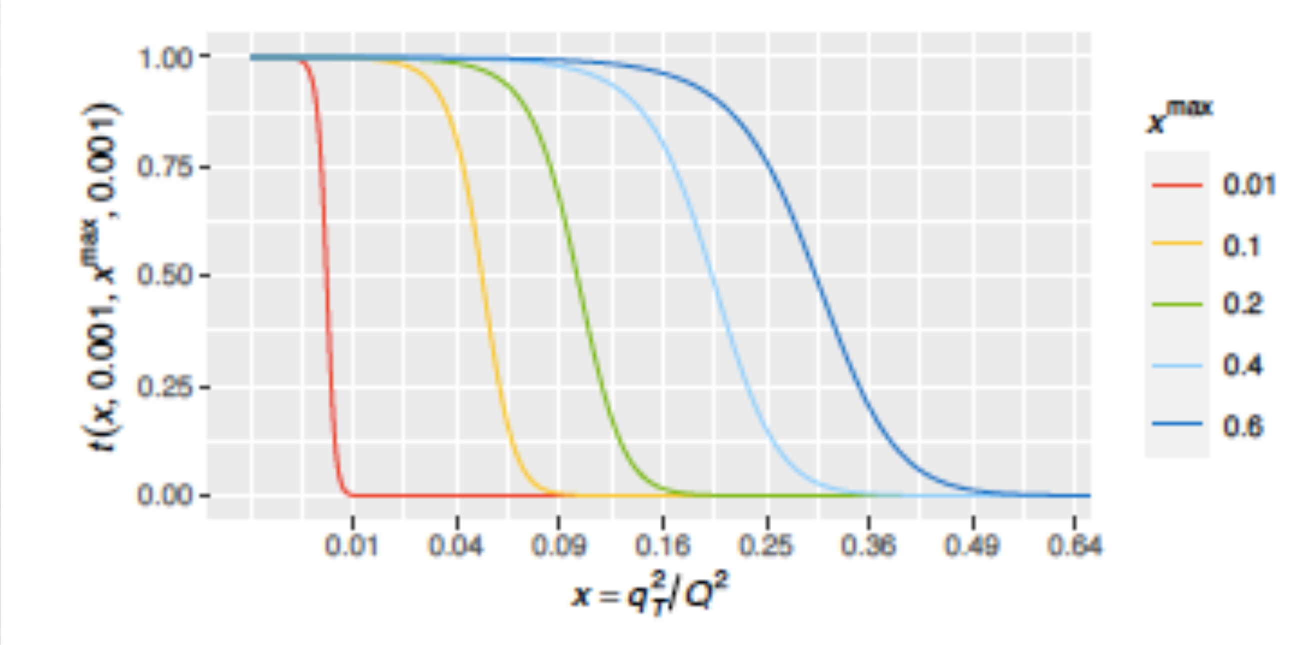
Collinear Anomaly

- In SCET the beam functions and the soft function have light-cone divergences which are not regulated by dimensional regularization;
- These are not soft divergences; they are due to gluons at large rapidity;
- This requires an additional regulator, which can be removed at the end of the calculation;
- However a vestige of this regulator remains. The product of the two beam functions depends on the large scale of the problem, Q ;
- This has been called the "collinear factorization anomaly" of SCET. Quantum effects modify a classical symmetry, $p \to \lambda p, \bar{p} = \bar{\lambda}\bar{p}$ with only $\lambda\bar{\lambda} = 1$ unbroken.

Matching to fixed order

$$\left.\frac{d\sigma^{\mathrm{N^3LL}}}{dq_T}\right|_{\text{naively matched to NNLO}} = \frac{d\sigma^{\mathrm{N^3LL}}}{dq_T} + \Delta\sigma, \quad \text{where } \Delta\sigma = \left[\frac{d\sigma^{\mathrm{NNLO}}}{dq_T} - \frac{d\sigma^{\mathrm{N^3LL}}}{dq_T}\right]_{\text{expanded to NNLO}}$$

- Fixed order result recovered up to higher order terms, (which can induce unphysical behavior).



- Also problems at small q_T, introduce cutoff q_0;
- So we need to implement a transition function, and choose its parameters on a case-by-case basis.

$$\left.\frac{d\sigma^{\mathrm{N^3LL}}}{dq_T}\right|_{\text{matched to NNLO}} = t(x)\left[\frac{d\sigma^{\mathrm{N^3LL}}}{dq_T} + \Delta\sigma\big|_{q_t>q_0}\right] + (1-t(x))\,\frac{d\sigma^{\mathrm{NNLO}}}{dq_T}$$

Becher and Neumann, 2009.11437

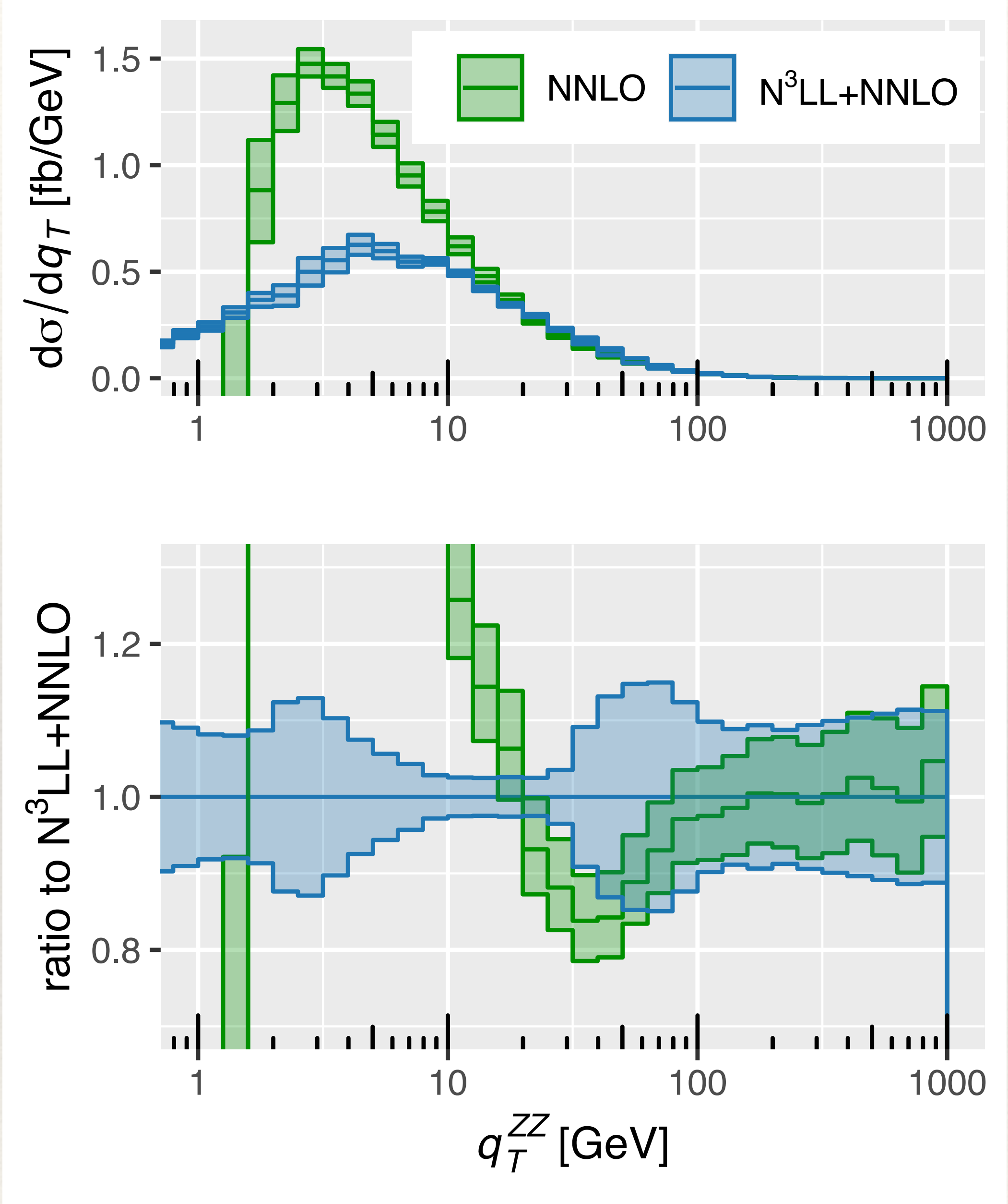
Error estimate

- We estimate the perturbative truncation uncertainty by varying the renormalization, factorization and resummation scales by the multipliers $(k_F; k_R) \in \{(2,2), (0.5,0.5), (2,1), (1,1), (0.5,1), (1,2), (1,0.5)\}$.
- For fixed order we use $\mu_F = k_F \hat{Q}$, $\mu_R = k_R \hat{Q}$
- $q^* = Q^2 \exp\left(-\pi / C_i / \alpha_s(q^*)\right)$ is characteristic scale at small q_T
- To set the resummation scale, we first calculate q^* for every event and then set $\mu = \max\{k_F \times q_T + q^* \exp(-q_T/q^*), 2\ \mathrm{GeV}\}$ so that for small q_T, μ approaches q^* and it remains in the perturbative region.

Becher, Neumann, 2009.11437

Vector boson pair production at small q_T

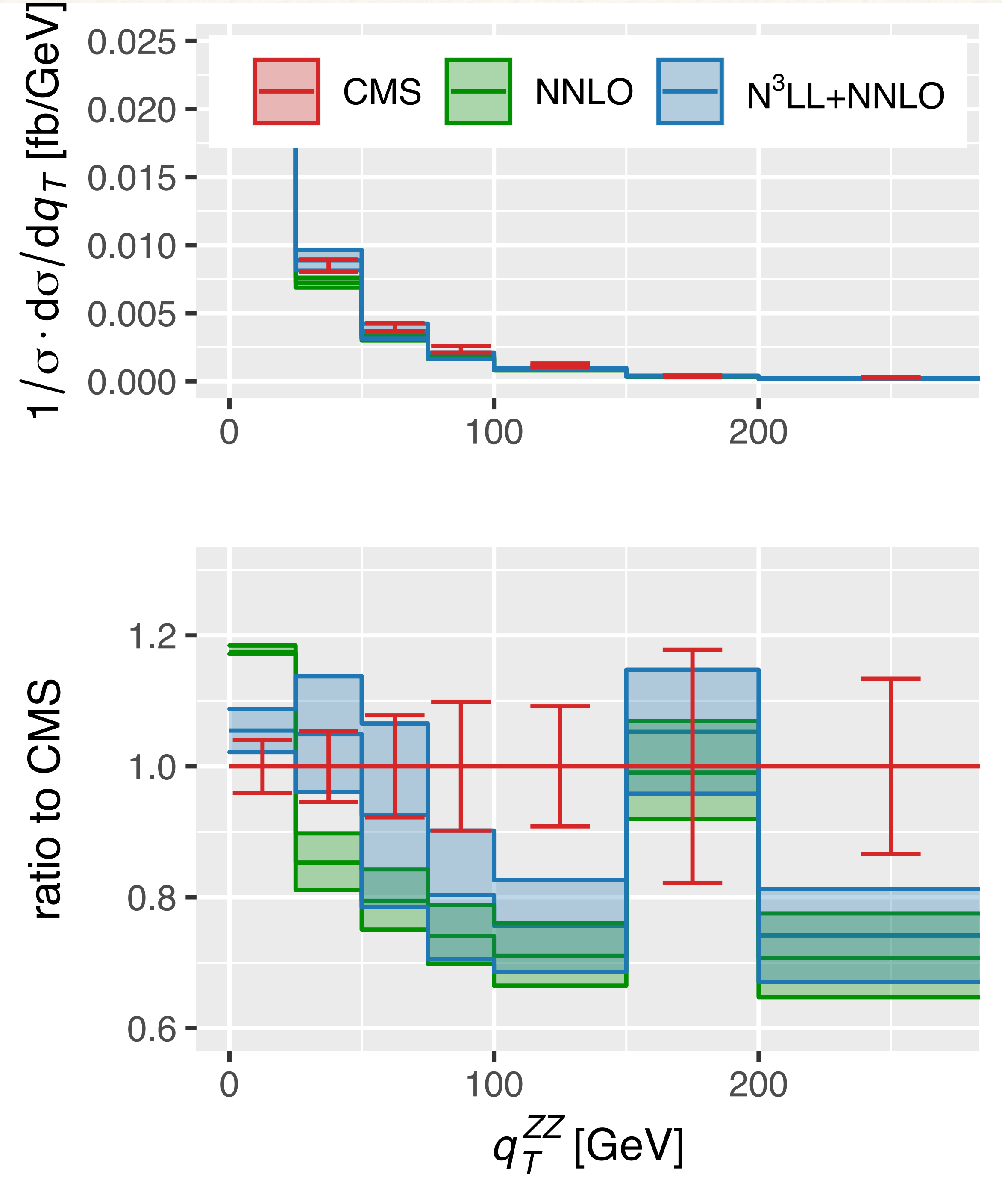
- Resummation effects are potentially more important for vector boson pair production at the same q_T since Q is larger.
- Resummation at N³LL+NNLO becomes important below $\sim 50 - 100$ GeV.



Transverse momentum distribution of the ZZ pair at NNLO and NNNLL+NNLO using CMS cuts at $\sqrt{s} = 13.6$ TeV

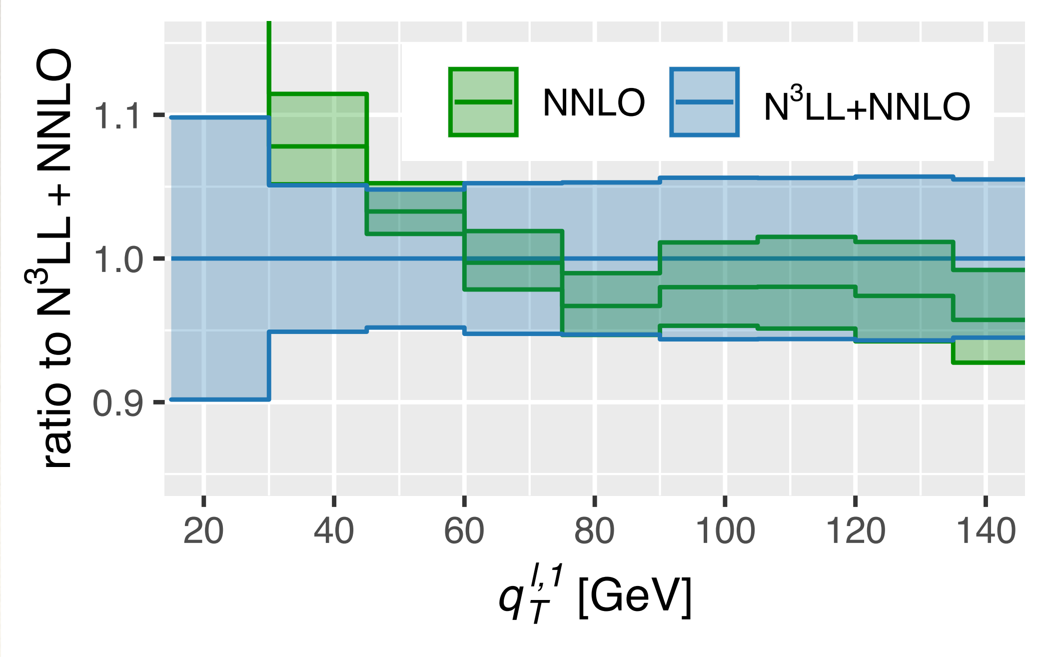
Comparison with CMS data at 13 TeV

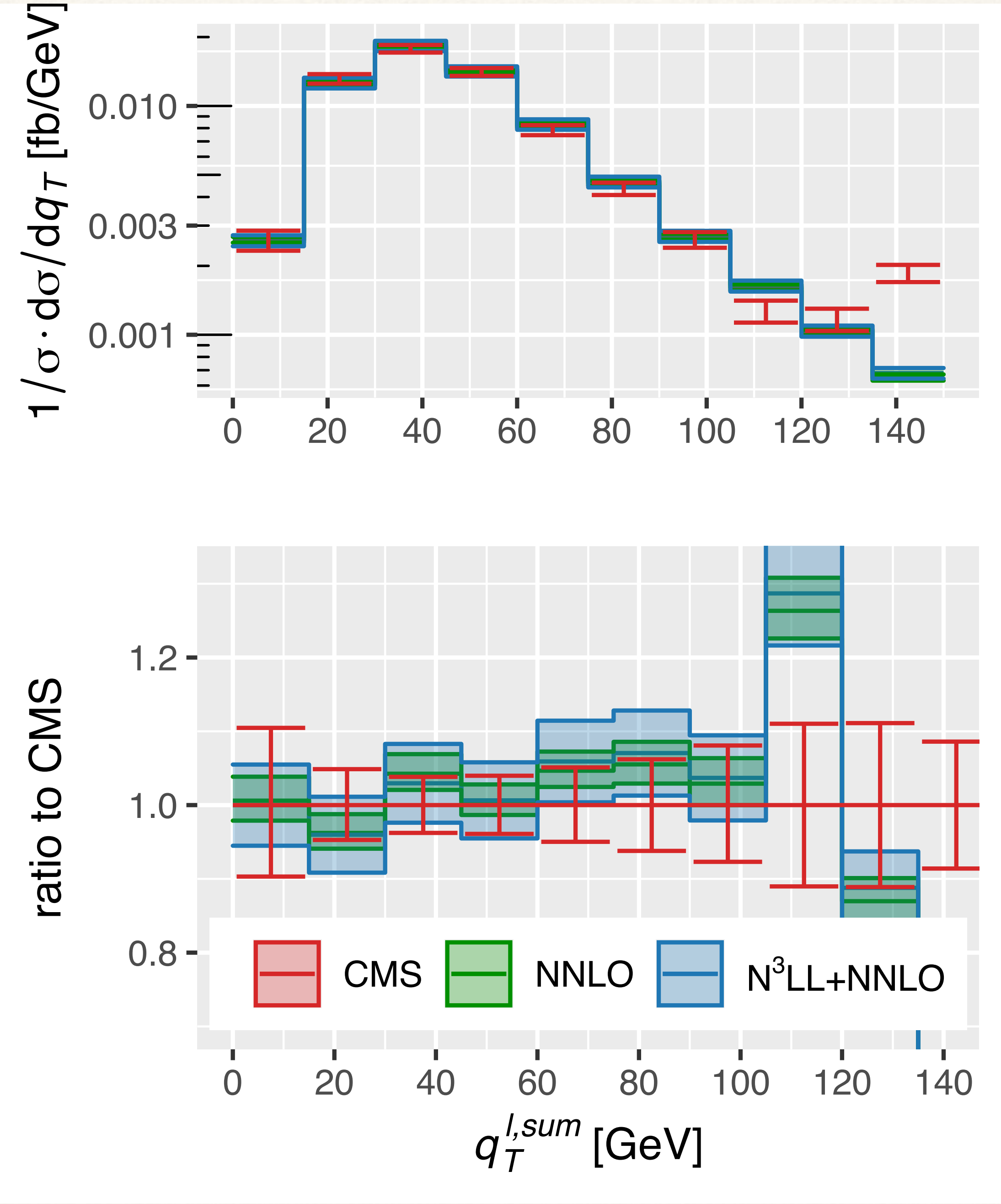
- We simplify the analysis, by applying the same cuts to both electrons and muons.
- We neglect identical particle effects.
- Resummation improves description below $q_T \sim 75$ GeV.
- More data will allow finer binning, so the resummation effects will be ever more necessary.



CMS results on lepton q_T in ZZ

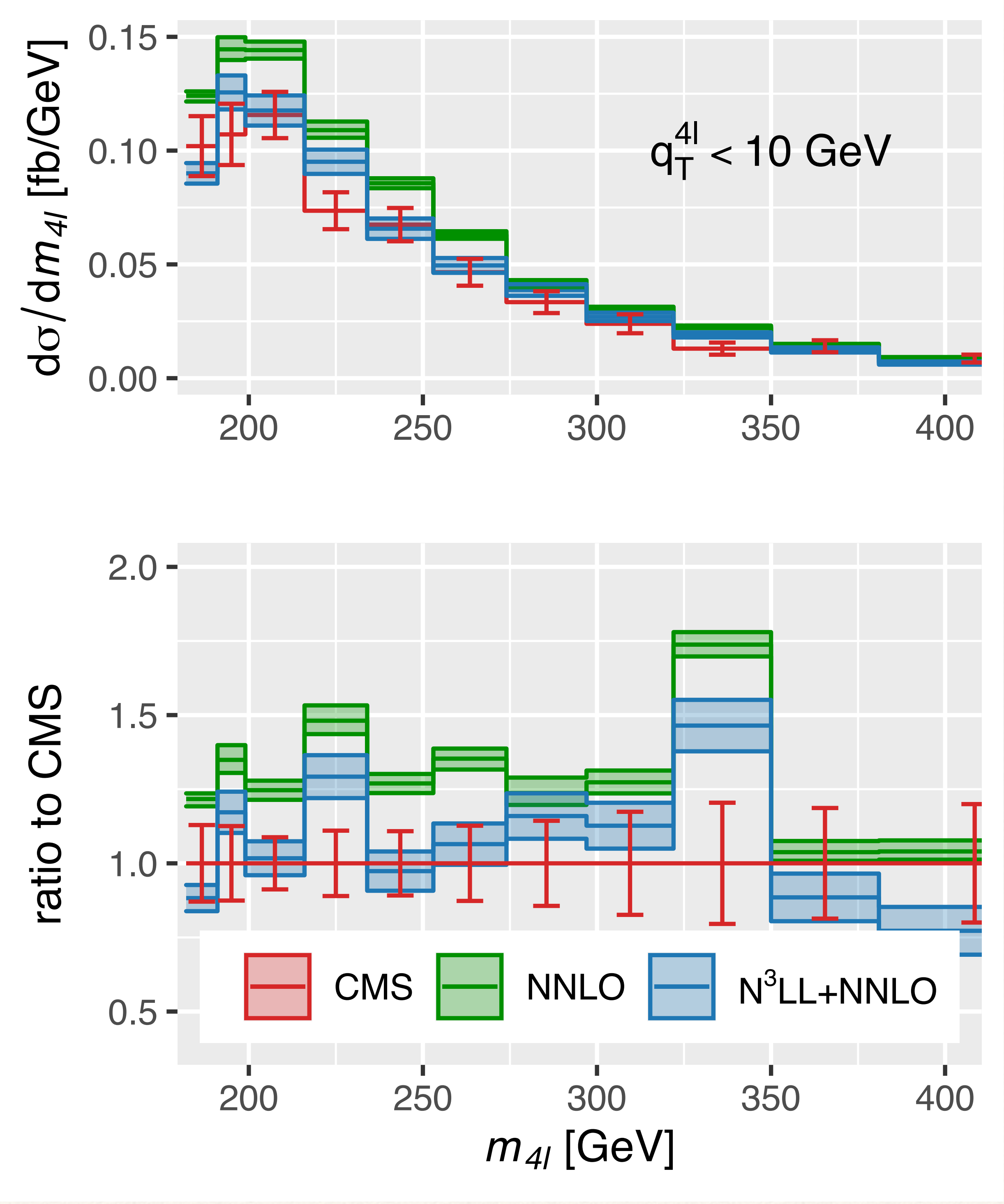
- CMS also present results on the lepton q_T^l (summed over all leptons). Here the effect of resummation is minimal
- However the $q_T^{l,1}$ of the leading lepton shows an effect.





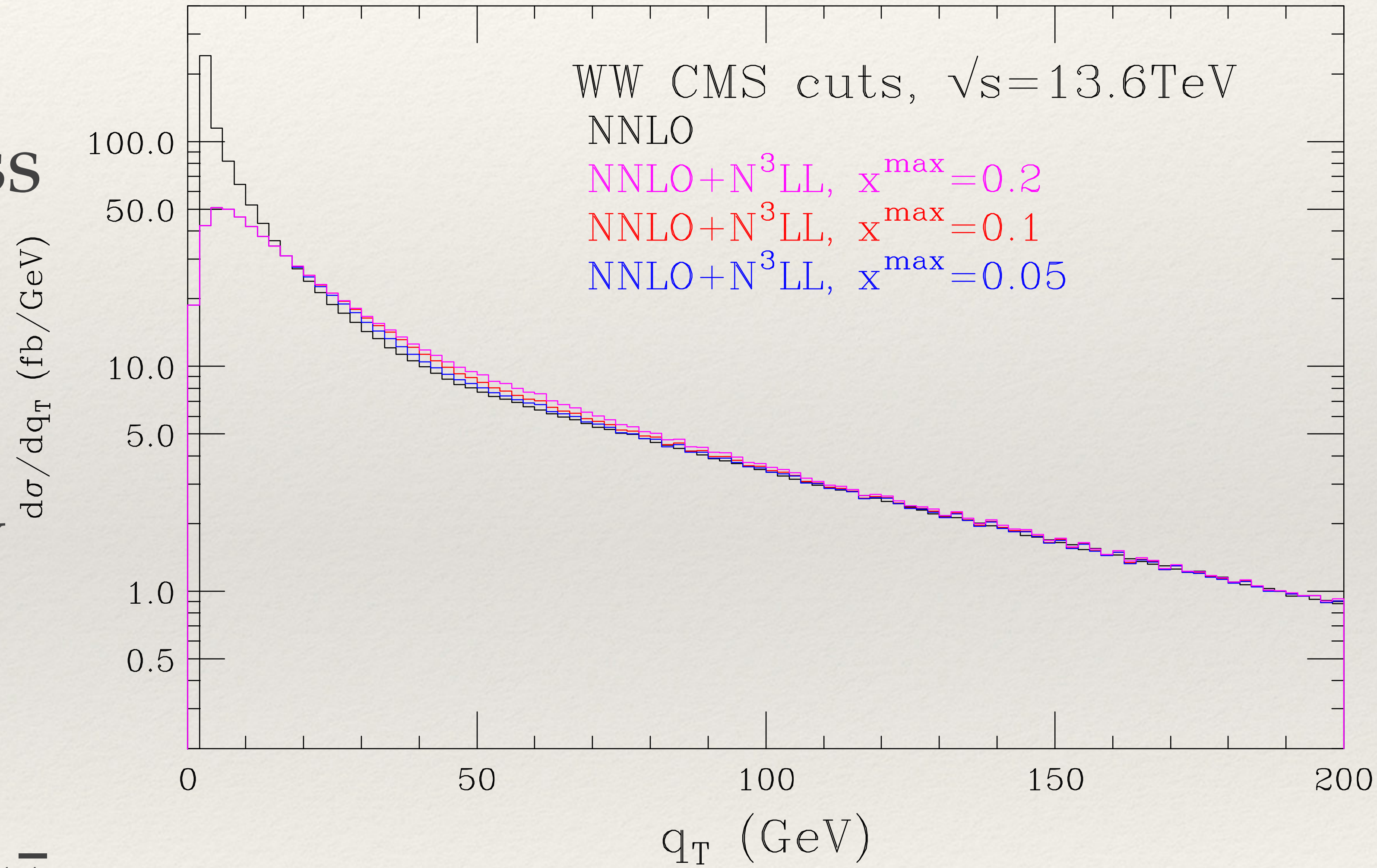
ATLAS data ZZ

- The ATLAS collaboration (2103.01918) performed measurements of the m_{4l} distribution in five slices of q_T^{4l}
- Expectation is that resummation should improve agreement with the data, as m_{4l} increases.



Truth WW cross section

- Here we show the truth $q_T(WW)$ cross section.
- Much more important for WW is the p_T^{veto} cross section to reduce background from $t\bar{t}$



Jet veto cross sections

see, for example, Becher et al, 1307.0025 , Stewart et al, 1307.1808

Jet veto cross section

- Jets defined using sequential recombination jet algorithms, (n=1(anti-k_T), n=0(Cambridge-Aachen) n=-1(k_T);
- Jet vetos also generate large logarithms, as codified in factorization formula; however logarithms are smaller if $p_T^{\rm veto} \sim 25$ GeV;
- Beam and Soft functions for leading jet p_T recently calculated at two-loop order using an exponential regulator by Abreu et al.
- Jet veto cross sections are simpler than the p_T resummed calculation (No b space).

$$d_{ij} = \min(p_{Ti}^n, p_{Tj}^n)\,\frac{\sqrt{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}}{R}\,, \qquad d_{iB} = p_{Ti}^n$$

Rapidity regulator ν

$$\frac{d^2\sigma(p_T^{veto})}{dM^2 dy} = \sigma_0 \left| C_V(-M^2,\mu)\right|^2$$
$$\left[\mathscr{B}_c(\xi_1, M, p_T^{veto}, R^2, \mu, \nu)\, \mathscr{B}_{\bar{c}}(\xi_2, M, p_T^{veto}, R^2, \mu, \nu) \times \mathscr{S}(p_T^{veto}, R^2, \mu, \nu)\right]$$

Beam functions Abreu et al, 2207.07037

Soft function Abreu et al, 2204.03987

$$\xi_{1,2} = (M/\sqrt{s})\, e^{\pm y} \qquad \sigma_0 = \frac{4\pi\alpha^2}{3N_c M^2 s}$$

Comments on Abreu et al

- Important step in making SCET results for almost complete N^3LL available to consumers, (such as us in MCFM).
- Unfortunately, in the ancillary materials for 2207.07037, the file BeamFunctionQQCACF.m contained a parameter R0, which should have been set to zero. (Thanks to Pier Monni for discussions - arXiv result will be updated after article is accepted for publication).
- Jets vetoed over all rapidity

The analytic two-loop soft function for leading-jet p_T

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Soft function
Abreu et al,
2204.03987

Prepared for submission to JHEP — CERN-TH-2022-118, ZU-TH 30/22

Quark and gluon two-loop beam functions for leading-jet p_T and slicing at NNLO

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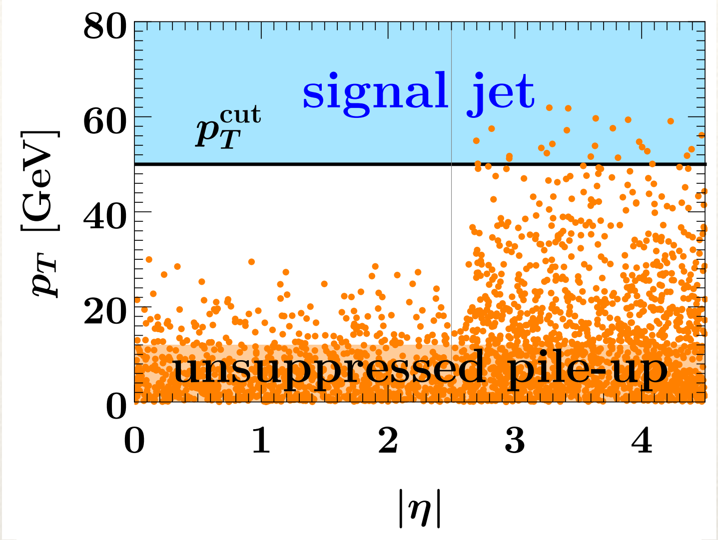
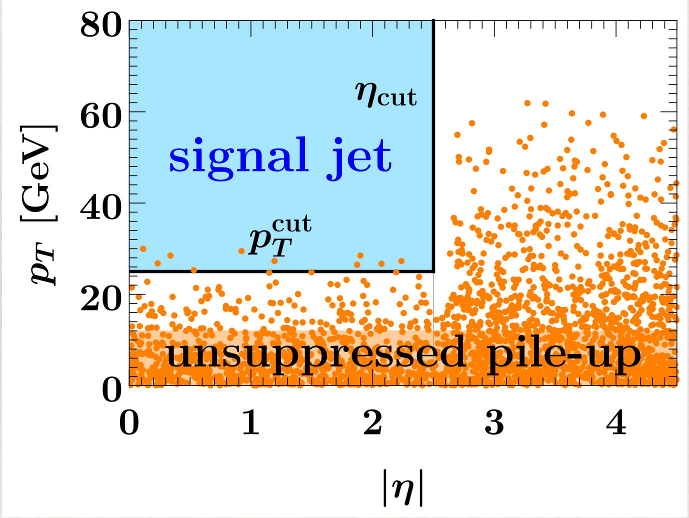
Beam functions
Abreu et al,
2207.07037

Jet veto cross sections in a limited rapidity range

- Formula so far are valid for jet cross sections which are vetoed for all values of rapidity $\eta_{\rm cut}$
- Experimental analyses perform jet cuts for $\eta < \eta_{\rm cut}$
- In 1810.12911, three theoretical regions are identified
  - $\eta_{\rm cut} \gg \ln(Q/p_T^{\rm veto})$ (standard jet veto resummation)
  - $\eta_{\rm cut} \sim \ln(Q/p_T^{\rm veto})$ ($\eta_{\rm cut}$-dependent beam functions)
  - $\eta_{\rm cut} \ll \ln(Q/p_T^{\rm veto})$ (collinear non-global logs)



Current theory calculation



Typical Experimental cuts

Strategy: determination where resummation is potentially important, before considering limited rapidity range resummation

Refactorize à la Becher-Neubert

$$\left[\mathscr{B}_c(\xi_1, Q, p_T^{veto}, R^2, \mu, \nu)\, \mathscr{B}_{\bar{c}}(\xi_2, Q, p_T^{veto}, R^2, \mu, \nu\ )\mathscr{S}(p_T^{veto}, R, \mu, \nu)\right]_{q^2=Q^2}$$

$$= \left(\frac{Q}{p_T^{veto}}\right)^{-2F_{qq}(p_T^{veto}, R, \mu)} e^{2h^F(p_T^{veto}, \mu)}\, \bar{B}_q(\xi_1, p_T^{veto}, R)\, \bar{B}_{\bar{q}}(\xi_2, p_T^{veto}, R)$$

"Collinear anomaly"

- In the perturbative region $\bar{B}_i(\xi, p_T^{veto}, R) = \sum_{j=g,q,\bar{q}} \int_\xi^1 \frac{dz}{z}\, \bar{I}_{ij}(z, p_T^{veto}, R, \mu)\, \phi_{j/P}(\xi/z, \mu)$
- The product of reduced beam functions is independent of the factorization scale thorough the calculated order.
- In our case this means $\frac{d}{d\ln\mu}\left[\bar{B}_q(\xi_1, p_T^{veto}, R)\, \bar{B}_{\bar{q}}(\xi_2, p_T^{veto}, R)\right] = O(\alpha_s^3)$

Coefficient of Collinear Anomaly for $q\bar{q}$ case

$$F_{qq}(p_T^{\rm veto},\mu) = a_S F_{qq}^{(0)} + a_S^2 F_{qq}^{(1)} + a_S^3 F_{qq}^{(2)} + \dots\ , \quad a_S = \frac{\alpha_S}{4\pi}$$

$$F_{qq}^{(0)} = \Gamma_0^F L_\perp + d_1^{\rm veto}(R,F)$$

$$L_\perp = 2\ln\frac{\mu}{p_T^{\rm veto}}$$

$$F_{qq}^{(1)} = \frac{1}{2}\Gamma_0^F\beta_0 L_\perp^2 + \Gamma_1^F L_\perp + d_2^{\rm veto}(R,F)$$

$$F_{qq}^{(2)} = \frac{1}{3}\Gamma_0^F\beta_0^2 L_\perp^3 + \frac{1}{2}(\Gamma_0^F\beta_1 + 2\Gamma_1^F\beta_0)L_\perp^2 + (\Gamma_2^F + 2\beta_0 d_2^{\rm veto}(R,F))L_\perp + d_3^{\rm veto}(R,F)$$

Full N³LL will require knowledge of $d_3^{\rm veto}(R,F)$

$$d_1^{\rm veto}(R,F) = 0$$

$$d_2^{\rm veto}(R,B) = d_2^B - 32C_B f(R,B)$$

$$f(R,B) = C_B\left(-\frac{\pi^2R^2}{12} + \frac{R^4}{16}\right) + C_A\left(c_L^A\ln R + c_0^A + c_2^A R^2 + c_4^A R^4 + \dots\right) + T_F n_f\left(c_L^f\ln R + c_0^f + c_2^f R^2 + c_4^f R^4 + \dots\right),$$

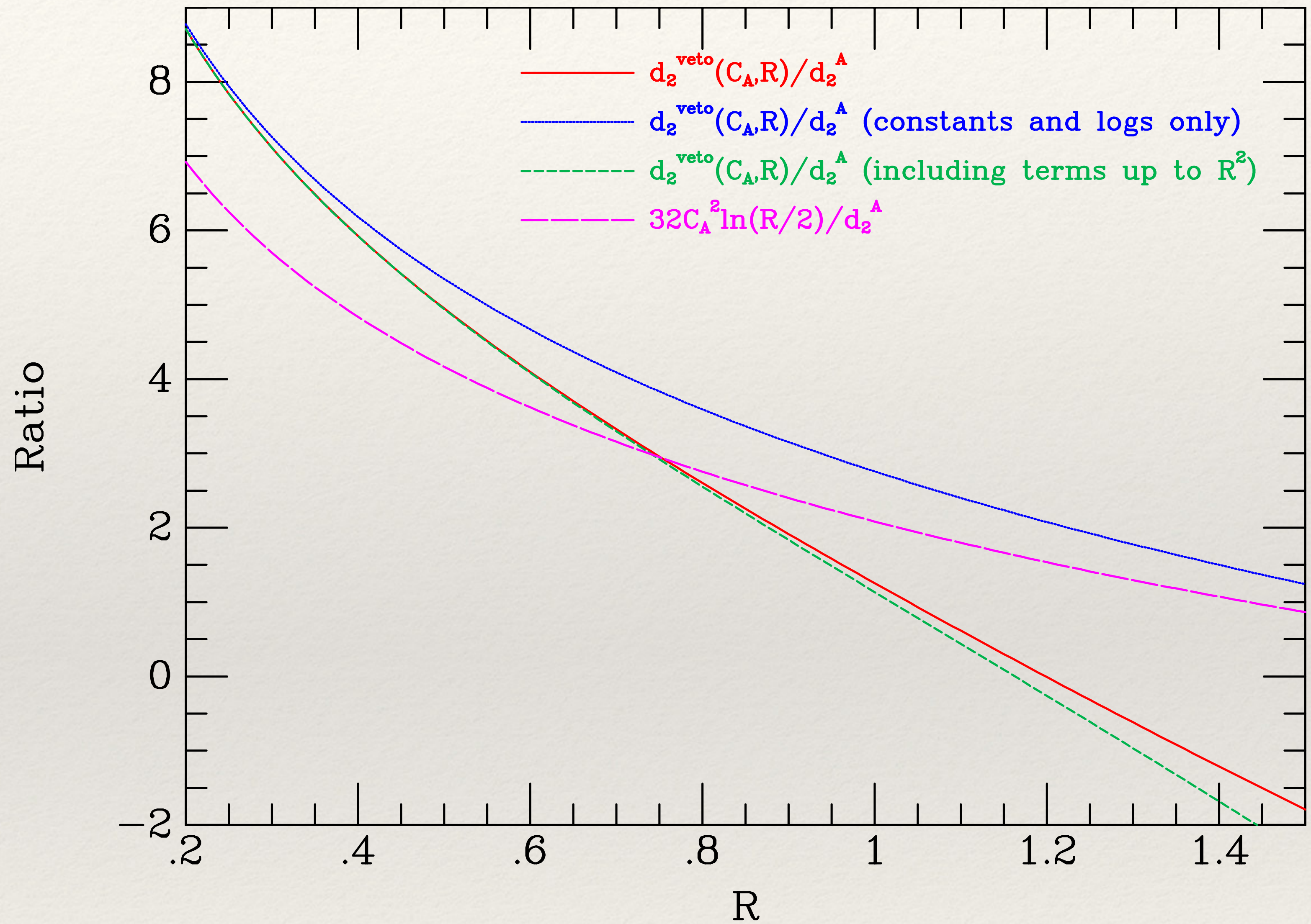
Coefficients c_i^A and c_i^f for $i < 10$, see 1307.0025

$$d_3^{\rm veto} \sim -8.3\times 64 C_B\ln^2(R/R_0) + O(\ln(R))$$

Log enhanced terms of $d_3^{\rm veto}$, see 1511.02886

Approximations to $d_2^{\rm veto}$

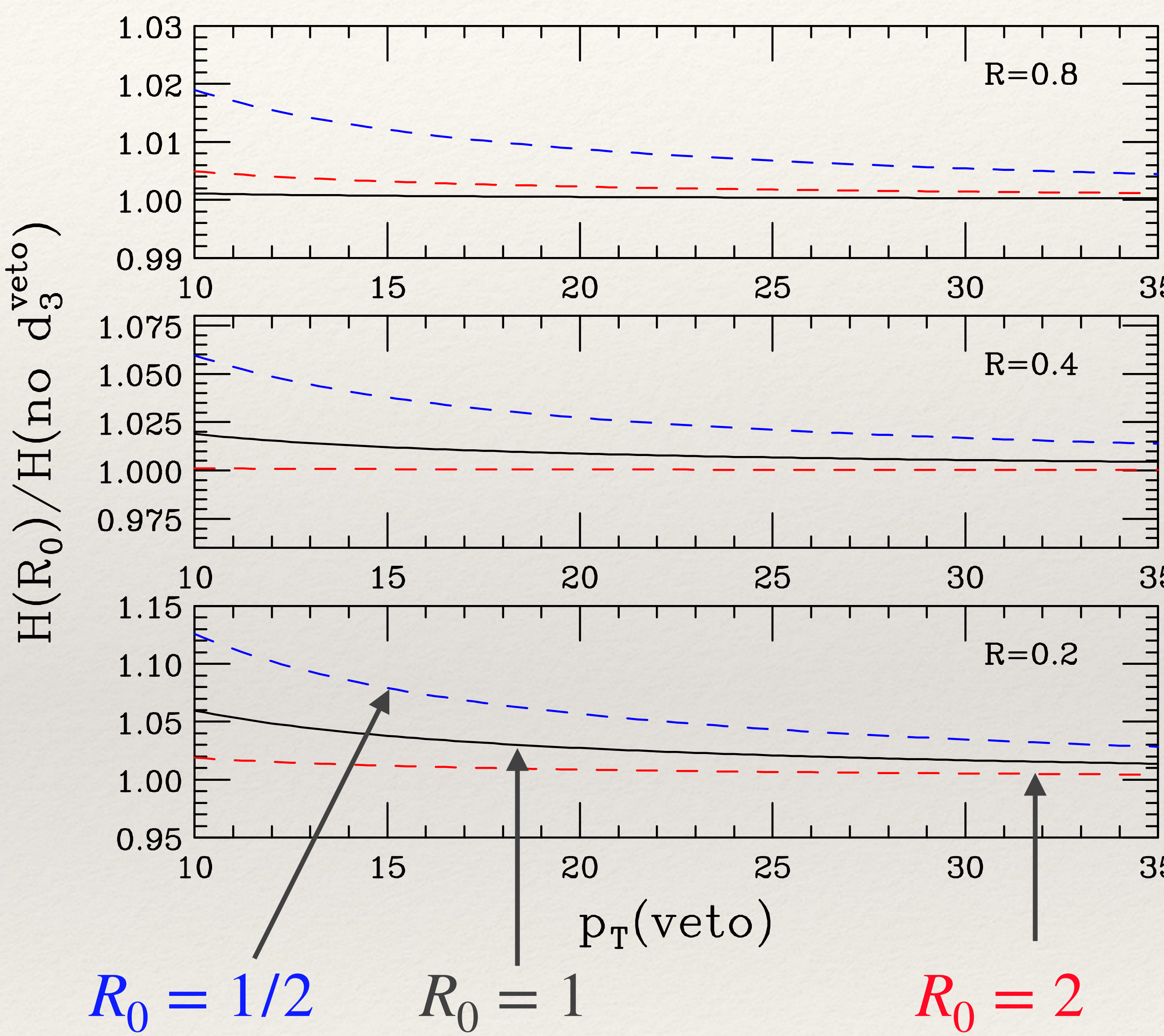
- Range of validity is $\frac{p_T^{\rm veto}}{Q} \ll R \ll \ln\left(\frac{Q}{p_T^{\rm veto}}\right)$
- At too small R terms of order $\ln^n R$ which are not covered by the factorization formula.
- At too large R, factorization formula breaks down.
- Results are presented as power series in R
- At $R \sim 0.4$ logarithmic approximation is about 25% too low.
- Results should be valid in a range around the experimentally preferred $R \sim 0.4 - 0.5$



Rescaled $d_2^{\rm veto}$ showing that limited number of terms in expansion is quite adequate for $R < 1$.

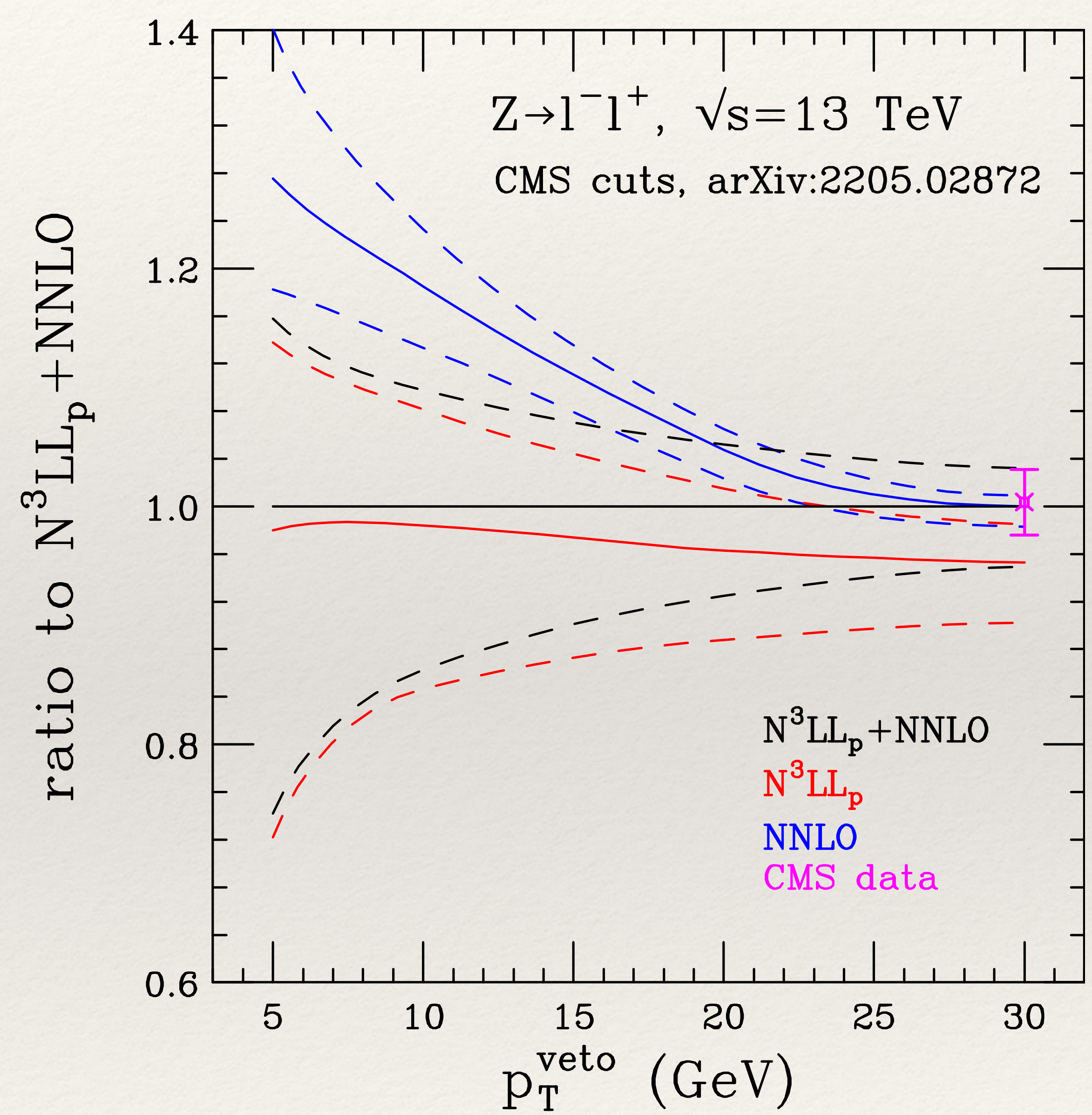
Estimated dependence on approximate $d_3^{\rm veto}$

- Effect of R_0 dependence in approximate form for $d_3^{\rm veto}$
- $d_3^{\rm veto} \sim -8.3 \times 64 C_B \ln^2(R/R_0)$
- $\left(\frac{m_H}{p_T^{\rm veto}}\right)^{-2\frac{\alpha_s(\mu)}{4\pi} d_3^{\rm veto}}$
- In this approximation, $d_3^{\rm veto}$ increases the cross section.
- Estimate ~ ≤ 2.5 % at $p_T^{\rm veto}$=25 GeV and $R = 0.4$

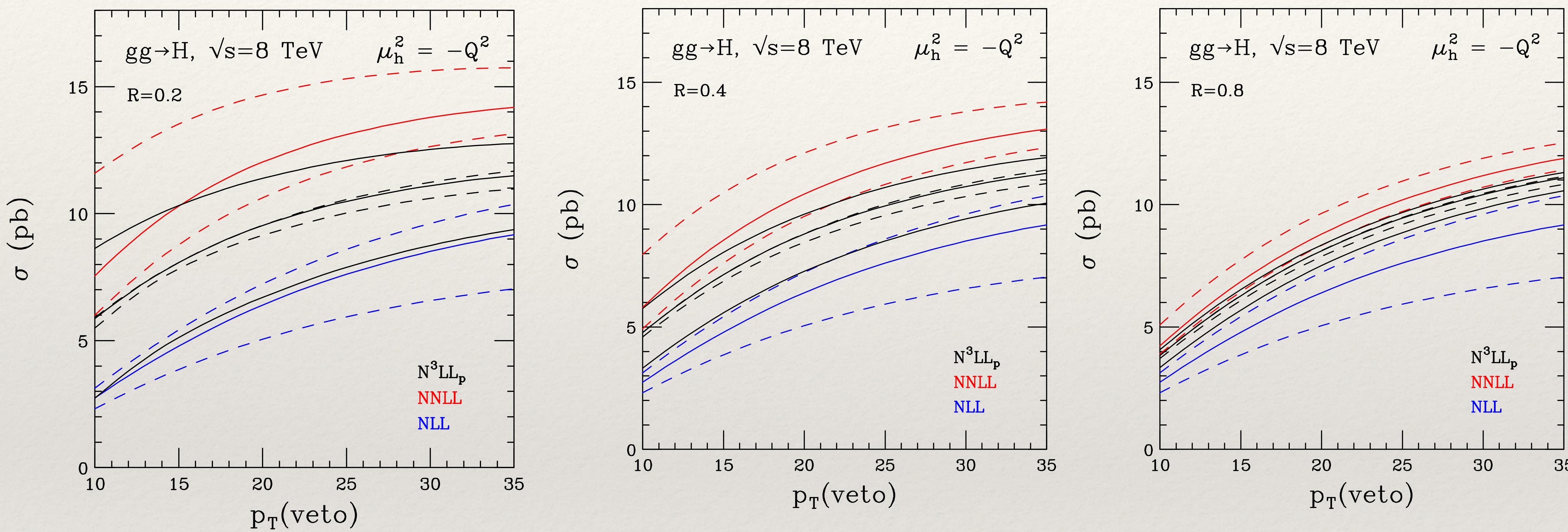


Jet veto in Z production

- At $p_T^{\text{veto}} \sim 25 - 30$ all calculations agree within errors.
- However error estimates differ between NNLO and N^3LL +NNLO.
- For $p_T^{\text{veto}} = 30$ GeV, $(\ln(Q/p_T^{\text{veto}} = 1.1) \ll (\eta_{\text{cut}} = 2.4)$
- As expected at (unphysically) small p_T^{veto} resummed calculations show deviations from fixed order.
- Jet veto resummation probably not so necessary here.



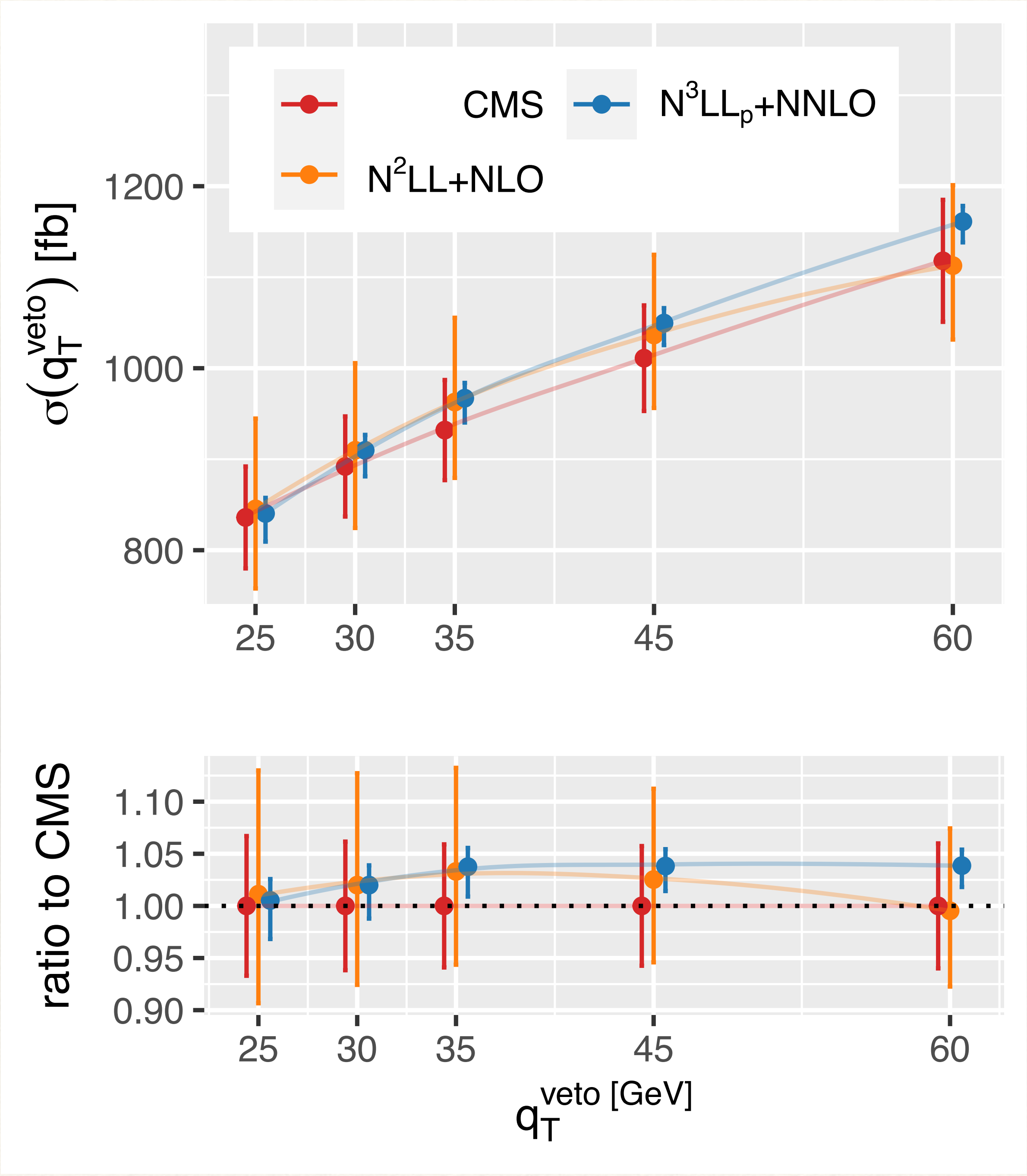
Jet-veto in Higgs production



- Uncertainties estimated by varying renormalization and factorization and rapidity scales by $2, \frac{1}{2}$ and adding in quadrature.
- In the main the perturbative series is well-behaved at moderate R and successive orders lie with in the band of the preceding order
- Summation appears needed in this case;

Jet veto in W^+W^- production

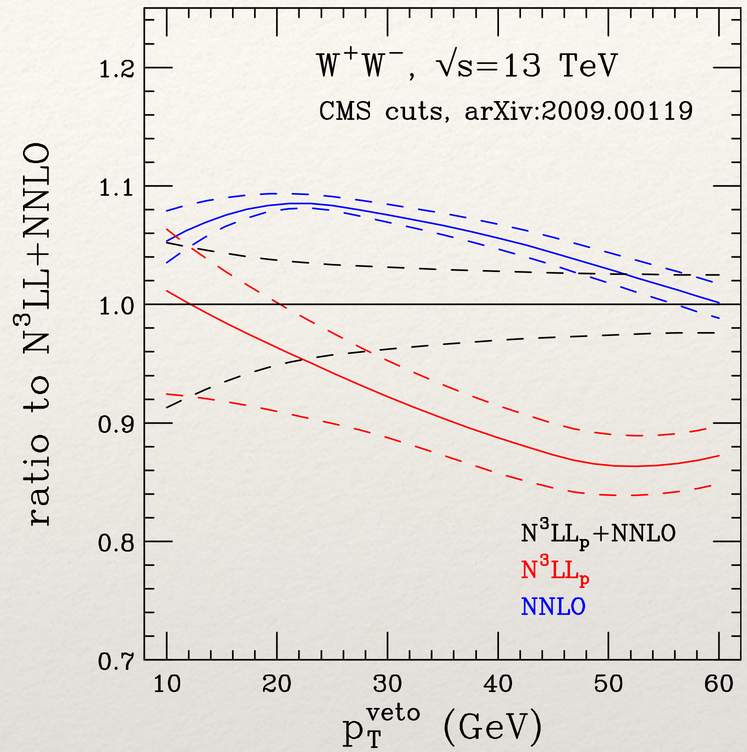
- Errors improve going from N^2LL +NNLO to N^3LL+NNLO
- Theoretical errors better than experimental.
- CMS data taken from 2009.00119



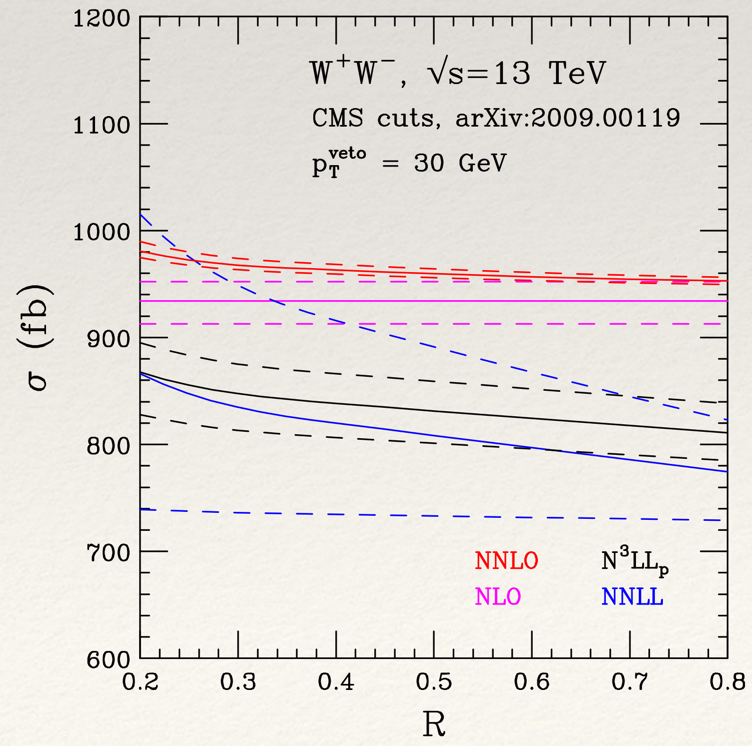
2210.10724

Jet veto in W^+W^- production

- Evidence that neither NNLO nor N^3LL is sufficient, especially around $p_T^{\rm veto} = 25 - 30 \mathrm{GeV}$



- R dependence is modest.
- $|\eta_{\rm cut}| < 4.5$, so we can argue that $(\ln(Q/p_T^{\rm veto}) = 1.3 - 2.2) \ll 4.5$



Conclusion

- Calculations at NNLO show mainly smaller power corrections for qT slicing than for zero-jettiness slicing. Calculation times roughly equal.
- The small q_T resummation in CuTe-MCFM has been extended to all color singlet final states with pairs of massive vector bosons — public release soon;
- The fine-grained experimental study of vector boson pair processes where the resummation effects will be crucial is, in the main, still to come;
- We have compared our predictions with the available data;
- We have also resummed cross sections at N^3LL_p +NNLO for all color singlet final state processes and for a p_T^{veto} at all rapidities. Necessary for Higgs production and for vector boson pair production.

Backup

Solution to RGE equations

$$\frac{d}{d\ln\mu} C(Q\,\mu) = \left[\Gamma_{\text{cusp}}(\mu)\ \ln\frac{Q^2}{\mu^2}\right] C(Q\,\mu)$$

- Traditional solution to the LL equation

$$C(Q,\mu) = \exp\left[2S(Q,\mu)\right]C(Q,Q) \qquad \frac{d}{d\ln\mu}S(Q,\mu) = -\Gamma_{\text{cusp}}(\alpha_S(\mu))\ln\frac{\mu}{Q}$$

$$S(Q,\mu) = -\int_Q^\mu \frac{d\mu'}{\mu'}\,\Gamma_{\text{cusp}}(\alpha_S(\mu'))\ \ln\frac{\mu'}{Q}$$

- We can write solution in terms of running coupling

- $$S(Q,\mu) = -\int_{\alpha_S(Q)}^{\alpha_S(\mu)} d\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)}\int_{\alpha_S(Q)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')} \qquad \frac{d\alpha_S}{d\ln\mu} = \beta(\alpha_S)$$

$$S(Q,\mu) \to \frac{\Gamma_0}{4\pi k_0^2}\frac{1}{\alpha_S(Q)}\left(\frac{r - r\ln r - 1}{r}\right) \text{ where } r = \alpha_S(\mu)/\alpha_S(Q)$$

- We recover the double log, setting

$$\beta(\alpha_S) = -k_0\alpha_S^2 \text{ and } \frac{1}{r} = 1 - k_0\alpha_S(Q)\ln(Q/\mu)$$

Second comment on 2207.07037

$$\bar{I} \otimes f = \int_x^1 \frac{dz}{z}\, \bar{I}(z)\, f(x/z) \sim \frac{1}{x}\int_x^1 dz\, \bar{I}(z)$$

- If we assume $f(x) \sim 1/x$
- However for any steeper function as $x \to 1$, the range of z sampled is close to $z = 1$.
- The result of 2207.07037 is almost entirely analytic; it is crucial that the behavior at $z = 1$ is accurately evaluated.
- Certain of the beam functions contain terms of order $\dfrac{f(z)\, R^n}{(1-z)^{n+1}}$ where $f(z)$ is a complicated function involving dilogarithms, trilogarithms etc.
- The singularity at $z = 1$ is only apparent, and the analytic forms must be expanded for numerical stability (easy to do…)