NNLO+PS for heavy-quark production

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Outline

- Introduction: heavy-quark production
- Transverse momentum resummation and MiNNLO$_{PS}$: extension to $Q\bar{Q}$
- MiNNLO$_{PS}$ for top-quark pair production
- Extension to $Q\bar{Q}$+colourless final state
- Summary and outlook

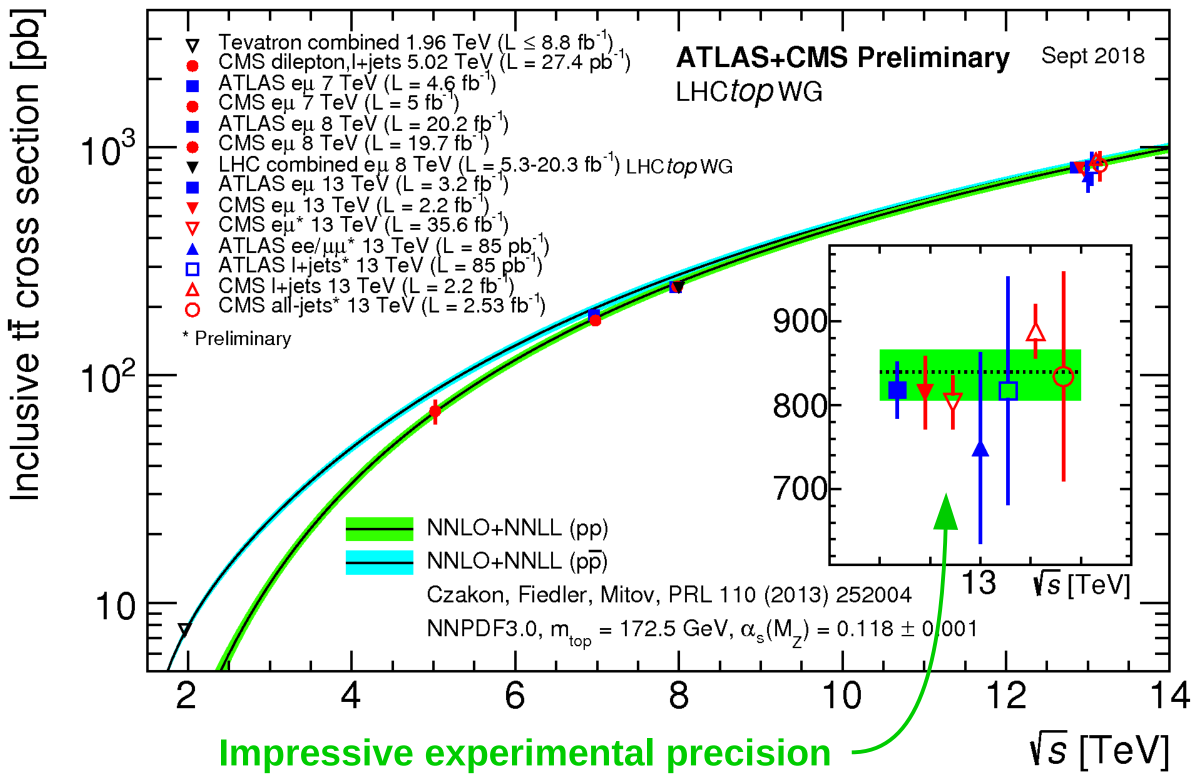
Introduction

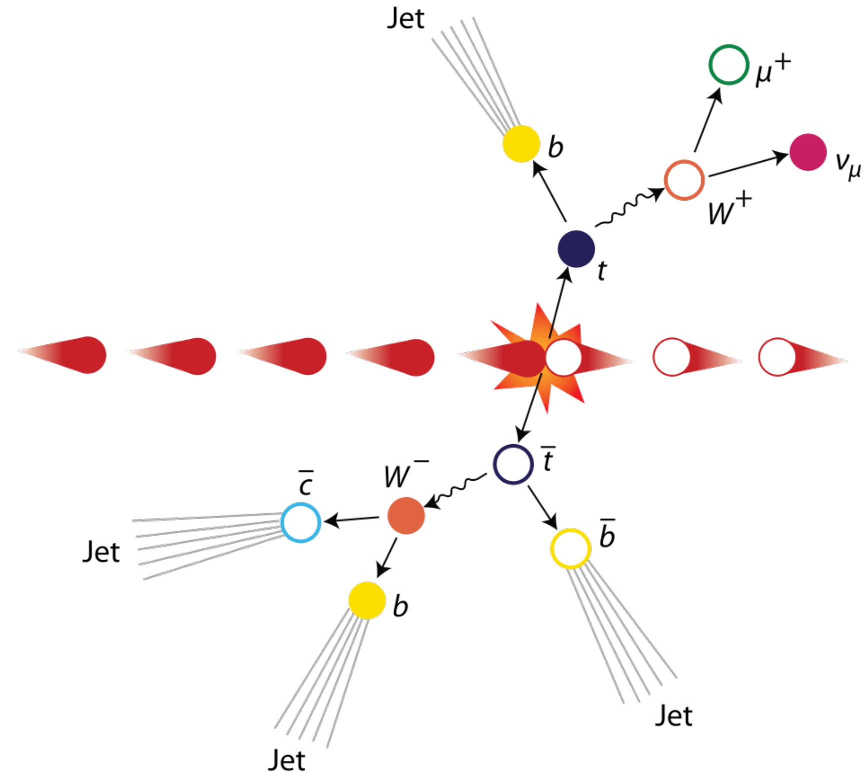
- Precise theoretical predictions are indispensable to fully exploit LHC data
- Higher-order (NNLO) QCD corrections are crucial to this end
- Event generators are a cornerstone of experimental analysis

NNLO-accurate event generators are needed

- Topic of this talk: heavy-quark production (+ colourless final state)
- Many important processes fall into this category

Top pair production





15 $t\bar{t}$ pairs produced per second at the LHC

- Precision tests of the SM (e.g. m_W, m_t and m_H relation)
- Vacuum stability
- BSM searches
- Ubiquitous background ⟶ 35% of all published ATLAS searches cite top++

$t\bar{t}H$ production → Direct sensitivity to top-quark Yukawa coupling
[see talk by Massimiliano]

$b\bar{b}$ production → Test of QCD and useful for PDFs determination
[see talk by Alessandro]

$b\bar{b}W$ and $b\bar{b}Z$ → Irreducible background to $W/Z+H \to b\bar{b}$
[see talk by Luca]

$b\bar{b}WW$ → $t\bar{t}$ with decays + non resonant contributions

$t\bar{t}W$ and $t\bar{t}Z$ → Important background to multi-lepton final states

$b\bar{b}H$ production → Bottom Yukawa sensitivity and background to HH

$c\bar{c}$ production → Applications to atmospheric neutrino flux

Framework to obtain (NNLO and) NNLO+PS predictions for these processes highly desirable!

Two main ingredients needed for this:

This talk:

Scattering amplitudes (two loop)

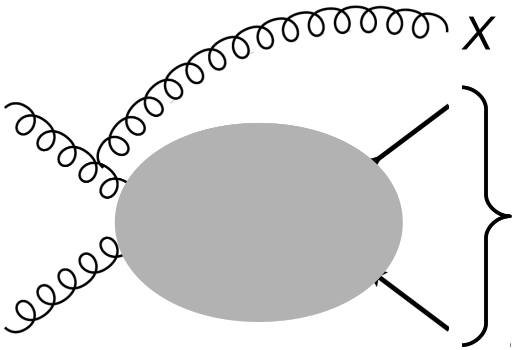
Matching procedure (NNLO+PS) → MiNNLO$_{PS}$

[Subtraction method (NNLO)] [q_T-subtraction]

Derivation of MiNNLO$_{PS}$ (and q_T-subtraction) based on low-q_T behaviour

↓

We focus on q_T-resummation and the difference between colour singlet and $Q\bar{Q}$



→ Divergent in the unresolved limits

↕

Soft or collinear emissions

↕

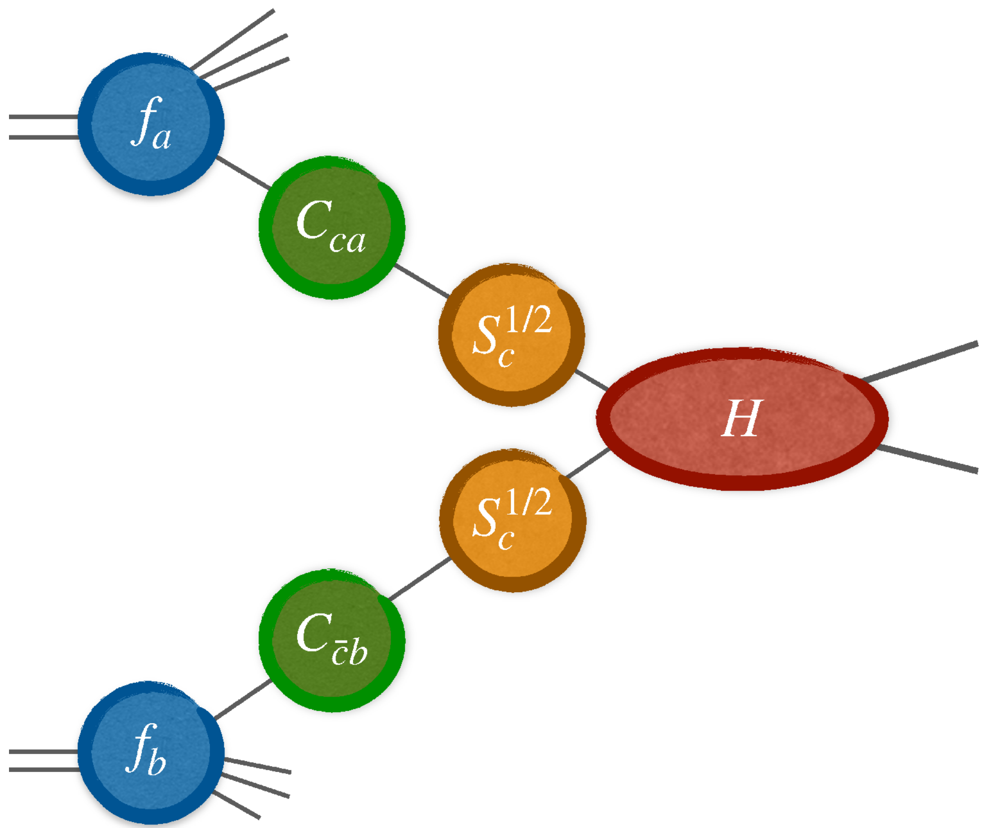
Low q_T limit for F

At N^{n}LO: $\mathrm{Log}^p(q_T/M) \equiv L^p$, with $1<p<2n$

Low transverse momentum behavior can be organized in an all-orders resummation formula

q_T resummation: color singlet

$$d\sigma^{(\text{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left(-S_c\right) \times \left[H C_1 C_2\right]_{c\bar{c};a_1 a_2} \times f_{a_1} f_{a_2}$$



Parton distribution functions

Collinear functions → hard-collinear emissions

Sudakov exponent → soft and flavor diagonal emissions

Universal

Hard function → hard process-dependent radiation

Resummed cross section physical (finite) when $p_T \to 0$

Can be computed at different logarithmic accuracy depending on which logs are included:

$$\text{LL: } \alpha_s^n L^{n+1}$$

$$\text{NLL: } \alpha_s^n L^n$$

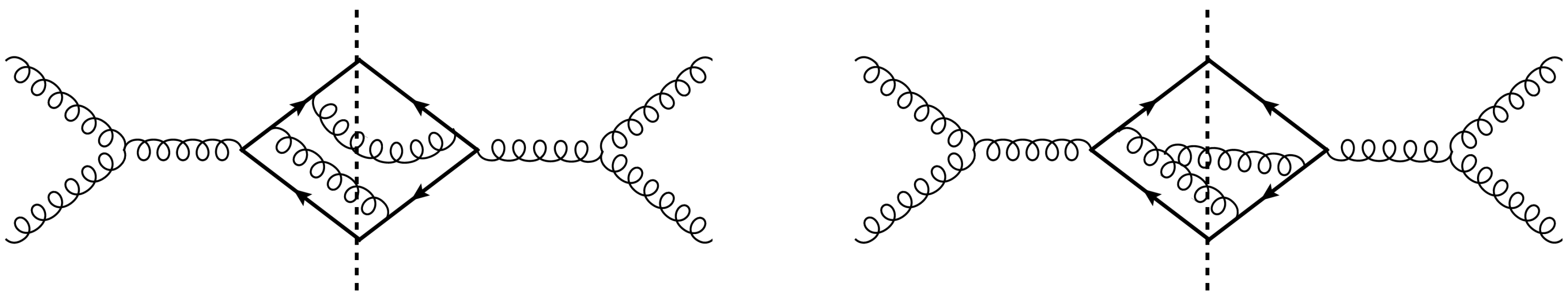
$$\text{NNLL: } \alpha_s^n L^{n-1}$$

Can also be 'matched' to the fixed order upon expansion in α_s:

NLL+NLO, NNLL+NLO, NNLL+NNLO

q_T resummation: heavy quark pairs

What changes for heavy-quark production? → Emission from **final state**



- Additional **divergencies** when FS emission becomes **soft**
- Obs: no new collinear divercencies since the mass of the quarks regulates them
- Presence of colored FS leads to **color interference** effects

↓

Colorless FS → only 2 hard partons → Color charge factors $\mathbf{T_i}.\mathbf{T_j}$ diagonal in color space

Heavy-quark production → 4 hard partons → unavoidable **color correlations**

q_T resummation: heavy quark pairs

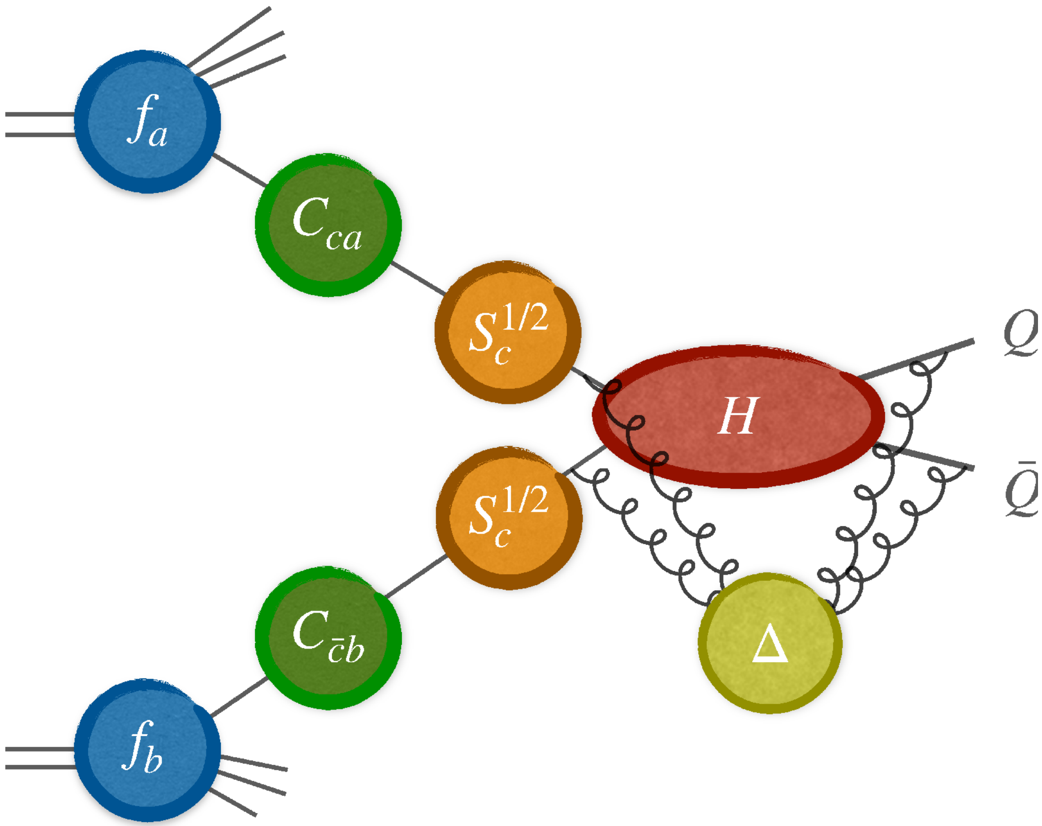
Bold: operator in color space

$|M\rangle$: vector in color space

$$d\sigma^{(\text{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left(-S_c\right) \times \left[H C_1 C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1} f_{a_2}$$

$$d\sigma^{(\text{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left(-S_c\right) \times \left[\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) C_1 C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1} f_{a_2}$$

Effects coming from soft emissions from the FS contained in operator **Δ**



In the colour singlet case, *H* is given by the (IR-subtracted) all-orders matrix element for $c\bar{c} \to F$

$$H = \mathrm{Tr}(\mathbf{H}) \sim \langle \mathcal{M} | \mathcal{M} \rangle$$

In the $t\bar{t}$ case, the presence of the operator **Δ** leads to non-trivial color correlations

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) \sim \langle \mathcal{M} | \boldsymbol{\Delta} | \mathcal{M} \rangle$$

$$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S_c(b)\right] \times \left[\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})C_1 C_2\right]_{c\bar{c};a_1 a_2} \times f_{a_1} f_{a_2}$$

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) \sim \langle \mathcal{M}|\boldsymbol{\Delta}|\mathcal{M}\rangle$$

IR regulated virtual corrections

$$\boldsymbol{\Delta} \sim \exp\left\{-\int_{b_0^2/b^2}^{M} \frac{dq^2}{q^2}\,\boldsymbol{\Gamma}(\alpha_s(q))\right\}^{\dagger} \mathbf{D}(\alpha_s(b_0/b),\phi)\,\exp\left\{-\int_{b_0^2/b^2}^{M} \frac{dq^2}{q^2}\,\boldsymbol{\Gamma}(\alpha_s(q))\right\}$$

Exponential of soft anomalous dimension matrix

Operator leading to azimuthal correlations

- Soft anomalous dimension encodes logarithmic behavior of soft wide-angle emissions
- **D** encodes the azimuthal dependence of the constant terms, with $\langle \mathbf{D} \rangle_{\Phi,\mathrm{av}} = 1$
- Even for q_T azimuthally-averaged cross sections, **D** contributes in the gluon channel due to the interference with the collinear coefficient functions (starting at NNLO)
- All the ingredients for NNLL+NNLO resummation are now known except for $\mathbf{D}^{(2)}$
- $\mathbf{D}^{(2)}$ contributes with a constant term at $O(\alpha_S^4)$ that vanishes upon azimuthal average
- Translation between virtual corrections and IR-regulated *M* highly non trivial!
  The correct finite part of subtraction operator needs to be explicitly computed

$$|\mathcal{M}\rangle = \left(1 - \tilde{\mathbf{I}}\right)|\mathcal{M}\rangle_{\mathrm{unreg}}$$

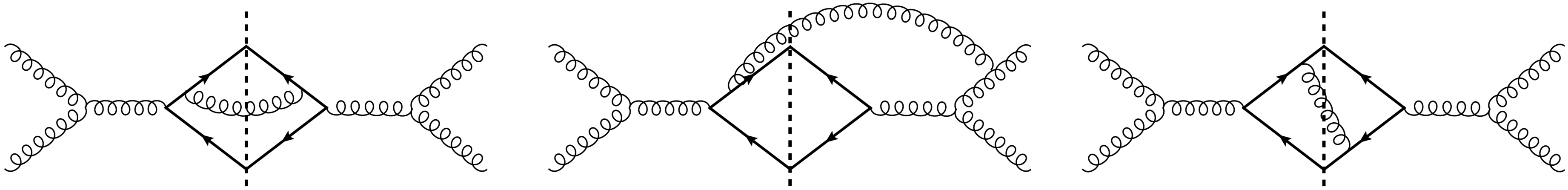
Extracted from integration of soft current at fixed q_T

Subtraction operator: NLO

- **I** operator can be extracted from computation of $d\sigma/d^2q_T$
- Only new soft singularities → integrate the (subtracted) **soft current**

E.g. at NLO:

$$-\mathbf{J}(k)^2|_{\rm sub} = \sum_{J=3,4}\left[\frac{p_J^2}{(p_J\cdot k)^2}\mathbf{T}_J^2 + \sum_{i=1,2}\left(\frac{p_i\cdot p_J}{p_J\cdot k} - \frac{p_1\cdot p_2}{(p_1+p_2)\cdot k}\right)\frac{2\,\mathbf{T}_i\cdot\mathbf{T}_J}{p_i\cdot k}\right] + \frac{2p_3\cdot p_4}{(p_3\cdot k)(p_4\cdot k)}\mathbf{T}_3\cdot\mathbf{T}_4$$



- After integration the following NLO subtraction operator can be obtained:

[Catani, Grazzini, Torre; 1408.4564]

Purely initial-state — New soft contributions

$$\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}\left(\epsilon,\frac{M^2}{\mu_R^2}\right) = -\frac{1}{2}\left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon}\left\{\left(\frac{1}{\epsilon^2}+i\pi\frac{1}{\epsilon}-\frac{\pi^2}{12}\right)(\mathbf{T}_1^2+\mathbf{T}_2^2)+\frac{2}{\epsilon}\gamma_c - \frac{4}{\epsilon}\,\mathbf{\Gamma}_t^{(1)}(y_{34}) + \mathbf{F}_t^{(1)}(y_{34})\right\}$$

Pole structure agrees with studies on one-loop amplitudes

[Catani, Dittmaier, Trocsanyi, 0011222]

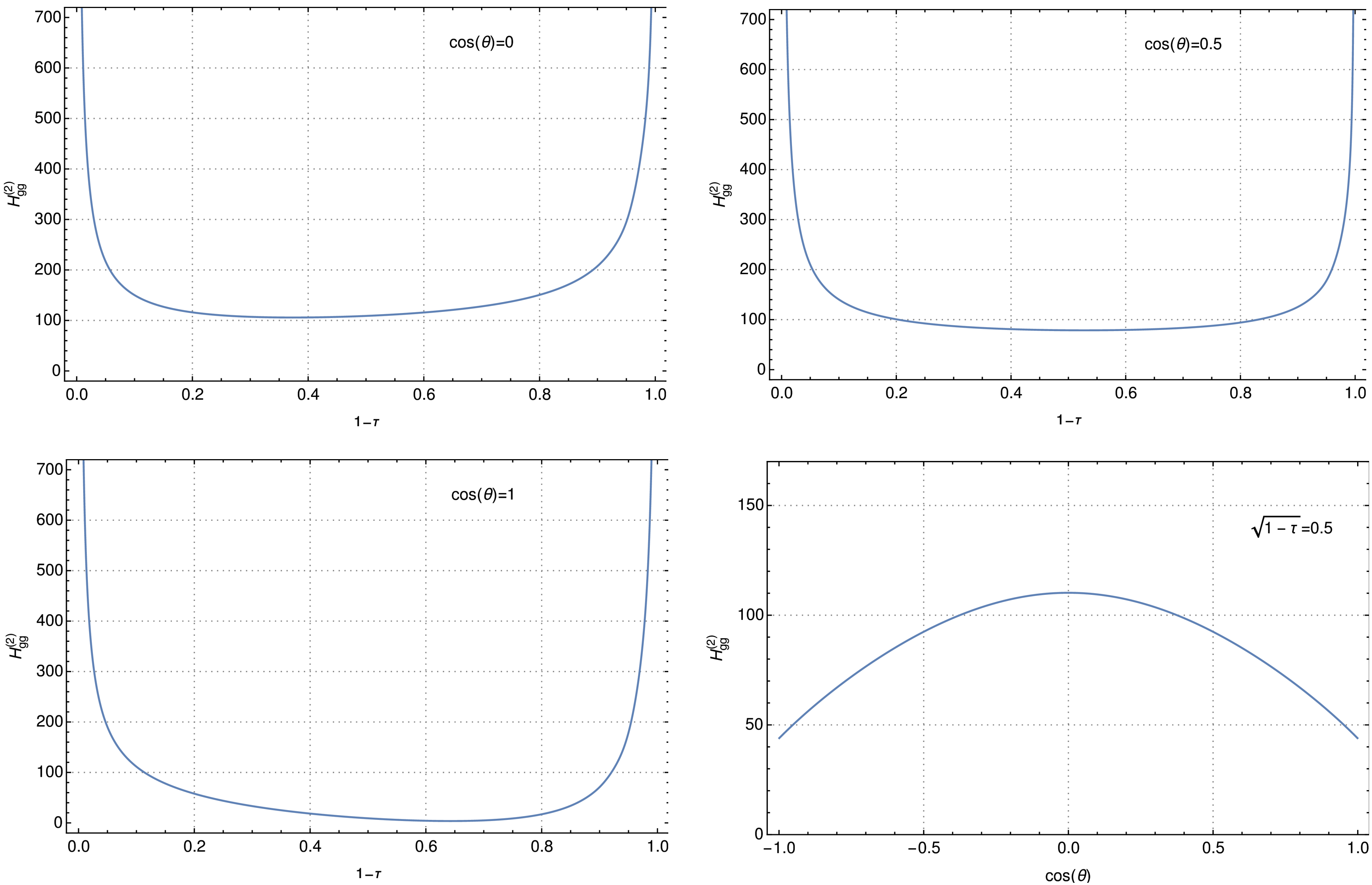
Finite piece: only from direct computation

- We had to **extend** the above results **to NNLO**

Subtraction operator

$\tau = 4m^2/s$, $\cos\theta$ scattering angle

- Computation finished few years ago   Catani, Devoto, Grazzini, JM (in prep), see also Angeles-Martinez, Czakon, Sapeta (18')
- Last missing ingredient for $O(\alpha_s^4)$ fixed-order expansion of resummation formula
- Results mostly analytical, numerical integration for some pieces



Extending MiNNLO for $t\bar{t}$

- Having the low-qT factorization formula available to the desired accuracy, we are in a position to extend the MiNNLO method to $Q\bar{Q}$

- However, the more complicated colour structure doesn't allow to follow the colour-singlet derivation

- More specifically, since the $t\bar{t}$ factorization formula does **not** take the simple form:

$$d\sigma^{(\mathrm{sing})} \sim \exp\left[-S_c(p_T)\right] \times \mathcal{L}(p_T)$$

used to describe the NNLO cross section as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} = \frac{\mathrm{d}}{\mathrm{d}p_{\mathrm{T}}}\left\{\exp[-\tilde{S}(p_{\mathrm{T}})]\mathcal{L}(p_{\mathrm{T}})\right\} + R_f(p_{\mathrm{T}}) \qquad (1)$$

However, in eq. (1) we are only interested in the fixed order accuracy

⬇

We can modify the $t\bar{t}$ factorization formula as long as we keep NNLO accuracy (and LL in view of the matching with the shower)

⬇

We can take it into a shape that resembles the colorless final state case

⬇

Connection to MiNNLO derivation becomes simpler

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) \sim \langle\mathcal{M}|\boldsymbol{\Delta}|\mathcal{M}\rangle \longrightarrow \text{“Sudakov”} \times \langle\mathcal{M}|\mathcal{M}\rangle + \mathrm{h.o.}$$

Extending MiNNLO for $t\bar{t}$

We can arrive to the following expression keeping NNLO accuracy:

Sudakov with modified $B^{(2)}$ accounting for $\langle M^{(0)}|\,\Gamma^{(2)}|\,M^{(0)}\rangle$ and $\langle M^{(1)}|\,\Gamma^{(1)}|\,M^{(0)}\rangle$ terms

Computed by diagonalizing $\Gamma^{(1)}$ → Sum of complex exponentials

$$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \, \exp\left[-S''_c(b)\right] \, \langle \mathcal{M}^{(0)}| \exp\left[\int \frac{dq^2}{q^2}\frac{\alpha_s(q)}{2\pi}(\boldsymbol{\Gamma}^{(1)} + \boldsymbol{\Gamma}^{(1)\dagger})\right] |\mathcal{M}^{(0)}\rangle \, [\mathrm{Tr}(\mathbf{H}\,\mathbf{D})C_1C_2]^{\phi}_{c\bar{c};a_1a_2} \, f_{a_1}f_{a_2}$$

Of the form $\sum_i \exp\left[-S(B \to B_i)\right]$

More precisely, each term is an 'usual' Sudakov form factor with an effective (complex) value of $B^{(1)}$ and $B^{(2)}$

$$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S_c(b)\right] \times [\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})C_1C_2]_{c\bar{c};a_1a_2} \times f_{a_1}f_{a_2}$$

$$\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) \sim \langle \mathcal{M}|\boldsymbol{\Delta}|\mathcal{M}\rangle$$

$$\boldsymbol{\Delta} \sim \exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\boldsymbol{\Gamma}(\alpha_s(q))\right\}^{\dagger} \mathbf{D}(\alpha_s(b_0/b),\phi) \exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\boldsymbol{\Gamma}(\alpha_s(q))\right\}$$

- First ever **NNLO+PS** for a **colored final state** in hadronic collisions

Extending MiNNLO for $t\bar{t}$

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Of the form $\sum_i \exp\left[-S(B \to B_i)\right]$

More precisely, each term is an 'usual' Sudakov form factor with an effective (complex) value of $B^{(1)}$ and $B^{(2)}$

Factorization formula was the starting point for color-singlet MiNNLO

Now we have a sum of colorless-final-state-like factorization formulas

↓

Follow MiNNLO color-singlet derivation for each of them and arrive to MiNNLO for $t\bar{t}$

- Method implemented in POWHEG-BOX-V2, code publicly available
- First ever **NNLO+PS** for a **colored final state** in hadronic collisions

Numerical results

Scale setting:

- Overall Born coupling: $\alpha_s(H_T/4)$
- MiNNLO scale setting: $\mu_R = \mu_F = m_{t\bar{t}}\, e^{-L}$, $Q = m_{t\bar{t}}/2$
- Scale uncertainties through 7-point variation
- No direct correspondence between MiNNLO scales and NNLO scales
- Upon integration over p_T they are of the order of $m_{t\bar{t}}$

Comparison to NNLO (computed with MATRIX) with $\mu_0 = m_{t\bar{t}}$ and $\mu_{LO} = H_T/4$

Modified logarithm:

$$L = \begin{cases} \log(Q/p_T) & \text{for } p_T<Q/2 \\ 0 & \text{for } p_T>Q \\ \text{Smooth interpolation in the middle} & \end{cases}$$

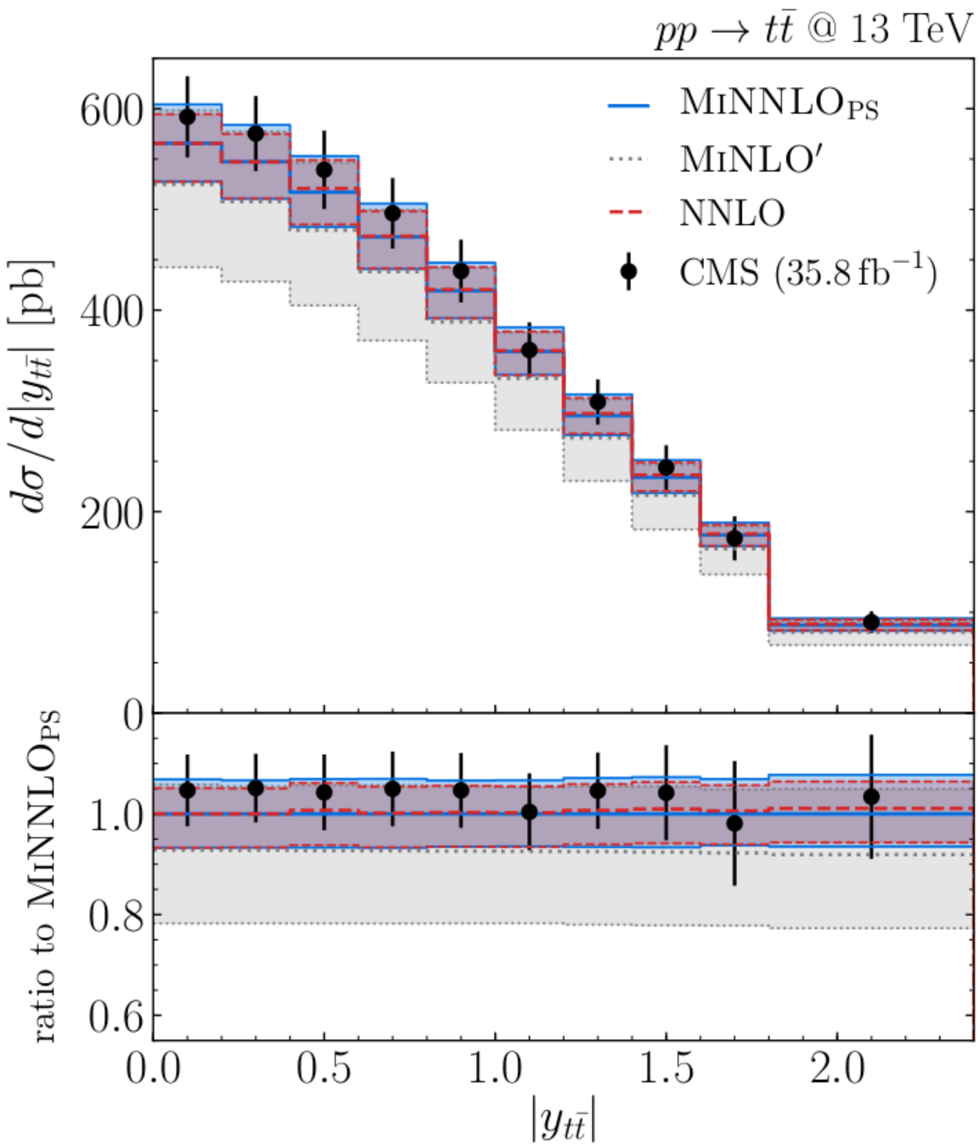
Showering:

We shower with Pythia8 (Monash 2013 tune).
For FO comparison (keeping top quarks stable) we do not include hadronization effects, MPI, or QED shower (for other results these are on)

Parton level results

[JM, Monni, Nason, Re, Wiesemann, Zanderighi]

MiNLO′	NNLO	MiNNLO$_{PS}$
$721.4^{+14\%}_{-16\%}$ pb	$782.0^{+5.2\%}_{-6.7\%}$ pb	$779.2^{+6.9\%}_{-6.6\%}$ pb

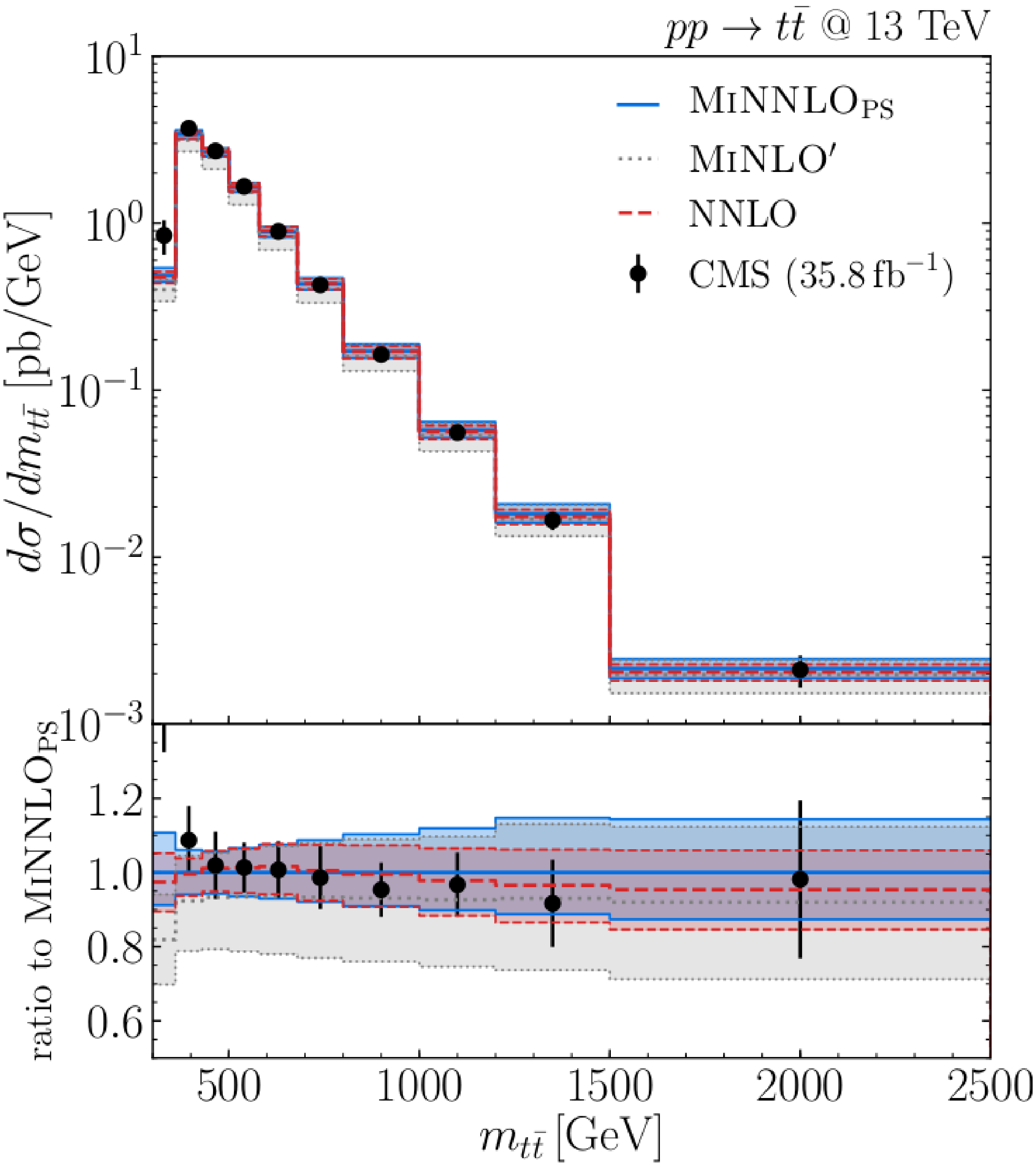


- Excellent agreement between MiNNLO and NNLO total cross sections, differences at the per-mille level
- Obs: even larger differences could be expected due to different scale settings and h.o. effects
- Similar size of uncertainties between MiNNLO and NNLO results
- Large reduction of scale uncertainties w.r.t. MiNLO' [Obs: MiNLO' for $t\bar{t}$ is also a new result]
- Excellent agreement in shape of rapidity distribution
- Excellent agreement with data*

*[data from CMS semileptonic analysis extrapolated to inclusive $t\bar{t}$ PS]

Parton level results

[JM, Monni, Nason, Re, Wiesemann, Zanderighi]



- Invariant mass of the $t\bar{t}$ system
- Full compatibility between MiNNLO and NNLO results in the whole range
- Small differences in shape expected
- Slightly larger uncertainties in the tail
- Good agreement with data except for region close to threshold

Coulomb resummation
Finite width effects
Issues in extrapolation to inclusive PS

See later particle level results!

Particle level results

- We now compare our event generator to particle-level data from:

Leptonic	Semi-leptonic	Fully hadronic
[ATLAS 1910.08819]	[CMS 1803.08856]	[ATLAS 2006.09274]

- Top decays included using ratio of tree-level decayed and undecayed MEs
  [As implemented in POWHEG ttbarj, Alioli, Moch, Uwer 1110.5251]

$$dP(\Phi_{\text{dec.}}|\Phi_{\text{undec.}}) = \frac{1}{\text{BR}(t \to b\ell\nu)\,\text{BR}(\bar{t} \to \bar{b}\ell\bar{\nu})} \frac{\mathcal{M}_{\text{dec.}}(\Phi_{\text{undec.}}, \Phi_{t\to b\bar{\ell}\nu}, \Phi_{\bar{t}\to\bar{b}\ell\bar{\nu}})}{\mathcal{M}_{\text{undec.}}(\Phi_{\text{undec.}})} d\Phi_{t\to b\bar{\ell}\nu}\, d\Phi_{\bar{t}\to\bar{b}\ell\bar{\nu}}$$

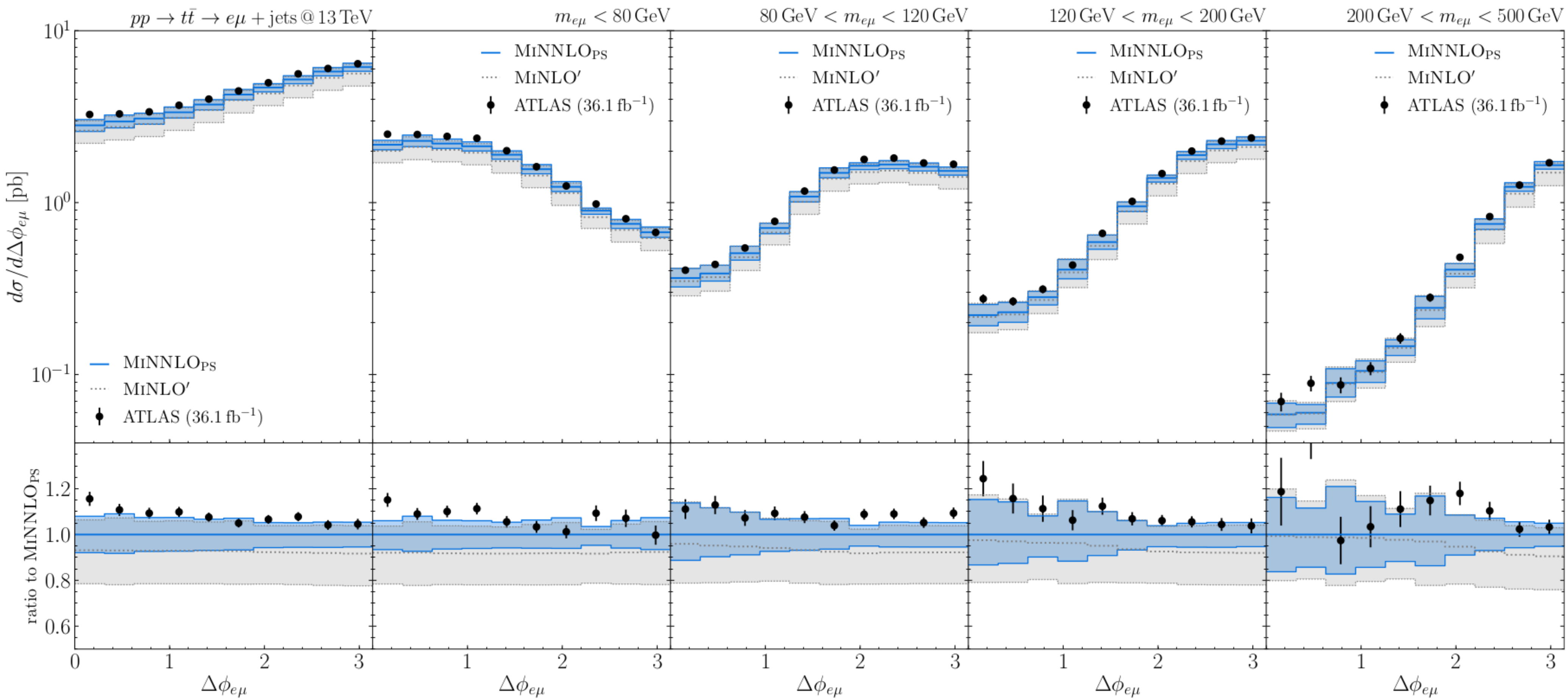
- Simple and fast procedure, though only LO accurate
  (obs: LO accuracy in tt, ttJ and ttJJ observables)

- Top-quarks and W bosons kept on-shell in what follows, though inclusion of off-shellness possible within the code

- Alternative: generate tt events and use MadSpin to decay them. Results compatible within uncertainties

Particle level results: leptonic

[JM, Monni, Nason, Re, Wiesemann, Zanderighi]
[ATLAS 1910.08819]

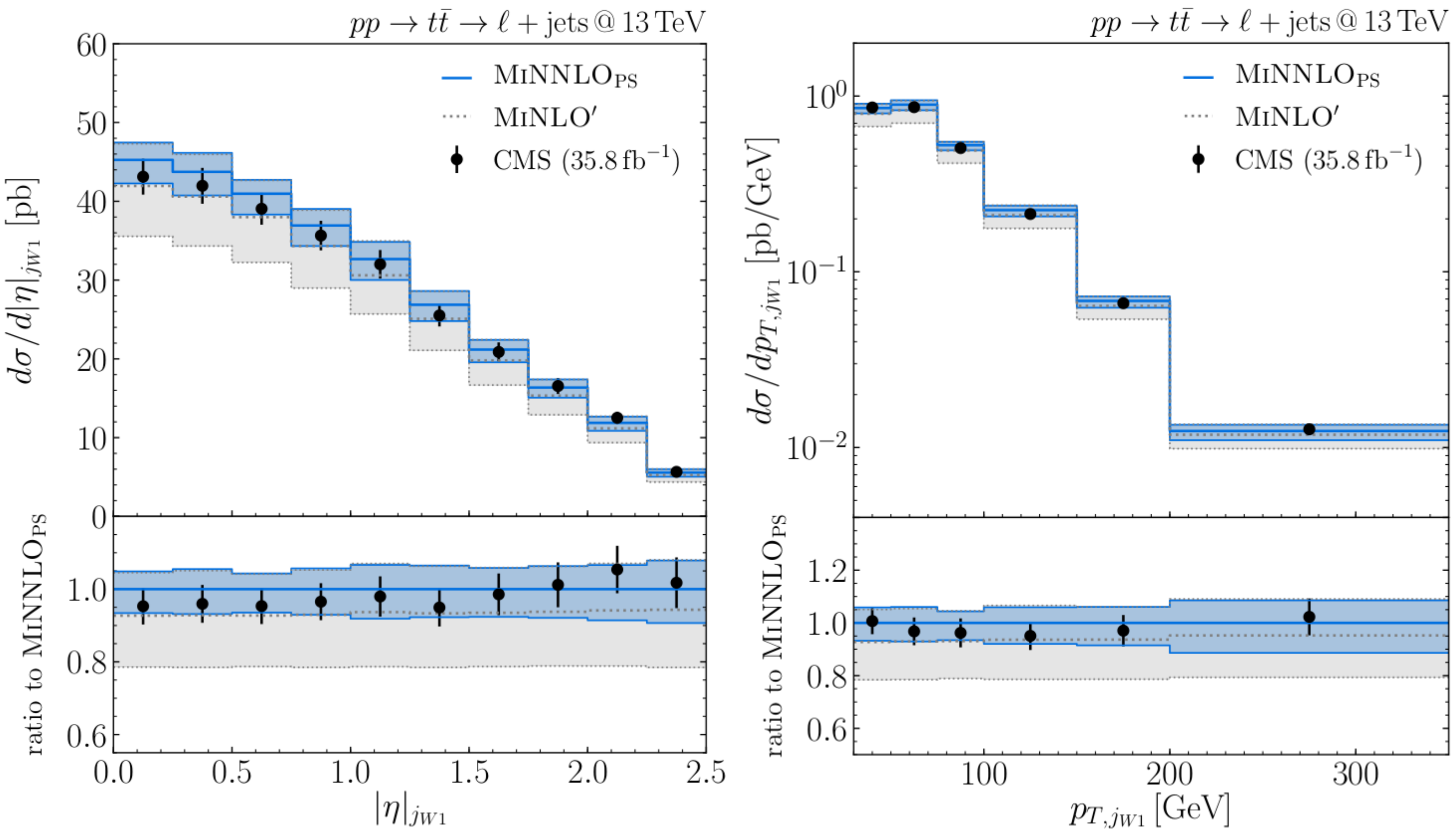


- Azimuthal angle between leptons → sensitivity to spin correlations in top-quark decays
- Very good agreement with data in all invariant mass slices
  (despite spin correlations in decay being only considered at LO)
- Data close to upper band of the MiNNLO prediction (also in other distributions)

Obs: total XS slightly smaller than ‘usual’ value (top++)
due to resummation effects and different scale settings

Particle level results: semi-leptonic

[JM, Monni, Nason, Re, Wiesemann, Zanderighi]
[CMS 1803.08856]

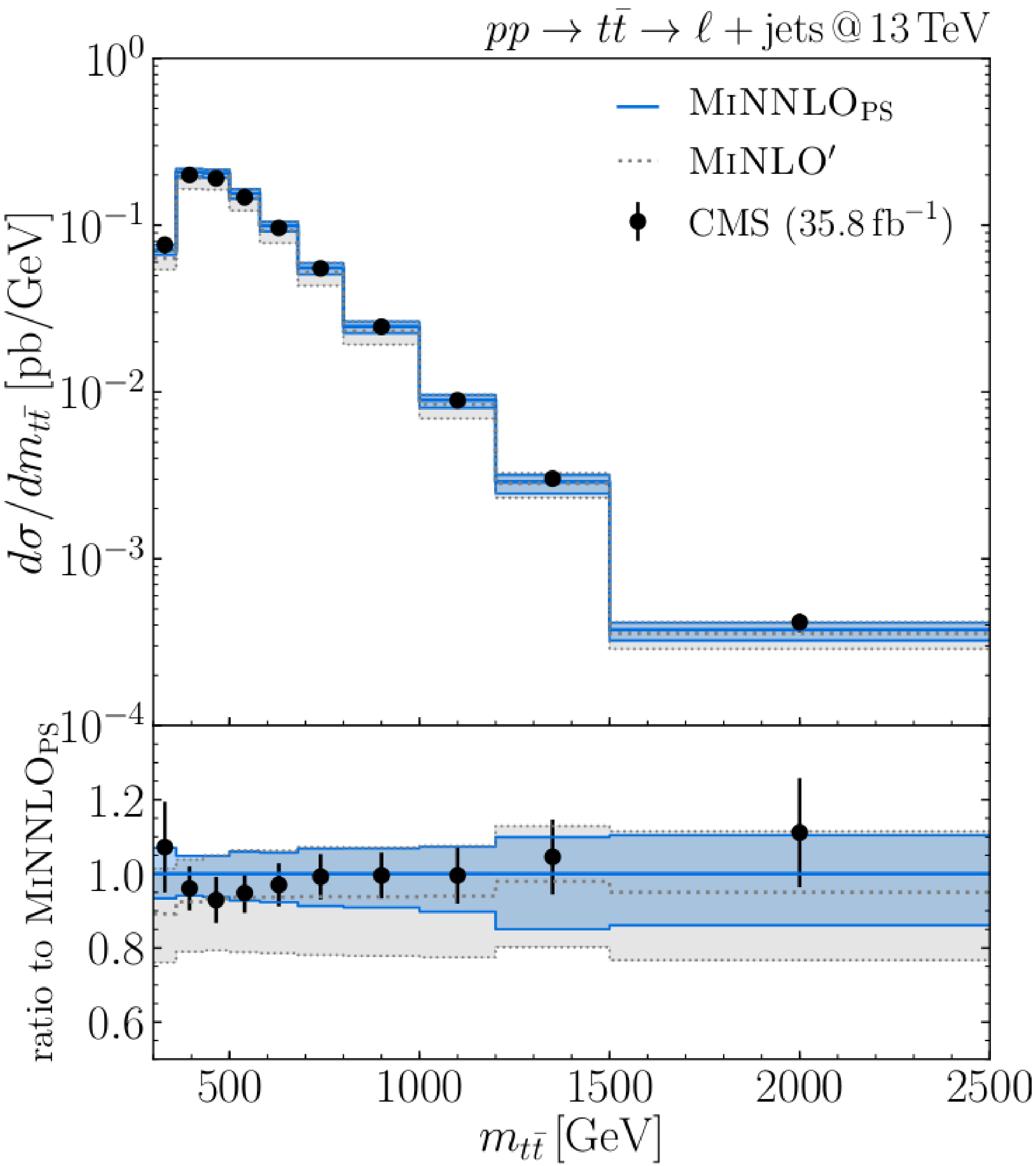


- Pseudo-rapidity and transverse momentum of the leading jet coming from hadronic W decay
- Excellent description both in shape and normalization
- Large reduction of uncertainties w.r.t. NLO-accurate generator

Particle level results: semi-leptonic

[JM, Monni, Nason, Re, Wiesemann, Zanderighi]

[CMS 1803.08856]

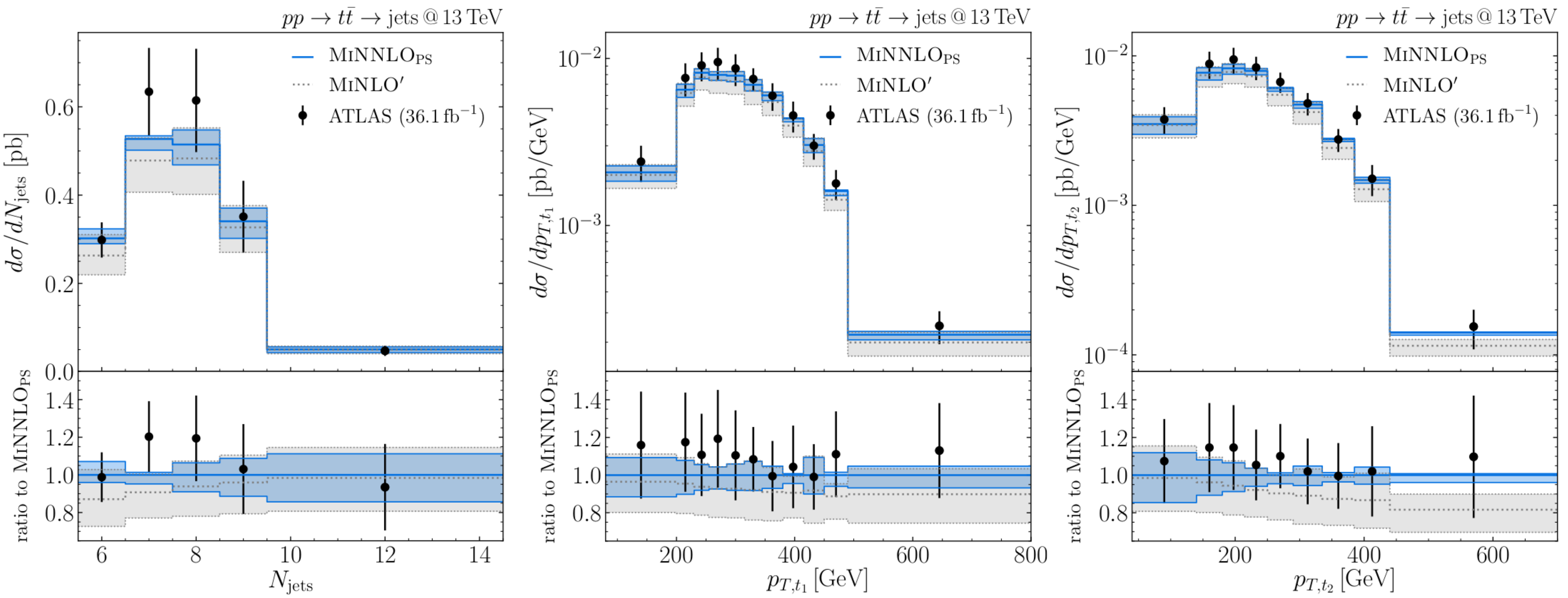


- Invariant mass of the reconstructed top-quark-pair system
- Slight shape difference compared to data, but excellent agreement within uncertainties
- Agreement even in the first bin, in variance with inclusive extrapolated results
- Obs: more effects included in the shower in this case (QED, MPI, hadronization) which might account for this difference
- Highlights the importance of doing data-theory comparison in fiducial PS

Particle level results: fully hadronic

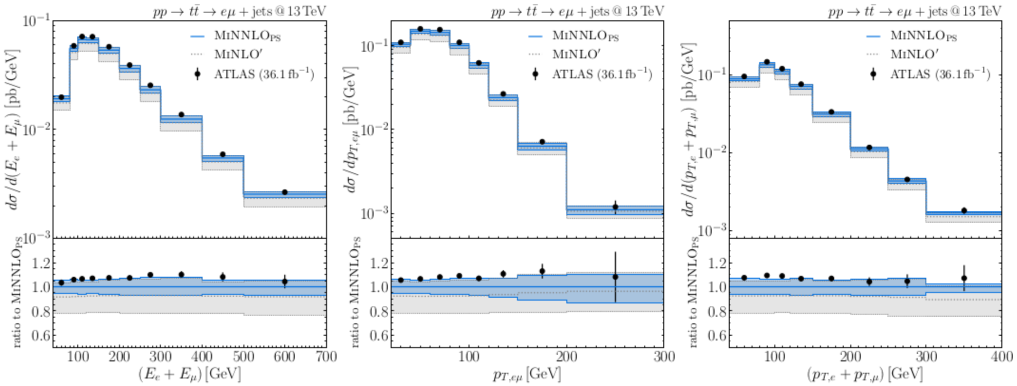
[JM, Monni, Nason, Re, Wiesemann, Zanderighi]

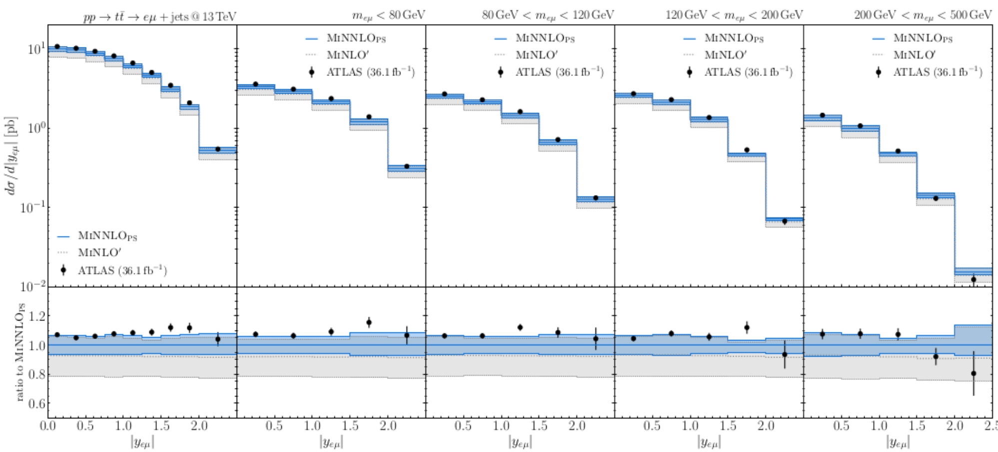
[ATLAS 2006.09274]

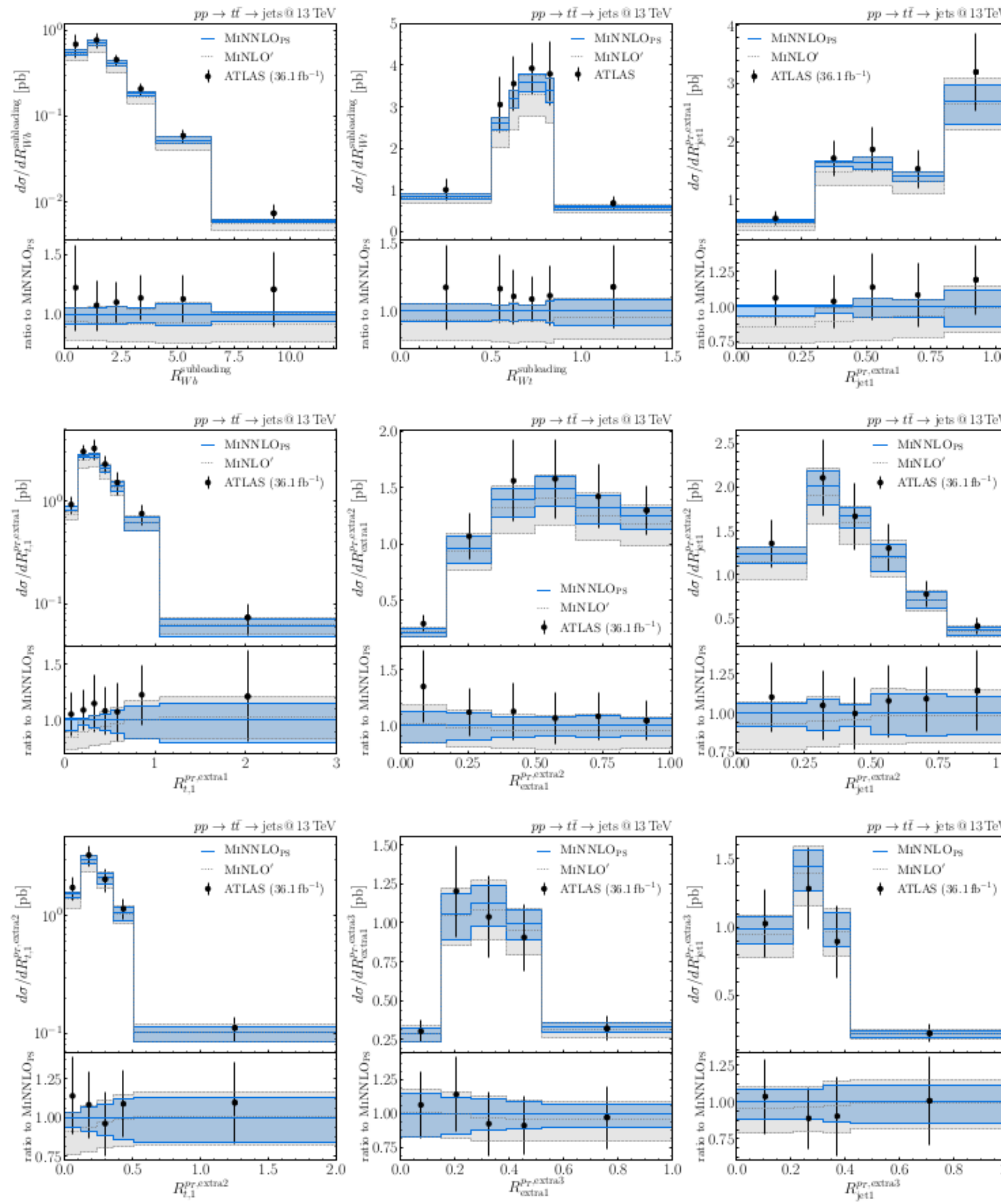


- Good agreement in fully hadronic final state, though experimental uncertainties much larger
- Obs: inclusion of MPI has a large impact in normalization (~10% effect)
- Strong reduction of uncertainties w.r.t. NLO+PS in regions inclusive in additional radiation
- Similar uncertainties e.g. for large N_{jets}, where NNLO accuracy is not met
- Shape of p_T distributions much better described at NNLO+PS

Particle level results







Many more distributions in the appendix of 2112.12135

Extension to $Q\bar{Q}F$

- No additional conceptual complication in low-q_T structure, nor in MiNNLO$_{PS}$
- However, perturbative ingredients need to be available for general kinematics

Finite piece of two-loop subtraction operator
(i.e. NNLO soft contributions)

- Old results: mostly analytic
  assumption $Q\bar{Q}$ back-to-back at LO
  grid+interpolation
- New results: extension to $Q\bar{Q}F$ kinematics
  [Devoto, JM] more pieces computed numerically
  on-the-fly numerical integration

- Implementation in library easily linked to POWHEG, MATRIX

Soft function for Heavy quark production in ARbitrary Kinematics
[Devoto, JM]

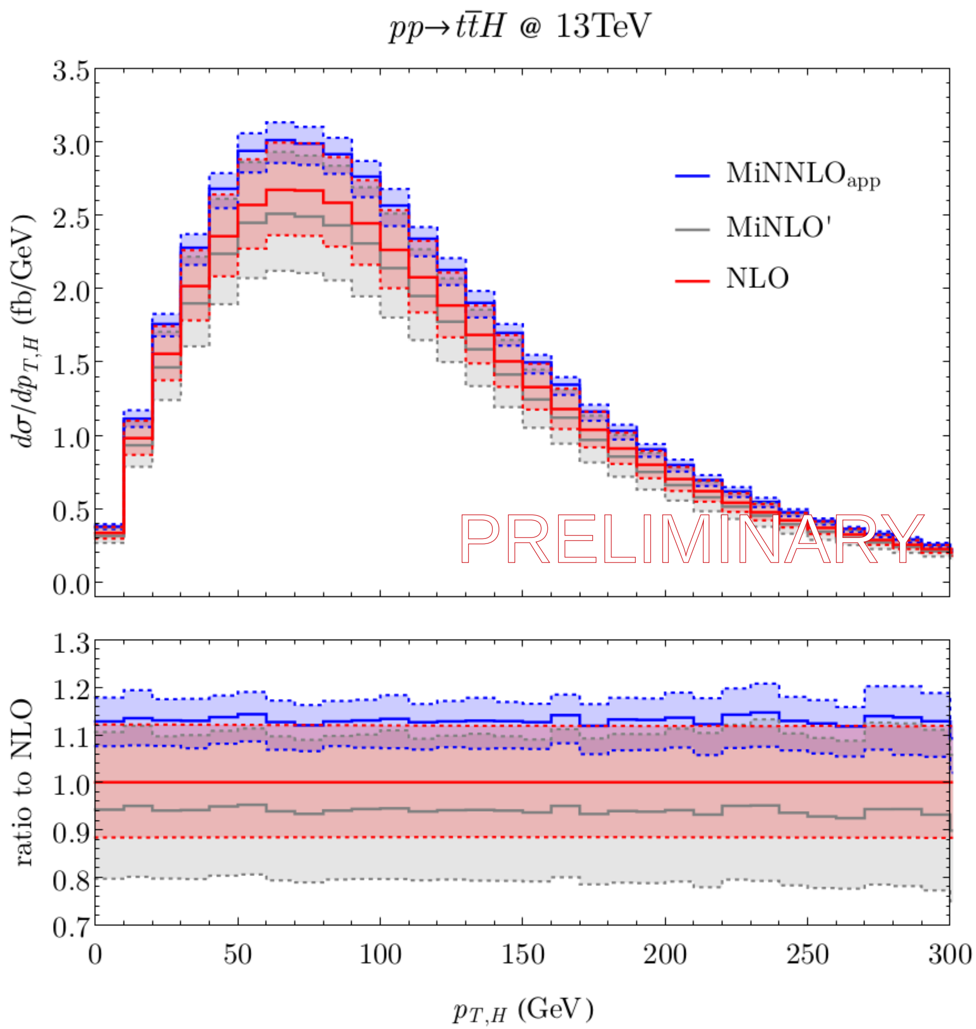
- Working towards a completely general MiNNLO$_{PS}$ implementation for $Q\bar{Q}F$
  [JM, Wiesemann]

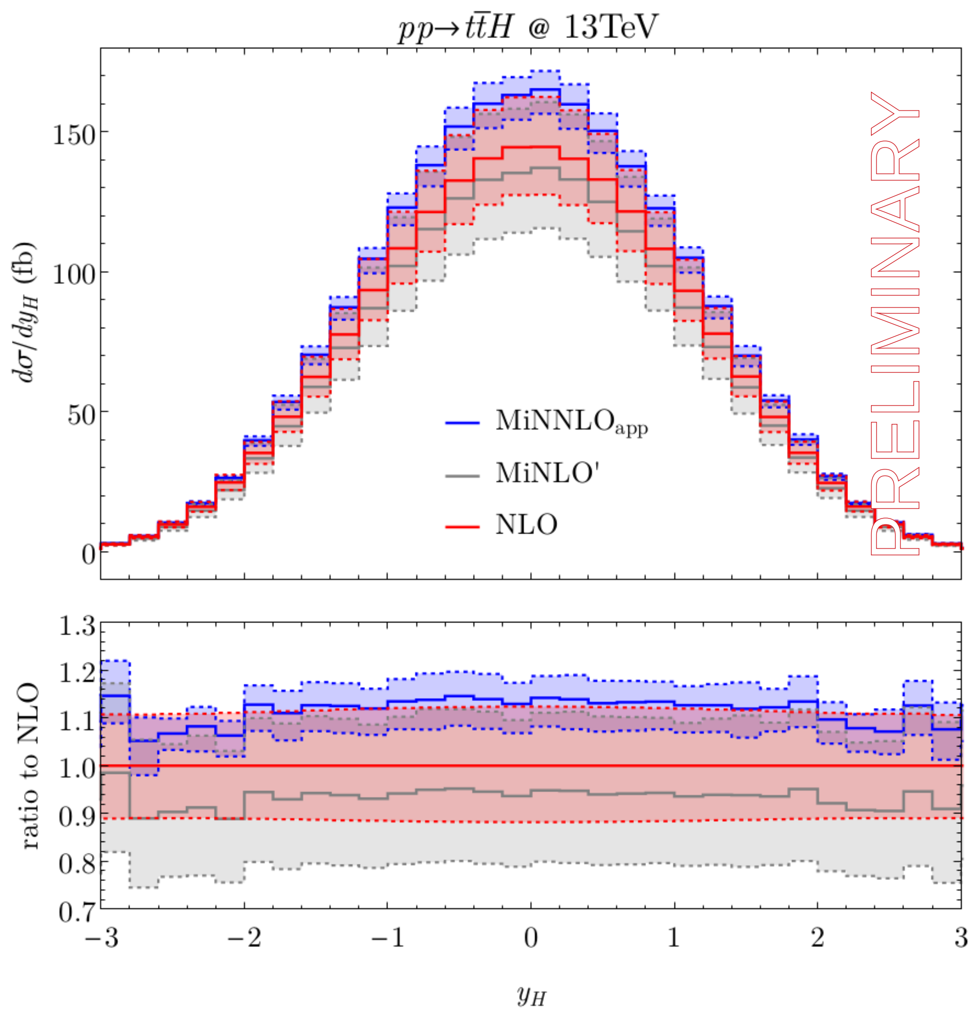
- Goal: framework with only needed input being the two-loop virtual corrections
  (tree and one-loop amplitudes: general interface to OpenLoops)

Extension to $Q\overline{Q}F$

- We have generated some **VERY PRELIMINARY** distributions for $t\bar{t}H$

[JM, Wiesemann]





- MiNNLO_{PS} results are exact except for two-loop, for which we set $H^{(2)} = 0$

Only per-mille-level effect in total XS

[see Massimiliano's talk]

- Note that MiNLO' results are a also a new result for this type of processes

Summary and Outlook

- Many processes involving heavy-quarks are central in LHC phenomenology
- Accurate predictions and event generators are crucial
- NNLO+PS generator for $t\bar{t}$ publicly available in POWHEG
- First NNLO-accurate event generator for heavy-quark production
- Low-q_T factorization now complete (up to NNLO) for $Q\bar{Q}$+colourless
- Fixed-order applications discussed later today (talks by Luca and Massimiliano)
- Extension of MiNNLO$_{PS}$ to heavy-quark + colourless underway
- Stay tuned!

Thanks!

Backup slides

$$d\sigma^{(\text{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S_c(b)\right] \times \left[\text{Tr}(\mathbf{H}\boldsymbol{\Delta})C_1C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1}f_{a_2}$$

$$S = \int_{b_0/b}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{M^2}{q^2} + B(\alpha_s(q))\right] \qquad \text{Tr}(\mathbf{H}\boldsymbol{\Delta}) \sim \langle\mathcal{M}|\boldsymbol{\Delta}|\mathcal{M}\rangle$$

$$\boldsymbol{\Delta} \sim \exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\boldsymbol{\Gamma}(\alpha_s(q))\right\}^{\dagger}\mathbf{D}(\alpha_s(b_0/b),\phi)\exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\boldsymbol{\Gamma}(\alpha_s(q))\right\}$$

- We can simplify the $\boldsymbol{\Gamma}^{(2)}$ contribution:

$$\boldsymbol{\Gamma}_t = \frac{\alpha_s}{2\pi}\boldsymbol{\Gamma}_t^{(1)} + \frac{\alpha_s^2}{(2\pi)^2}\boldsymbol{\Gamma}_t^{(2)} + \dots$$

already an $(\alpha_s)^2$ prefactor

- Up to NNLO it will only enter at the lowest possible order

$$\boldsymbol{\Delta}_{\text{NLL}} \equiv \boldsymbol{\Delta}(\boldsymbol{\Gamma}^{(2)} \to 0)$$

$$\langle\mathcal{M}|\boldsymbol{\Delta}|\mathcal{M}\rangle \sim \langle\mathcal{M}|\boldsymbol{\Delta}_{\text{NLL}}|\mathcal{M}\rangle - \int\frac{dq^2}{q^2}\frac{\alpha_s^2(q)}{(2\pi)^2}\langle\mathcal{M}^{(0)}|\boldsymbol{\Gamma}^{(2)} + \boldsymbol{\Gamma}^{(2)\dagger}|\mathcal{M}^{(0)}\rangle$$

Same kind of term generated by $B^{(2)}$

- We can actually include it via the replacement

$$B^{(2)} \to B'^{(2)} = B^{(2)} + \frac{\langle\mathcal{M}^{(0)}|\boldsymbol{\Gamma}^{(2)} + \boldsymbol{\Gamma}^{(2)\dagger}|\mathcal{M}^{(0)}\rangle}{|\mathcal{M}^{(0)}|^2}$$

Already projected over *M*, this is now just a number

$$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S_c(b)\right] \times \left[\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta})C_1C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1}f_{a_2}$$

$$S = \int_{b_0/b}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{M^2}{q^2} + B(\alpha_s(q))\right] \qquad \mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}) \sim \langle\mathcal{M}|\boldsymbol{\Delta}|\mathcal{M}\rangle$$

$$\boldsymbol{\Delta} \sim \exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\,\boldsymbol{\Gamma}(\alpha_s(q))\right\}^{\dagger}\mathbf{D}(\alpha_s(b_0/b),\phi)\exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\,\boldsymbol{\Gamma}(\alpha_s(q))\right\}$$

Now we have

$$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S'_c(b)\right] \times \left[\mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}_{\mathrm{NLL}})C_1C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1}f_{a_2}$$

$$S' = \int_{b_0/b}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{M^2}{q^2} + B'(\alpha_s(q))\right] \qquad \mathrm{Tr}(\mathbf{H}\boldsymbol{\Delta}_{\mathrm{NLL}}) \sim \langle\mathcal{M}|\boldsymbol{\Delta}_{\mathrm{NLL}}|\mathcal{M}\rangle$$

$$\boldsymbol{\Delta}_{\mathrm{NLL}} \sim \exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\frac{\alpha_s(q)}{2\pi}\,\boldsymbol{\Gamma}^{(1)}\right\}^{\dagger}\mathbf{D}(\alpha_s(b_0/b),\phi)\exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\frac{\alpha_s(q)}{2\pi}\,\boldsymbol{\Gamma}^{(1)}\right\}$$

We could do something similar for $\boldsymbol{\Gamma}^{(1)}$ and absorb it in $B^{(1)}$, however we would generate wrong $(B^{(1)})^2$ terms (already at NNLO) that can be corrected with a modified $A^{(2)}$.

We did not follow this approach as using a 'wrong' $A^{(2)}$ might potentially affect the shower accuracy

$$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \times \exp\left[-S'_c(b)\right] \times \left[\mathrm{Tr}(\mathbf{H}\mathbf{\Delta}_{\mathrm{NLL}})C_1C_2\right]_{c\bar{c};a_1a_2} \times f_{a_1}f_{a_2}$$

$$S' = \int_{b_0/b}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_s(q))\log\frac{M^2}{q^2} + B'(\alpha_s(q))\right] \qquad \mathrm{Tr}(\mathbf{H}\mathbf{\Delta}_{\mathrm{NLL}}) \sim \langle\mathcal{M}|\mathbf{\Delta}_{\mathrm{NLL}}|\mathcal{M}\rangle$$

$$\mathbf{\Delta}_{\mathrm{NLL}} \sim \exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\frac{\alpha_s(q)}{2\pi}\mathbf{\Gamma}^{(1)}\right\}^{\dagger}\mathbf{D}(\alpha_s(b_0/b),\phi)\exp\left\{-\int_{b_0^2/b^2}^{M}\frac{dq^2}{q^2}\frac{\alpha_s(q)}{2\pi}\mathbf{\Gamma}^{(1)}\right\}$$

- Now we do the following approximation:

$$\langle\mathcal{M}|\mathbf{\Delta}_{\mathrm{NLL}}|\mathcal{M}\rangle \to \frac{\langle\mathcal{M}^{(0)}|\mathbf{\Delta}_{\mathrm{NLL}}|\mathcal{M}^{(0)}\rangle}{|\mathcal{M}^{(0)}|^2} \times \langle\mathcal{M}|\mathcal{M}\rangle$$

Factorized form, but not NNLO accurate

Instead of

$$\langle\mathcal{M}^{(1)}|\mathbf{\Gamma}^{(1)} + \mathbf{\Gamma}^{(1)\dagger}|\mathcal{M}^{(0)}\rangle + \mathrm{c.c.}$$

We have

$$\frac{\langle\mathcal{M}^{(0)}|\mathbf{\Gamma}^{(1)} + \mathbf{\Gamma}^{(1)\dagger}|\mathcal{M}^{(0)}\rangle}{|\mathcal{M}^{(0)}|^2} \times \left(\langle\mathcal{M}^{(1)}|\mathcal{M}^{(0)}\rangle + \mathrm{c.c.}\right)$$

- It can be shown, by performing the F.O. expansion, that this mistake can also be absorbed (up to NNLO) with an additional redefinition of $B^{(2)}$

$$B'^{(2)} \to B''^{(2)} = B'^{(2)} + \frac{1}{|\mathcal{M}^{(0)}|^2}\left\{\langle\mathcal{M}^{(1)}|\mathbf{\Gamma}^{(1)} + \mathbf{\Gamma}^{(1)\dagger}|\mathcal{M}^{(0)}\rangle + \mathrm{c.c.} - \frac{\langle\mathcal{M}^{(0)}|\mathbf{\Gamma}^{(1)} + \mathbf{\Gamma}^{(1)\dagger}|\mathcal{M}^{(0)}\rangle}{|\mathcal{M}^{(0)}|^2} \times \left(\langle\mathcal{M}^{(1)}|\mathcal{M}^{(0)}\rangle + \mathrm{c.c.}\right)\right\}$$

- Finally we have **D**, which I overlooked for the moment

- We will consider the azimuthally averaged case:
  - ► NNLO contribution from $\mathbf{D}^{(2)}$ vanishes upon azimuthal average
  - ► Contribution from $\mathbf{D}^{(1)}$ starts at NNLO with a constant term coming from the interference with collinear splittings

$$[\mathrm{Tr}(\mathbf{HD})C_1 C_2]^{\phi} \sim [HC_1C_2]^{\phi} + \underbrace{\alpha_s^2 \langle \mathcal{M}^{(0)} | \mathbf{D}^{(1)} \times G^{(1)} | \mathcal{M}^{(0)} \rangle^{\phi}}_{\text{New term easily taken into account in MiNNLO method}}$$

We arrived therefore to the desired expression keeping NNLO accuracy!

Computed by diagonalizing $\mathbf{\Gamma}^{(1)}$ ⟶ Sum of complex exponentials

$$d\sigma^{(\mathrm{sing})} \sim d\sigma^{(0)}_{c\bar{c}} \exp\left[-S''_c(b)\right] \langle \mathcal{M}^{(0)}| \exp\left[\int \frac{dq^2}{q^2}\frac{\alpha_s(q)}{2\pi}(\mathbf{\Gamma}^{(1)}+\mathbf{\Gamma}^{(1)\dagger})\right]|\mathcal{M}^{(0)}\rangle \, [\mathrm{Tr}(\mathbf{H}\,\mathbf{D})C_1 C_2]^{\phi}_{c\bar{c};a_1a_2} \, f_{a_1} f_{a_2}$$

Of the form $\sum_i \exp\left[-S(B \to B_i)\right]$

More precisely, each term is an 'usual' Sudakov form factor with an effective (complex) value of $B^{(1)}$ and $B^{(2)}$

Factorization formula was the starting point for color-singlet MiNNLO

Now we have a sum of colorless-final-state-like factorization formulas

⬇

Follow MiNNLO color-singlet derivation for each of them and arrive to MiNNLO for $t\bar{t}$

- Method implemented in POWHEG-BOX-V2, code publicly available
- First ever **NNLO+PS** for a **colored final state** in hadronic collisions