MAX-PLANCK-INSTITUT
FÜR PHYSIK

Non-factorisable contribution to t-channel single-top production

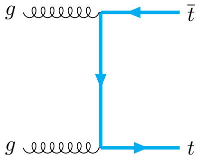
Based on 2108.09222 and 2204.05770 with Christian Brønnum-Hansen, Kirill Melnikov, Jérémie Quarroz & Chiara Signorile-Signorile.

Chen-Yu Wang | 2022-11-03 | Workshop on Tools for High Precision LHC Simulations

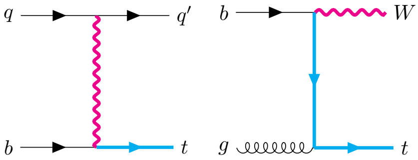
Outline

Motivation

- Top quark is the heaviest particle of the Standard Model:
  - enables better understanding of EW symmetry breaking.
  - probes new physics that involves top quarks.
- Top quarks are mainly produced through **top pairs**, but **single-top** production is also sizable:



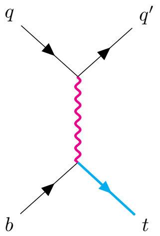
top pair production: $70 \sim 80\%$



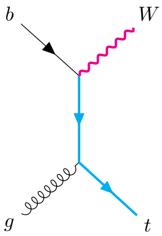
single-top production: $20 \sim 30\%$

- single-top production involves Wtb vertex:
  - determine CKM matrix element $|V_{tb}|$ / probe anomalous coupling.
  - indirect determination of top quark width Γ_t and mass m_t.
  - constrain bottom quark PDF.

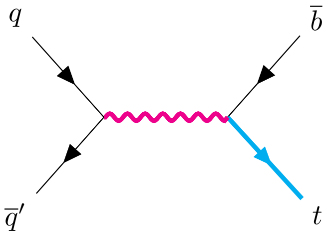
Motivation: single-top quark production



t-channel $\sim \frac{1}{t - m_W^2} \sim 70\%$



tW channel $\sim \frac{1}{t - m_t^2} \sim 25\%$



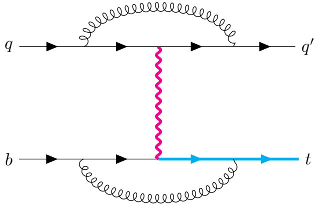
s-channel $\sim \frac{1}{s - m_W^2} \sim 5\%$

CMS measurement at 13 TeV: $\sigma_{t\text{-ch},t} = 130 \pm 1 \pm 19$ pb, $\sigma_{t\text{-ch},\bar{t}} = 77 \pm 1 \pm 12$ pb. *Sirunyan et al. 2020.*

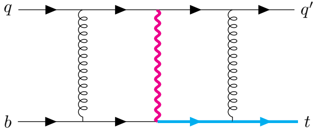
t-channel is the main production mode.

Higher order corrections to t-channel production

- NLO QCD and EW corrections has been known a while ago *Harris et al. 2002; Campbell, Ellis, et al. 2004; Sullivan 2004; Cao and Yuan 2005; Sullivan 2005; Schwienhorst et al. 2011*
- NNLO QCD corrections are only know for **factorisable** contributions *Brucherseifer et al. 2014; Berger et al. 2016; Campbell, Neumann, et al. 2021*
- We need two-loop **non-factorisable** contributions to complete the NNLO QCD correction.



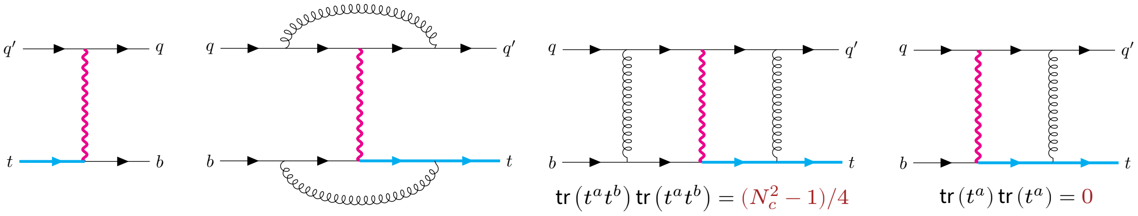
factorisable



non-factorisable

Factorisable approximation

- Non-factorisable contributions are **colour-suppressed** at NNLO $A^{\text{LO}} \otimes A^{\text{NNLO}}$.
  - but also **forbidden by colour** at NLO.



$\text{tr}\left(t^a t^a\right) \text{tr}\left(t^b t^b\right) = (N_c^2 - 1)^2/4$

$\text{tr}\left(t^a t^b\right) \text{tr}\left(t^a t^b\right) = (N_c^2 - 1)/4$

$\text{tr}\left(t^a\right) \text{tr}\left(t^a\right) = 0$

- Non-factorisable contributions **first appear** at NNLO. Are they negligible?

Non-factorisable contributions

It is not obvious that non-factorisable contributions are in fact negligible:

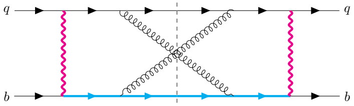
- Factorisable NNLO QCD contributions are **small** (few %) *Campbell, Neumann, et al. 2021.*
- **Possible π^2 enhancement** due to Glauber phase *Glauber 1959; Cheng and Wu 1969:*
  - It is an effect that shows up in the **virtual correction**.
  - Have been shown in non-factorisable contributions to VBF in the eikonal approximation *Liu, Melnikov, et al. 2019.*

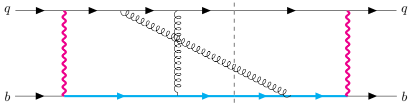
This factor $\pi^2 \sim 10$ could **compensate** the colour suppression $N_c^2 - 1 = 8$.

Purposes of this work

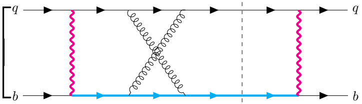
To calculate the non-factorisable contributions to single-top production at NNLO QCD.

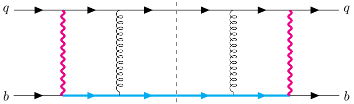
$$\mathrm{d}\hat{\sigma}_{\mathrm{nf}}^{\mathrm{NNLO}} = \mathrm{d}\hat{\sigma}_{RR} + \mathrm{d}\hat{\sigma}_{RV} + \mathrm{d}\hat{\sigma}_{VV}$$

$\mathrm{d}\hat{\sigma}_{RR} =$ 

$\mathrm{d}\hat{\sigma}_{RV} =$ 

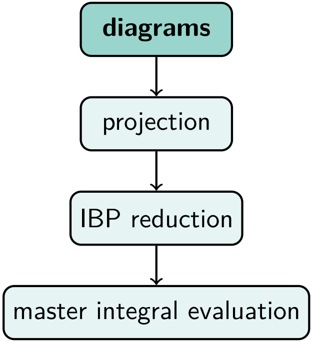
$+$ c.c.

$\mathrm{d}\hat{\sigma}_{VV} = \Bigg[$ 

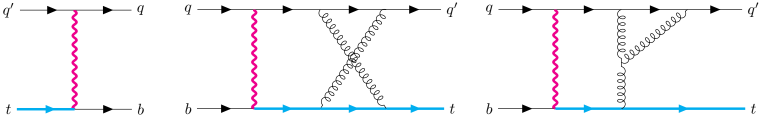
$+$ c.c. $\Bigg] +$ 

- Keep the **exact dependence** on kinematic invariants, m_t and m_W.
- Two-loop master integrals are evaluated numerically using the **auxiliary mass flow** method *Liu, Ma, and Wang 2018; Liu, Ma, Tao, et al. 2020; Liu and Ma 2021.*

Diagrams



- Diagrams are generated with `QGRAF` *Nogueira 1993* and processed in `FORM` *Vermaseren 2000; Kuipers et al. 2015; Ruijl et al. 2017.*
- Only the **Abelian** part of the amplitude contributes:

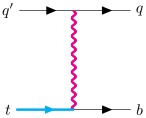


$\delta_{ij}\delta_{kl}$ $\quad (t^a t^b)_{ij}(t^b t^a)_{kl} \to (N_c^2 - 1)/4$ $\quad f^{abc}(t^a t^b)_{ij}(t^c)_{kl} \to 0$

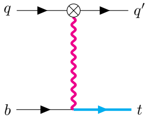
- 18 non-vanishing diagrams, only the symmetric part of the colour structure contributes after contracting with A^{LO}.

UV & IR structure

- Non-factorisable contributions are **UV-finite** at NNLO:



$\delta_{ij}\delta_{kl}$



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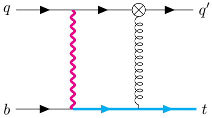
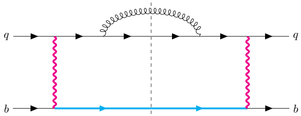


$t^a_{ij}t^a_{kl}$

- Non-factorisable contributions have **no collinear singularities**.

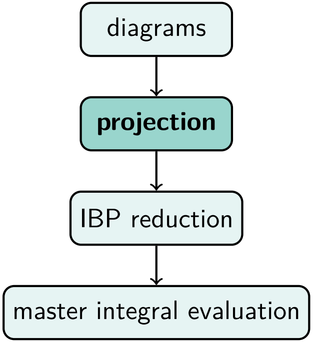


$\in$ factorisable contributions

Remaining IR divergence can be subtracted with the help of Catani operators.

Catani 1998; Catani et al. 2001; Becher and Neubert 2009a,b,c

Spinor structures and γ_5 scheme



- Rewrite the amplitude in terms of form factors and spinor structures:

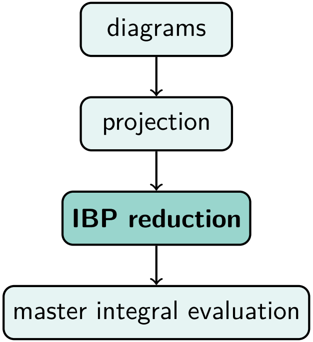
$$A(\{p_i\},\{u_j\},m_t,m_W) = \sum_{I=1}^{11} A_I(s,t,m_t,m_W) S_I(\{p_i\},\{u_j\})$$

- The spinor structures where $\gamma_7 = 1 - \gamma_5$ *Assadsolimani et al. 2014*

$$S_1 = \bar{t}(p_4)\,\gamma_7\,b(p_2) \times \bar{q}'(p_3)\,\not{p}_4\gamma_7\,b(p_1)$$
$$S_2 = \bar{t}(p_4)\,\not{p}_1\gamma_7\,b(p_2) \times \bar{q}'(p_3)\,\not{p}_4\gamma_7\,b(p_1)$$
$$\ldots$$

- Side note: only 4 spinor structures are independent in $d = 4$.

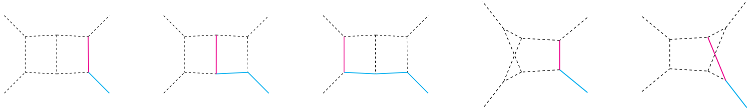
IBP reduction



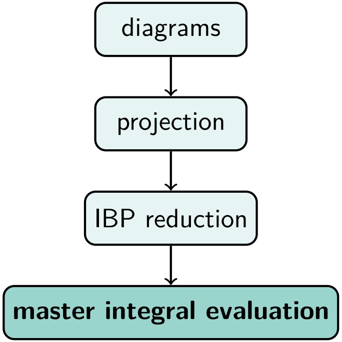
- Reduction performed **analytically** with `KIRA` 2.0: *Klappert, Lange, et al. 2020* and `FireFly` *Klappert and Lange 2020; Klappert, Klein, et al. 2021:*

$$\langle A^{(0)} | A^{(2)}_{\text{nf}} \rangle = \sum_{i=1}^{428} c_i(d, s, t, m_t, m_W) I_i$$

- Analytic reduction with four scales $(s,\ t,\ m_t,\ m_W)$: most complicated family took 4 days on 20 cores.
- 428 master integrals I_i in 18 families.
- File size of the simplified coefficients c_i: $\mathcal{O}(1\ \text{MB})$.



Master integral evaluation

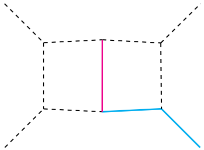


- Based on the **auxiliary mass flow** method *Liu, Ma, and Wang 2018; Liu, Ma, Tao, et al. 2020; Liu and Ma 2021:*

$$I \propto \lim_{\eta \to 0^+} \int \prod_{i=1}^{2} \mathrm{d}^d l_i \prod_{a=1}^{9} \frac{1}{[q_a^2 - (m_a^2 - i\eta)]^{\nu_a}}$$

- Add an imaginary part to the W **boson mass**:

$$m_W^2 \to m_W^2 - i\eta$$



- Solve differential equations at each kinematic point:

$$\partial_x I = M I, \quad x \propto -i\eta$$

with boundary condition $x \to -i\infty \Rightarrow$ physical point at $x = 0$.

Master integral evaluation

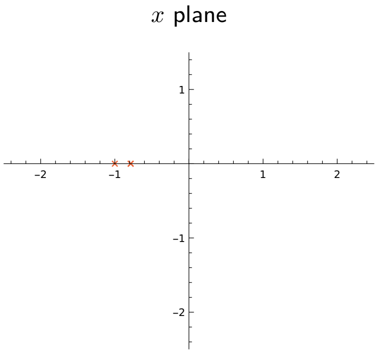
- Equation $\partial_x I = MI$ contains **singularities**.
- Expand I around **boundary** in variable $y = x^{-1} = 0$:

$$I = \sum_j^M \epsilon^j \sum_k^N \sum_l c_{jkl} y^k \ln^l y + \ldots$$

- Evaluate and expand around **regular points**:

$$I = \sum_j^M \epsilon^j \sum_{k=0}^N c_{jk} x'^k + \ldots$$

- Evaluate at the **physical point** $x = 0$.
- **Path** is fixed by **singularities** and desired precision.



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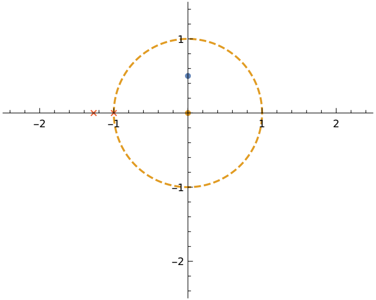
y plane



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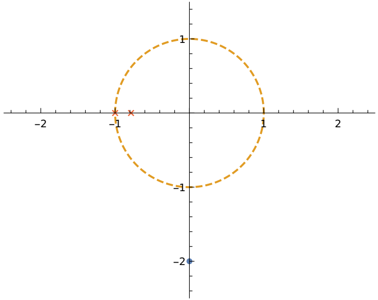
x plane



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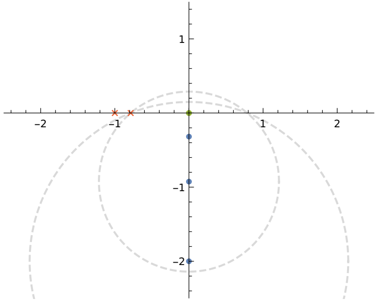
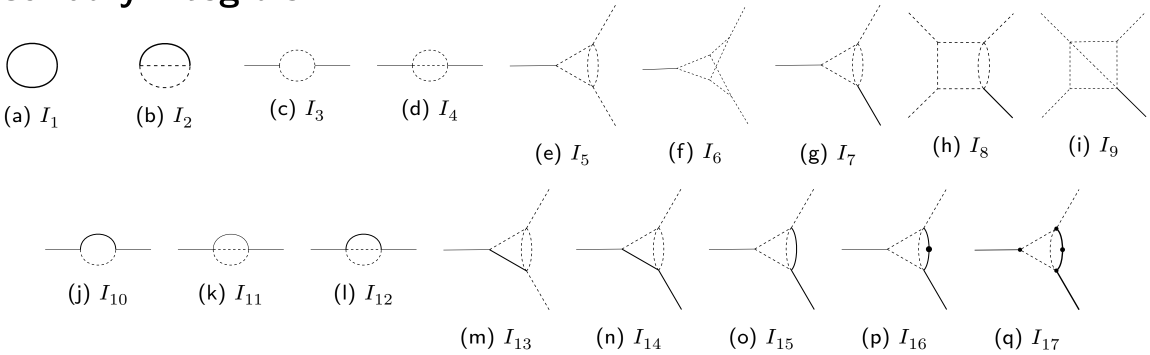


Boundary integrals



- Some boundary conditions are known analytically *'t Hooft and Veltman 1979; Chetyrkin et al. 1980; Scharf and Tausk 1994; Gehrmann and Remiddi 2000; Gehrmann, Huber, et al. 2005*
- Some are not available or don't have enough ϵ order
  $\Rightarrow$ calculated numerically by solving differential equations w.r.t. m_t^2.

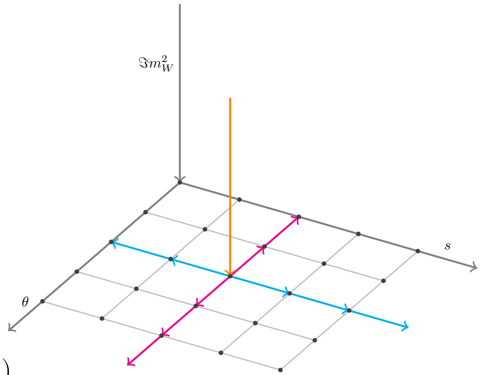
Master integral evaluation

- For each kinematic point (s, t), we can solve the differential equation w.r.t. m_W^2 to compute the master integrals.
- We can also use the differential equation w.r.t s and t to transport to different kinematic point.
- Solving differential equation in each direction:

$$(s_1, t_1) \xrightarrow{s} (s_2, t_1) \xrightarrow{t} (s_2, t_2)$$

- This also serves as a consistency check:

$$(s, t, m_W^2 + x) \xrightarrow{x} (s, t, m_W^2) \sim (s', t', m_W^2) \xrightarrow{s,t} (s, t, m_W^2)$$



Double virtual correction

- Comparison of poles at a typical phase space point $s \approx 104337$ GeV^2 and $t \approx -5179.68$ GeV^2.

	ϵ^{-2}	ϵ^{-1}
$\langle A^{(0)} \vert A^{(2)}_{\text{nf}} \rangle$	$-229.0940408654660 - 8.978163333241640i$	$-301.1802988944764 - 264.1773596529505i$
IR poles	$-229.0940408654665 - 8.978163333241973i$	$-301.1802988944791 - 264.1773596529535i$

- The cross section is evaluated with a **Vegas integrator**.
- 10 sets of 10^4 points extrated from a grid prepared on the **Born squared amplitude**.
- The 10 different sets give an estimation of the error on σ_{VV} about $\mathcal{O}(2\%)$.

All **428** two-loop master integrals evaluated to 20 digits in < 30 min on a single core.

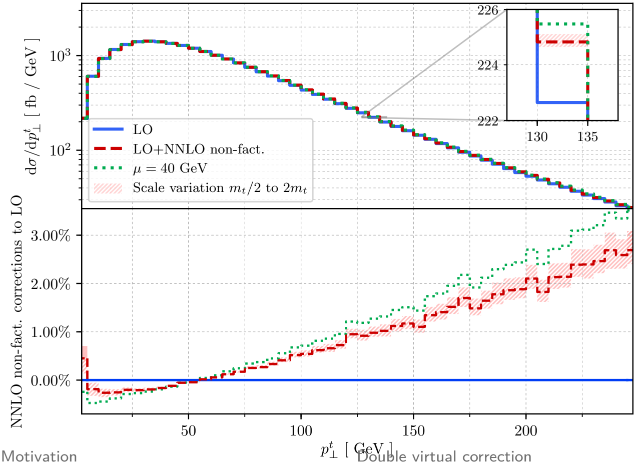
Results

- The non-factorisable correction to the LO cross section at 13 TeV and $\mu_F = m_t$:

$$\frac{\sigma_{pp\to X+t}}{1\ \text{pb}} = 117.96 + 0.26\left(\frac{\alpha_s(\mu_R)}{0.108}\right)^2$$

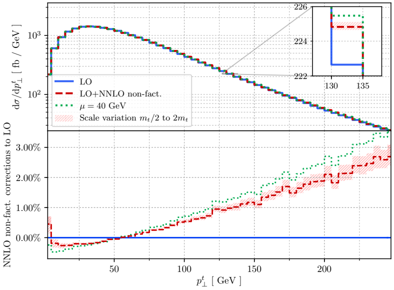
- Non-factorisable correction is about $0.22^{-0.04}_{+0.05}\%$ for $\mu_R = m_t$.
- Non-factorisable correction appears for the **first time** at NNLO; for this reason, they are **independent** of LO, NLO, and NNLO factorisable correction.
  $\Rightarrow$ No indication of a good scale choice.
- At $\mu_R = 40$ GeV (typical transverse momentum of the top quark):
  - Non-factorisable correction closes to 0.35%.
  - NNLO factorisable correction to NLO cross section is about 0.7%.

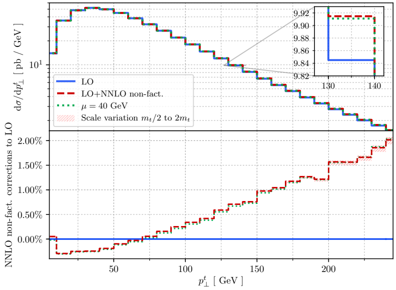
Top-quark transverse momentum distribution at 13 TeV



- There is a **significant** $p_\perp^t$-dependence of the non-factorisable corrections.
- Non-factorisable correction vanishes around 50 GeV, while the factorisable correction vanishes around 30 GeV.
- In some part of the phase space at low $p_\perp^t$, around the peak of the distribution, non-factorisable correction is **dominant** compared to factorisable correction.

Top-quark transverse momentum distribution at 100 TeV





- At 100 TeV and $\mu_F = m_t$: $\dfrac{\sigma_{pp\to X+t}}{1\text{ pb}} = 2367.0 + 3.8\left(\dfrac{\alpha_s(\mu_R)}{0.108}\right)^2$
- Non-factorisable correction is about 0.16% for $\mu_R = m_t$ and 0.25% for $\mu_R = 40$ GeV.
- The shape of the distribution is similar, the NNLO non-factorisable correction changes the sign around 70 GeV instead of 50 GeV at 13 TeV.

Conclusion

- We computed the missing piece to complete NNLO QCD corrections to the t-channel single-top production: the **non-factorisable contributions**.
- The **auxiliary mass flow** methd has been used for integral evaluation. It is prove to be sufficiently **robust** to produce results relevant for phenomenology.
- Non-factorisable contributions are smaller than, but **quite comparable** to, the factorisable ones.
- If a percent-level precsion in single-top studies can be reached, **the non-factorisable contributions will have to be taken into account**.

Thank you for your attention!

Master integral evaluation

- Add an imaginary part to the **internal top quark mass**:

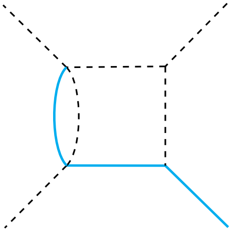
$$m_t^2 \to m_t^2 - i\eta.$$

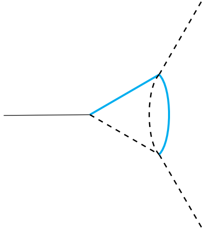
- Boundary condition: $\eta \to \infty \Rightarrow$ Physical point: $\eta \to 0$.
- Since top quark no longer on-shell, complicate behaviors arise in the limit $\eta \to 0$

$$I = \sum_j^M \epsilon^j \sum_k^N \sum_l c_{jkl} \eta^k \ln^l \eta + \dots$$

- We only need to know one region:

$$I = \eta^0 (c_{1,0} + c_{1,1}\eta + \cdots) + \eta^{-\epsilon}(c_{2,0} + c_{2,1}\eta + \cdots) + \eta^{-2\epsilon}(c_{3,0} + c_{3,1}\eta + \cdots)$$

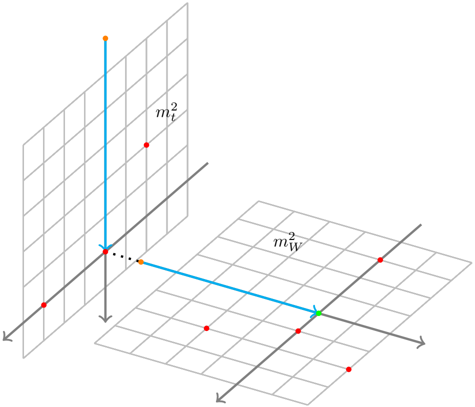




Master integral evaluation

- Boundary conditions are simpler:

- In this way, we are able to compute all boundary conditions for m_W^2 equation, and then solve the m_W^2 equation to reach the physical point.



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